Optimal production planning for a manufacturing system: an approach based on PA

Turki Sadok, Hajej Zied, Rezg Nidhal

LGIPM / Université de Lorraine, Ile du Saulcy, 57045 Metz Cedex, France
(Tel: (+33)(0)3-8731547458; e-mail: Sadok.Turki@univ-lorraine.fr - zied.hajej@univ-lorraine.fr - nidhal.rezg@univ-lorraine.fr)

Abstract:
In this paper, a manufacturing system composed by a single-product machine, a buffer and a stochastic demand is considered. A discrete flow model is adopted to describe the system and to take into account the lost demands. The objective of this paper is to determine the optimal production planning taken into account service level. This optimal production planning minimizing the sum of production, inventory, and lost sales costs. Perturbation analysis method is used for optimizing the proposed system. Using the discrete flow model, the trajectories of production rate, buffer level and lost demands are studied and the perturbation analysis estimators are evaluated. These estimators are shown to be unbiased and then they are implanted in an optimization algorithm which determines the optimal production planning in the presence of service level.

Keywords: Manufacturing system, discrete flow model, perturbation analysis, production planning.

1. INTRODUCTION

Over the past few decades, the determination of an optimal production plan which minimizes the total cost including production, inventory and lost demands is one of the first actions of a hierarchical decision making process. Sethi et al. (1997) determined the optimal production planning for stochastic manufacturing system consisting of a single machine that is failure prone and facing a constant demand, the authors found the rate of production over time in order to minimize the average cost of production and surplus. Dobos (2003) determined the optimal production rate and inventory stores levels in a reverse logistics system with constant demand and delay, the objective is to minimize the sum of the holding costs in the stores and costs of the manufacturing, remanufacturing and disposal. Dahane et al. (2012) considered a single randomly failing and repairable manufacturing system producing two products. By using a genetic algorithm, the authors determined simultaneously the optimal production rate of the first product during each period $k$ ($k$ periods over a finite horizon) and the optimal duration of the production interval of the second product in order to maximize the total expected profit. In this paper, we will determine the optimal production planning under hypotheses of service level Hajej et al. (2009). Indeed, this optimal production planning allows respecting the service level. Thus, an optimal production planning should be determined based on the relationship with the production planning and the service level.

However, we use a discrete flow model (Mahut (2000), Cassandras and Lafortune (2007), Hazelton (2010), Vasic and Ruskin (2011)) to describe the system and to integrate the service level. Indeed, discrete flow model is considered more realistic for discrete manufacturing systems than stochastic flow model (Markou and Panayiotou (2007)). However, it allows tracking individual parts part by part either in performance evaluation or real-time flow control and is generally easier to simulate. Under this discrete flow model, we will develop a stochastic optimization problem which seeks to minimize the total expected cost via a simulation based optimization.

Perturbation Analysis (PA) to optimize the system is proposed (Ho et al. (1979)) (Yu and Cassandras (2004)). Indeed, PA is a technique which allows obtaining sample path derivatives of a random variable with respect to some parameters of interest (e.g., buffer level, production rate ...). The most important advantage of PA method is that the simulation based on PA allows reducing the simulation time comparing to a classical simulation method. This advantage is explained by the fact that the optimization algorithm based on PA computes at every step the gradient estimators which corresponds to the new value of a parameter of interest. Therefore, the value of gradient estimator allows orienting quickly the algorithm to the optimal value. Yu and Cassandras (2004) applied perturbation analysis method to a stochastic flow model and then derived gradient estimators of throughput and buffer overflow metrics with respect to production control parameters, thereafter they used them as approximate gradient estimators in simple iterative schemes for adjusting thresholds in order to optimize an objective function that trades off throughput and buffer overflow costs. However, the authors showed that these gradient estimators
are unbiased before using them in the optimization algorithm. Indeed, the unbiasedness is the principal condition for making the application of PA useful in practice, since it enables the use of the sample PA derivative in control and optimization methods that employ stochastic gradient-based techniques. Then, these estimators could be used in stochastic approximation algorithm. Unfortunately to show the unbiasedness of the estimators in the discrete setting (discrete flow model) is more complicate than in the stochastic fluid model setting. Despite this disadvantage, we will use the discrete flow model to modeling our system by the fact this model is very realistic and more precise than stochastic fluid model which sometime does not maintain the identity of some important parameters of manufacturing systems (e.g., service level, delivery time).

