ABSTRACT

The loan market has contributed to the success and failure of economies. Examples of such failures are the US subprime mortgage crisis as well as the global economic meltdown that followed. Many factors influence the loan market, making it volatile and vulnerable. As such, it is important to understand the extent of its vulnerability. Such uncertainties emerge from asymmetric information in the loan market that may lead to credit rationing. Many studies have been devoted to exploring theoretical aspects of the credit market. However, before delving into the theory, it is important to understand and analyze empirical data. Having said that, the literature has yet to provide reliable methodologies for analyzing the empirical data of the loan market. Therefore, given an empirical survey, this study provides a model describing borrowers' behavior in the loan markets. Borrowers are faced with a variety of loan contracts with different terms and conditions from different banks. Logit models can be used to capture the borrowers' choice of bank. Credit is not easily available rather it is rationed and borrowers compete to obtain their required credit via best suited banks offers. The competition is guaranteed by developing a mathematical programming formulation (an objective function subject to constraints) integrated with the logit models for which a solution algorithm using Successive Coordinate Descent was developed. Numerical results of the methodology are presented. Loan terms and conditions as well as the borrowers characteristics and preferences are captured in the logit models as explanatory variables. The methodology allows sensitivity analysis on the explanatory variables demonstrating the fluctuation and vulnerability of credit flow.

Keywords: Credit Rationing, Loan Market, Logit Model, Mathematical Programming, Vulnerability
1. INTRODUCTION

Credit flow is the blood of a vibrant economy, hence carefully watching and ensuring its smoothness is essential. Banks sit at cross points on the path of credit flow. Money is deposited at banks and it is then lent to borrowers. Many factors affecting the credit market have made it highly volatile and vulnerable. Such that the current global economic crisis stemmed from the US subprime mortgage crisis of 2008. Therefore it is important to deepen our understanding of the bank-borrower relationship as well as the factors contributing in the vulnerability and uncertainty of the credit market.

On one hand borrowers need credit to fund their projects seeking loans with convenient terms and conditions such as low interest rate, low collateral requirements, low application fee etc. On the other hand banks seek responsible borrowers who pay back installments on time with no risk of default. From the borrowers perspective the terms and conditions of the loans are clear for each bank. As such, borrowers can compare different banks and choose the best suited one(s). However, from the banks’ view there is no absolute mechanism to assess credit riskiness of the borrowers. Compared to the borrowers, lenders have less information about the projects making information asymmetric or imperfect.

Asymmetric information may rise from imperfect conditions such as risk, uncertainty and vulnerability and is not limited to the credit market. In fact, it is expected to appear wherever there are flow, human decision and competition involved such as transportation (Bagloee et al., 2013), Supply Chain (Ajmal & Kristianto, 2010), Stock Market (Joseph & Mazouz, 2010), negotiation (Neves & Nakhai 2011), insurance market (Hanafizadeh et al., 2013), search algorithms on internet (Nagurney et al., 2007), information technology (Di Caprio & Santos-Arteaga 2011), manufacturing (Stiglitz and Weiss (1981) analyzed several models of credit rationing in markets with imperfect information that won the Nobel Prize for Stiglitz in 2001. Since then, many studies have been dedicated to this topic, some with conflicting results due to lack of reliable methods to analyze empirical data (Bester, 1985; Riley, 1987; De Meza & Webb, 1987; Coco, 1997; Lensink & Sterken, 2002; De Meza & Webb, 2006; Arnold & Riley, 2009; Arnold 2012).

Therefore in an attempt to fill the void in this study, we develop a methodology to model bank-borrower behavior. The methodology is based on a mathematical programming framework (an objective function subject to constraints) in which the borrowers compete to obtain best loan offer available with banks collateral. Those tools are used to compensate the asymmetric information. Interest rate and collateral are aimed to hedge banks against unknown risks associated with the borrowers’ projects, while the risks are better known to the borrowers (and the borrowers are not willing to disclose the extent of the risks to the banks).

Interest rate is the price of credit flow in the credit market. In general when demand increases, price will increase to balance supply and demand. In the credit market, not every borrower gets its credit demand met, even when there is no credit supply shortage and the borrowers is willing to pay higher interest rates. On one hand default by riskier borrowers brings down banks’ revenue, on the other hand riskier borrowers are willing pay higher interest rates. Therefore the interest rate stabilizes at a certain level, below which banks are not lending causing credit to become rationed. Credit rationing implies some (good or bad) borrowers may not get their credit demand fully met. Credit rationing is a stigma of the market that is supposed to be perfect and competitive but it is not.

In addition to imperfect (or asymmetric) information, there are other potential causes to the imperfect market such as bankruptcy costs, agency conflict, transaction costs and taxes asymmetry. The recent financial crisis has shown how far from perfect financial markets can be (Ćorić, 2010).

