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Abstract

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JEL Code: D43, L13, L88, L86

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Privacy Regulation, Cognitive Ability, and Stability of Collusion

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1 Introduction

With the advent of 'big data' and machine learning techniques, firms can gain access to vast amount of data on consumers' personal information. It is argued that firms often make use of these data to pursue their own business objectives (Dengler and Prüfer, 2021; Colombo and Pignataro, 2022). The fact that firms have access to granular data on consumers' personal information can have implications to consumers' welfare.

From an ethical perspective, consumers should have the right to privacy. This is because, firms may use consumers' personal information to influence and manipulate consumers' decision-making in subtle ways that hurts consumers (Calo, 2013; Acquisti et al., 2016), and consumers often fear about such exploitation by firms (Tucker, 2015).[1] In fact, excessive (mis)use of consumers' personal information by firms has led to growing public concerns (Acquisti et al., 2016; Goldfarb and Tucker, 2011). This might have contributed in inducing governments of a number of countries to bring in acts and regulations to safeguard consumer privacy recently.[2,3] Although, from consumers' economic benefits point of view, it does not appear to be straightforward to assert whether privacy protection benefits or harms consumers.

Access to more refined consumer data may enable firms to customize products and services to meet consumers' needs more precisely, which has a positive effect on consumers'

[1]For example, in a survey by Pew Research Centre, 86% of Internet users have taken measures to protect their privacy and leave little digital traces (Rainie et al., 2013).

[2]The General Data Protection Regulation (GDPR), enacted by the European Union in 2018, codifies consumers' rights to privacy and control over their individual information (Baik et al., 2022). The Government of India enacted Digital Personal Data Protection (DPDP) Act in 2023 to protect privacy.

[3]Several studies have attempted to examine the implication of GDPR in a variety of scenarios. See, for example, Aridor et al. (2023), Godinho de Matos and Adjerid (2022), and Goldberg et al. (2024).

surplus (Tucker, 2012). At the same time, it may also alter firms' pricing strategies, the implication of which on consumers' economic benefits is likely to depend on the underlying market structure and the type of consumer data accessible by firms. For instance, Montes et al. (2019) show that it is never optimal for any consumer to opt for privacy protection under duopolistic price competition, even when consumers can masquerade their personal information fully without incurring any private cost. The reason is, access to consumer data enables firms to engage in first-degree price discrimination, which intensifies competition for each consumer sufficiently. Belleflamme and Vergote (2016) and Dengler and Prüfer (2021) also find similar results, albeit in different scenarios. Bergemann et al. (2022) argues that consumers' control over their personal data, which is mandated by privacy regulations, may restrict efficient use of information. On the contrary, in an experimental study, Calvano et al. (2020) observes that in the era of 'big data' algorithmic pricing may serve as an instrument for firms to engage in hard-to-detect anti-competitive practices at the expense of consumers.[4] Prima facie evidence of anti-competitive practices against e-commerce firms, which are often alleged for misusing their access to consumer data, is also found in some cases.[5] However, it is not well understood whether access to consumer data helps firms to sustain tacit collusion. To the best of our knowledge, the existing theoretical literature on consumer privacy has ignored possible implications of privacy protection to stability of tacit collusion among firms. The question is: is there any trade-off between consumer privacy protection and the level of product market competition? If yes, how does it relate to scope and effectiveness of privacy regulation?

[4]See Harrington (2018), Miklós-Thal and Tucker (2019) and Mehra (2016) for further discussions.

[5]The European Commission launched an inquiry in 2015 to identify anti-competitive behavior in e-commerce markets.[Antitrust: Commission launches e-commerce sector inquiry] In 2016, the Competition and Markets Authority, UK, found evidence of collusion by sellers in e-commerce platforms. [Guardian]

Further, given the complex nature of big data and its applications, consumers are often less equipped than firms to process relevant information and make decisions rationally. Due to their constrained cognitive abilities, consumers often rely on simplified mental models to make decisions (Acquisti et al., 2007; Dengler and Prüfer, 2021). In such scenarios, it seems reasonable to assume that firms outperform consumers in terms of the level of strategic sophistication a la Nagel (1995), Stahl II and Wilson (1994) and Dengler and Prüfer (2021). Thus, in the era of 'big data' firms have comparative advantages over consumers, due to limited cognitive ability of the latter, which may enhance firms' ability to exploit consumers for furtherance of their selfish business interests. In this context, it is important to ask the following questions. Does consumers' cognitive ability matter for the stability of tacit collusion among firms? Does the implication of privacy regulation, if any, to product market collusion depend on consumers' cognitive ability?

This article attempts to address the above mentioned issues by analyzing the role of privacy regulation on sustainability of tacit collusion in an infinite horizon differentiated products duopoly with heterogeneous consumers. It compares and contrasts the effects of consumers' privacy protection on collusion sustainability under two alternative privacy regulations, which differ in terms of scope, in two different scenarios. In the first scenario, firms as well as consumers have unlimited cognitive abilities as in standard models; whereas, in the second scenario, consumers have limited cognitive ability. The scope of privacy regulation refers to the number of dimensions of consumers' personal information that falls under its purview.

Suppose that consumers live for two periods and, in each period, a consumer has a unit demand. Further, in each period, there are two sets of consumers, new and old, each of unit mass. New consumers' personal information is not accessible to anyone. Whereas, firms can access old consumers' personal information, unless their privacy is protected. The broad scope privacy regulation aims to prevent firms from accessing consumers' personal information regarding their preferences as well as purchase history, whereas only the information regarding consumers' preferences falls under the purview of the narrow scope privacy regulation. The scope of the privacy regulation is predetermined and is common knowledge.

Given the privacy regulation, old consumers can choose whether to opt for privacy protection or not. However, to opt for privacy protection, each consumer needs to incur a fixed private cost, which is invariant to scope of privacy regulation and is exogenous. This private cost of privacy protection can be thought of as efforts or payments that consumers need to make for privacy protection.[6] The higher the private cost of privacy protection, lower is the effectiveness of the privacy regulation.

In each period, firms and consumers play a three stage sequential move game; wherein (i) old consumers first decide whether to opt for privacy by incurring the private cost or not, (ii) in the second stage firms set their prices, and (iii) finally, consumers make their purchase decisions and payoffs are realized in the third stage. In stage 2 of each period, firms may either engage in tacit-collusion or compete in terms of price. In case firms engage in tacit-collusion in any period, they continue to do so in subsequent periods until one firm deviates. Following deviation, if any, firms compete in subsequent periods.

It follows that firms can set only uniform prices for new consumers; whereas, they can price discriminate and offer personalized prices to those old consumers who have not opted for

[6] Greater difficulties in erasing browser cookies increases this cost (Montes et al., 2019). Regulatory authorities can reduce this cost by mandating firms to set 'full privacy' as the default option. If firms are allowed to store and trade consumer data, private cost to protect privacy is likely to be high.

privacy protection. If old consumers opt for privacy protection under the broad scope privacy regulation, they become completely anonymous and indistinguishable from new consumers. In contrast, under the narrow scope of privacy regulation, by opting for privacy protection old consumers can hide their preferences but not their purchase history, which enables firms to charge them a price different from that for new consumers.[7] Therefore, there are two separate markets - a 'personalized market' in which firms set personalized prices for each consumer, and an 'anonymous market' ('(partially)anonymous market') in which each firm sets a uniform price for all (each firm can set different uniform prices for new and old consumers who opted for privacy protection) under the broad (narrow) scope privacy regulation. Old consumers take privacy decisions on the basis of their expectations regarding firms' pricing strategies. This article characterizes the equilibrium of the game and demonstrates the following results.

First, under the broad scope privacy regulation, a less effective privacy regulation makes tacit collusion harder to sustain. Interestingly, although consumers' surplus increases due to an increase in effectiveness of privacy protection in case of collusion, a totally ineffective privacy regulation (i.e., the prohibitively high private cost of privacy protection) may result in the maximum possible consumers' surplus. This is because, (a) collusion is least likely to be stable in case the privacy regulation is totally ineffective, and (b) consumers' surplus is always greater under competition than that under collusion. That is, there is an unintended consequence of the broad-scope privacy regulation due to its competition dampening effect. These results hold true regardless of whether consumers have limited cognitive abilities or not. The reason is, even when firms are strategically more sophisticated, they cannot exploit consumers' naivety under the broad scope privacy regulation. Further, the competition

[7]Implicitly, we assume that no arbitrage is possible, following the existing the literature on behaviour-based price discrimination.

dampening effect of the broad scope privacy regulation seems to be qualitatively similar in case of consumer heterogeneity in terms of intrinsic taste for privacy a la Dengler and Prüfer (2021) or if the market demand changes over time a la Vasconcelos (2008).

Second, the narrow scope privacy regulation, regardless of its effectiveness, does not have any impact on stability of tacit collusion in case consumers' cognitive ability is unlimited. This is because, in such a scenario, (a) firms can distinguish between old and new consumers, even if old consumers opt for privacy protection, and (b) consumers can correctly infer firms' pricing strategy and by not opting for privacy protection they can avoid a higher price in the partially-anonymous market. As a result, no consumer opts for privacy protection in the equilibrium under the narrow scope privacy regulation, regardless of the magnitude of private cost for privacy protection and whether firms compete or collude. This result is in contrast to that under the broad scope privacy regulation. Moreover, when consumers' cognitive ability is unlimited, the narrow scope privacy regulation is weakly dominated by the broad scope privacy regulation in terms of competition dampening effect of privacy regulation.

Third, if consumers' cognitive ability is constrained, stability of tacit collusion increases with the effectiveness of the narrow scope privacy regulation, as in the case of the broad scope privacy regulation. However, unlike as under the broad scope privacy regulation, a lower level of strategic sophistication of consumers makes collusion more likely to be stable under the narrow scope privacy regulation.

Related literature

The present analysis is primarily related to three strands of literature: (a) consumer privacy, (b) sustainability of collusion, and (c) behavior-based price discrimination.

First, the issue of consumer privacy has received renewed attention from researchers in recent decades. The core questions asked in the literature on privacy are: is there a demand for privacy even if consumers do not have an exogenous taste for privacy? What are the competitive and welfare effects of consumer privacy? Taylor (2004) shows that firms benefit by using consumers' information with naive consumers but not with fully rational consumers. When consumers fully anticipate the sale of information, some strategically refuse to buy from the first firm. In that case, the first firm may commit to a privacy regime to not sell consumers' information to the other firm. This result is also confirmed by Acquisti and Varian (2005), who consider firms with access to tracking technologies, and consumers with access to anonymizing technologies to erase their digital traces. Considering imperfect tracking ability of firms and allowing consumers to hide from firms' databases at a cost, Belleflamme and Vergote (2016) find that consumers may be collectively better off absent the hiding technology. More recently, Dengler and Prüfer (2021) analyze a model of limited strategic sophistication, considering a monopoly. They show that some consumers opt for privacy at a cost, even if they do not intrinsically value privacy, in case they are not too sophisticated. Taylor and Wagman (2014) argue that privacy regulations' competitive and welfare effects depend on the specific economic setting under consideration. Closely related to our framework, Montes et al. (2019) endogenize consumers' privacy choices and analyze the optimal strategy of a data broker. They find that the data broker sells consumers' information to only one firm in competitive markets. Although this stream of literature offers valuable insights to understand the implications of consumer privacy in different scenarios, to the best of our knowledge, the issue of (tacit) collusion has been sidestepped in this discourse.

Second, the issue of firms' anti-competitive behavior is of perennial interest of policymakers and scholars alike. Recently, Heywood et al. (2020), considering a delivered-pricing model, shows that price discrimination may facilitate collusion more than uniform pricing. There is growing literature on firms' incentives to collude in the context of online data and price discrimination. For example, Colombo and Pignataro (2022) investigate the impact of information accuracy on the sustainability of collusion and show that banning price discrimination may facilitate collusion sustainability. On the other hand, Peiseler et al. (2022) show that prohibiting price discrimination may prevent collusion if firms have sufficiently noisy signals about consumers' preferences. This article contributes to this literature by analysing implications of privacy regulation and cognitive abilities of consumers and firms on stability of (tacit) collusion.

Third, the present analysis is also related to the vast literature on behavior-based price discrimination (Villas-Boas, 1999; Fudenberg and Tirole, 2000). In behavior-based price discrimination, firms use the previous purchase behavior of consumers to infer their preferences and price accordingly. There is a size-able literature on behavior-based dynamic pricing under different market environments. These studies focus on the profitability issues of behavior-based price discrimination and firms' pricing strategies. We depart from this stream of literature as follows. Following Taylor and Wagman (2014) and Montes et al. (2019), we assume that consumers' data is available to the firms unless consumers choose to protect their privacy, i.e., we do not explicitly model firms' acquisition of consumers' information. In period 1 of their lives, consumers' preferences are unknown, whereas, in period 2, their information is available to the firms unless the consumers opt for privacy. This allows us to explicitly analyze the implications of consumer privacy on the sustainability of collusion, which, to the best of our knowledge, has not been studied in the literature so far.

The remainder of the analysis is as follows. Section 2 describes the baseline model. Section 3 deals with the impact of broad scope privacy regulation on collusion sustainability. Section 4 deals with the case of narrow scope privacy regulation. Section 5 provides concluding remarks.

2 The Model

We consider two symmetric competing firms, A and B, which produce differentiated goods. Firms A and B are located on the unit line at the extreme ends 0 and 1, respectively. Fixed costs and the marginal cost of production are assumed to be zero. Consumers are uniformly distributed over the linear city $[0, 1]$. Consumers have unit demand, and they derive the highest gross utility $v\,(> 0)$ if they buy from their most preferred variety, which is given by their location θ on the unit line; otherwise, they incur a linear transportation cost $t\,(> 0)$ per unit distance. Note that a consumer's location θ denotes her relative preference for firm B's product over that of firm A. More specifically, if firm i sets price p_i, a consumer with location θ derives the following utility.

$$u(\theta) = \begin{cases} v - p_A - t\,\theta & \text{if she buys from firm } A \\ v - p_B - t(1-\theta) & \text{if she buys from firm } B \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

We assume that v is sufficiently large such that $v \geq \frac{5t}{2}$.[8]

Time is discrete, and is given as $m = 0,\ 1,\ 2...,\ \infty$. We assume that all consumers

[8]This assumption ensures full market coverage in all the cases considered, i.e., collusion, deviation, and non-cooperation (unless stated otherwise). Additionally, under this assumption, collusive profits are always greater than non-cooperative profits.

purchase for two consecutive periods. In the 0^{th} period, mass one of consumers makes a purchase. In the next period, consumers from the previous period make another purchase, and mass one of new consumers enter the market. Therefore, in each period from the first period onward, there are two sets of consumers, "old" and "new"; the mass of each set is unity.[9]

Firms cannot obtain information on new consumers, and hence, they set uniform prices, independent of θ, for this set of consumers. On the other hand, firms may have access to old consumers' personal information regarding their purchase histories and, perhaps, also regarding their preferences, which depends on the feasibility and associated cost to protect consumers' privacy. In case firms are able to obtain (partial/full) access to an old consumer's private information, firms may offer that old consumer personalized prices.

We consider that the competent authority may bring privacy regulation in order to restrict firms from obtaining consumers' private information. The privacy regulation is considered to have two aspects – scope and effectiveness. The scope refers to the set of consumers' private information that falls under the umbrella of privacy regulation. In the present context, the scope may include only preferences of consumers or both preferences and purchase histories. On the other hand, effectiveness of privacy regulation refers to the extent of consumers' easiness to protect their privacy, given the scope of the regulation.

Definition 1 (Broad Scope and Narrow Scope of Privacy Regulation): The scope of privacy regulation is said be broad if the regulation aims to protect consumers' privacy regarding their preferences as well as purchase histories. Otherwise, if the privacy regulation aims to protect consumers' privacy regarding their preferences only, but not purchase histories, then

[9]The distinction between old and new consumers is similar to Montes et al. (2019).

the scope of privacy regulation is said to be narrow.

Definition 2 (Effectiveness of Privacy Regulation): The privacy regulation, given its scope, is said to be less effective, if consumers need to incur a higher private cost to protect their privacy.