The first contribution of this paper is to apply the PA method on the discrete flow model and to study the trajectories of production rate, buffer level and lost demands in order to derive gradient estimators. The second contribution is to show that these gradient estimators are unbiased and to use them in optimization algorithm for determining the optimal production planning.

The paper is organized as follows. The discrete flow model with service level is presented in section 2. In section 3 the PA method is applied on the discrete flow model. Numerical results are presented in section 4. Finally, the last section concludes the paper and gives some perspectives to our work.

2. MODEL AND PROBLEM FORMULATION

The studied manufacturing system is composed by a single-product machine \( M \) and buffer \( B \). The customer demand which is denoted by \( d(k) \) is random and given by a Normal distribution. The demand \( d(k) \) is satisfied from the buffer \( B \) with inventory service level \( \alpha \). (see Fig. 1).

The objective of this paper is to determine the optimal production planning taken into account service level.

The following parameters are used in the model formulation:

- \( \Delta t \): length of a production period
- \( H \): number of production periods in the planning horizon
- \( H, \Delta t \): length of the finite planning horizon
- \( u(k) \): production rate of machine \( M \) during period \( k \) \((k=0,1,\ldots,H-1)\)
- \( U=\{u(0),u(1),\ldots,u(H-1)\} \)
- \( \hat{d}(k) \): average demand during period \( k \) \((k=0,1,\ldots,H)\)
- \( \hat{x}(k) \): buffer level at the end of period \( k \) \((k=0,1,\ldots,H)\)
- \( \hat{x}_0(k) \): initial buffer level
- \( L(k) \): number of unsatisfied demands at the end of period \( k \)
- \( cp \): unit production cost of machine \( M \)
- \( cs \): inventory holding cost of one product unit during one period at the buffer \( B \).
- \( cs^- \): unit lost sales cost.
- \( U_{max} \): maximal production rate of machine \( M \)
- \( U_{min} \): minimal production rate of machine \( M \)
- \( \alpha \): probability index related to customer satisfaction and expressing the service level.
- \( u_{\alpha}(k) \): minimum cumulative production quantity during the period \( k \).

The buffer level at the period \( k+1 \) equals to the buffer level at the period \( k \) plus the production rate of machine \( M \) during period \( k \), minus the delivery rate during period \( k \). Therefore, the buffer level at the period \( k+1 \) is given by the following equation:

\[
x(k + 1) = x(k) + u(k) - d(k)
\]  

(1)

The service level requirement constraint for each period is expressed by the following constraint:

\[
PROB \ (x(k + 1) \geq 0) \geq \alpha
\]  

(2)

The following constraint defines an upper and lower bounds on the production level during each period \( k \):

\[
U_{min} \leq u(k) \leq U_{max}
\]  

(3)

The service level constrain: Another important transformation changes the service level constraint into equivalent, but deterministic, inequalities by specifying through the following lemma a minimum cumulative transported quantity depending on the service level requirements.

Lemma 1:

\[
PROB \ (x(k + l) \geq 0) \geq \alpha \Rightarrow (u(k) \geq u_{\alpha}(k)) \quad k=0,1,\ldots,H
\]

(4)

Proof of lemma 1:

\[
x(k + 1) = x(k) + u(k) - d(k)
\]

\[
\Rightarrow PROB \ (x(k + 1) \geq 0) \geq \alpha
\]

\[
\Rightarrow PROB \ (x(k) + u(k) - d(k) \geq 0) \geq \alpha
\]

\[
\Rightarrow PROB \ (x(k) + u(k) \geq d(k)) \geq \alpha
\]

\[
\Rightarrow PROB \ \left( \frac{x(k) + u(k) - \hat{d}(k)}{V_{d,k}} \geq \frac{d(k) - \hat{d}(k)}{V_{s,k}} \right) \geq \alpha \quad (A)
\]

Note that \( \frac{d(k) - \hat{d}(k)}{V_{s,k}} \) is a Gaussian random variable with an identical distribution as \( d(k) \).
It is possible from (A) to determine a lower bound for the control variable, assuming that \( \varphi \) is a probability distribution function and \( f \) a probability density function. Hence,

\[
(A) \implies \varphi_{d, k} \left( \frac{x(k) + u(k) - \hat{d}(k)}{V_{d, k}} \right) \geq \alpha
\]  

(B)

Since \( \lim_{\alpha \to 0} \varphi_{d, k} \to 0 \) and \( \lim_{\alpha \to 1} \varphi_{d, k} \to 1 \) we conclude that \( \varphi_{d, k} \) is strictly increasing. We note that \( \varphi \) is indefinitely differentiable, so we conclude that \( \varphi_{d, k} \) is invertible.