Traditionally, banks have had two major tools to screen borrowers: interest rate and collateral. Those tools are used to compensate the asymmetric information. Interest rate and collateral are aimed to hedge banks against unknown risks associated with the borrowers’ projects, while the risks are better known to the borrowers (and the borrowers are not willing to disclose the extent of the risks to the banks).

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Therefore in an attempt to fill the void in this study, we develop a methodology to model bank-borrower behavior. The methodology is based on a mathematical programming framework (an objective function subject to constraints) in which the borrowers compete to obtain best loan offer available with banks
each with different terms and conditions. With respect to credit supply we considered two important constraints explicitly: (i) Banks have a limited amount of available credit, we call it simply “banks capacity”. Banks cannot give loans beyond their capacity. (ii) As described earlier, due to imperfect information banks put a cap on the maximum amount of credit offered to borrowers which is termed credit rationing (different borrowers may face different rationing rates in approaching different banks). Which bank a borrower chooses is very important in the imperfect credit market (Ćorić, 2010). The way a borrower chooses banks is modeled with logit models.

The logit models capture the utility of loans terms and conditions perceived by the borrowers as well as their own characteristics (Hellmann & Stiglitz, 2000). The borrowers choose loan offers in a way that maximizes their utility. Loans terms and conditions cover a wide range of factors such as interest rate, collateral requirement, application fee and repayment plan which are embedded into utility functions of the logit model. The logit models are known as behavioral models, therefore behaviors of the borrower in case of different combinations of factors in the utility function (such as interest rate, collateral and etc) can be assessed.

Utility functions of the logit models are the core components of the methodology which require to be calibrated. Calibration means estimating weights of factors in the utility functions. The calibration is conducted using an empirical dataset containing records of previously extended loans which include loans terms and condition as well the borrowers’ profile. The choices that the borrowers had made in the empirical dataset reflect the behavior of them which is captured in the calibrated utility functions. Sometimes small changes in the factors affecting credit transactions can have drastic effects (Mankiw, 1986; Temin & Voth, 2008) which are captured in the weights. Accordingly the methodology has the potential to conduct a sensitivity analysis on the explanatory variables in the utility functions to see the fluctuation and vulnerability of credit flow.

In this article, we shall elaborate the theoretical foundations of the methodology. In order to numerically evaluate the proposed methodology, the outputs of application of an artificial dataset are presented and discussed.

The rest of the article is organized as follows. In Section 2, we explore relevant literature to highlight important matters in the subject of credit rationing. The methodology is defined and formulated in Section 3. We lay out the algorithm of the methodology in Section 4. Section 5 is dedicated to numerical results. Finally Section 6 concludes the paper.

2. LITERATURE REVIEW

The first reference to credit rationing dates back to usury ceiling law in 18th century (Smith, 1776; Viner, 1937; Temin and Voth, 2008; Bellier et al., 2012). In the aftermath of great depression, Keynes (1930) postulated the cause as: “loans had been extended to bad borrowers at the prevailing interest rate”. It took two decades before some theories to analyze the Keynes’s notion appeared in the literature (Roosa, 1951; Scott 1957a, b; Parker, 1972; Lindbeck, 1962). In such studies, the credit rationing is linked to banks’ limitation to attract more deposits and the impact of demand characteristics is ignored. Therefore in the early studies, neither the significance of interest rates nor that of collateral to balance supply-demand is present. Later on, borrowers’ risk of default was taken into consideration (Hodgman, 1960; Jaffee & Modigliani, 1969; Smith, 1972; Jaffee & Russell, 1976). At this point it was well established that a rise in interest rate increases the probability of default.

The knowledge accumulated up to this point paved the way to considering the complexity of the borrower-bank relationship especially the asymmetry of information. That led to the formulation of credit rationing by Stiglitz and Weiss (1981). Stiglitz-Weiss model explains how the adverse selection occurs: Banks have to quote a single interest rate for all borrowers, because it is not possible to differentiate the risk of different borrowers. As interest rate increases
good (low-risk) borrowers will no longer apply for loan, instead riskier borrowers will remain in the market, and banks adversely will select amongst them. An increase in interest rates causes a decrease in the probability of repayment (Waller & Lewarne, 1994). Consequently the effect of adverse selection tends to reduce the expected profit of the bank because the mix the credit-worthiness of borrowers deteriorates as the interest rate increases (Wijkander 1992, Tse 1997). In the case of quoted (not competitive) interest rates, credit demand may exceed supply hence some borrowers will not get credit even though they are willing to pay higher interest rates. It may also be the case that among a group of identical borrowers some receive loans while others do not (Williamson 1987).

Stiglitz and Weiss (1981); Bester and Hellwig (1987) consider a situation in which the borrowers may work less hard once the credit is granted, hence the probability of default will increase. This situation is called moral hazard which is also rooted in the asymmetric information. To motivate the borrower to work harder, banks require collateral from borrower in the loan contracts (Boot et al., 1991). Thus the collateral is intended to mitigate the problems of moral hazard as well as adverse selection. However Stiglitz and Weiss (1981) and Wette (1983) showed that an increase in collateral can lead to adverse selection that decreases the banks’ return on loan. More collateral lead to undertaking less-risky project and hence less return. Besanko and Thakor (1987) showed that bad (riskier) borrowers are in favor of loans with high interest rate and low collateral.

