STABILIZATIONS OF CONTINUOUS TIME SYSTEMS BY FIRST ORDER CONTROLLERS

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Abstract

In this paper we consider the problem of stabilizing a given but arbitrary linear time invariant continuous time system with transfer function $P(s)$, by a first order feedback controller $C(s) = \frac{x_1 + x_2}{s + x_3}$. The complete set of stabilizing controllers is determined in the controller parameter space $[x_1, x_2, x_3]$; this includes an answer to the existence question of whether $P(s)$ is “first order stabilizable” or not. The set is shown to be computable explicitly, for fixed $x_3$ by solving linear equations and the three dimensional set is recovered by sweeping over the scalar parameter $x_3$. This result is applicable to a) the simultaneous stabilization problem and b) the robust stabilization problem of a continuum of plants. The latter is illustrated by applying it to the stabilization of an interval family of transfer functions $P(s)$ which reduces to the stabilization of the Kharitonov vertex plants. In each case the solution is facilitated by the fact that linear equations are involved in the solution so that the intersection of sets can be found by adding more equations. Illustrative examples are included. They demonstrate that the shape of the stabilizing set in the controller parameter space is quite different and much more complicated compared to that of PID controllers despite the fact that both are “three term controllers”. It is remarkable that despite this complicated topology the set can be unravelled via “linear computations”. Extensions and applications to design are discussed.

1 Introduction

In the last 40 years, feedback control design problems have mainly been formulated and solved within the broad framework of optimal control theory. The power of optimal control theory is based on the YJBK characterization of all stabilizing controllers for a given plant [11]. Such a characterization allows us to effectively search for a controller that satisfies given optimality criteria. Unfortunately, this characterization (or parametrization) does not take account of the order of stabilizing controllers and often yields unnecessarily high order controllers. Due to this reason, the fixed order control design problem cannot take advantage of most of these powerful optimization based design approaches. In fact, there are only few results available to deal with the fixed order or fixed structure control problem [7, 4].

On the other hand, the majority of industrial control systems are based on simple and fixed structure controllers such as PID and first order controllers. De-
spite the availability of a great deal of research and extensive literature on the design and tuning of PID controllers (see [2] and references therein), modern optimal control methods and fundamental structure are largely ignored in this literature. An exception is the recent development reported in [6] where the complete set of stabilizing controllers was found and shown to be computable by solving a set of linear inequalities parametrized by the proportional (P) gain.

In this paper, we give a similar solution to the problem of finding all first order controllers $C(s) = \frac{x_1 s + x_2}{s + x_3}$ that stabilize a given linear time-invariant (LTI) plant $P(s)$ of order $n$. The method is based on determining regions in the parameter space wherein the numbers of open left half plane (LHP) and open right half plane roots of the closed loop characteristic polynomial remains invariant. These “invariant regions” can be determined by mapping the corresponding regions in the coefficient space to the parameter space. This type of decomposition in the coefficient space was originally proposed in [9] and is known as D-decomposition. An excellent account of D-decomposition and other parameter plane techniques is given in [10] where such parameter mappings are given for many design criteria. However to the best of our knowledge this technique of parameter mapping has not been applied to determine the set of first order controllers, and the present paper is the first complete solution of this problem.

In our problem the parameter space separates into disjoint open subsets and stabilization by a first order controller is possible if and only if there exists a region in $[x_1, x_2, x_3]$ space where $n + 1$ roots lie in the LHP. We show that for fixed $x_3$ these regions can be determined in the corresponding $x_1 - x_2$ plane by solving linear equations for the boundary crossing points as functions of the frequency in closed form and thus explicitly generating the root invariant regions. From these calculations the parameter space regions that consist of all first order stabilizing controllers are obtained by sweeping over the parameter $x_3$. The results can be displayed as sections in the $[x_1, x_2]$ plane for fixed $x_3$ or via 3-D graphics.

Comparing this solution to the recently obtained solution for stabilizing PID controllers we note that in the latter case it was shown (see [6] and [1]), that the transition from stability to instability could only occur through certain frequencies, for fixed proportional gain. This resulted in a simplification of the solution for the stabilization sets. Such a simplification does not occur in the first order case even though it is also a three parameter controller and in general roots can cross the boundary at all frequencies, even for fixed $x_3$. Nevertheless it is remarkable that the set can be unraveled through the solution of a nested set of linear equations. We also show how this solution extends to simultaneous stabilization of a set of plants $P_i(s), i = 1, 2 \cdots$ and by using certain vertex results (see [3] and [5]) to the problem of first order robust stabilization of a given interval plant family. Illustrative examples are given for each of these cases.