At the beginning of each period, old consumers can opt for privacy protection by incurring a private cost $c(\geq 0)$, the magnitude of which depends on the effectiveness of privacy regulation.Under broad (narrow) scope privacy regulation, firms treat an old consumer as a fully (partially) anonymous consumer, if that old consumer has opted for privacy protection by incurring cost c. In particular, if an old consumer with preference θ opts for privacy by paying c, and buys from firm i, the utility derived is $v - p_i - t\,|L_i - \theta| - c$, $L_A = 0,\ L_B = 1$.

Under broad scope of privacy regulation, firms cannot distinguish between new consumers and those old consumers who have opted for privacy protection. Thus, all new consumers and old consumers who have opted for privacy protection constitute *anonymous* market where firms charge uniform prices, independent of θ. On the other hand, old consumers who have not opted for privacy protection constitute *personalized* market where firms may set tailored prices to each consumer, depending on their preferences θ. Suppose that old consumers with $\theta \in [0, \theta_A]$ and $\theta \in [\theta_B, 1]$ opt for privacy protection, then Fig. 1 depicts the market structure in this case, following Montes et al. (2019).

Under narrow scope of privacy regulation, only the set of new consumers constitutes the *anonymous* market (the upper panel in Figure 1), whereas the set of old consumers who have opted for privacy protection constitutes the *partially anonymous* market (dotted line segments in the lower panel in Figure 1). The set of old consumers who have not opted for privacy protection constitutes *personalized* market (the thick line segment in the lower panel

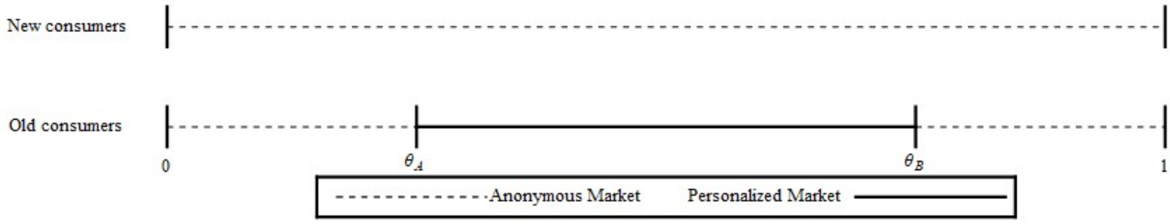

Fig. 1: Anonymous and personalized market under broad scope privacy regulation

in Figure 1).

We suppose that firms operate over the infinite horizon, and they discount their future profits by a common discount factor $\delta \in (0,1)$. In each period, firm and consumers play a sequential move game, stages of which are as follows.

Stage 1: Old consumers decide whether to pay for privacy or not.

Stage 2: Firms choose prices in the anonymous and personalized markets.

Stage 3: Consumers observe the prices in the market they are in and make their purchase decisions.

In each period, in Stage 2 firms may either compete in terms of prices or collude in the product market. In case firms decide to collude in any period, they continue to do so in subsequent periods until one firm deviates from the (implicit) collusive agreements. If either firm deviates in period m, punishment consists of reverting to static non-cooperative equilibrium in all future periods, from $m+1$ onward. That is, in the case of collusion, they adopt the *grim trigger* strategy (Friedman, 1971) as the punishment device. When colluding, firms maximize joint profits. If one firm deviates, it maximizes its profit, believing that the other maximizes the joint profits. In the punishment stage, each firm gets the non-cooperative payoff. We denote one period collusive, deviating, and non-cooperative profit of firm i as π_i^c,

π_i^d, and π_i^n, respectively.[10]

Suppose that the critical discount factor δ_{min} above which tacit collusion is made sustainable by the grim trigger strategy is $\delta_{min} = \max[\delta_A, \delta_B]$. As firms are symmetric, $\delta_A = \delta_B = \delta_{min}$. This is given by[11]

$$\delta_{min} = \frac{\pi_i^d - \pi_i^c}{\pi_i^d - \pi_i^n} \tag{2}$$

We examine the implications of effectiveness and scope of privacy regulation on stability of collusion in two alternative scenarios. The first scenario corresponds to the standard framework, in which players are rational with unlimited cognitive abilities. Whereas in the second scenario players are assumed to have limited cognitive abilities a la Dengler and Prüfer (2021), wherein consumers can anticipate only a limited number (k) of strategic interactions, whereas firms can anticipate one more number ($k + 1$) of strategic interactions than the consumers.

In what follows, we first analyze the role of effectiveness of privacy regulation and player's cognitive abilities in promoting competition under broad scope of privacy regulation in Section 3. In Section 4, we examine the impact of effectiveness of privacy regulation under narrow scope of privacy regulation, and draw the implication of the scope of privacy regulation by comparing the results under broad scope of privacy regulation with those under narrow scope of privacy regulation.

[10]Superscripts 'c', 'd', and 'n' denote collusion, deviation, and non-cooperative equilibrium throughout the analysis.

[11]This is a well-known result in the literature. See Belleflamme and Peitz (2010).

3 Broad Scope of Privacy Regulation

3.1 Unlimited cognitive abilities

Consider that each player has unlimited cognitive ability, but consumers are not forward looking. Consumers form expectations about prices, given the information available to them at that point in time, and consumers 'live' for two periods. They make their privacy decisions at the beginning of the second period before firms announce their prices. When firms collude, consumers' privacy decisions are based on the expected collusive prices, and their expectations are correct in equilibrium. Suppose that one firm deviates in the period, say, m. At the end of period m, new consumers observe that one firm has deviated. In light of this new information, consumers update their beliefs and correctly infer that firms will compete non-cooperatively from period $(m+1)$ onward (Petrikaitė, 2016; Schultz, 2005). The belief updation of old consumers does not matter as they exit the market in the second period of their lives.

We now turn to characterize the equilibrium under collusion, in the deviation period and in punishment phase, separately.

(a) Collusion

As utility derived from firm A's product decreases in θ, if it is optimal for an old consumer with $\theta_A \in [0, \frac{1}{2}]$ to opt for privacy protection by incurring private cost c in order to hide from the database of firm A, then it is also optimal for all old consumers with $\theta \in [0, \theta_A]$ to do the same. Likewise, if it is optimal for an old consumer with $\theta_B \in (\frac{1}{2}, 1]$ to opt for privacy protection by incurring private cost c in order to hide from the database of firm B, then it is also optimal for all old consumers with $\theta \in [\theta_B, 1]$ to opt for privacy protection. Therefore, suppose that a mass of $\theta_A \in [0, 1/2]$ and $(1-\theta_B)$ of old consumers, where $\theta_B \in (1/2, 1]$, incurs private cost c each to opt for privacy protection.

Suppose that consumers anticipate prices p^e_{iN} and $p^e_{iO}(\theta)$ in anonymous and personalized markets, respectively.[12] Then, a consumer with $\theta \in [0, \theta_A]$ opts for privacy protection if $v - p^e_{AN} - t\,\theta - c \geq v - p^e_{AO}(\theta) - t\,\theta$. The marginal consumer $\theta_A \in [0, \frac{1}{2}]$ who is indifferent between paying for privacy protection and not paying is given by $v - p^e_{AN} - t\,\theta_A - c = v - p^e_{AO}(\theta_A) - t\,\theta_A \implies p^e_{AO}(\theta_A) = p^e_{AN} + c$. Analogously, the marginal consumer $\theta_B \in (\frac{1}{2}, 1]$ is given by $p^e_{BO}(\theta_B) = p^e_{BN} + c$.

Assuming full market coverage, we now consider firms' pricing decisions. Consider, first, the anonymous market. Given $\theta_A \leq 1/2$ and $\theta_B > 1/2$, when firms collude, the maximum price they can charge in the anonymous market is $p^c_{iN} = v - t/2$, which is equal to the surplus of the consumer at location $\theta = 1/2$ in the market for new consumers. In the personalized market, firms charge monopoly prices at each location θ to extract the whole surplus of each consumer. Thus, the collusive prices charged by firm A and firm B in the personalized market are, respectively, $p^c_{AO}(\theta) = v - t\,\theta$ and $p^c_{BO}(\theta) = v - t\,(1 - \theta)$.

Consider the consumers' privacy decisions. As consumers' expectations about firms' prices are correct in equilibrium, in this case, we have $p^e_{iN} = p^c_{iN}$ and $p^e_{iO}(\theta) = p^c_{iO}(\theta)$. Therefore, $v - t/2 + c = v - t\,\theta_A$ and $v - t/2 + c = v - t\,(1 - \theta_B)$. From this, we have

$$\theta_A = \frac{1}{2} - \frac{c}{t} \text{ and } \theta_B = \frac{1}{2} + \frac{c}{t} \tag{3}$$

A strictly positive fraction of old consumers opt for privacy protection by incurring private cost c, if $\theta_A > 0$ or $\theta_B < 1$. From equation (3), this is the case when $c < t/2$. That is, if private cost of privacy protection is not too high, some old consumers prefer to opt for

[12] Subscripts 'iN'' and 'iO'' denote firm i's choices or outcomes in the anonymous and personalized markets, respectively.

privacy protection to be in the anonymous market in case of collusion under broad scope of privacy regulation and unlimited cognitive abilities. From the above, we have the following Lemma.

Lemma 1 *Suppose that the scope of privacy regulation is broad and players have unlimited cognitive abilities. Then, the following is true in the case of collusion.*

(i) A positive fraction of old consumers opts for privacy, if private cost of privacy protection is not prohibitively high, i.e., if $c < \frac{t}{2}$. Otherwise, none of the old consumers opts for privacy protection. Thus, personalized market exists only if the privacy regulation is not very ineffective.

(ii) In the personalized market, if it exists, in the equilibrium, firm A sets a price equal to $p^c_{AO}(\theta) = v - t\,\theta$, and firm B sets $p^c_{BO}(\theta) = v - t\,(1-\theta)$.

(iii) In the anonymous market, firm $i, (i = A, B)$ sets a price equal to $p^c_{iN} = v - \frac{t}{2}$.

Remark 1: As $\theta_A \leq \frac{1}{2}$ and $\theta_B > \frac{1}{2}$, firms' prices in the anonymous market depend on the surplus of the most distant consumer ($\theta = \frac{1}{2}$), and in the personalized market, firms set prices equal to the consumer's surplus at each location. Therefore, the optimal collusive price does not depend on effectiveness of privacy regulation, in any market - be it personalized or anonymous; however, the market division between anonymous and personalized markets does.

As firms are symmetric, both markets are divided evenly, and both firms earn equal profits in each market.[13] Accordingly, if $c < t/2$, collusive profits of firm A from anonymous

[13] In the present analysis, as firms are symmetric, therefore, the equal profit-sharing rule is consistent with both collusive allocation rules given in Friedman and Thisse (1993) as well as Jehiel (1992). See Matsumura and Matsushima (2011) for a comparison between two collusive allocation rules.

market (π^c_{AN}) and from personalized market (π^c_{AO}) are as follows.

$$\begin{aligned}
\pi^c_{AN} &= (\frac{1}{2}+\theta_A)\, p^c_{AN} = (1-\frac{c}{t})(v-\frac{t}{2}) \\
\pi^c_{AO} &= \int_{\theta_A}^{\frac{1}{2}} (v-t\,\theta)\; d\theta = \int_{(\frac{1}{2}-\frac{c}{t})}^{\frac{1}{2}} (v-t\,\theta)\; d\theta = \frac{(2v-t+c)c}{2t}
\end{aligned}$$

As firms are symmetric, the total collusive profit of firm i ($i = A,\ B$) is as follows, if $c < t/2$.

$$\pi^c_i = \pi^c_{iN} + \pi^c_{iO} = (v - \frac{t}{2} + \frac{c^2}{2t}) \tag{4}$$

It is easy to check from equation (4) that the collusive profit decreases with transportation cost t. This is because, an increase in transportation cost decreases the overall surplus of the consumers and thus leaves less for firms to extract from. However, the collusive profit increases with private cost of privacy protection c. The reason is, an increase in private cost of privacy protection shrinks the anonymous market, whereas it enlarges the personalized market by an equal amount. Because, in the personalized market, firms set monopoly prices at each location, the extra surplus extraction is more compared to the loss in the anonymous market.

Further, note that, if $c < \frac{t}{2}$, consumers' surplus in case of collusion is as follows.

$$\begin{aligned}
CS^c &= \underbrace{2\int_0^{\frac{1}{2}} (v-t\theta-p^c_{AN})\; d\theta}_{\text{New consumers}} + \underbrace{2\int_0^{\frac{1}{2}-\frac{c}{t}} (v-t\theta-c-p^c_{AN})\; d\theta}_{\text{Old consumers who paid for privacy}} + \underbrace{0}_{\text{Personalized market}} \\
&= \frac{t}{2} + \frac{c^2}{t} - c
\end{aligned} \tag{5}$$

Clearly, consumers' surplus decreases due to an increase in the private cost of privacy protection, whenever $c < \frac{t}{2}$.

(b) Optimal Deviation

Suppose that firm A, without loss of generality, deviates from the collusive agreement. Given consumers' beliefs that firms set collusive prices and that consumers make their privacy decisions *before* firms announce their prices, we have $\theta_A = \frac{1}{2} - \frac{c}{t}$ and $\theta_B = \frac{1}{2} + \frac{c}{t}$, if $c < \frac{t}{2}$. That is, whenever $c < \frac{t}{2}$, mass $\theta_A \in (0, 1/2)$ and $(1 - \theta_B) \in (0, 1/2)$ of old consumers have opted for privacy; otherwise, if $c \geq \frac{t}{2}$, none opts for privacy. Lets first consider that private cost of privacy protection is not prohibitively high ($c < \frac{t}{2}$).

In the personalized market, under collusion, firms set monopoly prices at each location such that each consumer is indifferent between the two firms and buys from the closer firm. Therefore, the deviating firm A slightly decreases its price (by sufficiently small $\epsilon > 0$) for the consumers closer to firm B ($\frac{1}{2} < \theta < \theta_B$), to capture the entire personalized market. Then,

$$\pi^d_{AO} = \pi^c_{AO} + \int_{\frac{1}{2}}^{(\frac{1}{2}+\frac{c}{t})} (v - t\,\theta - \epsilon)\; d\theta = \frac{(2v - t)c}{t} \quad \text{as} \quad \epsilon \to 0$$

In the anonymous market, let p^d_{AN} be the price set by deviating firm A, given firm B's collusive price $p^c_{BN} = v - \frac{t}{2}$. Further, given $\theta_A = \frac{1}{2} - \frac{c}{t}$ and $\theta_B = \frac{1}{2} + \frac{c}{t}$, let $\hat{\theta}$ be the preference of the marginal consumer who is indifferent between buying from firm A and firm B in the market of new consumers. Then,

$$v - p^d_{AN} - t\hat{\theta} = v - p^c_{BN} - t(1 - \hat{\theta}) \implies \hat{\theta} = \frac{1}{2} + \frac{p^c_{BN} - p^d_{AN}}{2t} \tag{6}$$

In the present market structure, firm A can have two optimal deviation strategies in the anonymous market, depending on the relative magnitudes of parameters v, c, and t: (*i*) firm A can undercut firm B's price in the anonymous market such that all the old consumers who prefer firm B and have opted for privacy protection still buy from firm B, i.e., $\hat{\theta} \leq \theta_B$, and (*ii*) firm A can undercut firm B's price in the anonymous market such that some of the old consumers who prefer firm B and have opted for privacy protection now buy from firm A, i.e.,

$\hat{\theta} > \theta_B$. Figure 2 illustrates the case (ii). In this case, consumers with $\theta \in [0, \hat{\theta}]$ in the market for new consumers, and with $\theta \in [0, \theta_A]$ and $\theta \in [\theta_B, \hat{\theta}]$ in the market for old consumers buy from the firm A. Note that, these consumers are part of the anonymous market. The first case arises when private cost of privacy protection is sufficiently high, although the second case arises when it is low, as we shall see next.