Thus \( (B) \Rightarrow \frac{x(k) + u(k) - \hat{d}(k)}{V_{d, k}} \geq \varphi_{d, k}^{-1}(\alpha) \)

\[
\Rightarrow x(k) + u(k) - \hat{d}(k) \geq V_{d, k} \cdot \varphi_{d, k}^{-1}(\alpha)
\]

\[
\Rightarrow u(k) \geq V_{d, k} \cdot \varphi_{d, k}^{-1}(\alpha) + \hat{d}(k) - x(k)
\]

Thus

\[
\text{Prob}(x(k + 1) \geq 0) \geq \alpha \Rightarrow \left( u(k) \geq V_{d, k} \cdot \varphi_{d, k}^{-1}(\alpha) + \hat{d}(k) - x(k) \right)
\]

Q.E.D.

The number of unsatisfied demands (lost) depends on the customer demand and the buffer level. Indeed, when the customer orders a demand \( d(k) \) and the buffer is empty, the demand will be not satisfied at all and will be lost. The number of unsatisfied demands during the period \( k \) is defined as follows:

\[
L(k) = \begin{cases} 
  u(k) - u_{\max} & \text{if } u(k) > u_{\max} \\
  u(k) - u_{\min}(k) & \text{if } u(k) < u_{\min}(k) \\
  0 & \text{otherwise}
\end{cases}
\]  

(4)

The cost function at the period \( k \), which is composed by the inventory cost and lost sales cost, is given by:

\[
C(k) = cp.u(k) + cs.x(k) + cs^{-}.L(k)
\]  

(5)

The total cost over the horizon \( H \) is given by:

\[
CT = \sum_{k=0}^{k=H-1} C(k)
\]  

(6)

In the following section, we will investigate the sample path trajectories of \( u(k), x(k) \) and \( L(k) \). This study will allow us to find the PA estimators and prove that these estimators are unbiased.

3. APPLICATION OF THE PA METHOD ON THE DISCRETE FLOW MODEL

We turn our attention in this section, to applying the PA method to the discrete flow model. The PA is an approach intended to estimate gradients of performances metric with respect to some parameters of interest. This method consists on observing and analyzing two sample paths, one is the nominal sample path \( (u(k)) \), and the other is the perturbed sample path \( (u^\delta(k)) \) (see Fig. 2). We assumed that the production rate during period \( k \) is increased by a perturbation, denoted by \( \delta \). In this paper, we consider \( \delta > 0 \) and we evaluate the resulting changes in buffer level \( x(k) \) and number of unsatisfied demands \( L(k) \) using geometric arguments (similar results could be easily obtained for \( \delta < 0 \)).

**Fig. 2. Production rate of the perturbed and nominal trajectories.**

The following assumptions are considered:

- For comparing the both sample paths, the same distribution of random variables (demand customer) is used.
- The maximal production rate, minimal production rate and unit costs are the same for both sample trajectories.
- The initial buffer level for the nominal and perturbed trajectories are equals.

The following notations are used:

- \( x^\delta(k) \): buffer level during the period \( k \) for the perturbed trajectory;
- \( u^\delta(k) \): production rate during the period \( k \) for the perturbed trajectory;
- \( L^\delta(k) \): number of unsatisfied demands during the period \( k \) for the perturbed trajectory;
- \( u^\min(k) \): minimum cumulative production quantity during the period \( k \) for the perturbed trajectory.

3.1 Study of buffers level trajectory

The inventory level during the period \( k \) for the perturbed trajectory is defined as follows:

\[
x^\delta(k + 1) = x^\delta(k) + u^\delta(k) - d(k)
\]

**Lemm2 :** \( x^\delta(k) = x(k) + k.\delta \) for all \( k = 0, 1, ..., H \)

**Proof :**

\[
x(k) = x(k - I) + u(k - I) - d(k - I) = x(0) + \sum_{i=0}^{k-1} u(i) - \sum_{i=0}^{k-1} d(i)
\]

\[
x^\delta(k) = x^\delta(k - I) + u^\delta(k - I) - d(k - I) = x^\delta(0) + \sum_{i=0}^{k-1} u^\delta(i) - \sum_{i=0}^{k-1} d(i)
\]
We have $x^g(0) = x(0)$ (assumption) and 
\[
\sum_{i=0}^{k-1} u^g(i) = \sum_{i=0}^{k-1} (u(i) + k) = \sum_{i=0}^{k-1} u(i) + k \delta ,
\]
then 
\[
x^g(k) = x(0) + \sum_{i=0}^{k-1} u(i) + k \delta = x(k) + k \delta .
\]