2 Computation of Root Distribution Invariant Regions

Consider an arbitrary LTI plant and a first order controller (see Figure 1)

![Figure 1: A unity feedback system](image)

Given by

Plant : $P(s) := \frac{N(s)}{D(s)}$

Controller : $C(s) := \frac{x_1 s + x_2}{s + x_3}$

We naturally assume that the plant $P(s)$ is stabilizable, by a controller of some order, not necessarily first order. Let us use the standard even-odd decomposition of polynomials:

\begin{align*}
N(s) &:= N_e \left( s^2 \right) + s N_o \left( s^2 \right) \\
D(s) &:= D_e \left( s^2 \right) + s D_o \left( s^2 \right).
\end{align*}

The characteristic polynomial of the closed loop system is

$$\delta(s) = D(s) \left( s + x_3 \right) + N(s) \left( x_1 s + x_2 \right)$$
\[ D_e \left( s^2 \right) + s D_0 \left( s^2 \right) \] (s + x_3) \\
+ N_e \left( s^2 \right) + N_0 \left( s^2 \right) \] (x_1 s + x_2) \\
= \left[ s^2 D_e \left( s^2 \right) + x_3 D_e \left( s^2 \right) + x_2 N_e \left( s^2 \right) \right] \\
+ x_1 s N_0 \left( s^2 \right) \\
+ s \left[ D_e \left( s^2 \right) + x_3 D_0 \left( x^3 \right) + x_2 N_0 \left( s^2 \right) \right] \\
+ x_1 N_e \left( s^2 \right) \] .

With \( s = j \omega \), we have
\[
\delta(j \omega) = \left[ -\omega^2 N_0 \left( -\omega^2 \right) x_1 + N_e \left( -\omega^2 \right) x_2 \right. \\
+ D_e \left( -\omega^2 \right) x_3 - \omega^2 D_0 \left( -\omega^2 \right) \right] \\
+ j \omega \left[ N_e \left( -\omega^2 \right) x_1 + N_0 \left( -\omega^2 \right) x_2 \right. \\
+ D_0 \left( -\omega^2 \right) x_3 + D_e \left( -\omega^2 \right) \right] .
\]

The complex root boundary is given by
\[
\delta(j \omega) = 0, \omega \in (0, +\infty) \tag{3}
\]
and the real root boundary is given by
\[
\delta(0) = 0, \delta_{n+1} = 0 \tag{4}
\]
where \( \delta_{n+1} \) denotes the leading coefficient of \( \delta(s) \).

Thus, the complex root boundary is given by:
\[
-\omega^2 N_0 \left( -\omega^2 \right) x_1 + N_e \left( -\omega^2 \right) x_2 + D_e \left( -\omega^2 \right) x_3 \\
-\omega^2 D_0 \left( -\omega^2 \right) = 0 \tag{5}
\]
\[
\omega \left[ N_e \left( -\omega^2 \right) x_1 + N_0 \left( -\omega^2 \right) x_2 + D_0 \left( -\omega^2 \right) x_3 \\
+ D_e \left( -\omega^2 \right) \right] = 0. \tag{6}
\]

Note that at \( \omega = 0 \) eq. (6) is trivially satisfied and eq. (5) becomes
\[
N_e(0) x_2 + D_e(0) x_3 = 0 \tag{7}
\]
which coincides with the condition \( \delta(0) = 0 \).

The condition \( \delta_{n+1} = 0 \) translates to
\[
d_n + x_1 n_n = 0. \tag{8}
\]
where \( d_n, n_n \) denote the coefficients of \( s^n \) in \( D(s) \) and \( N(s) \) respectively.

