Simulation of free turbulent particle-laden jet: evaluation of first-order turbulence models and stochastic dispersion models

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Abstract: This paper is devoted to the numerical and theoretical study of free turbulent glass particle-laden jets. This study is based on results from a hybrid Euler/Lagrange simulation. Five two-equation turbulence models (STD k-ε, RNG k-ε, k-ω (88), k-ω (98) and MTS) were investigated and applied to the carried phase field simulation. Two stochastic particle dispersion models (Eddy Interaction Model and Time Correlated Model) were employed for the prediction of the particulate phase. Numerical results were compared with available experimental measurements. It was found that MTS and k-ε models predictions agree better with experimental measurements than those of the other models. The results show also that the Time Correlated Model provides improved fluctuating results in comparison to the Eddy Interaction Model. In contrast, the EIM provides better results for simulating mean particle velocity.

Keywords: free particle-laden jet; turbulence model; stochastic dispersion model.


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1 Introduction

A particle or droplet-laden turbulent free jet is a key parameter in several industrial processes such as combustion systems (diesel engine, gasoline engine with direct injection, gas turbine, industrial furnace ...), spray drying (food, detergents, drugs), or spray scrubber for gas cleaning and particle separation (cyclone separator). Therefore, an accurate prediction of droplets or particles motion and coupling between the two phases are of great importance. Consequently, the objective of the researchers is to understand, predict and control all the phenomena existing within a two-phase jet. Although it appears more and more difficult to separate modelling and measurement problems, we are here only concerned with numerical simulations which remain a challenging research field.

Classically, two theoretical approaches are developed for modelling two-phase flows: the Eulerian approach (Euler Euler) and Lagrangian approach (Euler Lagrange). Extensive discussions on both approaches have been proposed by Shirolkar et al. (1996), Gouesbet and Berlemont (1999), Loth (2000), among others. It is important to mention that both approaches have advantages as well as disadvantages, and are therefore more complementary rather than strongly opposed to each other. For instance, any progress achieved in one approach may help making progress in the other.

The Eulerian approach treats both phases as two interpenetrating continua which are governed by a set of differential equations representing conservation laws (Picart et al., 1986; Rizk and Elghobashi, 1989; Simonin et al., 1993).

In the Lagrangian approach, a large number of particle trajectory realisations must be considered and, by averaging these realisations, the required quantities such as particle mean and fluctuating velocities vs. space and time may be evaluated (Gosman and Ioannides, 1981; Berlemont et al., 1990; Pozorski and Minier, 1998). The Lagrangian models are based on two steps: the first describes the continuous phase, and the second step consists in following particulate trajectories by taking into account the flow characteristics.

Any approach to describe the turbulence field may then be used for the first step. Methods based on Direct Numerical Simulation (DNS) provide unique opportunities to study specific effects like particle trapping in eddies (Elghobashi and Truesdel, 1993; Eaton and Fessler, 1994) but are limited to small Reynolds number flows. Large Eddy Simulation (LES) can handle more complex flows (Boivin et al., 2000; Almeida and Jaberi, 2008) but the best quality/price ratio is, up to now in engineering problems, obtained by using a complete stochastic modelling (RANS) using a two-equation turbulence model, supplemented with algebraic relations to account for the anisotropy of the flow, or Reynolds Stress Model for a more accurate prediction of Reynolds stresses.

The $k$-$\varepsilon$ model focused the attention of most authors who worked on this subject. Among them, we can cite Sommerfeld and Qui (1993) who adopted the same model in order to predict a confined swirling particulate two-phase flow, they announced that fluctuating velocity of the carrying phase and of the particulate phase are badly predicted and that the RSM model can correct this dissatisfaction. Shirolkar and McQuay (2001) tested two turbulence models ($k$-$\varepsilon$ and Multiple-Time-Scale) to calculate a swirling particulate two-phase flow. They estimated that the MTS model agrees more with the experimental results but they did not consider energy exchange between both phases. Later, and since they estimated that the $k$-$\varepsilon$ model provides undesirable results because of the assumption of turbulence isotropy, Chen and Pereira (2000) used a second order model (RSM) to predict the dispersion of nickel particles in a turbulent flow with the aim of modelling industrial diphasic flow. Also, Sijercic et al. (2007) used a second order model (RSM) in their calculation for the prediction of a two-phase free jet. In the same way, Beishuizen et al. (2007) used a modified Reynolds Stress Model for the simulation of turbulent dispersed two-phase flows including two-way coupling. Moreover, Fairweather and Hurn (2008) adopted the RSM model to validate an anisotropic model of particle-laden jet.

Concerning particle dispersion, three principal Lagrangian models can predict the particulate phase. The first approach largely discussed in the literature and extensively used in engineering calculations is called the Eddy Interaction Model (Gosman and Ioannides, 1981). In this approach, the discrete particle is assumed to interact with a succession of eddies. Each eddy is characterised by a
fluctuating velocity, a time scale (lifetime) and a length scale (size). The second approach, called Time Correlated Model or two-steps model (Zhou and Leschziner, 1991), considers two particles (fluid and discrete particles) and is based on the simultaneous realisation of each particle trajectory and considers them interactions. Because of the drawbacks of such two-step methods, as identified and discussed by Pozorski and Minier (1998), the latter ‘mixed’ the Lagrangian step and the Eulerian step of the previous method to give a simpler stochastic scheme.

In this work, we chose the Lagrangian approach (Euler/Lagrange) for the simulation of a turbulent free particle-laden jet. However, very little research has been done to compare several first order turbulence models for the simulation of the continuous phase to be used for particle dispersion calculations. Consequently, the following contribution objective is to test several first-order turbulence models (STD k-ε, RNG k-ε, k-ω (88), k-ω (98) and MTS). To test the performances of these models, we use, as bases of comparison, the experimental results of Prevost et al. (1996). These results are more detailed in his thesis (Prevost, 1994).

Once the turbulence model is fixed for the simulation of the continuous phase, two stochastic Lagrangian models, the Eddy Interaction Model (Gosman and Ioannides, 1981) and the Time Correlated Model (Berlemont et al., 1990) were utilised for the prediction of the particulate phase. These models take into consideration the effect of turbulence anisotropy on particle dispersion. This assumption is proved by applying algebraic relations deduced from second order closing to estimate the Reynolds tensor.