According to (4) we have 9 cases

Proof:

We have
\[
\sum \sum \sum \delta = \frac{1}{2} \sum \sum \sum \delta = \frac{1}{2} \sum \sum \sum \delta .
\]

3.2 Study of number of unsatisfied demands trajectory

The number of unsatisfied demands during the period $k$ for the perturbed trajectory is defined as follows:

\[
L^g(k) = \begin{cases} 
|u^g(k) - u_{\text{max}}| & \text{If } u^g(k) > u_{\text{max}} \\
|u^g(k) - u^a_{\text{max}}(k)| & \text{If } u^g(k) < u^a_{\text{max}}(k) \\
0 & \text{otherwise}
\end{cases}
\]

Lemma 3: for all $k = 0, 1, \ldots, H$ we have

\[
L^g(k) - L(k) = \begin{cases} 
\delta & \text{if } u^g(k) > u_{\text{max}} \text{ and } u^g(k) > u_{\text{max}} \\
\delta + (u^g(k) - u_{\text{max}}) & \text{if } u^g(k) > u_{\text{max}} \text{ and } u^g(k) < u_{\text{max}} \\
\delta - (u^g(k) - u_{\text{max}}) & \text{if } u^g(k) < u_{\text{max}} \text{ and } u^g(k) > u_{\text{max}} \\
0 & \text{otherwise}
\end{cases}
\]

Proof:

According to (4) we have 9 cases

- Case 1: if $u^g(k) > u_{\text{max}}$ and $u^g(k) > u_{\text{max}}$, then 
  \[
  L^g(k) = |u^g(k) - u_{\text{max}}| = |u(k) + \delta - u_{\text{max}}| \text{ and }
  L(k) = |u(k) - u_{\text{max}}|, \text{ thus we have } L^g(k) - L(k) = \delta .
  \]

- Case 2: if $u^g(k) > u_{\text{max}}$ and $u^g(k) < u_{\text{max}}$, we have 
  \[
  L^g(k) = |u^g(k) - u_{\text{max}}| = u^g(k) - u_{\text{max}} \text{ and }
  L(k) = |u(k) - u_{\text{max}}| = u_{\text{max}} - u(k) \text{ then }
  L^g(k) - L(k) = u^g(k) - u_{\text{max}} - u_{\text{max}} + u(k) = (u^g(k) + u(k)) - (u_{\text{max}} + u_{\text{max}})
  \]

- Case 3: if $u^g(k) > u_{\text{max}}$ and $u^g(k) \leq u(k) \leq u_{\text{max}}$, we have 
  \[
  L^g(k) = |u^g(k) - u_{\text{max}}| \text{ and } L(k) = 0 \text{ then }
  L^g(k) - L(k) = u^g(k) - u_{\text{max}} = u^g(k) - u_{\text{max}}
  \]

- Case 4: if $u^g(k) < u^a_{\text{max}}(k)$ and $u^g(k) > u_{\text{max}}$, we have 
  \[
  L^g(k) = |u^g(k) - u^a_{\text{max}}(k)| = u^a_{\text{max}}(k) - u^g(k) \text{ and }
  L(k) = |u(k) - u_{\text{max}}| = u_{\text{max}} - u(k) \text{ then }
  L^g(k) - L(k) = u^a_{\text{max}}(k) - u_{\text{max}} - u_{\text{max}} + u(k) = (u^a_{\text{max}}(k) + u(k)) - (u_{\text{max}} + u_{\text{max}})
  \]

- Case 5: if $u^g(k) < u^a_{\text{max}}(k)$ and if $u^g(k) < u_{\text{max}}$, we have 
  \[
  L^g(k) = |u^g(k) - u^a_{\text{max}}(k)| = u^a_{\text{max}}(k) - u^g(k) \text{ and }
  L(k) = |u(k) - u_{\text{max}}| = u_{\text{max}} - u(k) \text{ then }
  L^g(k) - L(k) = u^a_{\text{max}}(k) - u_{\text{max}} - u_{\text{max}} + u(k) = (u^a_{\text{max}}(k) + u(k)) - (u_{\text{max}} + u_{\text{max}})
  \]