For \( \omega > 0 \) we have
\[
-\omega^2 N_0 \left( -\omega^2 \right) x_1 + N_e \left( -\omega^2 \right) x_2 + D_e \left( -\omega^2 \right) x_3 \\
-\omega^2 D_0 \left( -\omega^2 \right) = 0 \tag{9}
\]
\[
N_e \left( -\omega^2 \right) x_1 + N_0 \left( -\omega^2 \right) x_2 + D_0 \left( -\omega^2 \right) x_3 \\
+ D_e \left( -\omega^2 \right) = 0. \tag{10}
\]

Rewrite the above in matrix form:
\[
\begin{bmatrix}
\omega^2 N_0 \left( -\omega^2 \right) - N_e \left( -\omega^2 \right) \\
N_e \left( -\omega^2 \right) - N_0 \left( -\omega^2 \right)
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
= \begin{bmatrix}
D_e \left( -\omega^2 \right) x_3 - \omega^2 D_0 \left( -\omega^2 \right) \\
-\omega^2 D_0 \left( -\omega^2 \right) x_3 - D_e \left( -\omega^2 \right)
\end{bmatrix} \cdot \begin{bmatrix}
A(\omega) \tag{11}
\end{bmatrix}
\]

We now consider the case when \( |A(\omega)| \neq 0 \) for all \( \omega > 0 \). The case when \( |A(\omega)| = 0 \) will be discussed later. Then
\[
|A(\omega)| = \omega^2 N_0 \left( -\omega^2 \right) + N_e \left( -\omega^2 \right) > 0, \ \forall \ \omega > 0.
\]

Therefore, for every \( x_3 \) eq. (11) has a unique solution \( x_1 \) and \( x_2 \) at each \( \omega > 0 \) given by:
\[
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
= \frac{1}{|A(\omega)|} \begin{bmatrix}
N_0 \left( -\omega^2 \right) & N_e \left( -\omega^2 \right) \\
N_e \left( -\omega^2 \right) & \omega^2 N_0 \left( -\omega^2 \right)
\end{bmatrix}
\cdot \begin{bmatrix}
D_e \left( -\omega^2 \right) x_3 - \omega^2 D_0 \left( -\omega^2 \right) \\
-\omega^2 D_0 \left( -\omega^2 \right) x_3 - D_e \left( -\omega^2 \right)
\end{bmatrix} \tag{12}
\]

In other words,
\[
x_1(\omega) = \frac{1}{|A(\omega)|} \left( \begin{bmatrix}
N_0 \left( -\omega^2 \right) D_e \left( -\omega^2 \right) \\
- N_e \left( -\omega^2 \right) D_0 \left( -\omega^2 \right)
\end{bmatrix} x_3 \\
- \omega^2 N_0 \left( -\omega^2 \right) D_0 \left( -\omega^2 \right) - N_0 \left( -\omega^2 \right) D_e \left( -\omega^2 \right) \right)
\]
\[
x_2(\omega) = \frac{1}{|A(\omega)|} \left( \begin{bmatrix}
- N_0 \left( -\omega^2 \right) D_e \left( -\omega^2 \right) \\
- \omega^2 N_0 \left( -\omega^2 \right) D_0 \left( -\omega^2 \right)
\end{bmatrix} x_3 \\
+ \omega^2 N_0 \left( -\omega^2 \right) D_0 \left( -\omega^2 \right) + N_0 \left( -\omega^2 \right) D_e \left( -\omega^2 \right) \right)
\]

For a fixed value of \( x_3 \), let \( \omega \) run from 0 to \( \infty \). The above equations trace out a curve in the \( x_1 - x_2 \) plane.
corresponding to the complex root space boundary. These curves along with the straight lines eq. (7) and eq. (8) partition the parameter space into a set of open root distribution invariant regions. By sweeping over $x_3$, we find these regions.

To complete our discussion let us consider the possibility $|A(\omega)| = 0$ for some $\omega \neq 0$. We will show that the assumption of stabilizability of the plant rules out this possibility. Let

$$|A(\omega)| = \omega^2 N^2_0 (\omega^2) + N^2_e (-\omega^2) = 0,$$  \hspace{1cm} (13)

for some $\omega \neq 0$. Since $N^2_0 (\omega^2), N^2_e (-\omega^2) \geq 0$, eq. (13) holds if and only if

$$N_0 (\omega^2) = N_e (-\omega^2) = 0.$$ \hspace{1cm} (14)

From eq (11) it follows that,

$$D_e (-\omega^2) x_3 - \omega^2 D_0 (-\omega^2) = 0$$

$$-D_0 (-\omega^2) x_3 - D_e (-\omega^2) = 0$$

and therefore

$$\omega^2 D_0^2 (-\omega^2) + D_0^2 (-\omega^2) = 0.$$ \hspace{1cm} (15)

Since $D_0^2 (\omega^2), D_e^2 (-\omega^2) \geq 0$, eq. (15) holds if and only if

$$D_0 (\omega^2) = D_0 (-\omega^2) = 0.$$ \hspace{1cm} (16)

From eqs. (14) and (16), it follows that eq. (13) has a solution for $\omega \neq 0$ if and only if $D(s)$ and $N(s)$ have a common factor $s^2 + \omega^2$ and this is ruled out by the assumption of stabilizability of the plant. Therefore, the case $|A(\omega)| = 0$ for some $\omega$ need not be considered.