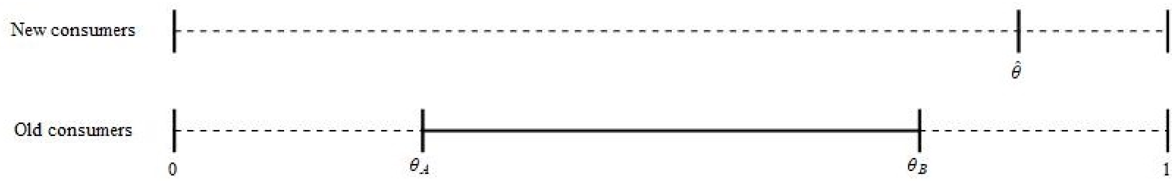

Fig. 2: Optimal Deviation

If firm i deviates from the collusive agreement, its optimal deviation pricing is summarized in the following lemma.

Lemma 2 *Suppose that the scope of privacy regulation is broad, players have unlimited cognitive abilities, and private cost of privacy protection is not prohibitively high. Then, if firm i deviates from the collusion, it is optimal for firm i to set the following prices.*

(i) In the personalized market, firm i sets

$$p_{iO}^d = \begin{cases} v - t\,\theta & \text{if } \theta \leq \frac{1}{2} \\ v - t\,\theta - \epsilon & \text{if } \theta > \frac{1}{2} \end{cases}$$

where $\epsilon > 0$ is sufficiently small.

(ii) In the anonymous market, when $v < \frac{7t}{2}$*, firm i sets*

$$p_{iN}^d = \begin{cases} \frac{v}{2} + \frac{3t}{4} - c & \text{if } c \geq \tilde{c}(v,t) \text{ and } t > (6 - 4\sqrt{2})v \\ \frac{v}{2} + \frac{t}{4} - c & \text{otherwise} \end{cases}$$

where $\tilde{c}(v,t) = \frac{v}{2} - \frac{(2\sqrt{2}+1)}{4}t$*.*

(iii) In the limiting case when $v \geq \frac{7t}{2}$*, firm i captures the entire anonymous market and sets a price*

$$p_{iN}^d = v - \frac{3t}{2}$$

Proof: See Appendix A.

Remark 2: In the personalized market, the deviation price is independent of effectiveness of broad scope privacy regulation as firm A sets a personalized price at each location. However, in the anonymous market, the deviation price decreases with the effectiveness of broad scope privacy regulation. The reason is as follows. As private cost of privacy protection increases, the size of the anonymous market decreases, and, hence, the total demand in the anonymous market decreases. Therefore, the profit-maximizing deviation price decreases with the effectiveness of broad scope privacy regulation.

As firms are symmetric, if firm i ($i = A,\ B$) deviates from the collusive agreement, then firm i's total profit from the deviation from both anonymous and personalized markets is (when $v < \frac{7t}{2}$)

$$\pi_i^d = \begin{cases} \frac{(2v+3t-4c)^2+32(2v-t)c}{32t}, & \text{if } c \geq \tilde{c}(v,t) \text{ and } t > (6-4\sqrt{2})v \\ \frac{(2v+t-4c)^2+16(2v-t)c}{16t}, & \text{otherwise} \end{cases} \tag{7}$$

In the limiting case, the deviating firm i captures the entire anonymous and personalized market (when $v \geq \frac{7t}{2}$), and earns the following profit.

$$\pi_i^d = 2v - 3t + 2c \ \text{ when } \ v \geq \frac{7t}{2} \tag{8}$$

The total deviation profit increases with private cost of privacy protection. The intuition is the same as in the case of collusion. Nevertheless, it's instructive to compare the effect of private cost of privacy protection under collusion and deviation. When $v < \frac{7t}{2}$,

$$\frac{\partial \pi_i^d}{\partial c} = \begin{cases} \frac{c}{t} + \frac{6v-7t}{4t} & \text{if } c \geq \tilde{c}(v,t) \ \text{ and } \ t > (6-4\sqrt{2})v \\ \frac{2c}{t} + \frac{2v-3t}{2t} & \text{otherwise} \end{cases}, \ \text{ and } \ \frac{\partial \pi_i^c}{\partial c} = \frac{c}{t} \tag{9}$$

Remark 3: It is easy to check that $\frac{\partial \pi_i^d}{\partial c} > \frac{\partial \pi_i^c}{\partial c} > 0$.

(c) Non-cooperative equilibrium

If any firm deviates in a period, firms compete non-cooperatively from the next period onward. Now, when both firms compete, competition intensifies in the personalized market for each consumer, which drives the tailored prices down enough so that no consumer has any incentive to opt for privacy protection (Montes et al., 2019). Suppose that firm $i; (i = A, \ B)$ sets prices p_{iN}^n and p_{iO}^n in the anonymous and personalized markets, respectively. Then, we have the following.

Lemma 3 *Suppose the privacy regulation is of broad scope, and players have unlimited cognitive abilities. Then, the non-cooperative equilibrium prices and profits are as follows.*

(i) In the personalized market,

$$p_{AO}^n(\theta) = \max\{t\,(1-2\theta), 0\}, \ \ p_{BO}^n(\theta) = \max\{t\,(2\theta-1), 0\}$$

(ii) In the anonymous market, firm i sets $p_{iN}^{n} = t$.

(ii) The equilibrium profit is $\pi_i^n = \frac{3t}{4}$.

Proof: See Appendix A.

3.1.1 Collusion sustainability

We, now turn to analyze the implication of effectiveness of privacy regulation on collusion sustainability, when the scope of privacy regulation is broad. Using equation (2), and profit expressions under collusion, deviation, and non-cooperative equilibrium, the critical discount factor can be obtained as follows. When $v < \frac{7t}{2}$,

$$\delta_{\min}^{B} = \begin{cases} 1 - \frac{8\left(2c^2+t(4v-5t)\right)}{16c^2+12v(4c+t)-56ct-15t^2+4v^2}, & \text{if } c \geq \tilde{c}(v,t) \text{ and } t > (6-4\sqrt{2})v \\ 1 - \frac{4\left(2c^2+t(4v-5t)\right)}{16c^2+4v(4c+t)-24ct-11t^2+4v^2}, & \text{otherwise;} \end{cases} \tag{10}$$

where $\tilde{c}(v,t) = \frac{v}{2} - \frac{(2\sqrt{2}+1)}{4}t$. Superscript 'B' denotes the case of broad scope privacy regulation.

In the limiting case when $v \geq \frac{7t}{2}$, the deviating firm captures the entire market and, in that case

$$\delta_{\min}^{B} = 1 - \frac{2c^2 + t(4v-5t)}{t(8c-15t+8v)}. \tag{11}$$

It turns out that in each of the above cases, $\frac{\partial \delta_{\min}^{B}}{\partial c} > 0$. Thus, the following proposition is immediate.

Proposition 1: *Suppose that the scope of privacy regulation is broad, players have unlimited cognitive abilities, and private cost of privacy protection is not prohibitive. Then, a less effective privacy regulation makes tacit collusion among firms less likely to be stable.*

Proof: See Appendix A.

The intuition behind Proposition 1 is as follows. The non-cooperative equilibrium profit (π_i^n) is independent of privacy cost c as no consumer opts for privacy under competition. When privacy cost is low, i.e., $c < \tilde{c}(v, t)$, as private cost of privacy protection increases, both incentive to collude ($\pi_i^c - \pi_i^n$) and incentive to deviate from the collusive agreement ($\pi_i^d - \pi_i^c$) increase. However, the incentive to deviate increases more than proportionately than the corresponding increase in the incentive to collude. Therefore, an increase in private cost of privacy protection makes it harder to sustain collusion. The same is true in case private cost of privacy protection is sufficiently high, i.e., $c \geq \tilde{c}(v, t)$, as well.

Therefore, an increase in private cost of privacy protection makes it harder for firms to sustain collusion.

Let us now examine the implications of effectiveness of broad scope privacy regulation on consumers' surplus. When firms collude, consumers' surplus decreases with an increase in private cost of privacy protection. However, collusion becomes less sustainable in case of higher private cost of privacy protection. On the other hand, under competition, consumers' surplus does not depend on private cost of privacy protection. Further, consumers' surplus under competition is always greater than that under collusion. It implies that the highest possible level of consumers' surplus can be obtained in case private cost of privacy protection is sufficiently high such that collusion is not sustainable, i.e., $\delta_{min}^B > \delta$ holds true.

It is important to note here that firms employ different deviation strategies depending on the underlying market structure, i.e., the extent of product differentiation (transportation cost t). Therefore, the critical discount factors obtained above are different in three cases given by equations (10) and (11), applicable depending upon the relative values of t, v and

c. However, in all cases, collusion becomes harder to sustain with an increase in private cost of privacy protection in the relevant parameter range.

To put this discussion into perspective, it is imperative that we discuss two extreme cases: when the private cost of privacy protection is prohibitively high, and when it is equal to zero.

(i) Totally Ineffective Broad Scope Privacy Regulation

Let us consider the case when private cost of privacy protection is prohibitively high, i.e., $c \geq \frac{t}{2}$, such that no consumer opts for privacy. In other words, we consider that the broad scope privacy regulation is *totally ineffective.* In this case anonymous and personalized markets coincide with markets for new and old consumers, respectively, and we have the following.

Corollary 1: *Suppose the privacy regulation is of broad scope, players have unlimited cognitive abilities, and private cost of privacy protection is prohibitively high. Then, the critical discount factor above which collusion is sustainable is as follows.*

$$\delta_{min}^{B}|_{c \geq \frac{t}{2}} = \begin{cases} 1 - \frac{4t(8v-9t)}{4v^2+36vt-39t^2}, & \textit{when } v < \frac{7t}{2} \\ \frac{8v-13t}{16v-22t}, & \textit{when } v \geq \frac{7t}{2} \end{cases}$$

Further, the critical discount factor $\delta_{min}^{B}|_{c \geq \frac{t}{2}}$ monotonically decreases as the transportation cost t increases.

Proof: See Appendix A.

Corollary 1 states that an increase in t facilitates collusion. That is, collusion is less likely to be stable in case the degree of product differentiation is higher. The reason is as follows. As transportation cost t increases, both incentive to collude $(\pi_i^c - \pi_i^n)$ and incentive to deviate from collusive agreement $(\pi_i^d - \pi_i^c)$ decrease. However, the incentive to deviate decreases more than the incentive to collude.

(ii) Fully effective Broad Scope Privacy Regulation

When private cost of privacy protection is zero, i.e., when the broad scope privacy regulation is *fully effective*, all old consumers opt for privacy under collusion and deviation. That is, the whole market is anonymous. However, under the punishment phase, no consumer opts for privacy even when the privacy cost is zero, as shown before.

Corollary 2: *Suppose the scope of privacy regulation is broad and it is fully effective (i.e., there is no private cost of privacy protection), and players have unlimited cognitive abilities. Then, the critical discount factor above which collusion is sustainable is as follows.*

$$\delta_{min}^B|_{c=0} = \begin{cases} 1 - \frac{4t(4v-5t)}{4v^2+4vt-11t^2}, & when \;\; v < \frac{7t}{2} \\ \\ \frac{4v-10t}{8v-15t}, & when \;\; v \geq \frac{7t}{2} \end{cases}$$

Further, $\frac{\partial \delta_{min}^B|_{c=0}}{\partial t} < 0$.

Proof: See Appendix A.

In this case as well, collusion is more likely to be stable in case products are more differentiated. The intuition is the same as that of Corollary 1.

It is crucial here to distinguish between the case of zero private cost of privacy protection and *uniform pricing* for all consumers. When private cost of privacy protection is zero, prices are indeed uniform under collusion and deviation. However, under the punishment phase, firms set personalized prices for old consumers.

From Corollary 1 and Corollary 2, the following is immediate.

Corollary 3: *Suppose the privacy regulation is of broad scope, and players have unlimited cognitive abilities. Then, regardless of whether the privacy regulation is fully effective or totally ineffective, collusion sustainability increases in the degree of product differentiation.*

Now, comparing the critical discount factors under the two extreme cases of broad scope privacy regulation, totally ineffective and fully effective, we get $\delta_{\min}^{B}|_{c=0} < \delta_{\min}^{FP}|_{c\geq\frac{t}{2}}$ (see Appendix A for details). It implies that collusion is less likely to be stable when broad scope privacy regulation is totally ineffective compared to that when the regulation is fully effective. Further, it can be checked that $\lim_{c\to\frac{t}{2}}\delta_{\min}^{B} = \delta_{\min}^{B}|_{c\geq\frac{t}{2}}$ and we have $\frac{\partial\delta_{\min}^{B}}{\partial c} > 0$, in each possible parametric configuration. Thus, $\delta_{\min}^{B}|_{c\geq\frac{t}{2}} > \delta_{\min}^{B}|_{c\in[0,\frac{t}{2})}$. It follows that, under broad scope of privacy regulation, collusion is least likely to be stable in case the privacy regulation is totally ineffective. Hence, it is always optimal from consumers' surplus point of view to set private cost of privacy protection at the the prohibitive level, i.e. to make the regulation totally ineffective.

Corollary 4: *Suppose the privacy regulation is of broad scope, and players have unlimited cognitive abilities. Then, product market competition among firms is best promoted in case the regulation is totally ineffective.*

3.1.2 Further issues

In this analysis, we have assumed that consumers do not have any explicit taste for privacy, and they make their privacy decisions purely from an economic point of view. However, in reality, consumers may have an intrinsic taste for privacy, which reduces the perceived cost of protecting privacy. To allow for such possibility, suppose that consumers value their privacy intrinsically and their intrinsic valuation for privacy is uniformly distributed over $[\underline{\mu}, \bar{\mu}]$.

We find that Proposition 1 holds qualitatively true in this case as well, at least for some parametric configurations (see Appendix A for details).[14]

We have also assumed that the market demand remains stable over time, which may not hold true always. Existing literature suggests that with a fixed number of market players, demand growth fosters collusion, whereas it becomes increasingly difficult to sustain collusion in shrinking markets (Vasconcelos, 2008). To gauge the implications of changes in market demand over time, it may be considered that in each period $m = 0, 1, 2, ...$, a mass α^m of new consumers enters the market, who becomes old in the next period. Therefore, in period m, there are α^m new consumers and α^{m-1} old consumers. The parameter α measures the change in demand. When $\alpha > 1$, total demand grows steadily at rate $(\alpha - 1)$, and when $0 < \alpha < 1$, total demand shrinks steadily at rate $(1 - \alpha)$.[15] That is, the market demand either always increases over time or its shrinks over time. By considering specific values of parameters, we show that Proposition 1 holds true in such cases as well (see Appendix A for details).[16]

3.2 Limited Cognitive Abilities

In Section 3.1, we have considered that all players, firms and consumers, have unlimited strategic sophistication. However, given the complex nature of means of protecting consumer privacy, it seems reasonable to consider that consumers' cognitive abilities are constrained, particularly in the context of online markets. In other words, it is plausible that consumers' ability to infer market outcomes correctly is rather limited. To model this, in this section, we

[14] It is easy to observe that results of this section are qualitatively similar to Proposition 1 in case consumers are identical in terms of their intrinsic valuation for privacy (i.e., when $\mu_j = \mu \forall j$) and $c \geq \mu$.

[15] Rate of change of demand from period m to $(m+1)$ is $\frac{\alpha^{m+1}+\alpha^m-\alpha^m-\alpha^{m-1}}{\alpha^m+\alpha^{m-1}} = \alpha - 1$.

[16] Formal analysis of implications of heterogeneous intrinsic valuations of privacy and changing market demand on collusion sustainability seems to be an interesting research agenda. However, this is beyond the scope of the present article.

assume that consumers have limited strategic sophistication, and firms outperform consumers in terms of strategic sophistication. More precisely, all consumers are assumed to have the same limited level of strategic sophistication, $k \in Z_0^+$. Following Dengler and Prüfer (2021), we assume that consumers with level-k ($k > 0$) of sophistication believe that every other consumer's level of sophistication is one level bellow k, i.e., at $k-1$. At the same time, they believe that firms share their level of sophistication. That is, a consumer with level-k believes that firms have level-k of sophistication and that firms respond optimally to level-$(k-1)$ of sophistication of other consumers. This is consistent as consumers are atomistic, and, therefore, an individual consumer is insignificant to firms' decisions. Note that level-0 thinking denotes absolute strategic naivety.