- Case 6: if $u^g(k) < u^a_{\text{max}}(k)$ and if $u^g(k) \leq u(k) \leq u_{\text{max}}$, then 
  \[
  L^g(k) = |u^g(k) - u^a_{\text{max}}(k)| = u^a_{\text{max}}(k) - u(k) \text{ and } L(k) = 0 \text{ then }
  L^g(k) - L(k) = u^a_{\text{max}}(k) - u_{\text{max}}
  \]

- Case 7: if $u^g(k) \leq u_{\text{max}}$ and $u^g(k) < u_{\text{max}}$, we have 
  \[
  L^g(k) = 0 \text{ and } L(k) = |u(k) - u_{\text{max}}| = u_{\text{max}} - u(k) \text{ then }
  L^g(k) - L(k) = u_{\text{max}} - u(k)
  \]

- Case 8: if $u^a_{\text{max}}(k) \leq u^g(k) \leq u_{\text{max}}$ and $u^g(k) > u_{\text{max}}$, then 
  \[
  u^g(k) \leq u_{\text{max}} < u(k) \text{ this case is impossible because we assumed that } u^g(k) = u(k) + \delta \text{ with } \delta > 0 \text{ then } u^g(k) < u(k) \text{ (impossible case)}
  \]

- Case 9: if $u^a_{\text{max}}(k) \leq u^g(k) \leq u_{\text{max}}$ and $u^g(k) \leq u_{\text{max}}$, we have 
  \[
  L^g(k) = 0 \text{ and } L(k) = 0 \text{ then } L^g(k) - L(k) = 0
  \]

Q.E.D.

In what follows, we will determine the PA estimators and prove their unbiasedness.

3.3 PA estimators

In this section we determine the estimators of the difference of each part (production, inventory and lost sales costs) of the expected cost.

The expected cost for the nominal trajectory is given by:

\[
CT_n = E \left[ \sum_{k=0}^{H} C(k) \right].
\]

Then

\[
CT_n = (cp.E \left[ \sum_{k=0}^{H} u(k) \right] + cs.E \left[ \sum_{k=0}^{H} x(k) \right] + cs.E \left[ \sum_{k=0}^{H} L(k) \right])
\]

The expected cost for the perturbed trajectory is given by:

\[
CT_{\delta} = E \left[ \sum_{k=0}^{H} C^\delta (k) \right]. \text{ With }
\]

\[
C^\delta (k) = cp.u^\delta (k) + cs.x^\delta (k) + cs.L^\delta (k)
\]
Then
\[ CT_{ex}^\delta = \left( cp.E \left[ \sum_{k=0}^{H-1} u^\delta(k) \right] + cs.E \left[ \sum_{k=0}^{H-1} x^\delta(k) \right] + cs^+.E \left[ \sum_{k=0}^{H-1} L^\delta(k) \right] \right) \frac{E \left[ \sum_{k=0}^{H-1} (L^\delta(k) - L^\delta(k)) \right]}{\Delta u} = E \left[ \sum_{k=0}^{H-1} (L^\delta(k) - L^\delta(k)) \right] \]

The difference between the expected cost for the perturbed and nominal trajectories is given by:
\[ \Delta CT_{ex} = cp.E \left[ \sum_{k=0}^{H-1} (u^\delta(k) - u(k)) \right] + cs.E \left[ \sum_{k=0}^{H-1} (L^\delta(k) - L(k)) \right] + cs^+.E \left[ \sum_{k=0}^{H-1} (x^\delta(k) - x(k)) \right] \]

According to lemmas 2 and 3, we have
\[ \Delta CT_{ex} = cp.E \left[ \sum_{k=0}^{H-1} (\delta) \right] + cs.E \left[ \sum_{k=0}^{H-1} (k.\delta) \right] + cs^+.E \left[ \sum_{k=0}^{H-1} (L^\delta(k) - L(k)) \right] \]

Then
\[ \Delta CT_{ex} = \frac{cp.E[H . \delta] + cs.E[(H.(H+1)/2) . \delta]}{\Delta u} + cs^+.E \left[ \sum_{k=0}^{H-1} (L^\delta(k) - L(k)) \right] \]

For making these estimators useful in practice, the unbiasedness should be proved.