Finally, the interaction between the two phases is carried out through source terms, to take into account the momentum exchanges. The turbulence modulation can be neglected, since it has a little impact on the turbulence of particles for dilute flows, as in the case considered here.

2 Mathematical model

2.1 Continuous phase

The simulation of a turbulent free jet requires the solution of the time-averaged equations for the conservation of mass (Eq. 1) and momentum (Eq. 2).

$$\frac{\partial}{\partial x_j}(\rho U_i) = 0$$

(1)

$$\frac{\partial(\rho U_i U_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_i} \left[ \mu \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \right] - \frac{\partial(\rho u_i u_j)}{\partial x_i}.$$  

(2)

The turbulence stress term $-\rho u_i u_j$ is a new unknown term which needs to be modelled in order to close the problem. The section below describes the different first order turbulence model closure relation-ships for the Reynolds stresses.

2.1.1 First-order turbulence models

The first order turbulence models are based on the Boussinesq (1877) assumption, which consists in modelling the Reynolds stresses using a turbulent viscosity and a mean velocity gradient.

$$-\rho u_i u_j = \mu_i \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \frac{2}{3} \rho k \delta_{ij}.$$  

(3)

The question is how to evaluate this new unknown term, which is the turbulent viscosity.

- **Standard k-ε model**

Since its creation in the 1970s (Launder and Spalding, 1974), the k-ε model has remained until nowadays the most popular turbulence model and it has played a central role in the calculation of many turbulent flows of scientific and engineering interest.

The eddy viscosity $\mu_i$ is related to the turbulent kinetic energy $k$ and to its dissipation rate $\varepsilon$ as:

$$\mu_i = \rho C_{\mu} \frac{k^2}{\varepsilon}.$$  

(4)

The modelled equations of $k$ and $\varepsilon$ are as follows:

$$\frac{\partial(\rho U_i k)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \frac{\mu + \mu_i}{\sigma_k} \right) \left( \frac{\partial k}{\partial x_j} \right) \right] + S_k$$

with

$$S_k = G - \rho \varepsilon.$$  

(5)

$$\frac{\partial(\rho U_i \varepsilon)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \frac{\mu + \mu_i}{\sigma_\varepsilon} \right) \left( \frac{\partial \varepsilon}{\partial x_j} \right) \right] + S_\varepsilon$$

with

$$S_\varepsilon = C_{\varepsilon 1} \frac{\varepsilon}{k} G - C_{\varepsilon 2} \rho \varepsilon^2.$$  

(6)

The turbulent kinetic energy production term is given by the following expression:

$$G = \mu_i \left[ \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right] \frac{\partial U_i}{\partial x_j}.$$  

(7)

According to Launder and Spalding (1974), the empirical constants used in the k-ε model are given in Table 1, including the Rodi (1972) correction for a turbulent free jet.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Empirical constants of the k-ε model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{\mu}$</td>
<td>$C_{\varepsilon 1}$</td>
</tr>
<tr>
<td>0.09–0.04f</td>
<td>1.44</td>
</tr>
</tbody>
</table>

with

$$f = \frac{Y}{2\Delta U} \left( \frac{\partial U_i}{\partial x} - \frac{\partial U_j}{\partial x} \right)^2.$$  

(8)
where $U_c$ is the longitudinal centreline mean velocity, $Y$ is the jet half-width and $\Delta U$ is the radial variation of the longitudinal mean velocity over $Y$.

**RNG $k$-$\varepsilon$ model**

The RNG $k$-$\varepsilon$ model was developed by Yakhot and Orszag (1986), and then revised by Yakhot et al. (1992). The structures of the turbulent kinetic energy equation and its scalar dissipation have the same forms as the STD $k$-$\varepsilon$ model. However, the RNG $k$-$\varepsilon$ model has a revised dissipation rate production term and modified empirical constants, derived from the RNG approach. The empirical constants for the RNG $k$-$\varepsilon$ model are given in Table 2.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Constants of the RNG $k$-$\varepsilon$ model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_\mu$</td>
<td>0.0845</td>
</tr>
<tr>
<td>$C_{\varepsilon}$</td>
<td>1.42–$R$</td>
</tr>
<tr>
<td>$C_{\varepsilon1}$</td>
<td>1.68</td>
</tr>
<tr>
<td>$\sigma_\varepsilon$</td>
<td>0.7179</td>
</tr>
<tr>
<td>$\sigma_\varepsilon$</td>
<td>0.7179</td>
</tr>
</tbody>
</table>

with 

$$R = \frac{1}{1 + \beta \eta^3}, \quad \eta = \sqrt{\frac{G}{C_\mu \rho \varepsilon}}$$

$\eta_0 = 4.38$ and $\beta = 0.015$.

**$k$-$\omega$ (88) model**

The $k$-$\omega$ model was proposed by Wilcox (1988) and is often used in modern CFD software tools. In this model, the second modelling parameter instead of $\varepsilon$ is $\omega$ which is defined as the turbulent frequency or the specific dissipation rate. This new variable is given by:

$$\omega = \frac{\varepsilon}{C_\rho k}$$

Turbulent viscosity is defined by:

$$\mu_t = \frac{\rho k}{\omega}$$

The modelled equations of $k$ and $\omega$ are as follows:

$$\frac{\partial (\rho U_i k)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \frac{\mu + \mu_t}{\sigma_k} \frac{\partial k}{\partial x_j} \right] + S_k$$

with 

$$S_k = G - \beta' pk \omega$$

$$\frac{\partial (\rho U_i \omega)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \frac{\mu + \mu_t}{\sigma_\omega} \frac{\partial \omega}{\partial x_j} \right] + S_\omega$$

with 

$$S_\omega = \alpha \frac{\omega}{k} G - \beta' \rho \omega^2.$$
The turbulent viscosity preserves the same form as the first-order turbulence models and are defined by:

\[ \nu = \rho \varepsilon \nu_k \]