The following example illustrates these computations.

**Example 1** Consider the following 13th order plant transfer function

$$P(s) = \frac{N(s)}{D(s)}$$

where

$$N(s) = s^{10} + 2s^9 + 3s^8 + 4s^7 + 10s^6 + 5s^5 + s^4 - 7s^3 + 4s^2 + s + 23$$

$$D(s) = s^{13} + 9s^{12} + 40s^{11} + 111s^{10} + 203s^9 + 115s^8 - 203s^7 + 60s^6 + 23s^5 + s^4 - 18s^3 + 21s^2 + 2s + 7.$$ Let a first order controller be

$$C(s) = \frac{x_1 s + x_2}{s + x_3},$$

then the characteristic polynomial evaluated at $s = j \omega$ becomes

$$\delta(j \omega) = \delta_T(\omega) + j \omega \delta_1(\omega)$$

where

$$\delta_T(\omega) = -\omega^{14} + (40 + 9x_3) \omega^{12}$$

$$+ (-203 - 2x_1 - x_2 - 111x_3) \omega^{10}$$

$$+ (-203 + 4x_1 + 3x_2 + 115x_3) \omega^8$$

$$+ (-25 - 5x_1 - 10x_2 - 60x_3) \omega^6$$

$$+ (-18 - 7x_1 + x_2 + x_3) \omega^4$$

$$+ (-2 - x_1 - 4x_2 - 21x_3) \omega^2$$

$$+ (23x_2 + 7x_3).$$

$$\delta_1(\omega) = (9 + x_3) \omega^{12} + (-111 - x_1 - 40x_3) \omega^{10}$$

$$+ (115 + 3x_1 + 2x_2 + 203x_3) \omega^8$$

$$+ (-60 - 10x_1 - 4x_2 + 203x_3) \omega^6$$

$$+ (1 + x_1 + 5x_2 + 25x_3) \omega^4$$

$$+ (-21 - 4x_1 + 7x_2 + 18x_3) \omega^2$$

$$+ (7 + 23x_1 + x_2 + 2x_3).$$

We now have

For $\omega = 0$, \hspace{1cm} $23x_2 + 7x_3 = 0$,

For $\omega > 0,$

$$x_1(\omega) = \frac{-9\omega^{22} + 120\omega^{20} - 181\omega^{18} - 452\omega^{16}}{\omega^{24} + 1429\omega^{14} - 1738\omega^{12} + 3355\omega^{10}} + \omega^{8} + 1586\omega^6 - 142\omega^4 + 504\omega^2 - 161$$

$$x_2(\omega) = \frac{-9\omega^{22} + 120\omega^{20} - 181\omega^{18} - 452\omega^{16}}{\omega^{24} + 1429\omega^{14} - 1738\omega^{12} + 3355\omega^{10}} - \omega^{8} + 1289\omega^6 - 225\omega^4 + 539\omega^2 - 154 \cdot x_3.$$
where
\[ |A(\omega)| = \omega^2 - 2\omega + 13\omega^16 - 26\omega^14 + 102\omega^12
- 117\omega^{10} + 281\omega^8 - 409\omega^6 + 76\omega^4
- 183\omega^2 + 529. \]

Figure 2 shows how these functions partition the controller parameter space. The numbers shown in the figure indicate the number of RHP roots of the polynomial $\delta(s)$ where the value $(x_1, x_2)$ is taken from the indicated region in the figure.

Figure 2: Root invariant regions (Example 1).

3 Computing All First Order Stabilizing Controllers: An Example

It is clear from the development shown in the previous section that the stabilizing regions in the controller parameter space can be easily found. This is shown in the example below.