Formally, suppose that a consumer θ_l has sophistication level $k > 0$. Then, $E_{\theta_l}(k_{\theta_m \neq \theta_l}) = (k-1)$, where θ_m is a consumer, and $E_{\theta_l}(k_i) = k$, where $i = A, B$ is a firm. However, firms are more sophisticated in their strategic reasoning than consumers, and they are correct in their beliefs about the level of sophistication of consumers. For simplicity, we also make an additional assumption that a firm is correct in its belief about sophistication level of the other firm. That is, $E_i(k_{j \neq i}) = (k+1)$, $i, j = A, B$, and $E_i(k_{\theta_l}) = k$, θ_l being a consumer. Because games with level-k models are solved recursively (Dengler and Prüfer, 2021), we first solve for level-0 consumers and level-1 firms, then level-1 consumers and level-2 firms, and so on.

We find that, in the present framework, consumers' privacy choices and firms' equilibrium behavior are not altered even when firms outperform consumers in terms of strategic sophistication under broad scope of privacy regulation. The results are summarized in the following proposition. The details and proofs of this section are presented in Appendix A.

Proposition 2: *Suppose that there is broad scope privacy regulation. Also, suppose that* *consumers and firms have, respectively, level-k and level-$(k+1)$ strategic sophistication. Then, the critical discount factor above which collusion is sustainable remains the same as that in the case of unlimited cognitive abilities of each player.*

Proof: See Appendix A.

Proposition 2 implies that even when firms are more strategically sophisticated than consumers, Proposition 1 holds true. Under broad scope privacy regulation, old consumers who opt for privacy become completely anonymous, and firms can not distinguish between new consumers and those old consumers. Therefore, they charge a single uniform price in the anonymous market consisting of those old consumers who opted for privacy protection and all new consumers. Under collusion, the maximum price firms can charge in the market for new consumers is equal to the surplus of the most distant consumer, i.e., equal to $(v - \frac{t}{2})$. Old consumers with level-0 thinking, who opt for privacy, naively expect firms to set the regular collusive price in the anonymous market, which is equal to $(v - \frac{t}{2})$. As consumers make their privacy decisions before firms announce their prices, private cost incurred by them for privacy protection becomes sunk at the price setting stage. Therefore, in principle, firms, being more strategically sophisticated than consumers, can charge a premium equal to the private cost of privacy protection over the expected price (i.e., $(v - \frac{t}{2} + c)$) for those old consumers. However, if firms set this price, they will lose out on demand from the set of new consumers which is contrary to the assumption of full market coverage. Because firms can not distinguish between these two groups of consumers, they set the price equal to consumers' expected price. In other words, consumers' expectations turn out to be correct. This also implies that the equilibrium price will remain the same with further iterations of strategic thinking. Therefore, even when firms have higher strategic sophistication than consumers, Proposition 1 holds true.

Proposition 1 and Proposition 2 together imply the following.

Proposition 3: *If there is broad scope privacy regulation, it is harder for firms to sustain collusion when the privacy regulation is less effective, regardless of whether both consumers and firms have unlimited cognitive abilities or consumers' have limited strategic sophistication and firms outperform consumers in terms of strategic sophistication.*

4 Narrow Scope of Privacy Regulation

We now turn to analyse implications of effectiveness of narrow scope privacy regulation on stability of collusion, and the role of cognitive abilities of players, if any. In this case, by opting for privacy protection, an old consumer can hide her preference, but she cannot disguise herself as a new consumer. Firms can distinguish between old consumers who have opted for privacy protection and the set of new consumers. Such situation may arise in reality (for example, in the health care industry), if privacy regulations are such that individual characteristics are well protected, and at the same time, the flow of information is required to facilitate high-quality services to the consumers (Shy and Stenbacka, 2016). Now, in this case, in the personalized market, firms set an individual price for each consumer. However, unlike in Section 3, in the anonymous market firms can potentially offer different prices to new consumers and old consumers who have opted for privacy protection by incurring private cost c.

4.1 Unlimited Cognitive Abilities

We first consider that each player has unlimited cognitive ability. Suppose that, in the case of collusion, firm i ($i = A, B$) sets price $p^c_{iO}(\theta)$ in the personalized market, $p^c_{iN_1}$ in the market for new consumers, and $p^c_{iN_2}$ in the market for old consumers who opt for privacy. Analysing the optimal pricing strategies of firms in the case of collusion, we obtain the following.

Lemma 4 *Suppose that the privacy regulation has a narrow scope and players have unlimited cognitive abilities. Then, firms set the following collusive prices in the equilibrium.*

(i) In the personalized market, firm A sets price equal to $p^c_{AO}(\theta) = v - t\,\theta$*, and firm B sets price equal to* $p^c_{BO}(\theta) = v - t\,(1-\theta)$*.*

(ii) In the market for new consumers, firm $i, (i = A, B)$ *sets a price equal to* $p^c_{iN_1} = v - \frac{t}{2}$*.*

(iii) In the market for old consumers, who have opted for privacy privacy protection, firm $i, (i = A, B)$ *sets a price equal to* $p^c_{iN_2} = v - c$*.*

Further, no old consumer opts for privacy.

Proof: See Appendix B.

This result is in sharp contrast to those under broad scope privacy regulation. The reason is as follows. Under narrow scope privacy regulation, firms can perfectly distinguish between old consumers who opt for privacy protection and the set of new consumers. As firms gain the highest profit in the personalized market under collusion, it is optimal for firms to set a sufficiently high price for those old consumers who may opt for privacy protection otherwise, such that no old consumer opts for privacy.

Next, we consider the case when one firm deviates from the collusive agreement. Suppose that firm i $(i, j = A, B;\ i \neq j)$, without loss of generality, deviates from the collusion. As consumers can not predict if a firm is going to deviate, and they make their privacy decisions before firms announce their prices, we have $\theta_A = 0$ and $\theta_B = 1$, as in the case of collusion. In this case, suppose that firm i sets prices p_{iN}^d and $p_{iO}^d(\theta)$ in the market for new consumers and that for old consumers, respectively. Note that this case becomes equivalent to the case of totally ineffective broad scope privacy regulation analysed in Section 3.1. The optimal deviation prices set by firm i are given in the following lemma.

Lemma 5 *Suppose that there is narrow scope privacy regulation. Then, if firm i deviates from the collusive agreement, it sets the following prices in different markets.*

(i) In the market for old consumers, equilibrium deviation prices are as follows.

$$p_{iO}^d(\theta) = \begin{cases} v - t\,\theta & \text{if } \theta \leq \frac{1}{2} \\ v - t\,\theta - \epsilon & \text{if } \theta > \frac{1}{2} \end{cases}$$

where $\epsilon > 0$ is sufficiently small.

(ii) In the market for new consumers, firm $i, (i = A, B)$ sets a price equal to

$$p_{iN}^d = \begin{cases} \frac{v}{2} + \frac{t}{4} & \text{if } v < \frac{7t}{2} \\ v - \frac{3t}{2} & \text{if } v \geq \frac{7t}{2} \end{cases}$$

Proof: Follows from the proof of corollary 1.

The intuition is straightforward and similar to the case of totally ineffective broad scope privacy regulation.

Finally, we consider the game in which firms compete non-cooperatively. Similar to the case of broad scope privacy regulation, no old consumer opts for privacy in this case as well.

The reason is that in the personalized market, firms compete aggressively at each location which drives down prices to the difference in transportation costs. Then, it is optimal for the old consumers to remain in the personalized market.

Lemma 6 *Suppose that the privacy regulation is of narrow scope and players have unlimited cognitive abilities. Then the non-cooperative equilibrium prices are given as follows.*

(i) In the market for old consumers, equilibrium prices are as follows.

$$p^n_{AO}(\theta) = \max\{t\ (1-2\theta), 0\},\ \ p^n_{BO}(\theta) = \max\{t\ (2\theta-1), 0\}$$

(ii) In the market for new consumers, firm $i, (i = A, B)$ *sets a price equal to* $p^n_{iN} = t$.

Proof: See Appendix B.

Although the mechanism driving consumers' privacy choices and the equilibrium in the case of collusion under narrow scope privacy regulation is different from those under broad scope privacy regulation, the equilibrium outcomes in terms of prices and profits under narrow scope privacy regulation are the same as that in the case of prohibitive privacy cost under broad scope privacy regulation. Let δ^N_{min} denote the critical discount factor above which collusion is sustainable under narrow scope privacy regulation. Then, $\delta^N_{min} = \delta^B_{min}|_{c\geq \frac{t}{2}}$. Superscript 'N' denotes the case of narrow scope privacy regulation.

From the above discussion, we state the following proposition.

Proposition 4: *Suppose that the scope of privacy regulation is narrow and players have unlimited cognitive abilities. Then, the following is true.*

(i) No old consumer opts for privacy protection even when firms collude.

(ii) The critical discount factor above which collusion is sustainable is as follows.

$$\delta^N_{min} = \begin{cases} 1 - \frac{4t(8v-9t)}{4v^2+36vt-39t^2} & when \ \ v < \frac{7t}{2} \\ \\ \frac{8v-13t}{16v-22t} & when \ \ v \geq \frac{7t}{2} \end{cases}$$

Proof: The proof follows from Lemma 4 and Corollary 1.

As $\delta^N_{min} = \delta^B_{\min}|_{c \geq \frac{t}{2}}$ and $\delta^B_{\min}|_{c \geq \frac{t}{2}} > \delta^B_{\min}|_{c < \frac{t}{2}}$, the following is true.

Proposition 5: *Suppose that players have unlimited cognitive abilities. Then, sustainability of collusion does not depend on the scope of privacy regulation, if broad scope privacy regulation is totally ineffective. Otherwise, if broad scope privacy regulation is not totally ineffective, narrow scope privacy regulation makes collusion less likely to be stable compared to broad scope privacy regulation.*

Proposition 5 states that narrow scope privacy regulation weakly dominates broad scope privacy regulation in terms of promoting competition among firms, when consumers and firms have unlimited cognitive abilities.

4.2 Limited Cognitive Abilities

Let us now consider that consumers have limited strategic sophistication, due to their limited cognitive abilities, compared to firms as in Section 3.2. However, unlike as in Section 3.2, the privacy regulation is of narrow scope. We have seen in Section 4.1 that, under narrow scope privacy regulation, no consumer opts for privacy protection in case consumers and firms both have unlimited strategic sophistication. However, this is no longer true in case consumers have limited strategic sophistication with respect to firms. Consider, for instance, the case of collusion. An old consumer with sophistication level-0 expects the regular collusive price $(v - \frac{t}{2})$ if she opts for privacy protection. On the other hand, consumers can perfectly anticipate the prices in the personalized market. As firms having more sophistication can distinguish between old consumers who opted for privacy protection and new consumers, they can price differently to these two groups. In particular, for level-0 consumers, firms can charge a price equal to $(v - \frac{t}{2} + c)$, and all consumers who opted for privacy will still buy the product as private cost of privacy protection is sunk at the pricing stage. As the level of sophistication increases, firms will be able to charge higher to the old consumers who opted for privacy protection. As in Section 3, no consumer opts for privacy protection if $c \geq \frac{t}{2}$, regardless of their level of strategic sophistication. Moreover, the number of consumers who opt for privacy decreases as the sophistication level increases. Thus, unlike as in Section 3, if consumers' sophistication level k is greater than a threshold, i.e., if $k \geq \bar{k} = \frac{t}{2c} - 1$, no consumer opts for privacy even when $c < \frac{t}{2}$ (see Appendix B for details). Therefore, when consumers are sufficiently sophisticated or private cost of privacy protection is at least as high as $\frac{t}{2}$, then the analysis in the present scenario becomes the same as in Section 4.1.

Let $\delta^N(k)$ denote the critical discount factor above which collusion is sustainable under narrow scope of privacy protection when consumers have limited cognitive abilities. Then, $\delta^N(k) = \delta^N_{\min}$ if $c \geq \frac{t}{2}$ or $k \geq \bar{k}$. Otherwise, if $c < \frac{t}{2}$ and $k < \bar{k}$, $\delta^N(k)$ is as follows (see Appendix B for details).

$$\delta^N(k) = \begin{cases} \frac{16c^2(k+1)^2+16c(k+1)(t-2v)-(3t-2v)^2}{32c^2(k+1)^2-32c(k+1)v+31t^2-20tv-4v^2} & \text{when } v < \frac{7t}{2} \\ \frac{2c^2(k+1)^2+2c(k+1)(t-2v)+t(5t-2v)}{4c^2(k+1)^2-4c(k+1)v+2t(5t-3v)} & \text{when } v \geq \frac{7t}{2} \end{cases} \tag{12}$$

In the latter case, the impact of effectiveness of privacy regulation on collusion sustainability for level-k of consumers can be summarized in the following proposition.

Proposition 6: *Suppose that the privacy regulation is of narrow scope. Also, suppose that* *consumers and firms have, respectively, level-k and level-$(k+1)$ strategic sophistication. Then, the critical discount factor $\delta^N(k)$ increases with an increase in private cost of privacy protection c, unless $c \geq \frac{t}{2}$ or $k \geq \bar{k}$.*

Proof: See Appendix B.

Unlike Proposition 4, the critical discount factor in the present scenario depends on private cost of privacy protection if it is not prohibitively high (i.e. $c < \frac{t}{2}$) and consumers are not sufficiently sophisticated (i.e. $k < \bar{k}$). In this case, the incentive to deviate is always higher than that to collude, and it increases at a greater rate with an increase in private cost of privacy protection. Therefore, the critical discount factor monotonically increases with an increase in private cost of privacy protection.

Clearly, when consumers are less sophisticated than firms and the scope of privacy regulation is narrow, it is harder for firms to sustain collusion in case the privacy regulation is less effective, as under broad scope privacy regulation. However, from Proposition 2, Proposition 4 and Proposition 6, we get the following.

Proposition 7: *Cognitive ability of consumers plays a crucial role in determining sustainability of collusion under narrow scope of privacy regulation. Limited cognitive ability of consumers compared to firms makes collusion more likely to be stable under narrow scope of privacy regulation, unless the regulation is totally ineffective and consumers' level of strategic sophistication is sufficiently high.*

Further, when products are sufficiently differentiated, effectiveness of privacy regulation is less than a critical level and consumers' strategic sophistication is less than a threshold, collusion is less likely to be stable under narrow scope privacy regulation than under broad scope privacy regulation.[17] However, in case of other plausible parametric configurations, it turns out that sustainability of collusion may be higher or lower under narrow scope privacy regulation compared to that under broad scope privacy regulation, unlike as in case of unlimited cognitive abilities.

5 Concluding Remarks

In an infinite-horizon differentiated products duopoly with consumer heterogeneity in terms of preferences and purchase histories, we have examined the implications of privacy regulation on stability of tacit collusion among firms. We have considered two different scenarios based on consumers' cognitive abilities, and focused on two key aspects of privacy regulation – scope and effectiveness. Our analysis contributes to the growing debate on privacy regulations in the present era of 'big data' and artificial intelligence, and highlights potential negative implications of privacy regulation, via its effects on firms' product market conducts, on consumers' economic benefits.

We have shown that, although firms (consumers) always obtain higher (lower) profits (economic benefits) under collusion than that under competition, the possibility of collusion to be sustained in the equilibrium crucially depends on (a) the scope and effectiveness of the privacy regulation in force and (b) whether consumers' cognitive ability is limited or unlimited. If there is broad scope privacy regulation, which aims to restrict firms' access to consumers' data on preferences as well as purchase histories, collusion is more likely to be stable in case

[17] From equations (10), (11), and (12), when $\frac{2v}{3+2\sqrt{2}} < t < \frac{2v}{5}$, $\frac{v}{2} - t\frac{2\sqrt{2}+1}{4} < c < \frac{t}{2}$ and $0 \leq k < \frac{t}{2c} - 1$, we have $\delta^B_{\min} = 1 - \frac{8\left(2c^2+t(4v-5t)\right)}{16c^2+12v(4c+t)-56ct-15t^2+4v^2}$ and $\delta^N(k) = \frac{16c^2(k+1)^2+16c(k+1)(t-2v)-(3t-2v)^2}{32c^2(k+1)^2-32c(k+1)v+31t^2-20tv-4v^2}$. It can be checked that, at $k = 0$, $\delta^N(k) > \delta^B_{\min}$. Further, $\frac{\partial \delta^N(k)}{\partial k} > 0$ and $\frac{\partial \delta^B_{min}}{\partial k} = 0$. Therefore, if when $\frac{2v}{3+2\sqrt{2}} < t < \frac{2v}{5}$, $\frac{v}{2} - t\frac{2\sqrt{2}+1}{4} < c < \frac{t}{2}$ and $0 \leq k < \frac{t}{2c} - 1$, we have $\delta^N(k) > \delta^B_{min}$.

the regulation is more effective, i.e., when consumers need to incur a lower private cost to opt for privacy protection. In this case, the stability of collusion does not depend on the level of consumers' cognitive ability. On the contrary, under the narrow scope privacy regulation, wherein consumers cannot hide their purchase history, the effectiveness of privacy protection does not have any impact on collusion stability in case consumers have unlimited cognitive ability. However, if consumers' cognitive ability is limited, collusion is more likely to be stable in case (a) the narrow scope privacy regulation is more effective, as in the case of broad scope privacy regulation, and/or (b) consumers' level of strategic sophistication is less.