The modelled equations for \( k, \varepsilon \) are as follows:

\[ \frac{\partial (\rho U, k)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_k}{\sigma_k} \right) \left( \frac{\partial k}{\partial x_j} \right) \right] + S_k \]

with \( S_k = \rho (\varepsilon_k - \varepsilon) \) \hspace{1cm} (17)

\[ \frac{\partial (\rho U, \varepsilon)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_k}{\sigma_k} \right) \left( \frac{\partial \varepsilon}{\partial x_j} \right) \right] + S_\varepsilon \]

with \( S_\varepsilon = \frac{c_1}{\sigma_\varepsilon} \frac{\rho \varepsilon^2}{k} + \frac{c_2}{\sigma_\varepsilon} \frac{\rho \varepsilon}{k} - \frac{c_3}{\sigma_\varepsilon} \frac{\rho \varepsilon^2}{k} \) \hspace{1cm} (18)

\[ \frac{\partial (\rho U, \varepsilon)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_k}{\sigma_k} \right) \left( \frac{\partial \varepsilon}{\partial x_j} \right) \right] + S_\varepsilon \]

with \( S_\varepsilon = \frac{c_1}{\sigma_\varepsilon} \frac{G^2}{\rho k} + \frac{c_2}{\sigma_\varepsilon} \frac{G \varepsilon}{k} - \frac{c_3}{\sigma_\varepsilon} \frac{\rho \varepsilon^2}{k} \) \hspace{1cm} (19)

The spectral assumption of balance between the region of transfer and dissipation gives:

\[ \varepsilon = \varepsilon_k \quad \text{and} \quad k = k_i + k_p. \] (21)

Empirical constants of the MTS model are given in Table 5.

**Table 5**  
Empirical constants of the MTS model

<table>
<thead>
<tr>
<th>( C_\mu )</th>
<th>( C_{pl} )</th>
<th>( C_{p1} )</th>
<th>( C_{p2} )</th>
<th>( C_{p3} )</th>
<th>( C_{pl} )</th>
<th>( \sigma_\mu )</th>
<th>( \sigma_\varepsilon )</th>
<th>( \sigma_k )</th>
<th>( \sigma_\varepsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.09</td>
<td>0.21</td>
<td>1.24</td>
<td>1.84</td>
<td>0.29</td>
<td>1.28</td>
<td>1.66</td>
<td>0.75</td>
<td>0.75</td>
<td>1.15</td>
</tr>
</tbody>
</table>

### 2.1.2 Reynolds tensor

The first-order turbulence model predicts the turbulent kinetic energy \( k \), but does not give any precise information on its re-partition according to the directions. There are several ways to better estimate the anisotropic structure of turbulence. One way is to use the Reynolds Stress Model, which is difficult to manage due to the stability problem. An intermediate solution is to estimate the Reynolds stress from algebraic relations deduced from a second order closure (Rodi, 1979). These relations supplement the first-order turbulence models and are defined by:

\[ \frac{u_i u_j}{\nu} = k \left[ \frac{2}{3} \frac{\delta_{ij}}{\eta} + \frac{1}{G - \varepsilon + c_\varepsilon \varepsilon} \right] \]

\[ \left[ (1 - \gamma_j) \left( \frac{P_j - \frac{2}{3} \delta_{ij} G}{\eta} \right) - \gamma_j \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \right] \] (22)

where \( k, \varepsilon, G \) and the gradients are calculated with the aid of the first-order turbulence model. Tensors \( P_j \) and \( D_j \) are given by:

\[ P_j = \left[ \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right] \] (23)

\[ D_j = \left[ \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right] \] (24)

Constants \( c_\varepsilon, \gamma_1, \gamma_2 \) and \( \gamma_3 \) are respectively given by 1.8, 0.76, 0.18 and 0.2 (Picart et al., 1986).

### 2.2 Dispersed phase

The main goal in the Lagrangian approach is to statistically represent the particle history in a given flow field. The starting point is the fundamental law of dynamics:

\[ m_p \frac{d\bar{v}_p}{dt} = \sum \bar{F}_i - \bar{m}_p \]

where \( \bar{F}_i \) stands for the resulting forces on the particle.

#### 2.2.1 Particle momentum equation

Since the density ratio of the gas to glass particles is very small, the main forces acting on a particle are the drag and gravity-floatability. All other forces, such as the lift and Basset forces, can be neglected. Note that it is assumed that the particles can be considered as point-particles. Moreover, it is assumed that particle-particle interaction is negligible since the mass loading ratio in our case is about 8%.

The final particle trajectory equation can be written as:

\[ \frac{d\bar{v}_p}{dt} = \frac{1}{\tau_p} (u_{ij} - \bar{u}_p) + g \left( 1 - \frac{\rho_p}{\rho} \right) \] (26)

where \( u_{ij} \) is the instantaneous fluid velocity along the path of the discrete particles and \( \tau_p \) is the particle response time scale given by:

\[ \tau_p = \frac{4}{3} \mu C_p R_p. \] (27)

The mathematical expression for the Reynolds particle number is defined as:
\[ R_{\text{ep}} = \rho_f D \frac{u_f - u_i}{\mu_f} \]  

(28)

The drag coefficient \( C_D \) is calculated by using the standard empirical correlations for a rigid sphere as:

\[
C_D = \begin{cases} 
\frac{24}{R_{\text{ep}}} & \text{for } R_{\text{ep}} < 1 \\
\frac{24}{R_{\text{ep}}} \left(1 + 0.15 \frac{R_{\text{ep}}}{1000}\right) & \text{for } 1 < R_{\text{ep}} < 1000, \\
0.44 & \text{for } R_{\text{ep}} > 1000 
\end{cases}
\]

(29)

Two problems arise, the determination of time steps required for the integration of particles momentum and position equations, and the determination of the fluid velocity ‘seen’ by the particles (fluid instantaneous velocity along the path of discrete particles).

### 2.2.2 Eddy interaction model

Historically, the first approach which has been developed and widely used in engineering calculations is based on the Eddy Life-Time concept. First described by Gosman and Ioannides (1981), the method has been adapted by Shuen et al. (1983), Faeth (1983), Chen and Pereira (2000), among others.

In this model, the particle is assumed to interact with the eddy for a time which is the minimum of the eddy life time \( t_e \) and the eddy transit time \( t_t \). Thus, the expression of the time step is given by:

\[ t_{\text{int}} = \Delta t = \min(t_e, t_t). \]  

(30)

The time and the length scales for an eddy are determined as follows:

\[ L_s = C_\mu^{3/4} \frac{k^{3/2}}{\varepsilon} \quad \text{and} \quad t_e = \frac{3}{2} C_\mu^{1/4} \frac{k}{\varepsilon}. \]  

(31)

The eddy transit time, the time a particle will take to cross an eddy, is expressed through the following expression:

\[ t_t = -\tau_p \ln \left( 1 - \frac{L_e}{\tau_p|v_{\text{rel}}|} \right) \]  

(32)

where \( v_{\text{rel}} \) is the relative velocity between the two phases.