Banks can also adjust other terms of the loan contract to screen borrowers such as maturity of the loan (Stiglitz and Weiss, 1983), or add loan covenants (Berlin & Mester, 1992; Carey et al., 1993).

3. THE PROBLEM OF LOAN MODELLING

Let \( G_p \) be loan demand for borrower \( p \in P \) who have to find a suitable bank \( k \in K \). Borrower \( p \) approaches an available bank that provides maximum utility. Utility is the core concept of discrete choice models, including logit models. Interest rates, collateral, socio-economic requirements, credit limit, processing fee, and other terms and conditions are the most important factors that determine the utility. Utilities are usually defined as a linear function of the important factors contributing to choice, and this equation is called the utility function. The factors coefficients indicate the relative importance of each factor in the utility function.

The utility of a bank \( k \in K \) is perceived by the borrower. Borrowers, depending on their socio-economic characteristics and their financial needs, perceive different levels of utility from the same bank. For instance, less risky borrowers with sounds business plan have more concern about the interest rates and repayment plan. Therefore, we denote the utility function as \( u_{pk} \) which is a function of borrower \( k \) as well as bank \( p \).

Regardless of any credit limit, the logit bank choice model may be formulated as follows:

\[
\text{LBC; Logit Bank Choice:}
\]

\[
g_{pk} = \frac{G_p \cdot e^{u_{pk}}}{\sum_k e^{u_{pk}}} \quad (1)
\]

where \( g_{pk} \) is the amount of credit acquired by borrower \( p \) from bank \( q \) out of the borrower’s total credit demand \( G_p \). Constraints for credit limits are introduced here:

\[
\text{BCC; Bank Capacity Constraints:}
\]

\[
\sum_p g_{pk} \leq C_k \quad k \in K \quad (2)
\]

\[
\text{CRC: Credit Rationing Constraints:}
\]

\[
g_{pk} \leq F_{pk} \quad p \in P, k \in K \quad (3)
\]

In Equation 3, credit available to borrower with bank \( k \) is rationed. It means that regardless of the borrowers’ credit demand, bank \( k \) has set a ceiling on the maximum
credit available to borrower \( p \) which is denoted
by \( F_{pk} \) which is itself subject to availability of
credit with the bank.

Simply put, the loan modeling is converted
to solving the following problem:

\[
\text{LBC subject to BCC and CRC (or Equation 1}
\text{ s.t. Equation 2,3).}
\]

Figure 1 is a graphic illustration of the
problem structure. The primary output is
credit flow \( (g_{pk}) \); the amount of credit acquired
by borrower \( p \) from bank \( k \). The inputs of the
problem are summarized as follows:

\[
G_p: \text{credit demand of borrower } p
\]

\[
F_{pk}: \text{credit rationing; maximum credit}
\text{offered by bank } k \text{ to borrower } p
\]

\[
C_k: \text{capacity or maximum available credit}
\text{with bank } k
\]

\[
U_{pk}: \text{utility of applying to bank } k \text{ perceived}
\text{by borrower } p
\]

\[
\theta_{pk}: \text{Shadow price corresponding to credit}
\text{rationing imposed by bank } k \text{ for borrower } p
\]

\[
\beta_k: \text{Shadow price corresponding to available credit}
\text{at bank } k
\]

\[
h_k = e^{-\beta_k}
\]

4. METHODOLOGY

4.1. Logit-Based Mathematical
Programming for Loan Modeling

Spiess (1996) was the first scholar to solve the
above problem without the rationing constraint.

By applying the Kuhn-Tucker optimality
conditions (Hess et al., 1984), Spiess (1996) proved that the problem of Equation 1 s.t. Equation 2 is equivalent to the following minimization problem:

\[
\text{Min } \sum_{p} \sum_{k} g_{pk} (\log g_{pk} - 1 + u_{pk})
\]

s.t.: \[
\sum_{k} g_{pk} = G_p \quad p \in P
\]

Spiess (1996) then added BCC (Equation 2)) to the above problem. Thus, the Logit bank choice (LBC) problem considering explicit banks’ capacity constraint (BCC) became:

\[
\text{Min } \sum_{p} \sum_{k} g_{pk} (\log g_{pk} - 1 + u_{pk})
\]

s.t.: \[
\sum_{k} g_{pk} = G_p \quad p \in P
\]

\[
\sum_{p} g_{pk} \leq C_k \quad k \in K
\]

We call the above problem (Equation 6, 7, 8) the Spiess problem. Note that Equation 7) ensures that the outcome \((g_{pk})\) meets the demand \((G_p)\) and Equation 8) is the bank’s capacity constraint (BCC).