We have also shown that, when consumers' cognitive ability is unlimited, the broad scope privacy regulation makes collusion more likely to be stable compared to the narrow scope privacy regulation. This result continues to hold true in case consumers' level of strategic sophistication is less than that of firms, albeit under certain conditions. Our analysis suggests that a more effective and broad scope privacy regulation is more likely to dampen product market competition, and thus reduce consumers' economic benefits.

Our analysis has the following testable implications. First, firms are more likely to engage in tacit collusion in case they are mandated to place 'privacy' as the default option for consumers compared to that in case 'no privacy' is set as the default option. Second, the scope of privacy regulation has a positive effect on likelihood of tacit collusion. Third, the intensity of product market competition is less in more sophisticated markets, where firms adopt more complex and harder to infer product market strategies of firms. Findings of Calvano et al. (2020) seem to lend some support to the third hypothesis.

Our results also highlight the importance of designing effective competition policy and its stricter enforcement, particularly in the context of digital markets. This is because, strict enforcement of effective competition policy reduces firms' incentive to collude, and thus can help avoiding the unintended consequence of privacy regulation, at least partially.

In this article we have considered a duopoly market, where firms are symmetric and they acquire consumer data at zero cost. Intuitively, we can say that the implication of privacy regulation on stability of collusion in case of oligopoly is likely to be similar to that in case of duopoly. However, if firms are asymmetric in terms of marginal cost of production, sharing of collusive profits may follow different mechanisms (Friedman and Thisse, 1993; Jehiel, 1992), which may have implications to stability of collusion in the context of privacy regulation as well. Further, due to the presence of growing business of data intermediaries, it seems to be interesting to extend the present analysis by allowing for endogenous acquisition of consumers' information by firms from data intermediaries, as in Montes et al. (2019) and Bounie et al. (2021).

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Appendix A

Proof of Lemma 2

(a) Interior Solution

Suppose that the marginal consumer in the anonymous market who is indifferent between buying from firms A and B is given by $\hat{\theta}$. In the anonymous market, the deviating firm A can have two deviation strategies: (i) firm A can undercut firm B's price in the anonymous market such that all the old consumers who prefer firm B and have paid a private cost for the privacy still buy from firm B, i.e., $\hat{\theta} \leq \theta_B$, and (ii) firm A can undercut firm B's price in the anonymous market such that some of the old consumers who prefer firm B and have paid a private cost for the privacy now buy from firm A, i.e., $\hat{\theta} > \theta_B$. Now given the collusive price by firm B, $p^c_{BN} = v - \frac{t}{2}$, suppose that firm A deviates and sets a price p^d_{AN} in the anonymous market. Then,

$$v - p^d_{AN} - t\hat{\theta} = v - p^c_{BN} - t(1 - \hat{\theta}) \implies \hat{\theta} = \frac{1}{2} + \frac{p^c_{BN} - p^d_{AN}}{2t}$$

In what follows, we derive the deviation prices and profits in each case.

Case (i): Suppose that $\hat{\theta} \leq \theta_B$.

Demand for firm A in anonymous market is $(\theta_A + \hat{\theta})$. Firm A chooses p^d_{AN} to maximize the following objective function.

$$\max_{p^d_{AN}} \pi^d_{AN} = (\theta_A + \hat{\theta})p^d_{AN} = (1 - \frac{c}{t} + \frac{p^c_{BN} - p^d_{AN}}{2t})p^d_{AN}$$

Given $p^c_{BN} = v - \frac{t}{2}$, first-order condition of the above maximization problem yields $p^d_{AN} = \frac{v}{2} + \frac{3t}{4} - c$. The equilibrium profit of firm A from the deviation in the anonymous market is $\pi^d_{AN} = (\frac{v}{2} + \frac{3t}{4} - c)(\frac{v}{4t} + \frac{3}{8} - \frac{c}{2t})$.

The total profit of firm A from the deviation is, therefore, $\pi_A^d = \pi_{AO}^d + \pi_{AN}^d$. As firms A and B are symmetric, if either firm i ($i = A,\ B$) deviates from the collusion, it earns the following profit

$$\pi_i^d = \frac{(2v + 3t - 4c)^2 + 32(2v - t)c}{32t}$$

The condition $\hat{\theta} \leq \theta_B$ is satisfied when $c \geq \frac{v}{2} - \frac{5t}{4}$. Further, $c \leq \frac{t}{2}$. Therefore, $\frac{v}{2} - \frac{5t}{4} \leq \frac{t}{2} \implies v \leq \frac{7t}{2}$. Case (i) is applicable when $c \geq \frac{v}{2} - \frac{5t}{4}$ and $v \leq \frac{7t}{2}$.

Case (ii): Suppose that $\hat{\theta} > \theta_B$.

Now, demand for firm A in the anonymous market is $(\theta_A + \hat{\theta} + \hat{\theta} - \theta_B)$. Firm A chooses p_{AN}^d to maximize the following objective function.

$$\max_{p_{AN}^d} \pi_{AN}^d = (\theta_A + \hat{\theta} + \hat{\theta} - \theta_B)p_{AN}^d = (1 - \frac{2c}{t} + \frac{p_{BN}^c - p_{AN}^d}{t})p_{AN}^d$$

Given $p_{BN}^c = v - \frac{t}{2}$, profit maximization yields the equilibrium price of firm A, $p_{AN}^d = \frac{v}{2} + \frac{t}{4} - c$. The equilibrium profit of firm A from deviation in the anonymous market is $\pi_{AN}^d = \frac{(2v+t-4c)^2}{16t}$.

Total profit of firm A from deviation is $\pi_A^d = \pi_{AO}^d + \pi_{AN}^d$. As firms are symmetric, if either firm i deviates from the collusion, it earns the profit

$$\pi_i^d = \frac{(2v + t - 4c)^2 + 16(2v - t)c}{16t}$$

The condition $\hat{\theta} > \theta_B$ is satisfied when $c < \min\{\frac{v}{2} - \frac{3t}{4}, \frac{t}{2}\}$. For full market coverage, we must have $v \geq \frac{5t}{2}$. Therefore, the condition $\hat{\theta} > \theta_B$ is satisfied when $c < \frac{t}{2}$.

Further, for interior solution, $\hat{\theta} < 1$. This implies that $v + 2c < \frac{7t}{2}$. Considering the possibility of zero privacy cost, for interior solution, we must have $v < \frac{7t}{2}$.

When $c < \frac{v}{2} - \frac{5t}{4}$, firm A employs the deviation strategy as in case (ii). Now, consider the case when $\frac{v}{2} - \frac{5t}{4} \leq c < \frac{t}{2}$. Given that $\frac{5t}{2} \leq v \leq \frac{7t}{2}$, comparing the total deviation profits under case (i) and case (ii), we have

$$\frac{(2v+3t-4c)^2+32(2v-t)c}{32t} \geq \frac{(2v+t-4c)^2+16(2v-t)c}{16t} \text{ if } c \geq \frac{v}{2}-\frac{(2\sqrt{2}+1)}{4}t$$
$$\text{and } t > (6-4\sqrt{2})v$$

L.H.S in the first inequality is the total deviation profit in case (i), and R.H.S. is the total deviation profit in case (ii).

From the above, if firm i $(i = A,\ B)$ deviates from the collusive agreement, its optimal deviation pricing strategy in the anonymous market is as follows.

$$p_{iN}^d = \begin{cases} \frac{v}{2}+\frac{3t}{4}-c & \text{if } c \geq \tilde{c}(v,t) \text{ and } t > (6-4\sqrt{2})v \\ \frac{v}{2}+\frac{t}{4}-c & \text{otherwise} \end{cases}$$

where $\tilde{c}(v,t) = \frac{v}{2} - \frac{(2\sqrt{2}+1)}{4}t$.

Total profit of firm i $(i = A,\ B)$ from the deviation from both personalized and anonymous markets is

$$\pi_i^d = \begin{cases} \frac{(2v+3t-4c)^2+32(2v-t)c}{32t} & \text{if } c \geq \tilde{c}(v,t) \text{ and } t > (6-4\sqrt{2})v \\ \frac{(2v+t-4c)^2+16(2v-t)c}{16t} & \text{otherwise} \end{cases}$$

(b) Limiting case

Suppose that $v \geq \frac{7t}{2}$. Then, $\hat{\theta} = 1$ from case (ii) above. That is, firm A captures the entire anonymous market as well. Suppose that firm A sets a price p_{AN}^d in the anonymous market, then under limit pricing

$$v - t - p_{AN}^d = v - p_{BN}^c \implies p_{AN}^d = v - \frac{3t}{2}$$

Profit from the anonymous market is $(v-\frac{3t}{2})(2-\frac{2c}{t})$. Total deviation profit is $\left(2-\frac{2c}{t}\right)\left(v-\frac{3t}{2}\right)+\frac{c(2v-t)}{t}$.

[QED]

Proof of Lemma 3 Suppose that old consumers with $\theta \in [0, \theta_A]$ and $\theta \in [\theta_B, 1]$ have opted for privacy protection. Then, in the personalized market firms compete at each location such that prices are driven down to the difference in transportation costs. Thus, firms' optimal prices in the personalized market (Montes et al., 2019; Taylor and Wagman, 2014) are as follows.

$$p^n_{AO} = \max\{t(1-2\theta), 0\},\ p^n_{BO} = \max\{t(2\theta-1), 0\}$$

Now, suppose that firm i sets a price p^n_{iN} in the anonymous market. Firm A solves the following profit maximization problem in the anonymous market.

$$\max_{p^n_{AN}} \pi^n_{AN} = (\theta_A + \frac{1}{2} + \frac{p^n_{BN} - p^n_{AN}}{2t})\, p^n_{AN}$$

Firm B solves an analogous problem. Solving problems of firm A and firm B, we get the following.

$$p^n_{AN} = t + \frac{2t(2\theta_A + (1-\theta_B))}{3} > t \text{ and } p^n_{BN} = t + \frac{2t(\theta_A + 2(1-\theta_B))}{3} > t$$

Now, consider consumers' privacy decisions. Suppose that consumers anticipate prices p^e_{iN} and $p^e_{iO}(\theta)$ in the anonymous and personalized markets, respectively. Then, a consumer with $\theta \in [0, \theta_A]$ opts for privacy protection, if $v - p^e_{AN} - t\theta - c \geq v - p^e_{AO}(\theta) - t\theta$. Because consumers' expectations are correct in equilibrium, i.e., $p^e_{iN} = p^n_{iN}$ and $p^e_{iO}(\theta) = p^n_{iO}(\theta)$ holds true in the equilibrium, we have $p^n_{AN} \leq t(1-2\theta) - c < t$, which is a contradiction for a positive value of θ_A. A similar argument applies to the privacy decisions of consumers with $\theta \in [\theta_B, 1]$. Therefore, when both firms compete non-cooperatively to maximize their own profits, no consumer opts for privacy protection by incurring cost c. The anonymous and personalized markets in this case correspond to the markets for new and old consumers, respectively.

Consider the market for new consumers. Firm i sets price p^n_{iN} to maximize its own profit. The marginal consumer is given by

$$v - p^n_{AN} - t\,\bar{\theta} = v - p^n_{BN} - t\,(1-\bar{\theta}) \implies \bar{\theta} = \frac{1}{2} + \frac{p^n_{BN} - p^n_{AN}}{2t}$$

Profit maximization for both firms yields equilibrium prices as $p^n_{iN} = t$, and equilibrium profits are $\pi^n_{iN} = \frac{t}{2}$ from the market for new consumers.

The equilibrium prices in the market for old consumers are

$$p^n_{AO}(\theta) = \max\{t\;(1-2\theta), 0\} \text{ and } p^n_{BO} = \max\{t\;(2\theta-1), 0\}$$

Each firm earns $\pi^n_{iO} = t/4$ from the set of old consumers. Therefore, total non-cooperative equilibrium profit of firm i ($i = A,\ B$) from both markets is as follows.

$$\pi^n_i = \pi^n_{iN} + \pi^n_{iO} = \frac{3t}{4}$$

[QED]

Proof of Proposition 1

(i) Consider case (i) in equation (10). Then, $\delta^B_{\min} = 1 - \frac{8\left(2c^2+t(4v-5t)\right)}{16c^2+12v(4c+t)-56ct-15t^2+4v^2}$. Differentiating $\delta^{FP}_{\min}$ with respect to c, we have

$$\frac{\partial \delta^B_{\min}}{\partial c} = \frac{32\left(4c^2(7t-6v) - c(5t-2v)^2 + 2t\left(35t^2 - 58tv + 24v^2\right)\right)}{\left(16c^2 - 56ct + 48cv - 15t^2 + 12tv + 4v^2\right)^2}$$

Given $v \geq \frac{5t}{2}$, it is straightforward to check that $\frac{\partial \delta^B_{\min}}{\partial c} > 0$.

(ii) Next, consider case (ii) in equation (10). Then, $\delta^B_{\min} = 1 - \frac{4\left(2c^2+t(4v-5t)\right)}{16c^2+4v(4c+t)-24ct-11t^2+4v^2}$. Differentiating $\delta^B_{\min}$ with respect to c, we have

$$\frac{\partial \delta^B_{\min}}{\partial c} = \frac{16\left(4c^2(3t-2v)+c\left(-29t^2+28tv-4v^2\right)+2t\left(15t^2-22tv+8v^2\right)\right)}{\left(16c^2-24ct+16cv-11t^2+4tv+4v^2\right)^2}$$

Given $v \geq \frac{5t}{2}$, it is straightforward in this case too to check that $\frac{\partial \delta^B_{\min}}{\partial c} > 0$.

(iii) Next, consider the case when $v \geq \frac{7t}{2}$ (equation (11)). Then,

$$\delta^B_{\min} = \frac{2\left(c^2-4ct+t(5t-2v)\right)}{t(-8c+15t-8v)}$$

Differentiating $\delta^B_{\min}$ with respect to c, we have $\frac{\partial \delta^B_{\min}}{\partial c} > 0$.

[QED]

Proof of Corollary 1

When $c \geq \frac{t}{2}$, no old consumer opts for privacy. In that case, collusive profit of firm i is $\pi^c_i|_{c\geq\frac{t}{2}} = (v-\frac{t}{2})/2 + \int_0^{1/2}(v-t\theta)d\theta = v - \frac{3t}{8}$.

In the personalized market, the deviating firm decreases its price slightly for consumers closer to the rival firm at each location $\theta \in [\frac{1}{2}, 1]$ to capture the entire market, and earns profit $= \int_0^1 (v-t\theta)\, d\theta = v - \frac{t}{2}$.