Equation (32) does not have solutions when \( L_e > \tau_p|v_{\text{rel}}| \).

In such a case, the particle is assumed to be trapped by the eddy and the interaction time will be the eddy life time \( t_e \).

\[
\begin{cases} 
|v_{\text{rel}}| \leq \frac{L_e}{\tau_p} t_{\text{int}} = t_e \\
|v_{\text{rel}}| > \frac{L_e}{\tau_p} t_{\text{int}} = \min(t_e, t_t) 
\end{cases}
\]

(33)

During that interaction, the fluid fluctuating velocity is kept constant and the discrete particle is moved with respect to its momentum equation. Then, a new fluctuating fluid velocity is sampled and the process is repeated.

The fluctuating velocity used for each eddy is randomly sampled with a Gaussian PDF:

\[ u_{f,j} = \zeta \sqrt{u_{f,j}^2} \]  

(34)

where \( \zeta \) is the random Gaussian variable having zero mean values and unity deviation.

### 2.2.3 Time correlated model

This approach, also called a two-step model, is similar to the Eddy Life Time concept in a sense that the focus is also to solve the particle trajectory given by Equation (26). The difference lies in how the fluid fluctuating velocity is determined along the particle trajectory.

The method is based on the simultaneous realisation of a fluid trajectory and a particle trajectory. Originally developed by Ormancey and Martinon (1984), the method has been adapted by Zhou and Leschziner (1991), Lu et al. (1993), Burry and Bergeles (1993) or Chen and Pereira (1995) and extended by Berlemont et al. (1990).

A fluid particle instantaneously owns the velocity of the surrounding fluid and the simulation of its trajectory relies on a quite simple equation such as:

\[ x_{f,j}(t + \Delta t) = x_{f,j}(t) + u_{f,j}(x_{f,j}(t)) \Delta t. \]  

(35)

The instantaneous fluid velocity \( u_{f,j} \) is decomposed into a mean part, which is known (interpolated on particle location for instance from turbulence model predictions) and a fluctuating part \( u' \), which is generated with a Gaussian PDF and with respect to the Lagrangian time correlation.

The discrete particle trajectory is calculated with its motion equation. Differences appear between the two trajectories. Thus, the fluid velocity is transferred from the fluid position to the particle position with respect to Eulerian correlation, and the process is repeated.

The integration time must be selected in order to collect all the events undergone by the particle during its course. Desjonquères (1987) has showed that \( \Delta t \) is about:

\[ \Delta t = \min \left( \frac{\tau_p}{10}, \frac{\tau_L}{5} \right) \]  

(36)

where \( \tau_L \) is the Lagrangian integral time scale given by (Berlemont et al., 1990):

\[ \tau_L = \frac{|u_f|^2}{\varepsilon}. \]  

(37)

This approach has been extended by Berlemont et al. (1990), who include the correlation matrix method for fluid trajectories in order to handle any kind of correlation. Also, he defined a spherical correlation domain around the fluid particle \( L_D \). When the distance between the two particles is greater than the correlation length scale \( L_D \), a new fluid particle is sampled on the discrete particle location and the process is repeated.
\( L_D \) is defined as the arithmetic mean value between the normal scales:
\[
L_D = \frac{L_{z}\xi + L_z\eta}{2}.
\]
(38)

The Eulerian scale is expressed as:
\[
L_{\xi \eta} = \sqrt{\frac{\tau_{\xi \eta}}{\rho u}}.
\]
(39)

In our calculations, we expressed the Lagrangian and Eulerian correlations through a Frenkiel family of functions as described by Berlemont et al. (1990).

### 2.3 Two-way coupling

The interaction between the continuous and dispersed phases is introduced by treating particles as sources of momentum, and are calculated for each Eulerian cell of the continuous phase through the following expression:
\[
\left\{ S_{\text{pu}} \right\} = -\frac{1}{V_{\text{cell}}} \sum_{i} m_{i} N_{i} \Delta t_{i} \times \sum_{n} \left( \frac{\left[ u_{p,n}^{u} \right]^{1/3} - \left[ u_{p,n}^{e} \right]^{1/3}}{\Delta t_{i}} \right) - g_{i} \left( 1 - \frac{\rho_{f}}{\rho_{p}} \right)
\]
(40)

where the sum over \( k \) is related to the number of computational particles passing through the considered control volume with the volume \( V_{\text{cell}} \) and the sum over \( n \) indicates averaging along the particle trajectory (time averaging). \( N_{i} \) is the number of real particles in one computational particle and \( m_{i} \) is the mass of an individual particle. Furthermore, \( \Delta t_{i} \) is the Lagrangian time step which is used in the solution of particles momentum and position equations.

### 3 Boundary conditions

#### 3.1 Continuous phase

In the case of an axisymmetric jet and for reasons of symmetry, only half of the physical fields are considered as a computational domain with the following considerations:

- At the inlet, and in order to overcome as much as possible the influence of the jet injection, the axial mean and fluctuating RMS velocity profiles were calculated by extrapolating the measured values at \( x = 0.5d \) (Fig. 1), and are calculated from the following relations:
  \[
  U_{\text{inlet}} = U_{p,0} (1 - 2Y/d)^{1/5} \quad \text{for} \quad x = 0 \quad \text{and} \quad 0 \leq y < d/2
  \]
  (41)

  \[
  k_{\text{inlet}} = [U_{p,0} (0.04 + 0.1Y/d)]^{2} \quad \text{for} \quad x = 0 \quad \text{and} \quad 0 \leq y < d/2.
  \]
  (42)

  The \( k \) profile at the jet exit was prescribed assuming:
  \[
  u_{\text{r}}^{2} = v_{\text{r}}^{2} \quad \text{and} \quad w_{\text{r}}^{2} = 0.
  \]

  For the MTS model, a highly non-equilibrium condition was used to estimate the inlet turbulence quantities (Chen, 1985):
  \[
  k_{p} = k_{i} = 0.5k \quad \text{and} \quad \epsilon_{p} = 0.5\epsilon_{i} = 0.5\epsilon.
  \]
  (43)

  It should be noted that the radial velocity is null and that no experimental information is available on the dissipation rate of \( k \) at the emission section.