It is required to ensure the feasibility of the solutions to the Spiess problem; by providing adequate credit to meet the entire demand as follows:

\[
\sum_{p} G_p \leq \sum_{k} C_k
\]

Clearly, in the case of a credit shortage, we can assume a dummy bank with infinite credit capacity and minimum utility. Minimum utility of the dummy bank ensures that the dummy bank will be the last option for the borrowers. Since Equation 9) can be treated beforehand, it was excluded from the formulation of the Spiess problem. Fewer constraints make the problem more tractable.

The Spiess problem can be extended to accommodate credit rationing constraints (CRC) as follows:

\[
\text{(Loan Modeling-LM Problem)}
\]

\[
\text{Min } \sum_{p} \sum_{k} g_{pk} (\log g_{pk} - 1 + u_{pk})
\]

s.t.: \[
\sum_{k} g_{pk} = G_p \quad p \in P
\]

\[
\sum_{p} g_{pk} \leq C_k \quad k \in K
\]

\[
g_{pk} \leq F_{pk} \quad p \in P, k \in K
\]

We call the above problem the Loan Modeling (LM) problem. The presence of the CRC (i.e. Equation 13) in the above problem leads to a feasibility discussion similar to the Spiess problem. Ensuring the feasibility of the LM problem solutions requires providing adequate rationing rates \((F_{pk})\) with respect to available credit capacity \((C_k)\) and also meeting the entire demand \((G_p)\) as follows:

\[
C_k \leq \sum_{p} F_{pk} \quad k \in K
\]

\[
G_p \leq \sum_{k} F_{pk} \quad p \in P
\]

Similarly we can ensure the feasibility of the solution beforehand by providing a dummy bank \(k'\) with infinite capacity \((C_{k'} = \infty)\) and minimum utility \((u_{pk'} < \min\{u_{pk} | p \in P, k \in K - \{k'\}\})\) with no rationing \((F_{pk'} = \infty)\). It is evident that to avoid problems of degeneracy we assume \(C_k > 0 \quad \forall k \in K\).

4.2. Solution Algorithm

As for establishing a solution algorithm for the LM problem, it is important to note all the constraints in the LM problem are linear. In order to have a more tractable problem we can eliminate the constraint by deriving the dual format of the problem. To do so, let us introduce
the dual variables $\alpha_p$ for constraint Equation 11) and $\beta_k \geq 0$ and $\theta_{pk} \geq 0$ for constrains E(12) and Equation 13) respectively. The Kuhn-Tucker optimality condition is established as follow:

$$L = \sum_{p} g_{pk} \left( \log g_{pk} - 1 + u_{pk} \right) + \alpha_p \left( \sum_{k} g_{pk} - G_p \right) + \beta_k \left( \sum_{p} g_{pk} - C_k \right) + \theta_{pk} \left( g_{pk} - F_{pk} \right)$$

(16)

Hence first order Kuhn-Tucker optimality conditions may be written as:

$$\nabla L_{g_{pk}} = 0 \quad \Rightarrow \quad \log g_{pk} - 1 + u_{pk} + g_{pk} \cdot \frac{1}{g_{pk}} + \alpha_p + \beta_k + \theta_{pk} = 0$$

(17)

Subjecting the credit flow $(g_{pk})$ derived from Equation 17) to the constraints of the LM problem (Equation 11,12,13)) results in a new problem as follows:

$$g_{pk} = e^{-n_{pk} - \alpha_p - \beta_k - \theta_{pk}}$$

(18)

s.t: Equation 11,12,13

Now we can establish the dual format of LM problem as follow:

$$D = \min_{\alpha, \beta, \theta} \sum_{p} \sum_{k} e^{-n_{pk} - \alpha_p - \beta_k - \theta_{pk}} + \alpha_p G_p + \beta_k C_k + \theta_{pk} F_{pk}$$

(19)

s.t. $\beta_k \geq 0$, $k \in K$ and $\theta_{pk} \geq 0$, $p \in P, k \in K$

Again by establishing first order Kuhn-Tucker optimality conditions for the dual problem we have:

$$\nabla D_{\alpha_p} = 0 \quad \Rightarrow \quad \sum_k e^{-n_{pk} - \alpha_p - \beta_k - \theta_{pk}} = G_p \quad p \in P$$

(20)

$$\nabla D_{\beta_k} \geq 0 \quad \Rightarrow \quad \sum_p e^{-n_{pk} - \alpha_p - \beta_k - \theta_{pk}} \leq C_k \quad k \in K$$

(21)

$$\nabla D_{\theta_{pk}} \geq 0 \quad \Rightarrow \quad e^{-n_{pk} - \alpha_p - \beta_k - \theta_{pk}} \leq F_{pk} \quad p \in P, k \in K$$

(22)

Solving the above equations yields optimal values of the dual variables.

Spiess (1996) developed a solution algorithm using a Successive Coordinate Descent (SCD) method for his own dual problem based on the following argument: the dual problem was free of any explicit constraints therefore it could be solved by using an SCD method. The same argument is applied to our own dual problem Equation 19). Therefore, we developed a solution algorithm based on the SCD method as shall be discussed below.