In the anonymous market, given collusive price set by the rival firm, the deviating firm maximizes its own profit. Suppose that firm A, without loss of generality, deviates from the collusion. Given $p^c_{BN} = v - \frac{t}{2}$, firm A sets price p^d_{AN} to maximize its own profit. Let θ_d be the marginal consumer in the anonymous market who is indifferent between buying from firm A and firm B. Then, θ_d is given as

$$v - t\,\theta_d - p^d_{AN} = v - t\,(1-\theta_d) - p^c_{BN} = v - t\,(1-\theta_d) - (v - \frac{t}{2}) \implies \theta_d = \frac{1}{4} + \frac{v}{2t} - \frac{p^d_{AN}}{2t}$$

Firm A's objective function is $\max_{p^d_{AN}} \theta_d\ p^d_{AN}$. The first order condition of firm A's maximization problem yields $p^d_{AN} = (\frac{v}{2} + \frac{t}{4})$. Note that for $\theta_d \geq \frac{1}{2}$, we must have $v \geq \frac{5t}{2}$ and for interior solution, i.e., $\theta_d < 1$, we must have $v < \frac{7t}{2}$. As firms are symmetric, therefore, if firm i ($i = A, B$) deviates from the collusive agreement, it set a price $p^d_{iN} = (\frac{v}{2} + \frac{t}{4})$ in the anonymous market, and earns a profit equal to $(\frac{v}{4t} + \frac{1}{8})(\frac{v}{2} + \frac{t}{4})$. Therefore, total profit of the deviating firm i is $\pi^d_i|_{c \geq \frac{t}{2}} = \frac{1}{32}\left(\frac{4v^2}{t} - 15t + 36v\right)$. The non-cooperative profit of firm i is $\pi^n_i = \frac{3t}{4}$.

Hence, using equation (2), the critical discount factor, in this case, is obtained as follows

$$\delta^B_{\min}|_{c \geq \frac{t}{2}} = 1 - \frac{4t(8v - 9t)}{4v^2 + 36vt - 39t^2} \text{ when } v < \frac{7t}{2}$$

Further, differentiating $\delta^B_{\min}|_{c \geq \frac{t}{2}}$ with respect to t, we have $\frac{\partial \delta^B_{\min}|_{c \geq \frac{t}{2}}}{\partial t} < 0$.

In the limiting case, when $v \geq \frac{7t}{2}$, the deviating firm serves the entire market. In the anonymous market, the deviating firm i sets a price $p^d_i = v - \frac{3t}{2}$. The total deviation profit in this case is

$$\pi^d_i = (v - \frac{3t}{2}) + (v - \frac{t}{2}) = 2(v - t)$$

The critical discount factor is

$$\delta^B_{\min}|_{c \geq \frac{t}{2}} = \frac{8v - 13t}{16v - 22t} \text{ when } v \geq \frac{7t}{2}$$

Differentiating $\delta^B_{\min}|_{c \geq \frac{t}{2}}$ with respect to t, we have $\frac{\partial \delta^B_{\min}|_{c \geq \frac{t}{2}}}{\partial t} < 0$.

[QED]

Proof of Corollary 2

When privacy cost is zero, all old consumers opt for privacy under collusion and deviation. That is, the entire market is anonymous. Therefore, collusive and deviation profits of a firm

i are as follows ($v < \frac{7t}{2}$).

$$\pi_i^c|_{c=0} = v - \frac{t}{2}, \;\; \pi_i^d|_{c=0} = \frac{(2v+t)^2}{16t}$$

However, in the case of the non-cooperative game, no consumer opts for privacy even when the privacy cost is zero, and the profit, in that case, is $\pi_i^n = \frac{3t}{4}$.

Using equation (2), we get the critical discount factor in this case as follows.

$$\delta_{\min}^B|_{c=0} = 1 - \frac{4t(4v-5t)}{4v^2+4vt-11t^2} \;\; \text{when} \;\; v < \frac{7t}{2}$$

In the limiting case, when $v \geq \frac{7t}{2}$, the deviating firm serves the entire market. The deviating firm i sets a price $p_i^d = v - \frac{3t}{2}$. The total deviation profit in this case is

$$\pi_i^d = 2(v - \frac{3t}{2}) = 2v - 3t$$

Using equation (2), we get the following critical discount factor.

$$\delta_{\min}^B|_{c=0} = \frac{4v-10t}{8v-15t} \;\; \text{when} \;\; v \geq \frac{7t}{2}$$

Further, differentiating $\delta_{\min}^B|_{c=0}$ with respect to t in both cases, we obtain the following.

$$\frac{\partial \delta_{\min}^B|_{c=0}}{\partial t} < 0$$

[QED]

Comparison of the Critical Discount Factor under Totally Ineffective and Fully Effective Broad Scope Privacy Regulation

In both cases when $\frac{5t}{2} \leq v \leq \frac{7t}{2}$ and $v > \frac{7t}{2}$, comparing $\delta_{\min}^B|_{c=0}$ and $\delta_{\min}^B|_{c\geq\frac{t}{2}}$, we have

$$\delta_{\min}^B|_{c=0} - \delta_{\min}^B|_{c\geq\frac{t}{2}} < 0$$

[QED]

Consumers' Taste for Privacy (Section 3.1.2)

Consumers' taste for privacy μ_j is distributed uniformly over $[\underline{\mu}, \bar{\mu}]$. Therefore, the effective privacy cost faced by a consumer is $c_j = c - \mu_j$. Equivalently, consumers face heterogenous privacy costs c_j, uniformly distributed over $[\underline{c}, \bar{c}]$; $\underline{c} = c - \bar{\mu}$, $\bar{c} = c - \underline{\mu}$. We assume that $c \geq \bar{\mu}$, i.e., all consumers face non-negative effective privacy costs.

Collusion

Suppose that consumers anticipate prices p^e_{iN} and $p^e_{iO}(\theta)$ in anonymous and personalized markets, respectively. An old consumer with preference $\theta \in [0, \frac{1}{2}]$ and effective privacy cost c_j opts for privacy if $v - p^e_{AN} - t\theta - c_j \geq v - p^e_{AO}(\theta) - t\theta$. Similarly, an old consumer with preference $\theta \in (\frac{1}{2}, 1]$ and effective private privacy cost c_j opts for privacy if $v - p^e_{BN} - t(1-\theta) - c_j \geq v - p^e_{BO}(\theta) - t(1-\theta)$.

In the personalized market, collusive prices are $p^c_{AO}(\theta) = v - t\theta$ and $p^c_{BO}(\theta) = v - t(1-\theta)$, and in the anonymous market, collusive prices are $p^c_{iN} = v - \frac{t}{2}$, $i = A,\ B$.

Now, consider the consumers with $\theta \in [0, \frac{1}{2}]$. There are three segments.

(a) $0 \leq \theta \leq \frac{1}{2} - \frac{\bar{c}}{t}$

(b) $\frac{1}{2} - \frac{\bar{c}}{t} < \theta < \frac{1}{2} - \frac{\underline{c}}{t}$

(c) $\frac{1}{2} \geq \theta \geq \frac{1}{2} - \frac{\underline{c}}{t}$

Consumers in segment (a) always opt for privacy. Consumers in segment (c) never opt for privacy. In segment (b), however, consumers with higher taste for privacy opt for privacy whereas consumers with lower taste for privacy do not. Similarly, three segments are characterized for consumers with $\theta \in (0, \frac{1}{2}]$. For consumers with $\theta \in (\frac{1}{2}, 1]$, there are three segments as follows.

(i) $1 \geq \theta \geq \frac{1}{2} + \frac{\bar{c}}{t}$

(ii) $\frac{1}{2} + \frac{\underline{c}}{t} < \theta < \frac{1}{2} + \frac{\bar{c}}{t}$

(iii) $\frac{1}{2} \leq \theta \leq \frac{1}{2} + \frac{\underline{c}}{t}$

Consider the anonymous market. Demands of firm A from the set of new consumers and segment (i) of $[0, \frac{1}{2}]$ of old consumers are $\frac{1}{2}$ and $(\frac{1}{2} - \frac{\bar{c}}{t})$, respectively. Consider segment (ii) $\theta \in (\frac{1}{2} - \frac{\bar{c}}{t}, \frac{1}{2} - \frac{\underline{c}}{t})$. For given θ, an old consumer with taste for privacy c_j opts for privacy if $\theta < \frac{1}{2} - \frac{c_j}{t} \implies c_j < t(\frac{1}{2} - \theta)$. Therefore, demand for firm A in the anonymous market from segment (ii) is $\frac{1}{\bar{c}-\underline{c}} \int_{\frac{1}{2}-\frac{\bar{c}}{t}}^{\frac{1}{2}-\frac{\underline{c}}{t}} (t(\frac{1}{2} - \theta) - \underline{c})\ d\theta = \frac{\bar{c}-\underline{c}}{2t}$. Total profit of firm A from the anonymous market is, then,

$$\pi_{AN}^c = (\frac{1}{2} + \frac{1}{2} - \frac{\bar{c}}{t} + \frac{\bar{c} - \underline{c}}{2t})(v - \frac{t}{2}) = (1 - \frac{\bar{c} + \underline{c}}{2t})(v - \frac{t}{2})$$

Next, consider the personalized market for firm A. In segment (ii), for given $\theta \in \frac{1}{2} - \frac{\bar{c}}{t} < \theta < \frac{1}{2} - \frac{\underline{c}}{t}$, demand for firm A is $\frac{\bar{c}-t(\frac{1}{2}-\theta)}{\bar{c}-\underline{c}}$. Therefore, profit of firm A from the personalized market is

$$\pi_{AO}^c = \int_{\frac{1}{2}-\frac{\bar{c}}{t}}^{\frac{1}{2}-\frac{\underline{c}}{t}} (\frac{\bar{c} - t(\frac{1}{2} - \theta)}{\bar{c} - \underline{c}})(v-t\theta)\ d\theta + \int_{\frac{1}{2}-\frac{\underline{c}}{t}}^{\frac{1}{2}} (v-t\theta)\ d\theta = \frac{(\bar{c} - \underline{c})(6v - 3t + 2\bar{c} + 4\underline{c})}{12t} + \frac{(2v - t + \underline{c})\underline{c}}{2t}$$

Total collusive profit of firm A is $\pi_A^c = \pi_{AN}^c + \pi_{AO}^c$. As firms are symmetric, the total collusive profit of firm i ($i = A,\ B$) is

$$\pi_i^c = \frac{3t(2v - t) + \underline{c}^2 + \underline{c}\bar{c} + \bar{c}^2}{6t} \tag{A.1}$$

Optimal deviation

Suppose that firm A, without loss of generality, deviates from the collusive agreement. Consumers have already made their privacy choices based on the expectations of collusive prices.

In the personalized market, firm A decreases its price slightly for $\theta > \frac{1}{2}$ to capture the entire personalized market. Therefore, profit of firm A from deviation from the personalized market is

$$\pi^d_{AO} = \pi^c_{AO} + \int_{\frac{1}{2}}^{\frac{1}{2}+\frac{\underline{c}}{t}} (v - t\theta)\ d\theta + \int_{\frac{1}{2}+\frac{\underline{c}}{t}}^{\frac{1}{2}+\frac{\bar{c}}{t}} (\frac{\bar{c} - t(\theta - \frac{1}{2})}{\bar{c} - \underline{c}})(v - t\theta)\ d\theta = \frac{(2v - t)(\bar{c} + \underline{c})}{2t}$$

Next, consider the anonymous market. Suppose that firm A sets a price p^d_{AN} in the anonymous market. Let $\hat{\theta}$ be the indifferent consumer in the market for new consumers. Then, $v - p^d_{AN} - t\hat{\theta} = v - p^c_{BN} - t(1 - \hat{\theta})$. Given that $p^c_{BN} = v - \frac{t}{2}$, we have $\hat{\theta} = \frac{1}{4} + \frac{v - p^d_{AN}}{2t}$.

Suppose that $\hat{\theta} \leq \frac{1}{2} + \frac{\underline{c}}{t}$. Profit maximization problem of firm A in the anonymous market is, therefore,

$$\max_{p^d_{AN}} (\hat{\theta} + (\frac{1}{2} - \frac{\bar{c}}{t}) + \frac{\bar{c} - \underline{c}}{2t})p^d_{AN} = (\frac{3}{4} + \frac{v - (\bar{c} + \underline{c})}{2t} - \frac{p^d_{AN}}{2t})p^d_{AN}$$

From the first-order condition of the above maximization problem, we have

$$p^d_{AN} = \frac{3t}{4} + \frac{v - (\bar{c} + \underline{c})}{2},\ \hat{\theta} = \frac{v}{4t} - \frac{1}{8} + \frac{\bar{c} + \underline{c}}{4t}$$

The profit of firm A from deviation from the anonymous market is

$$\pi^d_{AN} = \frac{(2v + 3t - 2\underline{c} - 2\bar{c})^2}{32t}$$

Total deviation profit of firm A is $\pi^d_A = \pi^d_{AN} + \pi^d_{AO}$. As firms are symmetric, if firm i ($i = A,\ B$) deviates from the collusive agreement, its total deviation profit is

$$\pi^d_i = \frac{(2v - t)(\bar{c} + \underline{c})}{2t} + \frac{(2v + 3t - 2\underline{c} - 2\bar{c})^2}{32t} \tag{A.2}$$

Note that for $\hat{\theta} \leq \frac{1}{2} + \frac{\underline{c}}{t}$, we must have $t \geq \frac{2(v + \bar{c} - 3\underline{c})}{5}$.

Next, consider the case when $\hat{\theta} \geq \frac{1}{2} + \frac{\bar{c}}{t}$. Profit maximization problem of firm A in the anonymous market is

$$\max_{p_{AN}^d}(\hat{\theta} + (\frac{1}{2} - \frac{\bar{c}}{t}) + \frac{2(\bar{c} - \underline{c})}{2t} + \hat{\theta} - (\frac{1}{2} + \frac{\bar{c}}{t}))p_{AN}^d = (\frac{1}{2} + \frac{v - p_{AN}^d}{t} - \frac{\bar{c} + \underline{c}}{t})p_{AN}^d$$

First-order condition of the above maximization yields the following.

$$p_{AN}^d = \frac{v}{2} + \frac{t}{4} - \frac{\bar{c} + \underline{c}}{2},\ \hat{\theta} = \frac{v}{4t} + \frac{1}{8} + \frac{\bar{c} + \underline{c}}{4t}$$

The profit of firm A from the anonymous market is

$$\pi_{AN}^d = \frac{(2v + t - 2\bar{c} - 2\underline{c})^2}{16t}$$

As firms are symmetric, if firm i ($i = A,\ B$) deviates from the collusive agreement, its total profit from deviation is

$$\pi_i^d = \frac{(2v - t)(\bar{c} + \underline{c})}{2t} + \frac{(2v + t - 2\bar{c} - 2\underline{c})^2}{16t} \qquad (A.3)$$

For $\hat{\theta} \geq \frac{1}{2} + \frac{\bar{c}}{t}$, we must have $t \leq \frac{2(v + \underline{c} - 3\bar{c})}{3}$.

Non-cooperative equilibrium

As $\underline{c} \geq 0$, following the logic of the main analysis in Section 3.1.1, no old consumer opts for privacy. Therefore, non-cooperative profit of firm i ($i = A,\ B$) is, from Lemma 3

$$\pi_i^n = \frac{3t}{4} \qquad (A.4)$$

Now, let us consider following examples to illustrate the effect of private cost of privacy on the stability of collusion.

Example 1 Suppose that $v = 2$ and $t = \frac{3}{5}$. Further, suppose that $\underline{\mu} = 0$ such that consumers' taste for privacy is uniformly distributed over $[0, \bar{\mu}]$. In this example, the condition for $\hat{\theta} \geq$

$\frac{1}{2}+\frac{\bar{c}}{t}$ is always satisfied. Comparing the deviation profits from equations (A.2) and (A.3), we find that profit expression in (A.3) is greater than that in (A.2).