  The characteristic length scale and dissipation are calculated from the following relations:
  \[
  L_{n} = 0.005d \quad \text{and} \quad \epsilon_{\text{inlet}} = C_{p} k_{\text{inlet}}^{1.5/L_{n}}.
  \]
  (44)

  A numerical co-flow \((U_{\text{cof}} = 0.05 \text{ ms}^{-1} \text{ for } x = 0 \text{ and } y \geq d/2)\) was used to stabilise the computations. In fact, a jet with a ratio of co-flow velocity to inlet velocity less than 0.05 has developed to closely approximate free jet in a stagnant medium (Antonia and Bilger, 1973).

- At the outflow boundary, the gradient of dependent variables in the axial direction is set to zero, except for the axial velocity which undergoes a special treatment to check the principle of the mass conservation:
  \[
  \left( \frac{\partial \Phi}{\partial x} \right)_{\text{outlet}} = 0, \quad \Phi = V, k, \text{ or } \epsilon.
  \]
  (45)

- On the symmetry axis, the radial velocity and radial gradients of all variables are set to zero.
  \[
  \left( \frac{\partial \Phi}{\partial y} \right)_{y=0} = 0 \quad \text{and} \quad V = 0, \Phi = U, k, \text{ or } \epsilon.
  \]
  (46)

- In the free boundary parallel to the axis, we consider the following conditions:
  \[
  U = 0, \quad \frac{\partial V}{\partial y} = 0, \quad k = 0, \epsilon = 0.
  \]
  (47)

#### 3.2 Dispersed phase

- At the inlet, the particles mean that the longitudinal velocity was about 0.6 times the inlet fluid mean velocity \( U_{p,\text{inlet}} = 0.6 U_{\text{inlet}} \) (Fig. 2(a)). The particles’ longitudinal fluctuating RMS velocity was about three times that of the fluid \( U_{p,\text{inlet}}^{u} = 3 U_{\text{inlet}}^{u} \) (Fig. 2(b)), whereas the particles’ transversal fluctuating RMS velocity was equal to that of the fluid.

- The particle-wall and particle-symmetry axis collisions are modelled as simple reflections.

- When a particle leaves the calculation domain, its trajectory will be stopped and a new particle is launched.
4 Numerical procedure

The calculation procedure is based on the TEACH code (Gosman and Ideriah, 1976) for the simulation of the Eulerian phase, and on the PALAS code (PArticle Lagrangian Simulation; Berlemont et al., 1990) for the simulation of the Lagrangian phase. Initially,

1. the flow field is calculated until a partially converged solution is achieved
2. a large number of particles are tracked through the flow field
3. the source terms to the gas-phase equation are calculated
4. the flow field is recomputed under the influence of the particle source terms until a partially converged solution is achieved
5. repeat steps (2)–(4) until overall convergence is achieved.

Generally, a large number of coupling iterations between the Eulerian phase and the Lagrangian phase are required to achieve overall convergence, depending on the particles’ mass loading, particles’ diameter and the type of the flow. In our case, since the mass loading ratio is about 8%, three coupling iterations can give satisfactory results.

The Finite Volume Method (FVM) with a staggered grid as described by Patankar (1980) is used for the numerical resolution of the transport equations of the continuous phase. The formulation is elliptic convection-diffusion and the pressure velocity coupling is achieved through the SIMPLE algorithm. The convergence of the numerical solution was based on the sum of the absolute residual sources and must be less than 0.3%. The diffusion and the convection coefficients are discretised using a Hybrid scheme. Calculation is carried out with a structured, orthogonal, expanding grid, with an expansion factor of 1.05. In fact, the mesh in both directions becomes wider when one moves away from the emission section.
Simulation of free turbulent particle-laden jet of the jet. The considered calculation domain was 600 mm in the streamwise and 200 mm in the radial direction. Different grid sizes are tested to show that the results are independent of numerical influences for grids finer than $54 \times 54$ meshes.

The system of Equations (22)–(24) is solved using a Predictor-Corrector Algorithm (PCM) as recommended by Picart et al. (1986).

For the simulation of the behaviour of the particles, typically 25,000 trajectories for the class of size 40–45 $\mu$m were numerically tracked. The sizes of the particles are generated randomly so that their distribution corresponds to the experimental data. The initial position at the jet’s exit is also generated randomly. For the resolution of the particles’ momentum equation, we use a second-order Runge-Kutta scheme.

5 Results and discussion

The flow configuration is shown in Figure 3 and consists of a jet with glass beads issuing vertically downward from a pipe in still ambient air. The geometric and flow parameters used in the present computation are the same as those in the experimental works of Prevost (1994) and are summarised in Table 6, knowing that $\varphi$ presents the ratio of mass flux particles/fluid at injection. The exit Reynolds number based on the tube diameter ($d = 0.01$ m) is 13100, corresponding to a mean exit velocity of 20 m $\cdot$ s$^{-1}$ ($\approx 0.8 U_{\infty}$). The injection tube was 75 diameters long to provide a fully-developed turbulent pipe flow at the jet inlet.

Table 6 Operating conditions

<table>
<thead>
<tr>
<th>$U_{\infty}$ (m $\cdot$ s$^{-1}$)</th>
<th>$d/2$ (m)</th>
<th>$D_p$ ($\mu$m)</th>
<th>$\varphi$ (Kg $\cdot$ m$^{-3}$)</th>
<th>$\rho_p$ (Kg $\cdot$ m$^{-3}$)</th>
<th>Re</th>
<th>$Q_{\infty}$ (Ls$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>24.85</td>
<td>0.005</td>
<td>40–45</td>
<td>0.08</td>
<td>1.205</td>
<td>2460</td>
<td>13,100</td>
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</tbody>
</table>

The virtual origin obtained with the $k$-$\varepsilon$ model is smaller than the experimental data of Prevost (1994), and that obtained with $k$-$\omega$ (88) model is the greatest. Nevertheless, better predictions of $A$ and $B$ are obtained with the MTS model.