To reduce computational complexity, we eliminate the exponential terms in the optimality conditions Equation 20,21,22) through some simple substitutions: $a_p = e^{-\alpha_p}$, $b_k = e^{-\beta_k}$, $n_{pk} = e^{-\theta_{pk}}$ and $U_{pk} = e^{-\eta_{pk}}$. Hence the non-negativity conditions of the dual variables become $(a_p \geq 0)$ and $(0 \leq b_k; \ n_{pk} \leq 1)$. Now the optimality condition of the dual problem can be rewritten as:

$$\sum_k U_{pk} \cdot a_p \cdot b_k \cdot n_{pk} = G_p \quad p \in P$$

(23)

$$\sum_p U_{pk} \cdot a_p \cdot b_k \cdot n_{pk} \leq C_k \quad k \in K$$

(24)

$$U_{pk} \cdot a_p \cdot b_k \cdot n_{pk} \leq F_{pk} \quad p \in P, k \in K$$

(25)
The SCD is an iterative process and at the end of each iteration \((a^i_p, b^i_k, n^i_{pk})\) the values of the dual variables \((a_p, b_k, n_{pk})\) are updated for the next iteration denoted by \(i\). The maximum number of iterations is denoted by \(i_{\text{max}}\). Now the SCD algorithm of the dual problem Equation 19 can be written as follows in Box 1.

The above algorithm is simple and can be coded in any programming language. We used the Visual Basic (VB) language. To simplify the code’s use, the program interface is an MS-Excel file where the input data can be easily entered. The outputs are also reported in an MS-Excel format. The computer hardware used is a PC with 2.33GHz Intel(R) Xeon(R) CPU and 3.25GB of RAM. Section 4.3 discusses the numerical results of testing the algorithm.

### 4.3. Shadow Prices

There is a delicate interpretation of the dual variables on the creditsupplyside \((\beta_k, \theta_{pk})\).

According to Operational Research (OR) terminology; \(\beta_k, \theta_{pk}\) (beta and theta) are the shadow prices associated with bank capacity and credit rationing rates. With respect to the objective function, the shadow price \(\beta_k\) represents the value of one extra unit of credit added to bank disposal capacity \(k\).

Consider two identical banks \((k', k'')\) with only one difference: Bank \(k'\) offers a grace period of one month in case of default. Therefore it is expected to have \(\beta_{k'} > \beta_{k''}\). Banks with attractive loan offers will receive more requests than other banks. Borrowers compete for applying for the attractive loan offers. Hence, as a result of this competition the shadow price is the price that the market is willing to pay for one additional unit of credit in the accretive and fully depleted banks. Obviously, for underutilized banks, there is no competition and the corresponding shadow

### Box 1.

#### Step 0 (initialization and preparation)

Set \(i = 1\) and \(a^0_p = b^0_k = n^0_{pk} = 1\)

#### Step 1 Computation

\[
a^i_p = \sum_k G_{pq} \quad (26)
\]

\[
g^i_{pk} = U_{pk} \cdot a^i_p \cdot b^i_k \cdot n^i_{pk} \quad (27)
\]

#### Step 2 Stopping Criteria

\[
| \sum_p g^i_{pk} - C_k | \leq \epsilon_k \quad \text{and} \quad | \sum_p g^i_{pk} - F_{pk} | \leq \epsilon_{pk} \quad \text{or} \quad i = i_{\text{max}} \text{ then Stop}
\]

#### Step 3 Updating

\[
b^i_k = \min \{ 1, \frac{b^{i-1}_k \cdot C_k}{\sum_p g^i_{pk}} \} \quad (28)
\]

\[
n^i_{pk} = \min \{ 1, \frac{n^{i-1}_{pk} \cdot F_{pk}}{g^i_{pk}} \} \quad (29)
\]

#### Step 4 Continue

set \(i = i + 1\) and continue to Step 1.
prices are zero \( (\beta_k = 0) \). Intuitively, the shadow price is the answer to a variety of planning issues such as: (i) which bank is the most attractive one? The bank with maximum \( \beta_k \) is the most attractive one. (ii) How much additional money a bank has to obtain to meet the demand? For the bank with non-zero beta capacity of bank can be extended until \( \beta_k \) becomes zero.

A similar interpretation is applied to the shadow price of credit rationing \( (\theta_{pk})\). We will elaborate on the interpretation of shadow prices in section 5.3.

5. NUMERICAL RESULTS

5.1. Preparing the Case-Study

A case-study consisting of 100 borrowers, 10 banks was developed \(| P | = 100, | K | = 10\). Below, we define and specify the case-study so that it can be utilized by other researchers as a benchmark.

As discussed before (See Figure 1), there are four sets of inputs: \( G_p, U_{pk}, C_k, F_{pk} \). These inputs are specified stochastically (randomly); in order to avoid any biased results of applying the methodology to the case study.