Example 2 Consider the following 8th order plant:
\[
P(s) := \frac{N(s)}{D(s)}
\]

\[
\begin{align*}
-27\omega^{22} + 396\omega^{20} - 1257\omega^{18} \\
+ 2596\omega^{16} - 1527\omega^{14} + 1042\omega^{12} \\
- 2951\omega^{10} + 3303\omega^8 - 1364\omega^6 \\
+ 86\omega^4 - 518\omega^2 + 161
\end{align*}
\]

and a first order controller
\[
C(s) = \frac{x_1s + x_2}{s + x_3}.
\]

Then we have
\[
\begin{align*}
N_0(s) &= 99s^6 + 9255s^4 + 88656s^2 + 75600 \\
N_0'(s) &= 3s^6 + 1320s^4 + 37287s^2 + 120420 \\
D_0(s) &= s^8 + 131s^6 + 2017s^4 + 7896s^2 + 5670 \\
D_0'(s) &= 18s^6 + 625s^4 + 4753s^2 + 8919
\end{align*}
\]

and
\[
\delta(j\omega) = \delta_r(\omega) + j\omega\delta_i(\omega)
\]

where
\[
\begin{align*}
\delta_r(\omega) &= (18 + 3x_1 + x_3)\omega^8 \\
&+ (-625 - 1320x_1 + 99x_2 - 131x_3)\omega^6 \\
&+ (4753 + 37287x_1 + 9255x_2 + 2017x_3)\omega^4 \\
&+ (-8919 - 120420x_1 - 88656x_2 - 7896x_3)\omega^2 \\
&+ 75600x_2 + 5670x_3
\end{align*}
\]

\[
\delta_i(\omega) = \omega^8 + (-131 - 99x_1 - 3x_2 - 18x_3)\omega^6 \\
+ (2017 + 9255x_1 + 1320x_2 + 625x_3)\omega^4 \\
+ (-7896 - 88656x_1 - 37287x_2 - 4753x_3)\omega^2 \\
+ (5670 + 75600x_1 + 120420x_2 + 8919x_3)
\]

Note that the leading coefficient does not vanish in this example. Thus the real and complex root boundaries are given by:

For $\delta(0) = 0, \quad 75600x_2 + 5670x_3 = 0,$

For $\delta(j\omega) = 0,$

\[
\begin{align*}
45\omega^{14} + 3411\omega^{12} &+ -9681\omega^{10} + 634077\omega^8 \\
-1899129\omega^6 &- 69813\omega^4 + 25591140\omega^2 \\
-428652000
\end{align*}
\]

\[
x_1(\omega) = \frac{|A(\omega)|}{\delta_r(\omega)}
\]

\[
45\omega^{14} - 69\omega^{12} + 12207\omega^{10} \\
-159585\omega^8 + 220167\omega^6 - 6387621\omega^4 \\
-12203946\omega^2 + 8505000
\]

\[
\cdot x_3
\]
where

\[ x_2(\omega) = \frac{\omega^{16} + 69\omega^{14} - 12207\omega^{12} + 159585\omega^{10} - 220167\omega^8 + 6387621\omega^6 + 12203946\omega^4 - 8505000\omega^2}{-153\omega^{14} + 47859\omega^{12} - 301169\omega^{10} + 62911227\omega^8 - 526622253\omega^6 + 1809907839\omega^4 - 2173643100\omega^2 + 428652000} \cdot x_3. \]

Figure 4: Stability region for $-0.7 \leq x_3 \leq 0.5$ (Example 2).

PID controllers, the stability region in the first order controller parameter space is quite complicated and completely different from that of PID controllers.

Remark 1 It is noted that stable stabilizing first order controllers exist when the set is non-empty for $x_3 > 0$. Similarly, if the set exists for $\text{Sign} [x_1] = \text{Sign} [x_2]$, minimum phase stabilizing first order controllers exist. Controllers with these restrictions can be found by searching in the corresponding orthants of the three dimensional design space.

4 Simultaneous or Robustly Stabilizing First Order Controllers

The computation described in the previous section can be extended in a straightforward and obvious way to determine the set of first order controllers that simultaneously stabilize a number of plants $P_i(s)$. Explicit solutions for the stability boundary can be found for each plant, for fixed $x_3$, the stabilizing regions intersected and this process repeatedly done as $x_3$ is swept.