5.1 Continuous phase

The centreline axial velocity decay of the gaseous phase is plotted in Figure 4. It presents one of the most important characteristic parameters of the jet studies, since the decrease of this quantity is the consequence of the variation of all jet parameters, and it can be written as:

$$\frac{U_{\infty}}{U_{\infty}} = A \left( \frac{x}{d} - B \right)$$

(48)

where $A$ represents the mean streamwise velocity decay rate and $B$ represents the virtual origin.

Figure 4 Normalised axial velocity along the centreline of the jet (see online version for colours)

The values of the constant $B$ (reported in Table 7) show a considerable scatter. Such variations are not unexpected because the location of the virtual origin is sensitive to the variation of the Reynolds number, the velocity profiles and intensities at the jet exit and the density ratio of the injected fluid to the ambient fluid. Constants ($A$ and $B$) calculated from the $k$-$\omega$ (88) model are significantly greater than other reported data. Constants $A$ obtained from the $k$-$\varepsilon$ and $k$-$\omega$ (98) models are close to the value measured by Chassaing (1979).

Table 7 Mean streamwise velocity decay rates and virtual origins (continuous phase)

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>0.189</td>
<td>0.35</td>
<td>0.235</td>
<td>0.46</td>
<td>0.187</td>
<td>0.228</td>
</tr>
<tr>
<td>$B$</td>
<td>1.0</td>
<td>5.5</td>
<td>3.8</td>
<td>8.0</td>
<td>5.17</td>
<td>4.0</td>
</tr>
</tbody>
</table>

The virtual origin obtained with the $k$-$\varepsilon$ model is smaller than the experimental data of Prevost (1994), and that obtained with $k$-$\omega$ (88) model is the greatest. Nevertheless, better predictions of $A$ and $B$ are obtained with the MTS model.

It was proved that the RNG $k$-$\varepsilon$ model has been found to poorly predict the velocity profile for the plane jet flow and round jet flow due to its overestimation of the flow spreading rates (Wilcox, 1998).

Figure 5(a)–(c) show the radial profiles of the mean longitudinal velocity at several downstream locations.

For the first section, at $x/d = 10$, results reported by the MTS and the $k$-$\varepsilon$ models coincide and are in close agreement with the measurements value for $y/x < 0.1$. In contrast, for $y/x > 0.1$, results obtained by these two models predict higher velocities at the jet edge.
Figure 5  Radial variation of carrier phase mean velocity at: (a) $x/d = 10$; (b) $x/d = 20$ and (c) $x/d = 30$ (see online version for colours)

Furthermore, the results determined from the $k-\omega$ (88) and $k-\omega$ (98) models are smaller than the experimental values, and results obtained from the RNG $k-\varepsilon$ model are greater than the experimental values. However, at the edge of the jet, the $k-\omega$ (98) model predicts a higher velocity.

For the second and third sections, ($x/d = 20$ and $x/d = 30$), only MTS and $k-\varepsilon$ models predict reasonably the mean longitudinal velocity near the centreline of the jet while they predict slightly higher velocities at the jet edge. Differences between the simulation and the experimental data are most apparent near the edge of the jet, where the real flow is intermittently turbulent and laminar.

The deficiency of the results of the $k-\omega$ (88) model compared to other models could be partly attributed to neglecting the cross diffusion term

$$\frac{1}{\omega} \frac{\partial k}{\partial x} \frac{\partial \omega}{\partial x},$$

because its inclusion can lead to improving the model’s performance for predicting the free turbulent jet. Its primary role will be to increase $\omega$ and hence, reduce the kinetic energy $k$ by increasing the dissipation source term.

STD $k-\varepsilon$, RNG $k-\varepsilon$ and MTS models have the cross diffusion term in their $\varepsilon$ equations. The $k-\omega$ (98) model uses the revised coefficient $f'_{\beta}$ to provide the impact of the cross diffusion term on the $k$ equation. The action provided by $f'_{\beta}$ does not completely emulate the action of the diffusion term described in the $\varepsilon$ equation, since the $k-\omega$ (98) model did not perform as well as the STD $k-\varepsilon$ model.

The evolution of the streamwise $(U'_x/U_f)$ and spanwise $(V'_f/U_f)$ turbulence intensities along the centreline is shown in Figure 6(a) and (b). A rapid increase of these parameters is observed initially, and then, they tend asymptotically to a limit in the far field, because of the decreasing local streamwise velocities.

We note from Figure 6(a) and (b), that the MTS and $k-\varepsilon$ models predictions close with the experiments results of Prevost (1994) than the prediction of the other models (RNG $k-\varepsilon$, $k-\omega$ (88) and $k-\omega$ (98)), with a little advantage to the MTS model. Finally, we indicate that $k-\omega$ (98) model significantly underpredicts the experience and that $k-\omega$ (88) and RNG $k-\varepsilon$ models significantly overpredict the experimental data. The overprediction concerning the $k-\omega$ (88) model is also attributed to the neglecting of the cross diffusion term.

Figure 6  Variation of: (a) longitudinal velocity fluctuation along centreline and (b) transversal velocity fluctuation along centreline (see online version for colours)
The radial non-dimensional profiles of longitudinal and transversal fluctuating (RMS) velocity against \( y/x \) are presented in Figures 7–9 at three locations downstream from the injector \( (x/d = 10, \ x/d = 20 \) and \( x/d = 30 \)). According to these figures, we state that the MTS and \( k-\varepsilon \) models prove their capacities to predict well the radial evolution of the fluctuating (longitudinal and transversal) gaseous phase velocity. These figures indicate also that the longitudinal fluctuating velocity predictions obtained with the MTS model are better when compared to the results obtained with the \( k-\varepsilon \) model near the centreline region of the jet. However, at the jet edge, the \( k-\varepsilon \) model provides improved results in comparison to the MTS model.

**Figure 7** Radial profiles of: (a) longitudinal velocity fluctuation at \( x/d = 10 \) and (b) transversal velocity fluctuation at \( x/d = 10 \) (see online version for colours)

**Figure 8** Radial profiles of: (a) longitudinal velocity fluctuation at \( x/d = 20 \) and (b) transversal velocity fluctuation at \( x/d = 20 \) (see online version for colours)

**Figure 9** Radial profiles of: (a) longitudinal velocity fluctuation at \( x/d = 30 \) and (b) transversal velocity fluctuation at \( x/d = 30 \) (see online version for colours)
Furthermore the transversal fluctuating velocity results obtained with the MTS model are better compared to the \( k-\varepsilon \) model predictions for the two sections \( x/d = 20 \) and \( x/d = 30 \), but are less for the section \( x/d = 10 \). We indicate also that the overprediction of the mean and fluctuating velocity provided by the \( k-\omega \) (88) model is only seen in the following two sections: \( x/d = 20 \) and \( x/d = 30 \).