Tables 1 and 2 present the uniform random numbers used to compute the input data. With respect to substitution made for Equation 23,24,25), the exponential format of the utility function \( (U_{pk}) \) is defined as:

\[
U_{pk} = \exp(-1 * R_{pV_{pk}} * R_{kV_{pk}})
\]  

where \( U_{pk} \) is the utility perceived by borrower \( p \) of choosing bank \( k \) that depends upon the borrower’s preferences and the loan conditions offered by the bank, both are represented by random numbers \( (R_{pV_{pk}}, R_{kV_{pk}}) \). Characteristics impacting borrower preferences, such as socio-economic, demographic, sex, risk attitude, type of the business, and possession of collateral are all represented by \( R_{pV_{pk}} \). Also, \( R_{kV_{pk}} \) includes banks’ terms and conditions considered for loans such as interest rate, collateral requirement, various fees, minimum repayment and default terms.

We could define the case study utility based on a single random term such as \( U_{pk} = \exp(-1 * R_{pV_{pk}}) \), which must return 10,000 records of \( R_{pkV_{pk}} \) to populate the case-study. Instead, to define the case-study in a concise format: we use the product of two random terms for which we reported 100 and 10 records in Table 1 and 2. The same rationale is applied for defining the other inputs. To simulate an unbiased situation for the credit demand \( (G_p) \) and supply sides \((C_k, F_{pk})\), exponential types of random numbers are utilized which yield a highly unpredictable dataset (Leemis & Park, 2006; Bagloee & Reddick, 2011). An exponential random number \( (x) \) with expected value of \( \lambda \) can be simply computed as follows.

\[
x = -\lambda \cdot \log(R)
\]  

where \( 0 \leq R \leq 1 \) is a uniform random number. Now the demand and supply sides are defined as:

\[
G_p = 10,000 \cdot \log(R_{pG_p})
\]  

\[
C_k = -100,000 \cdot \log(R_{kC_k})
\]  

\[
F_{pk} = -G_p \cdot \log(R_{pF_p} \cdot R_{kF_k}) \cdot \min(C_k)
\]  

In Equation 32 we assumed 10,000 credit unit as the expected credit demand for each borrower, but this number will vary depending on a random numbers \( (R_{pG_p}) \) representing the conditions based on which, borrowers credit needs are determines. In total, the credit demand becomes \( \sum_p G_p = 1,037,611 \). In
Table 1. Random numbers corresponding to borrowers (p) used to populate the case-study

<table>
<thead>
<tr>
<th>No</th>
<th>$R_{p_{U_{pk}}}$</th>
<th>$R_{p_{G_{p}}}$</th>
<th>Gp</th>
<th>No</th>
<th>$R_{p_{U_{pk}}}$</th>
<th>$R_{p_{G_{p}}}$</th>
<th>Gp</th>
</tr>
</thead>
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<tr>
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<td>9,834</td>
<td>51</td>
<td>0.6520</td>
<td>0.8725</td>
<td>717</td>
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<td>52</td>
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<td>0.4016</td>
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</tr>
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</tr>
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</table>

*continued on following page*
Given the size of demand matrix (100 borrowers) with 10,000 as the expected credit demand for each and number of banks (10 banks), the expected credit capacity of each bank should be $E(C_k) = 100,000 = 100 \times 10,000 / 10$ (it is }

**Table 1. Continued**

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<tr>
<th>No</th>
<th>$R_{U_p}$</th>
<th>$R_{G_p}$</th>
<th>$G_p$</th>
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<th>$R_{U_p}$</th>
<th>$R_{G_p}$</th>
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</table>

*Total credit demand is $\sum G_p = 1,037,611$

**Table 2. Random numbers and inputs corresponding to banks(k) used to populate the case-study, as well as some outputs**

| No | $R_{U_p}$ | $R_{G_p}$ | $C'_p$; consumption of credit | $\beta$ Exponential. Shadow Price | $C'_p$; consumption of credit | Error; $|C'_p-C'_p|$ |
|----|-----------|-----------|-------------------------------|----------------------------------|-------------------------------|-----------------------------|
| 1  | 0.1916    | 0.9794    | 133,584                       | 0.3493                           | 133,584                       | 0                           |
| 2  | 0.7248    | 0.5980    | 26,919                        | 0.0819                           | 26,919                        | 0                           |
| 3  | 0.4181    | 0.9220    | 9,209                         | 0.0247                           | 9,209                         | 0                           |
| 4  | 0.1756    | 0.8519    | 74,830                        | 0.1803                           | 74,830                        | 0                           |
| 5  | 0.5889    | 0.2126    | 74,449                        | 0.2143                           | 74,449                        | 0                           |
| 6  | 0.1756    | 0.6088    | 48,258                        | 0.1163                           | 48,258                        | 0                           |
| 7  | 0.9820    | 0.6390    | 171,312                       | 0.5763                           | 171,312                       | 0                           |
| 8  | 0.4738    | 0.7668    | 61,400                        | 0.1684                           | 61,400                        | 0                           |
| 9  | 0.5890    | 0.6696    | 414,628                       | 1.0000                           | 347,459                       | 67,169                      |
| 10 | 0.4137    | 0.6767    | 90,192                        | 0.2411                           | 90,192                        | 0                           |
| Total | 1,104,780 | 12,853,476 | 1,037,611*                   |                                  |                               |                             |