Substituting $\underline{c}=c-\bar{\mu}$ and $\bar{c}=c-\underline{\mu}=c$, we have critical discount factor as

$$\hat{\delta}=\frac{3\left(600c^2+440c+121\right)-60(30c+11)\bar{\mu}+500\bar{\mu}^2}{3\left(800c^2+440c+421\right)-60(40c+11)\bar{\mu}+700\bar{\mu}^2}$$

Differentiating the above expression with respect to c and $\bar{\mu}$, we get the following.

$$(i)\frac{\partial\hat{\delta}}{\partial c}>0,\ (ii)\frac{\partial\hat{\delta}}{\partial\bar{\mu}}<0$$

Example 2 Suppose that $v=2$ and $t=\frac{3}{4}$. Further, suppose that $\underline{\mu}=0$ such that consumers' taste for privacy is uniformly distributed over $[0,\bar{\mu}]$. The deviation profit in equation (A.2) is greater than that in equation (A.3) if

$$\frac{1}{8}<\mu_2<\frac{3}{8}\left(\sqrt{2}-1\right)\ \text{and}\ \frac{1}{16}\left(8\mu_2+13\right)-\frac{3}{4\sqrt{2}}<c<\frac{1}{16}\left(7-8\mu_2\right)$$

In this case, both the deviation cases (A.2) and (A.3) are possible. The critical discount factor can be obtained as (using equations (2), (A.1 - A.4))

$$\hat{\delta}=\begin{cases}\frac{3\left(512c^2+864c+1\right)-48(32c+27)\bar{\mu}+448\bar{\mu}^2}{3(768c^2+864c+409)-144(16c+9)\bar{\mu}+704\bar{\mu}^2}, & \text{corresponding to deviation profit in equation (A.2)}\\ \frac{3\left(384c^2+224c+49\right)-48(24c+7)\bar{\mu}+320\bar{\mu}^2}{1536c^2-48(32c+7)\bar{\mu}+672c+448\bar{\mu}^2+759}, & \text{corresponding to deviation profit in equation (A.3)}\end{cases}$$

Differentiating the above expression with respect to c and $\bar{\mu}$, we get the following.

$$(i)\frac{\partial\hat{\delta}}{\partial c}>0,\ (ii)\frac{\partial\hat{\delta}}{\partial\bar{\mu}}<0$$

[QED]

Growing and Shrinking Markets (Section 3.1.2)

We first derive collusive, deviation, and non-cooperative equilibrium profits for general α, and then solve for growing and shrinking markets separately. Suppose that $\pi^c_{i,m}$, $\pi^d_{i,m}$ and $\pi^n_{i,m}$ denote firm i's collusive, deviation and non-cooperative profits, respectively, in period m.

Collusion

Note that, following the same logic as in the baseline analysis in Section 3.1, the collusive pricing remains the same as given by Lemma 1. If firm i ($i = A, B$) sets prices p^c_{iN} and p^c_{iO} in anonymous and personalized markets, respectively, then $p^c_{iN} = v - \frac{t}{2}$, $p^c_{AO} = v - t\theta$, and $p^c_{BO} = v - t(1-\theta)$. Further, the marginal old consumers $\theta_A \in [0, \frac{1}{2}]$ and $\theta_B \in (\frac{1}{2}, 1]$ are given as $\theta_A = \frac{1}{2} - \frac{c}{t}$ and $\theta_B = \frac{1}{2} + \frac{c}{t}$.

In period m, there are α^{m-1} old consumers and α^m new consumers. Profit of firm A from the personalized market in period m is

$$\alpha^{m-1}\int_{\frac{1}{2}-\frac{c}{t}}^{\frac{1}{2}} (v - t\theta)\, d\theta = \alpha^{m-1}\frac{(2v - t + c)c}{2t}$$

and profit of firm A from the anonymous market in period m is

$$(\alpha^{m-1}\theta_A + \frac{\alpha^m}{2})(v - \frac{t}{2}) = \alpha^{m-1}(\frac{1}{2} - \frac{c}{t} + \frac{\alpha}{2})(v - \frac{t}{2})$$

As firms are symmetric, total collusive profit of firm i ($i = A,\ B$) in period m is

$$\pi^c_{i,m} = \alpha^{m-1}\frac{(2v - t + c)c}{2t} + \alpha^{m-1}(\frac{1}{2} - \frac{c}{t} + \frac{\alpha}{2})(v - \frac{t}{2}) = \alpha^{m-1}\frac{(2v - t)(1 + \alpha)t + 2c^2}{4t}$$

That is,

$$\pi^c_{i,m} = \alpha^{m-1}\pi^c_i \text{ where } \pi^c_i = \frac{(2v - t)(1 + \alpha)t + 2c^2}{4t} \tag{A.5}$$

It is clear that π^c_i is independent of period m.

Optimal deviation

Suppose that firm A, without loss of generality, deviates from the collusive agreement. We have $\theta_A = \frac{1}{2} - \frac{c}{t}$ and $\theta_B = \frac{1}{2} + \frac{c}{t}$. In the personalized market, firm A slightly decreases its price for consumers with $\theta \in (\frac{1}{2}, \theta_B]$ and captures the entire personalized market. Firm A's profit from the personalized market is, therefore,

$$\pi^d_{AO,m} = \alpha^{m-1}\left[\int_{\frac{1}{2}-\frac{c}{t}}^{\frac{1}{2}} (v - t\theta)\, d\theta + \int_{\frac{1}{2}}^{\frac{1}{2}+\frac{c}{t}} (v - t\theta - \epsilon)\, d\theta\right] = \alpha^{m-1}\frac{(2v-t)c}{t} \quad \text{as } \epsilon \to 0 \quad \text{(A.6)}$$

Now, we consider the anonymous market. Given the collusive price of firm B, $p^c_{BN} = v - \frac{t}{2}$, firm A sets a price p^d_{AN} to maximize its own profit. Suppose that $\hat{\theta}$ is the marginal consumer in the market for new consumers, who is indifferent between buying from firms A and B. Then,

$$v - p^d_{AN} - t\hat{\theta} = v - p^c_{BN} - t(1-\hat{\theta}) \implies \hat{\theta} = \frac{1}{4} + \frac{v - p^d_{AN}}{2t}$$

As in the analysis in Section 3.1, there can be two optimal deviation strategies in the anonymous market, depending on the market characteristics, either $\hat{\theta} \le \theta_B$ or $\hat{\theta} > \theta_B$.

Case (i): $\hat{\theta} \le \theta_B$.

Profit maximization of firm A in the anonymous market in period m is

$$\max_{p^d_{AN}} \alpha^{m-1}[\frac{1}{2} - \frac{c}{t} + \alpha(\frac{1}{4} + \frac{v - p^d_{AN}}{2t})]p^d_{AN}$$

First-order condition of the above maximization problem yields the following deviation price and profit, respectively, of firm A in period m from anonymous market.

$$p^d_{AN} = \frac{v}{2} + \frac{t}{4} + \frac{t-2c}{2\alpha}, \quad \pi^d_{AN,m} = \alpha^{m-1}\frac{(2v\alpha + (2+\alpha)t - 4c)^2}{32t\alpha} \quad \text{(A.7)}$$

The preference of indifferent consumer in the anonymous market is

$$\hat{\theta} = \frac{v}{4t} + \frac{1}{8} - \frac{1}{4\alpha} + \frac{c}{2\alpha t}$$

For $\hat{\theta} \leq \theta_B$, we must have

$$c > \frac{2\alpha(\frac{v}{4} - \frac{t}{4\alpha} - \frac{3t}{8})}{2\alpha - 1} \equiv c_1 \text{ (say)} \tag{A.8}$$

Case (ii): $\hat{\theta} > \theta_B$.

Profit maximization of firm A in the anonymous market in period m is

$$\max_{p^d_{AN}} \left[\alpha^m \hat{\theta} + \alpha^{m-1}(\theta_A + \hat{\theta} - \theta_B)\right] p^d_{AN} = \alpha^{m-1}\left[\frac{-2c}{t} + (1+\alpha)(\frac{1}{4} + \frac{v - p^d_{AN}}{2t})\right] p^d_{AN}$$

First-order condition of the above maximization problem yields the following deviation price and profit, respectively, of firm A from the anonymous market in period m.

$$p^d_{AN} = \frac{v}{2} + \frac{t}{4} - \frac{2c}{1+\alpha}, \; \pi^d_{AN,m} = \alpha^{m-1}\frac{((2v+t)(1+\alpha) - 8c)^2}{32t(1+\alpha)} \tag{A.9}$$

The preference of indifferent consumer in the anonymous market is

$$\hat{\theta} = \frac{v}{4t} + \frac{1}{8} + \frac{c}{(1+\alpha)t}$$

For $\hat{\theta} > \theta_B$, we must have

$$c < (\frac{v}{4} - \frac{3t}{8})(\frac{1+\alpha}{\alpha}) \equiv c_2 \text{ (say)} \tag{A.10}$$

For interior solution, i.e., $\hat{\theta} < 1$, we must have $v < \frac{7t}{2} - \frac{4c}{1+\alpha}$.

Non-cooperative equilibrium

It can be easily shown that no old consumer opts for privacy in this case, similar to the analysis in Section 3.1.1. Suppose that mass θ_A and $(1 - \theta_B)$ of old consumers have opted for privacy in stage 1. Given θ_A and θ_B, firms A and B maximize the following profit functions, respectively, in the anonymous market.

$$\max_{p^n_{AN}} = \left[\alpha^{m-1}\theta_A + \alpha^m(\frac{1}{2} + \frac{p^n_{BN} - p^n_{AN}}{2t})\right] p^n_{AN}$$

$$\max_{p^n_{BN}} = \left[\alpha^{m-1}(1 - \theta_B) + \alpha^m(\frac{1}{2} + \frac{p^n_{AN} - p^n_{BN}}{2t})\right] p^n_{BN}$$

Solving the first-order conditions of the above maximization problems simultaneously, we get the following.

$$p^n_{AN} = t + \frac{2(2\theta_A + 1 - \theta_B)t}{3\alpha} > t, \;\; p^n_{BN} = t + \frac{2(2(1-\theta_B) + \theta_A)t}{3\alpha} > t$$

whereas in the personalized market, firms' prices are $p^n_{AO} = \max\{t(1-2\theta), 0\}$ and $p^n_{BO} = \max\{t(2\theta - 1), 0\}$. Now, consider consumers' privacy choices in stage 1. Suppose that consumers expect prices p^e_{iN} and $p^e_{iO}(\theta)$ in the anonymous and personalized markets, respectively. Then, a consumer closer to firm A opts for privacy if $v - t\theta - p^e_{AN} - c \geq v - t\theta - p^e_{AO}(\theta)$. As consumers' expectations are correct in equilibrium, this condition becomes $p^n_{AN} \leq t(1-2\theta) - c < t$, which is a contradiction. Therefore, no old consumer opts for privacy. This implies that the anonymous and personalized markets coincide with the markets for new and old consumers, respectively. Using Lemma 3, the non-cooperative profit of firm i ($i = A, B$) in period m is

$$\pi^n_{i,m} = \alpha^{m-1}\pi^n_i \;\text{ where }\; \pi^n_i = \frac{(1+2\alpha)t}{4} \tag{A.11}$$

In any period m, collusion is sustainable if the following incentive compatibility constraint holds.

$$\sum_{s=m}^{\infty} \pi^c_{i,s}\delta^{s-m} \geq \pi^d_{i,m} + \sum_{s=m+1}^{\infty} \pi^n_{i,s}\delta^{s-m}; \;\; i = A, \; B$$

After simplification, the above condition becomes

$$\delta \geq \frac{(\pi^d_i - \pi^c_i)}{\alpha(\pi^d_i - \pi^n_i)} = \delta_{min} \tag{A.12}$$

Note that the above condition is stationary, i.e., independent of time period m..

Now, let us discuss the growing and shrinking markets, separately.

We take a couple of examples to illustrate the effect of the private cost of privacy on collusion stability. We restrict the value of $\alpha \leq 3$ for tractability in case of growing markets.

Example 3 Suppose that $v = 2$ and $t = \frac{3}{5}$. Then c_1 and c_2 from equations (A.8) and (A.10) are

$$c_1 = \frac{6 - 11\alpha}{20 - 40\alpha}, \quad c_2 = \frac{11(\alpha + 1)}{40\alpha}$$

Growing markets: It can be checked that $c_1 < c_2$ and $c_2 > \frac{t}{2}$. Further, comparing the deviation profits from cases (i) and (ii) for $c_1 < c < c_2$, we get that firm A earns higher profit in case (ii) by deviating. Therefore, the deviation profit in this case is

$$\pi^d_{A,m} = \alpha^{m-1}\frac{(40c - 23(\alpha + 1))^2}{480(\alpha + 1)} + \frac{17c}{3} \tag{A.13}$$

Using equations (2), (A.5), (A.11) and (A.13), the critical discount factor is

$$\delta_{\min} = \frac{121(\alpha + 1)^2 - 400(\alpha - 3)c^2 + 880(\alpha + 1)c}{\alpha\,(385\alpha^2 + 842\alpha + 1600c^2 + 880(\alpha + 1)c + 457)}$$

It can be noted that $0 < \delta_{\min} < 1$. Differentiating above expression with c and α, we get the following.

$$\frac{\partial\delta_{\min}}{\partial c} > 0, \quad \frac{\partial\delta_{\min}}{\partial\alpha} < 0$$

Shrinking markets: Now, given $0 < \alpha < 1$, we have $c_2 > \frac{t}{2}$. Comparing the deviation profits, we find that the deviating firm earns more profit in case (ii). The critical discount factor $\delta_{\min}$ in this case is, thus, given by (using equations (2), (A.5), (A.9) and (A.11))

$$\delta_{\min} = \frac{121(\alpha + 1)^2 - 400(\alpha - 3)c^2 + 880(\alpha + 1)c}{\alpha\,(385\alpha^2 + 842\alpha + 1600c^2 + 880(\alpha + 1)c + 457)}$$

Note that $0 < \delta_{\min} < 1$. Differentiating the above expression with respect to c and α, we get

$$\frac{\partial\delta_{\min}}{\partial c} > 0, \quad \frac{\partial\delta_{\min}}{\partial\alpha} < 0$$

Example 4 Suppose that $v = 2$ and $t = \frac{3}{4}$. Then c_1 and c_2 from equations (A.8) and (A.10) are

$$c_1 = \frac{6 - 7\alpha}{16 - 32\alpha}, \quad c_2 = \frac{7(\alpha + 1)}{32\alpha}$$

Growing markets: It can be checked that $c_1 < c_2$. Now, the deviation profits in cases (i) and (ii) are

$$\frac{\left(4\alpha + \frac{3(\alpha+2)}{4} - 4c\right)^2}{24\alpha} + \frac{13c}{3} \quad \textbf{Case (i)}$$

$$\frac{\left(\frac{19(\alpha+1)}{4} - 8c\right)^2}{24(\alpha+1)} + \frac{13c}{3} \quad \textbf{Case (ii)}$$

Comparing the above expressions, we have

$$\frac{\left(4\alpha + \frac{3(\alpha+2)}{4} - 4c\right)^2}{24\alpha} + \frac{13c}{3} > (<) \frac{\left(\frac{19(\alpha+1)}{4} - 8c\right)^2}{24(\alpha+1)} + \frac{13c}{3}$$

$$\text{if } \; c > (<) \frac{19\alpha^2 + 13\alpha - 6}{16(3\alpha - 1)} - \frac{1}{16}\sqrt{\frac{361\alpha^4 + 95\alpha^3 - 217\alpha^2 + 49\alpha}{(3\alpha-1)^2}}$$

Let $\delta_{\min1}$ and $\delta_{\min2}$ are critical discount factors in cases (i) and (ii) respectively. Then,

$$\delta_{\min1} = \frac{(6-7\alpha)^2 - 256(\alpha-1)c^2 + 96(11\alpha-2)c}{\alpha\left(217\alpha^2 + 156\alpha + 256c^2 + 96(11\alpha-2)c + 36\right)}$$

$$\delta_{\min2} = \frac{49(\alpha+1)^2 - 256(\alpha-3)c^2 + 448(\alpha+1)c}{\alpha\left(217\alpha^2 + 506\alpha + 1024c^2 + 448(\alpha+1)c + 289\right)}$$

It can checked that $0 < \delta_{\min1},\ \delta_{\min2} < 1$. Differentiating the above expressions with respect to c and α, we have

$$\frac{\partial\delta_{\min1}}{\partial c} > 0, \quad \frac{\partial\delta_{\min1}}{\partial\alpha} < 0, \quad \frac{\partial\delta_{\min2}}{\partial c} > 0, \quad \frac{\partial\delta_{\min2}}{\partial\alpha} < 0$$

Shrinking markets: Given that $0 < \alpha < 1$, comparing the deviation profits, we have

$$\frac{\left(4\alpha + \frac{3(\alpha+2)}{4} - 4c\right)^2}{24\alpha} + \frac{13c}{3} > \frac{\left(\frac{19(\alpha+1)}{4} - 8c\right)^2}{24(\alpha+1)} + \frac{13c}{3}$$

if

$$\frac{\left(4\alpha + \frac{3(\alpha+2)}{4} - 4c\right)^2}{24\alpha} + \frac{13c}{3} > \frac{\left(\frac{19(\alpha+1)}{4} - 8c\right)^2}{24(\alpha+1)} + \frac{13c}{3}$$

if

$$\frac{7}{305}\left(23 + 4\sqrt{14}\right) < \alpha < 1 \text{ and}$$

$$\frac{19\alpha^2+13\alpha-6}{16(3\alpha-1)}-\frac{1}{16}\sqrt{\frac{361\alpha^4+95\alpha^3-217\alpha^2+49\alpha}{(3\alpha-1)^2}}<c<\frac{1}{32}(5\alpha+5)$$

Critical discount factors in cases (i) and (ii), respectively, are

$$\delta_{\min 1}=\frac{(6-7\alpha)^2-256(\alpha-1)c^2+96(11\alpha-2)c}{\alpha\left(217\alpha^2+156\alpha+256c^2+96(11\alpha-2)c+36\right)}$$

$$\delta_{\min 2}=\frac{49(\alpha+1)^2-256(\alpha-3)c^2+448(\alpha+1)c}{\alpha\left(217\alpha^2+506\alpha+1024c^2+448(\alpha+1)c+289\right)}$$

It can checked that $0<\delta_{\min 1},\ \delta_{\min 2}<1$. Differentiating the above expressions with respect to c and α, we have

$$\frac{\partial\delta_{\min 1}}{\partial c}>0,\quad \frac{\partial\delta_{\min 1}}{\partial\alpha}<0,\quad \frac{\partial\delta_{\min 2}}{\partial c}>0,\quad \frac{\partial\delta_{\min 2}}{\partial\alpha}<0$$

[QED]

Proof of Proposition 2

Under broad scope privacy regulation, old consumers who have paid for privacy and new consumers constitute anonymous market, and firms can not distinguish between these two groups.