**Theorem:** The estimators of the difference of each part of the expected average cost are unbiased, i.e.:
\[ E \left[ \frac{H}{\Delta u} \right] = E \left[ \frac{H}{\Delta u} \right], \quad E \left[ \frac{H.(H+1)/2}{\Delta u} \right] = E \left[ \frac{H.(H+1)/2}{\Delta u} \right] \]

**Proof:**

- For \( E[H/\Delta u] = E[H/\Delta u] \) we have \( H \) is a constant than
\[ E[H/\Delta u] = \frac{H}{\Delta u} \] and \( E[H/\Delta u] = \frac{H}{\Delta u} \) then \( E[H/\Delta u] = E[H/\Delta u] \)

- For \( E[H.(H+1)/2]/\Delta u \) we have
\[ E[H.(H+1)/2]/\Delta u = \frac{H.(H+1)/2}{\Delta u} \] and \( E[H.(H+1)/2]/\Delta u = \frac{H.(H+1)/2}{\Delta u} \)

- For \( E[\sum_{k=0}^{H-1} (L^\delta(k) - L(k))/\Delta u] \) we have \( \sum_{k=0}^{H-1} (L^\delta(k) - L(k)) \) is constant, then we have:
\[ \frac{\sum_{k=0}^{H-1} (L^\delta(k) - L(k))/\Delta u}{\Delta u} = E \left[ \sum_{k=0}^{H-1} (L^\delta(k) - L(k)) \right] \]

Remark: The fact that the estimators are statistically unbiased, mean that the estimated value equal to the real value.

4. **NUMERICAL EXAMPLE**

4.1 **Optimization algorithm**

Before presenting the numerical results, we present an algorithm (PA estimation algorithm) for determining the PA estimators which will be used thereafter in an optimisation algorithm.

Let \( S, R \) and \( W \) be the PA estimators (parameters) which will be used in the following algorithm. Indeed, for the first and second estimators are known and which are \( S=H \) and \( R=H.(H+1)/2 \) (see the previous section). For the third estimator we have this algorithm.
PA estimation algorithm

Beginning

We, 0, k=0, q=0, l=0, w=0 //Initialisation 
Do
  • W= W+ (L(k) - L(k))
  • Advance k.
While k<H 
(Third estimator=W)
End

In what follows, we present an optimisation algorithm which determines the optimal production rate in a period k. Indeed, we combined this algorithm with Nelder-Mead method under MATHEMATICA software for determining the optimal production plan over the horizon H.

Optimization algorithm

The optimisation algorithm allows us to determine the optimal production planning and is given by:

Beginning

u= U_{max} , u= U_{max} and u_{w}= u_{0(k)} //Initialisation 
Do

Step 1: u(k)= u

Step 2: Determine the estimators R, S et W which correspond to u(k)= u_{i} by using the PA estimation algorithm.

Step 3: Determine the difference estimation of the cost function V(u(k)) by using the estimators R, S and W, with

\[ V(u(k)) = ((cp \cdot R) + (cs \cdot S) + (cs^{-} \cdot W)). \]

Step 4: If V(u(k)) \leq 0 then u_{i} = u_{w} and u= U_{max} return to step 1, else go to step 5.

Step 5: u_{w}= \text{int} \left[(u_{i} + u_{w})/2 \right] then determine V(u_{w}).

Step 6: If V(u_{w}) \leq 0 then u= u_{w} else u= u_{w}.

Step 7: If u_{i} \leq u then return to step 1, else go to step 8.

Step 8: The optimal production rate for the period k is equal to u_{w}. 
End.

4.2 Numerical results

In this part we use the PA estimators in an optimization algorithm, which allows us to determine the optimal production planning. The following arbitrarily chosen input data are considered as an example to illustrate our approach:

H=20 months.

(i) Lower and upper boundaries of production capacities: \( U_{\min} = 4 \) and \( U_{\max} = 10 \).

\( cp = 3 \text{mu (monetary unit)/k}, \ cs = 2 \text{mu/k} \) and \( cs^{-} = 350 \text{mu/k} \).

(ii) The demand is assumed Gaussian with a standard deviation \( \sigma = 2.0164 \).

(iii) The customer satisfaction degree, associated with the stock constraint, is equal to 90% (\( \theta = 0.9 \)).

In this part we use the PA estimators in an optimization algorithm, which allows us to determine the optimal production plan (\( u^{\ast}(k) \)) in Table 1.

5. CONCLUSIONS

In this paper, a manufacturing system composed by a single-product machine, a buffer and a demand is considered. A discrete fluid model is adopted to describe the system and to take into account. The PA method is applied the discrete fluid model. The buffer level and lost demands trajectories is studied and analyzed. The perturbation analysis estimators are determined and shown to be unbiased. These estimators are then implemented in an optimization algorithm for determining the optimal production planning.

For future research, we will combine the production with maintenance and we will apply the PA method for determining the optimal production and maintenance plan.

REFERENCES