*it is also the total credit demand
comparable with the demand). Also the credit capacity varies across the banks due to the random number \( R_{kC_k} \). In Equation 34), the expected rates for credit rationing to proceed the demand is assumed as \( E(F_{pk}) = G_p \) while it does not exceed the credit capacity of the corresponding bank. Similarly, credit rationing rates may vary due to random numbers \( (R_{pCp}, R_{kCk}) \) representing different conditions at both ends (borrowers and banks).

To ensure the feasibility of the solution, the constraints set forth by Equation 9,14,15) must be met. Therefore in Eqs(33,34), \( C_k \) and \( F_{pk} \) have been specified conservatively greater than \( G_p \) (See Tables 1 and 2).

5.2. Executing the Algorithm

The algorithm described in Section 4 is applied to the data developed for the case-study, above. The stopping criterion is 100 iterations \( (i_{\text{max}} = 100) \). As shown in Figure 1, given four sets of input variables \( G_p, U_{pk}, C_k, F_{pk} \), the algorithm yields three sets of output variables which are:

1. \( g_{pk} \), credit obtained by borrower \( p \) from bank \( k \);
2. \( b_k = e^{-\beta_k} \), shadow price of bank \( k \), and
3. \( n_{pk} = e^{-\theta_{pk}} \), shadow prices of the credit rationing rates.

It is clear that given the credit flow \( g_{pk} \), it is easy to calculate \( C'_k \) credit consumption rates of the banks \( (\forall k \in K) \), and the \( F'_{pk} \) consumption of rationed credit: \( C'_k = \sum_p g_{pk} \), \( F'_{pk} = g_{pk} \).

Table 2 shows credit consumption rates for each banks computed for the last iteration along with the shadow prices. Table 3 presents the aggregate gaps between credit consumption rates and bank’s capacity \( (\sum_k |C_k - C'_k|) \) as well as between consumption rates and the credit rationing \( (\sum_k |F_{pk} - F'_{pk}|) \) for 100 successive iterations. Consecutive decreases in the gaps in Table 3 indicate the algorithm’s convergence behavior. Furthermore, an error index based on the gap between credit capacity and consumption is defined in Table 3 as \( \%\text{Err}_k = 100 \times \frac{\sum_k |C_k - C'_k|}{\sum_k C_k} \). Figure 2 graphically illustrates rapid decrease of the error index over 100 successive iterations.

As shown in Table 3 at iteration 39, the results converge to 100 percent accuracy. It is worth noting that the running time for the entire 100 iterations is 30 seconds.

5.3. Interpretation of the Algorithm Outputs

The methodology described in this study is meant to provide insight for bankers or regulators of borrowers’ behaviors. The shadow prices can be used for this purpose because they provide key information. Two sets of shadow prices related to the supply side can be interpreted: \( b_k = e^{-\beta_k} \), and \( n_{pk} = e^{-\theta_{pk}} \). Table 2 presents shadow prices in exponential form for the banks credit deposit or capacity \( (b_k) \). It is clear that after 100 iterations, bank 9 has untapped credit accounted to 67,169 credit unit. Bank 9 has a shadow price of 1 \( (b_9 = 1) \). For the remaining banks we have \( 0 < b_k < 1 \) or \( \infty > \beta_k > 0 \). A smaller \( b_k \) indicates a higher shadow price or beta \( (\beta_k) \) which implies that the corresponding bank is more attractive for the borrowers and the competition to get loan is more intense than other banks. Alternatively, loan’s terms and conditions can be adjusted. For example interest rate, processing fees, repayment plans can be managed to balance demand among the banks.

Similarly, with respect to the shadow price of credit rationing \( (\theta_{pk} \text{ or theta}) \): (i) Is the current rationing for a particular borrower efficient? The degree to which the rationing is efficient can be evaluated by how close to equal thetas are across the borrowers. (ii) How much more the credit rationing can be uplifted? The
Table 3. Convergence result of 100 successive iterations