In summary, the comparisons between our calculations and the experimental measurements of Prevost (1994) showed that the mean and fluctuating characteristics concerning the radial or axial evolutions of the dynamic parameters of the flow-fields are better predicted with the MTS and \( k-\varepsilon \) models. Nevertheless, the MTS model proves to be better than the \( k-\varepsilon \) model in predicting the mean flow dynamic properties at the jet centreline. However the \( k-\varepsilon \) model provides improved mean results in comparison to the MTS model at the jet edge.

5.2 Dispersed phase

The particle mean and fluctuating velocities are obtained using the ensemble averaging procedure for all the particle trajectories that cross the Eulerian control volume in question. The averages are obtained over particles’ momentum and position equations through times. Then, the ensemble means are averaged azimuthally. Therefore, an accurate prediction of the single phase flow fields is a prerequisite for an accurate prediction of dynamic parameters for the dispersed phase. Since the results of the MTS and \( k-\varepsilon \) models agree better than other models with experimental data for the gas phase, they are adopted in conjunction with the trajectory tracking. In addition, results of the \( k-\omega \) (88) model were also considered, in order to make out the influence of undesirable gas phase results on dispersed phase predictions.

Figure 10 shows the particles’ mean longitudinal velocity decay as a function of the normalised distance \( x/d \). The predicted results obtained from the two dispersion models (TCM and EIM) are nearly similar, with a little advantage to the EIM. Furthermore, predictions using the MTS model for the gas phase provide improved results in comparison to the other predictions which use the \( k-\varepsilon \) and \( k-\omega \) (88) models, similar to the corresponding gas phase results. These remarks are deduced from Table 8.

![Figure 10](image_url) Axial velocity of particles along centreline (see online version for colours)

The longitudinal particles phase velocity predictions did not differ significantly between all model predictions, in the case of Figure 11(a), and this fact is reflected in the predicted average gas phase velocities in Figure 5(a). We can see from Figure 11(b) that EIM predictions are closer to the experimental data than the TCM at the edge of the jet. The same comments can be extracted from Figure 11(c).

The non-dimensional profiles of longitudinal particles’ fluctuating (RMS) velocity are plotted in Figure 12(a)–(c) for three downstream sections, \( x/d = 10 \), \( x/d = 20 \) and \( x/d = 30 \). As can be seen, the numerical values of the two dispersion models underestimate significantly the experimental results. However, the Time Correlated Model provides better results than the Eddy Interaction Model.

![Figure 11](image_url) Radial variation of solid phase mean velocity at: (a) \( x/d = 10 \); (b) \( x/d = 20 \) and (c) \( x/d = 30 \) (see online version for colours) (continues on next page)

---

**Table 8** Mean streamwise velocity decay rates and virtual origins (dispersed phase)

<table>
<thead>
<tr>
<th>Model</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>TCM</td>
<td>0.125</td>
<td>12</td>
</tr>
<tr>
<td>TCM + ( k-\varepsilon )</td>
<td>0.15</td>
<td>14</td>
</tr>
<tr>
<td>TCM + MTS</td>
<td>0.31</td>
<td>18</td>
</tr>
<tr>
<td>TCM + ( k-\omega ) (88)</td>
<td>0.142</td>
<td>15</td>
</tr>
<tr>
<td>EIM</td>
<td>0.177</td>
<td>17</td>
</tr>
<tr>
<td>EIM + ( k-\varepsilon )</td>
<td>0.32</td>
<td>18</td>
</tr>
<tr>
<td>EIM + MTS</td>
<td>0.19</td>
<td>12</td>
</tr>
</tbody>
</table>

---

(a)  
(b)
Simulation of free turbulent particle-laden jet

**Figure 11** Radial variation of solid phase mean velocity at: (a) $x/d = 10$; (b) $x/d = 20$ and (c) $x/d = 30$ (see online version for colours) (continued)

In Figure 13(a)–(c), the radial profiles of particles’ transversal fluctuating (RMS) velocity normalised by the centreline value of particles’ mean longitudinal velocity vs. the radial distance normalised by $x$ for three downstream sections are plotted.

**Figure 12** Radial variation of particle longitudinal velocity fluctuation at: (a) $x/d = 10$; (b) $x/d = 20$ and (c) $x/d = 30$ (see online version for colours)

We note also, that the numerical results underpredict significantly the experimental measures for both models of dispersion. These discrepancies can be attributed to the initial conditions (Berlemont et al., 1990), first-order turbulence model (Sommerfeld and Qui, 1993) or experimental measurements (Berlemont et al., 1990; Chen, 2000). Nevertheless, in our simulations, although we used first-order turbulence models, only the fluctuating particles
velocity underpredict the experiment, whereas the flow fluctuating velocity presents a satisfactory agreement with the experiment Figures 6–9. Concerning the initial conditions, they are well-defined in the experiment and checked with our simulations. Consequently, these discrepancies are not only attributed to the initial conditions, first-order turbulence models or experimental measurements, but can also be attributed to the stochastic dispersion models. Therefore, we suggest further improvements for these models, especially for the prediction of the fluctuating particles’ velocity.

Table 9 compared our simulation results with the experiments results of Prevost (1994) and of Hardalupas (1989). It should be noticed that the Time Correlated Model provides better results than the Eddy Interaction Model, with regard to the particle fluctuating (RMS) velocity using MTS gas phase model predictions.

Furthermore, Figures 12(a) and 13(a) show the particle fluctuating (RMS) velocity predictions (both, longitudinal and transversal) for the section $x/d = 10$, using the $k$-$\varepsilon$ model for the gas phase, and provide improved results in comparison to the corresponding gas phase results.