To determine the robust stabilizability of a continuum of plants by first order controllers we consider specifically an interval plant family $P(s) = \frac{N(s)}{D(s)}$ where $N(s)$ and $D(s)$ are interval polynomials. Recall the result of [8] and the Vertex Theorem [5]...
which deals with a polynomial family of the form:

\[
d(s) = F_1(s) D(s) + F_2(s) N(s)
\]

where \(D(s)\) and \(N(s)\) are interval polynomials and

\[
F_i(s) = s^i (a_i s + b_i) U_i(s) Q_i(s), \quad i = 1, 2.
\]

where \(t_i \geq 0\) is an arbitrary integer, \(a_i\) and \(b_i\) are arbitrary real numbers, \(U_i(s)\) is an anti-Hurwitz polynomial, and \(Q_i(s)\) is an even or odd polynomial. For robust stability of such a family it is enough that \(\bar{F}(s) = (F_1(s), F_2(s))\) stabilizes the finite set of vertex polynomials \(\Delta^*(s):\)

\[
\Delta_*(s) := \left\{ \delta_i(s) : F_1(s) D_i(s) + F_2(s) N_j(s), \quad i, j = 1, 2, 3, 4 \right\}
\]

where \(D_i(s)\) and \(N_j(s)\) are the Kharitonov polynomials of \(D(s)\) and \(N(s)\), respectively.

As seen, the problem of robust stabilization with 1st order controllers is a special case of this result with

\[
F_1(s) = s + x_3 \quad \text{and} \quad F_2(s) = x_1 s + x_2.
\]

For the given interval plant

\[
P(s) := \left\{ P(s) = \frac{N(s)}{D(s)} : N(s) \in N(s), D(s) \in D(s) \right\},
\]

let its vertices be

\[
P_*(s) := \left\{ \frac{N_i(s)}{D_j(s)} : N_i(s) \in K_N(s), D_j(s) \in K_D(s) \right\}
\]

where \(K_N(s)\) and \(K_D(s)\) are the set of Kharitonov polynomials of \(N(s)\) and \(D(s)\), respectively. Let \(R_i\) be controller parameter space regions that consist of all first order stabilizing controllers for the \(i^{\text{th}}\) vertex system \(P_i(s)\). Then every first order stabilizing controller that robustly stabilizes the interval system \(P(s)\) belongs to

\[
\mathcal{R} := \cap_i R_i.
\]

Conversely, by the result of [8] and the Vertex Theorem of [5] every point in \(\mathcal{R}\) corresponds to a first order stabilizing controller for \(P(s)\). The following example illustrates this.

**Example 3** Consider the following interval plant.

\[
P(s) = \left\{ \frac{N(s)}{D(s)} : \begin{align*}
    N(s) &= \frac{n_7 s^7 + n_6 s^6 + n_5 s^5 + n_4 s^4 + n_3 s^3 + n_2 s^2 + n_1 s + n_0}{d_6 s^8 + d_7 s^7 + d_6 s^6 + d_5 s^5 + d_4 s^4 + d_3 s^3 + d_2 s^2 + d_1 s + d_0}
\end{align*}
\]

where the upper and lower bounds of the interval polynomials \(D(s)\) and \(N(s)\) are

\[
D^+ = \{2, 18.1, 131.1, 625.1, 2017.1, 4753.1, 7896.1, 8919.1, 5670.1\}
\]

\[
D^- = \{1, 17.9, 130.9, 624.9, 2016.9, 4752.9, 7895.9, 8918.9, 5669.9\}
\]

\[
N^+ = \{3.1, 99.1, 1320.1, 9255.1, 37287.1, 88656.1, 120420.1, 75600.1\}
\]

\[
N^- = \{2.9, 98.9, 1319.9, 9254.9, 37286.9, 88655.9, 120419.9, 75599.9\}
\]

We now construct the 16 vertex systems:

\[
P_*(s) = \left\{ \frac{N_i(s)}{D_j(s)} : N_i(s) \in K_N(s), D_j(s) \in K_D(s) \right\}
\]

By repeating the step given in Example 2 for all 16 systems, we can find stabilizing compensators for each of them. Figure 5 displays these for the case when \(x_3 = 0.5\).

Clearly, the intersection of these 16 figures is the region for robust stabilization. This is shown in Figures 6 and 7.

### 5 Concluding Remarks

A computation of all first order stabilizing controllers for a given LTI system is obtained in this paper. The result is based on determination of root invariant regions via D-decomposition and parameter mapping. The result is also extended to simultaneous stabilization and robust first order stabilization of interval plants. It is also possible to consider stabilizability with respect to shifted half plane by the same calculations with \(s\) replaced by \(s + \sigma\). Since PID and first order controllers are a major share of
today’s industrial controllers, we believe that the result given here along with the results in [6] is a significant step to improve industrial control in general. A control scheme that optimizes a given optimality criteria and the attainment of other performance criteria over these sets is currently under investigation. We anticipate the parameter plane techniques developed in [10] to be useful in this context.

References