| Itr* | $\sum_{k} |C_k - C_k'|$ | $\sum_{p} |F_{p} - F_{p}'|$ | Itr | $\sum_{k} |C_k - C_k'|$ | $\sum_{p} |F_{p} - F_{p}'|$ |
|------|----------------|----------------|------|----------------|----------------|
| 1    | 774,674        | 11,827,792     | 51   | 67,169         | 11,815,865     |
| 2    | 546,212        | 11,826,947     | 52   | 67,169         | 11,815,865     |
| 3    | 374,823        | 11,817,970     | 53   | 67,169         | 11,815,865     |
| 4    | 300,090        | 11,815,865     | 54   | 67,169         | 11,815,865     |
| 5    | 242,125        | 11,815,865     | 55   | 67,169         | 11,815,865     |
| 6    | 194,600        | 11,815,865     | 56   | 67,169         | 11,815,865     |
| 7    | 157,789        | 11,815,865     | 57   | 67,169         | 11,815,865     |
| 8    | 130,416        | 11,815,865     | 58   | 67,169         | 11,815,865     |
| 9    | 110,734        | 11,815,865     | 59   | 67,169         | 11,815,865     |
| 10   | 96,896         | 11,815,865     | 60   | 67,169         | 11,815,865     |
| 11   | 87,324         | 11,815,865     | 61   | 67,169         | 11,815,865     |
| 12   | 80,773         | 11,815,865     | 62   | 67,169         | 11,815,865     |
| 13   | 76,324         | 11,815,865     | 63   | 67,169         | 11,815,865     |
| 14   | 73,318         | 11,815,865     | 64   | 67,169         | 11,815,865     |
| 15   | 71,293         | 11,815,865     | 65   | 67,169         | 11,815,865     |
| 16   | 69,933         | 11,815,865     | 66   | 67,169         | 11,815,865     |
| 17   | 69,020         | 11,815,865     | 67   | 67,169         | 11,815,865     |
| 18   | 68,408         | 11,815,865     | 68   | 67,169         | 11,815,865     |
| 19   | 67,998         | 11,815,865     | 69   | 67,169         | 11,815,865     |
| 20   | 67,724         | 11,815,865     | 70   | 67,169         | 11,815,865     |
| 21   | 67,540         | 11,815,865     | 71   | 67,169         | 11,815,865     |
| 22   | 67,417         | 11,815,865     | 72   | 67,169         | 11,815,865     |
| 23   | 67,335         | 11,815,865     | 73   | 67,169         | 11,815,865     |
| 24   | 67,280         | 11,815,865     | 74   | 67,169         | 11,815,865     |
| 25   | 67,243         | 11,815,865     | 75   | 67,169         | 11,815,865     |
| 26   | 67,219         | 11,815,865     | 76   | 67,169         | 11,815,865     |
| 27   | 67,202         | 11,815,865     | 77   | 67,169         | 11,815,865     |
| 28   | 67,191         | 11,815,865     | 78   | 67,169         | 11,815,865     |
| 29   | 67,184         | 11,815,865     | 79   | 67,169         | 11,815,865     |
| 30   | 67,179         | 11,815,865     | 80   | 67,169         | 11,815,865     |
| 31   | 67,176         | 11,815,865     | 81   | 67,169         | 11,815,865     |
| 32   | 67,174         | 11,815,865     | 82   | 67,169         | 11,815,865     |
| 33   | 67,172         | 11,815,865     | 83   | 67,169         | 11,815,865     |
| 34   | 67,171         | 11,815,865     | 84   | 67,169         | 11,815,865     |
| 35   | 67,170         | 11,815,865     | 85   | 67,169         | 11,815,865     |
| 36   | 67,170         | 11,815,865     | 86   | 67,169         | 11,815,865     |
| 37   | 67,170         | 11,815,865     | 87   | 67,169         | 11,815,865     |
| 38   | 67,170         | 11,815,865     | 88   | 67,169         | 11,815,865     |
| 39   | 67,169         | 11,815,865     | 89   | 67,169         | 11,815,865     |
| 40   | 67,169         | 11,815,865     | 90   | 67,169         | 11,815,865     |

continued on following page
credit rationing rates can be increased up to the point that the thetas are equal for all the borrowers.

6. SUMMARY AND CONCLUSION

We developed a model for loan applications that addresses a variety of deficiencies identified in previous studies in the context of credit rationing. The borrower bank choices are considered within the structure of a Logit model. A mathematical programming problem is introduced to explicitly consider banks’ credit deposits (or capacities) and credit rationing constraints. Introduction of credit rationing along with other constraints is a unique aspect of this study. A solution algorithm using Successive Coordinate Descent was developed for the Logit-based mathematical programming. The algorithm was tested on an artificial case. The results show convergence of the algorithm.

The algorithm’s output includes shadow prices for constraints in the supply side. The shadow prices contain important information which is key to addressing a variety of credit issues, such as the identification of credit shortages, identification of fair application fee and etc.

Loans terms and condition as well the borrowers’ characteristics and preferences are captured in the Logit model as explanatory variables in the utility functions. The methodology has the potential to conduct sensitive analysis on the explanatory variables to see the fluctuation and vulnerability of credit flow.

This study addresses some shortcomings in the current state of loan modeling. Nonetheless it opens the door to additional study topics. Credit demand and Bank deposits are assumed to be fixed and exogenous data in the model. It is important to consider variation of credit demand and investors’ behaviors in the supply side. The application to real datasets has yet to be addressed.

ACKNOWLEDGMENT

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