Table 9  Comparison of particle fluctuating RMS velocities characteristics

<table>
<thead>
<tr>
<th>$x/d$</th>
<th>$U_p' / U_p$</th>
<th>$V_p' / U_p$</th>
<th>$U_p' / U_{k_s}$</th>
<th>$V_p' / U_{k_s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hardalupas (1989)</td>
<td>$y/x = 0$ 0.16 0.04 0.25 0.07 0.265 0.13</td>
<td>$y/x = 0.1$ 0.16 0.06 0.20 0.07 0.245 0.15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prevost (1994)</td>
<td>$y/x = 0$ 0.145 0.03 0.32 0.06 0.31 0.10</td>
<td>$y/x = 0.1$ 0.21 0.04 0.20 0.07 0.22 0.12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Simulation TCM + MTS</td>
<td>$y/x = 0$ 0.083 0.024 0.127 0.032 0.13 0.05</td>
<td>$y/x = 0.1$ 0.18 0.025 0.173 0.047 0.147 0.09</td>
<td></td>
<td></td>
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<tr>
<td>Simulation EIM + MTS</td>
<td>$y/x = 0$ 0.04 0.014 0.082 0.018 0.125 0.044</td>
<td>$y/x = 0.1$ 0.09 0.015 0.106 0.029 0.093 0.07</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5.3 Two-way coupling
The reciprocal of the jet centreline velocity is plotted in Figure 14(a), with and without particles, using the MTS model coupled with the TCM model. In Figure 14(b), the non-dimensional fluctuating (RMS) velocity of the carrier phase is plotted as a function of the non-dimensional distance from the jet exit. We noticed that the simulation results conform to the experimental results in the case of mean velocity decay and not for the fluctuant velocity profiles.

Figure 14  (a) Axial velocity of the gaseous phase with and without particles and (b) axial velocity fluctuation of the gaseous phase with and without particles (see online version for colours)

6 Conclusion
As stated earlier, the main objective of this study is to evaluate the performance of five first-order turbulence models in the prediction of local gas properties to be used for particle dispersion calculations. In addition, we investigated the capability of two stochastic Lagrangian dispersion models in predicting the velocity field of particles. The above exhaustive numerical research leads to the following conclusions:

Comparisons between the results obtained from many turbulence models and the results of the corresponding experimental investigations made previously, and taken from reference, have showed mostly a good agreement only with the STD $k$-$\varepsilon$ and MTS models.
The superiority of the behaviour of the MTS model is due to its repartition of the turbulence kinetic energy spectrum into two distinct time scale ranges, in order to predict better the process of production and gradual cascade transfer of turbulent kinetic energy \( k \), and consequently, to predict the physics of turbulence more accurately. However, the poorer performance of the \( k-\alpha \) \((88)\) model could be caused by the neglect of the cross diffusion term.

The particle fluctuating velocities (both, longitudinal and transversal) are underpredicted across the jet, with considerable discrepancies between predictions and measurements. These discrepancies could be attributed to the initial conditions, first-order turbulence models, experimental measurements and stochastic dispersion models. Furthermore, the TCM agrees better with experimental measurements than the EIM.

The particles’ mean velocities predictions using both dispersion models are nearly identical in the centreline region of the jet, with a little advantage to the EIM. In contrast, for the edge region of the jet, the EIM provides better results in comparison to the TCM.

Future work will focus on investigating different particles’ dispersion models in conjunction with the second-order closure for the gas phase.

References


Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$C_D$</td>
<td>Drag coefficient</td>
</tr>
<tr>
<td>$C_{mu}, C_{mp}, C_i$</td>
<td>Turbulence model constants</td>
</tr>
<tr>
<td>$d$</td>
<td>Nozzle diameter (m)</td>
</tr>
<tr>
<td>$D_p$</td>
<td>Particle diameter (m)</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravitational acceleration (m·s$^{-2}$)</td>
</tr>
<tr>
<td>$G$</td>
<td>Production term of $k$ (kg·m$^{-1}$·s$^{-3}$)</td>
</tr>
<tr>
<td>$k$</td>
<td>Turbulent kinetic energy (m$^2$·s$^{-2}$)</td>
</tr>
<tr>
<td>$L_c$</td>
<td>Eddy length scale (m)</td>
</tr>
<tr>
<td>$m_p$</td>
<td>Particle mass (kg)</td>
</tr>
<tr>
<td>$P$</td>
<td>Pressure (N·m$^{-2}$)</td>
</tr>
<tr>
<td>$Q_V$</td>
<td>Volume flux (l·s$^{-1}$)</td>
</tr>
<tr>
<td>$Re_g$</td>
<td>Reynolds number</td>
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<td>Source terms of the gas phase</td>
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<td>Source terms due to the particles phase</td>
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<tr>
<td>$t_c$</td>
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<td>$t_{int}$</td>
<td>Eddy interaction time (s)</td>
</tr>
<tr>
<td>$u$</td>
<td>Instantaneous velocity (m·s$^{-1}$)</td>
</tr>
<tr>
<td>$u'$</td>
<td>Fluctuating velocity (m·s$^{-1}$)</td>
</tr>
<tr>
<td>$U$</td>
<td>Mean velocity (m·s$^{-1}$)</td>
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<tr>
<td>$U'$</td>
<td>Root Mean Square velocity (m·s$^{-1}$)</td>
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<td>$x$</td>
<td>Axial coordinate (m)</td>
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<td>$y$</td>
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Greek symbols

<table>
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<tr>
<th>Symbol</th>
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<tr>
<td>$\delta_{ij}$</td>
<td>Kroneker delta (= 1, $i=j, 0, i \neq j$)</td>
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<td>$\Delta t$</td>
<td>Time step (s)</td>
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<td>$\epsilon$</td>
<td>Dissipation rate of $k$ (m$^2$·s$^{-3}$)</td>
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<td>$\Phi$</td>
<td>Generalised variable</td>
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<td>$\omega$</td>
<td>Specific dissipation rate of $k$ (s$^{-1}$)</td>
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Subscripts

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Superscripts

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<td>$-$</td>
<td>Time average</td>
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<td>$'$</td>
<td>Fluctuation</td>
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Acronyms

<table>
<thead>
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<th>Acronym</th>
<th>Description</th>
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<td>Eddy Interaction Model</td>
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<tr>
<td>STD</td>
<td>Standard</td>
</tr>
<tr>
<td>TCM</td>
<td>Time Correlated Model</td>
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