We first consider the case when firms collude.

Suppose that consumers anticipate prices p^e_{iN} and $p^e_{iO}(\theta)$ in anonymous and personalized markets, respectively. As in Section 3.1, as utility derived from firm A's product decreases in θ, if an old consumer with $\theta_A\in[0,\frac{1}{2}]$ pays for the privacy, then all old consumers with $\theta\in[0,\theta_A]$ pay for the privacy. Likewise, if an old consumer with $\theta_B\in[\frac{1}{2},1]$ pays for the privacy, then all old consumers with $\theta\in[\theta_B,1]$ pay for the privacy.

An old consumer with $\theta\in[0,\theta_A]$ and an old consumer with $\theta\in[\theta_B,1]$ opt for privacy if the following hold, respectively.

$$v - p^e_{AN} - t\,\theta - c \geq v - p^e_{AO}(\theta) - t\,\theta,\;\; v - p^e_{BN} - t\,(1-\theta) - c \geq v - p^e_{BO}(\theta) - t\,(1-\theta) \quad (A.14)$$

Consider, first, the personalized market. As firms know the preferences of all old consumers who have not paid for privacy, they divide the personalized market evenly and set monopoly prices at each location to extract the whole surplus of each consumer. Therefore, collusive prices in the personalized market are as follows.

$$p^c_{AO}(\theta) = v - t\,\theta,\;\; p^c_{BO}(\theta) = v - t\,(1-\theta) \quad (A.15)$$

Although consumers have limited strategic sophistication, they understand the nature of the personalized market. They can correctly infer that firms charge monopoly prices in the personalized market at each location. Therefore, irrespective of the level of their strategic sophistication, old consumers' beliefs about firms' pricing in the personalized market are correct. That is,

$$p^e_{AO}(\theta) = p^c_{A0}(\theta) = v - t\,\theta,\;\; p^e_{BO}(\theta) = p^c_{BO}(\theta) = v - t\,(1-\theta) \quad (A.16)$$

Using equations $(A.14)$ and $(A.16)$, an old consumer with $\theta \in [0, \theta_A]$ and an old consumer with $\theta \in [\theta_B, 1]$, respectively, choose privacy if

$$\max\{v - t\,\theta - p^e_{AN} - c, -c\} \geq 0,\;\; \max\{v - t\,(1-\theta) - p^e_{BN} - c, -c\} \geq 0 \quad (A.17)$$

From equation $(A.17)$, we get the marginal consumers θ_A and θ_B who are indifferent between opting for privacy and remaining in the personalized market.

$$v - t\,\theta_A = p^e_{AN} + c,\;\; v - t\,(1-\theta_B) = p^e_{BN} + c \quad (A.18)$$

Now, we characterize consumers' expectation formation in the anonymous market. We start with consumers with sophistication level $k = 0$. As consumers with level $k = 0$ are naive, they expect that firms will charge a regular collusive price equal to the surplus of the most distant consumer in the market for new consumers. That is, $p^e_{iN} = v - \frac{t}{2}$.

Firms have sophistication level $k = 1$. Therefore, they correctly infer that old consumers opt for privacy under the belief that firms set a price equal to $(v - \frac{t}{2})$. As consumers make their privacy decisions before firms set the prices, therefore, the cost paid for the privacy is sunk. Therefore, firms can charge a price equal to the surplus of marginal old consumer who opts for privacy, i.e., $p^e_{iN} + c = v - \frac{t}{2} + c$. However, in the present scenario, old consumers who have paid for privacy and new consumers constitute anonymous market. Therefore, if firms set prices equal to $v - \frac{t}{2} + c$, they will lose demand from the set of new consumers. Under the full market coverage assumption, the optimal prices of the firms in the anonymous market are $p^c_{iN} = v - \frac{t}{2}$. This implies that consumers' expectations about firms' prices in the anonymous market turn out to be correct, i.e., $p^e_{iN} = p^c_{iN}$. The result is, therefore, the same as the case when consumers have unlimited cognitive abilities. It also implies that considering further levels of strategic sophistication, i.e., level $k > 0$ of consumers and level $k + 1$ of firms, will yield the same equilibrium prices. From equation $(A.18)$ and the above discussion, we state the following.

Result *(Optimal collusive pricing with level-k under broad scope privacy regulation). Regardless of the cognitive abilities of consumers and firms, when firms collude, optimal pricing is as follows.*

1. *In the anonymous market, firm $i, (i = A, B)$ sets a price equal to $p^c_{iN} = v - \frac{t}{2}$.*

2. *In the personalized market, firm A sets a price equal to $p^c_{AO}(\theta) = v - t\,\theta$, and firm B*

sets $p^c_{BO}(\theta) = v - t\,(1-\theta)$.

Next, we consider the case when one firm, say A, deviates. In the personalized market, firm A decreases its price slightly to capture the entire market. In the anonymous market, from the same reasoning as in the case of collusion, the deviating firm's optimal pricing is given by Lemma 2, irrespective of the level of strategic sophistication (cognitive abilities) of consumers and firms.

When both firms compete non-cooperatively, old consumers with level $k = 0$ can correctly infer that firms set prices equal to the difference in the transportation costs. Therefore, no consumer opts for privacy (see Section 3.1). In that case, even if firms have higher cognitive abilities than consumers, they do not have any scope to use iterative thinking to price more. As a result, consumers' price expectations are correct in the equilibrium.

From the above, it is clear that in the case of broad scope privacy regulation, firms can not gain by having superior cognitive abilities than consumers, and we get the same equilibrium outcomes under collusion, deviation, and punishment as with unlimited cognitive abilities of firms and consumers alike.

Appendix B

Proof of Lemma 4

Suppose that consumers anticipate that firm i $(i = A, B)$ sets prices $p^e_{iO}(\theta)$, $p^e_{iN_1}$ and $p^e_{iN_2}$ in the personalized market, the market for new consumers, and the market for old consumers who opt for privacy, respectively.

Consider, first, the market for old consumers. As the utility of a consumer decreases with θ if she purchases from firm A and increases with θ if he purchases from firm B, let us suppose that consumers with $\theta \in [0, \theta_A]$, closer to firm A, and $\theta \in [\theta_B, 1]$, closer to firm B, opt for privacy.

An old consumer with $\theta \in [0, \theta_A]$ opts for privacy if

$$v - t\theta - p^e_{AN_2} - c \geq v - t\theta - p^e_{AO}(\theta)$$

In the personalized market, firms set monopoly prices at each location θ to extract the whole surplus of the consumers. Therefore, $p^c_{AO}(\theta) = v - t\theta$. As consumers' expectations are correct in the equilibrium, we have $p^e_{AO}(\theta) = p^c_{AO}(\theta) = v - t\theta$. Therefore, marginal consumer θ_A is given by

$$v - t\,\theta_A - p^e_{AN_2} - c = v - t\,\theta_A - (v - t\,\theta_A) = 0 \implies \theta_A = \frac{v - c - p^e_{AN_2}}{t}$$

For $\theta_A > 0$, we must have $p^e_{AN_2} < v - c$.

Suppose that firm i $(i = A, B)$ sets a price $p^c_{iN_2}$ in the market for old consumers who opt for privacy. As firms collude and the personalized market share depends on θ_A, therefore, firm A chooses price $p^c_{AN_2}$ so as to maximize its profit from the market for old consumers, $[0, \frac{1}{2}]$.

The profit expression for firm A in the market for old consumers is

$$\theta_A \, p^c_{AN_2} + \int_{\theta_A}^{\frac{1}{2}} (v - t\,\theta)\, d\theta = (\frac{v}{2} - \frac{t}{8}) + \frac{1}{2t}(v - c - p^e_{AN_2})(2p^c_{AN_2} - p^e_{AN_2} - (v - c))$$

As consumers' expectations are correct in equilibrium, i.e., $p^e_{AN_2} = p^c_{AN_2}$, and $p^e_{AN_2} \leq v - c$ for $\theta_A \geq 0$, the above profit expression is highest if $p^e_{AN_2} = p^c_{AN_2} = v - c$.

By a similar argument, firm B sets a price $p^c_{BO}(\theta) = v - t(1-\theta)$ in the personalized market, and $p^c_{BN_2} = v - c$ for old consumers who may opt for privacy. The marginal old consumer with $\theta_A \in [0, \frac{1}{2}]$ who is indifferent between paying and not paying for privacy, therefore, is given by $\theta_A = 0$. Similarly, the marginal old consumer with $\theta_B \in (\frac{1}{2}, 1]$ who is indifferent between paying and not paying for privacy, is given by $\theta_B = 1$. This implies that no old consumer opts for privacy, in this case.

Next, consider the market for new consumers. As firms collude and divide the market evenly, they set prices equal to the surplus of the most distant consumer, i.e., the consumer with $\theta = \frac{1}{2}$. Therefore, in the market for new consumers, firm i ($i = A, B$) sets price $p^c_{iN_1} = v - \frac{t}{2}$. Combining all the above cases, we get Lemma 4. [QED]

Proof of Lemma 6

In the market for new consumers, it easily follows that firm i will set a price $p^n_{iN_1} = t$, from Lemma 3.

Now, consider the market for old consumers. Suppose that consumers with $\theta \in [0, \theta_A]$ and $\theta \in [\theta_B, 1]$ pay for privacy.

Suppose that old consumers anticipate prices $p^e_{iN_2}$ and $p^e_{iO}(\theta)$ for consumers who pay for privacy and those who do not pay for privacy, respectively. Therefore, a consumer with

$\theta \in [0, \theta_A]$ opts for privacy if $v - p^e_{AN_2} - t\theta - c \geq v - p^e_{AO}(\theta) - t\theta$ and a consumer with $\theta \in [\theta_B, 1]$ opts for privacy if $v - p^e_{BN_2} - t(1-\theta) - c \geq v - p^e_{BO}(\theta) - t(1-\theta)$.

In the personalized market, firms compete for each consumer, and, therefore, prices are equal to the difference between transportation costs. That is, in the personalized market, firm A sets a price equal to $p^n_{AO}(\theta) = \max\{t(1-2\theta), 0\}$, and firm B sets a price equal to $p^n_{BO}(\theta) = \max\{t(2\theta-1), 0\}$. Consumers' expectations are correct in equilibrium; therefore, $p^e_{iO}(\theta) = p^n_{iO}(\theta)$. Suppose that firm i sets a price $p^n_{iN_2}$ in the market for consumers who do not opt for privacy. Now, marginal consumers θ_A and θ_B are given as follows.

$$\theta_A = \frac{1}{2} - \frac{p^e_{AN_2} + c}{2t}, \; \theta_B = \frac{1}{2} + \frac{p^e_{BN_2} + c}{2t}$$

In the market for old consumers who opt for privacy, firms act as monopolies. Therefore, firm A chooses the price $p^n_{AN_2}$ to maximize the profit $\theta_A \; p^n_{AN_2} + \int_{\theta_A}^{\frac{1}{2}} t(1-2\theta) \; d\theta$. Given that consumers' expectations are correct in equilibrium, the profit expression is highest when $p^e_{AN_2} = p^n_{AN_2} = (t-c)$. But a consumer with $\theta \in [0, \theta_A]$ opts for privacy if $p^e_{AN_2} \leq t(1-2\theta) - c$. As $t(1-2\theta) - c \leq (t-c)$, no consumer finds it optimal to opt for privacy. A similar argument applies for consumers with $\theta \in [\theta_B, 1]$. Therefore, under competition, no old consumer opts for privacy. This concludes the proof of Lemma 6. [QED]

Limited Cognitive Abilities (Section 4.2)

In the case of narrow scope privacy regulation, firms do not know the preferences of old consumers who have paid for privacy. However, firms can identify if a consumer is old or new. Therefore, firms can distinguish between the set of old consumers who have paid for privacy and the set of new consumers.

Collusion

We first consider the case when firms collude.

Suppose that consumers anticipate prices p^e_{iN1}, p^e_{iN2} and $p^e_{iO}(\theta)$ in the market for new consumers, the market for old consumers who opt for privacy and the personalized market, respectively.

Stage 3 (Consumers' purchasing decisions): A consumer with preference θ buys the product if $v - t\theta \geq p$, where p is the price faced by the consumer. We assume full market coverage such that every consumer buys in the equilibrium.

Stage 2 (Firms' pricing decisions): In the personalized market, firm i sets a personalized price $p^c_{iO}(\theta)$. The collusive personalized prices are $p^c_{AO}(\theta) = v - t\theta$ and $p^c_{BO}(\theta) = v - t(1-\theta)$. In the market for new consumers, firm i sets the collusive price $p^c_{iN1} = v - \frac{t}{2}$. In the market for old consumers, suppose that consumers with $\theta \in [0, \theta_A]$ and $\theta \in [\theta_B, 1]$ opt for privacy. Firms having higher strategic sophistication than consumers can correctly infer θ_A and θ_B. Suppose that firm i sets a price p^c_{iN2} for this segment of the consumers.

Stage 1 (Consumers' privacy decisions): An old consumer with preference $\theta \in [0, \frac{1}{2}]$ opts for privacy iff

$$v - p^e_{AN2} - t\theta - c \geq v - t\theta - p^e_{AO}(\theta)$$

and an old consumer with preference $\theta \in (\frac{1}{2}, 1]$ opts for privacy iff

$$v - p^e_{BN2} - t(1-\theta) - c \geq v - t(1-\theta) - p^e_{BO}(\theta)$$

Now, consumers understand the nature of the personalized market, and they correctly anticipate that the colluding firms set monopoly prices at each location in the personalized market. Therefore, $p^e_{AO}(\theta) = p^c_{AO}(\theta) = v - t\theta$ and $p^e_{BO}(\theta) = p^c_{BO}(\theta) = v - t(1-\theta)$. This implies that consumers with $\theta \in [0, \frac{1}{2}]$ and $\theta \in (\frac{1}{2}, 1]$ opt for privacy iff

$$v - p^e_{AN2} - t\theta - c \geq 0 \text{ and } v - p^e_{BN2} - t(1-\theta) - c \geq 0,$$

respectively.

Suppose that $\theta_A \in [0, \frac{1}{2}]$ and $\theta_B \in (\frac{1}{2}, 1]$ denote the preferences of marginal consumers who are indifferent between opting and not opting for privacy. Therefore,

$$\theta_A = \frac{v - p^e_{AN2} - c}{t}, \;\; \theta_B = 1 - \frac{v - p^e_{BN2} - c}{t} \tag{B.1}$$

Stage 2 - Firms' pricing (Revisited): As firms are more sophisticated than consumers, they correctly infer θ_A and θ_B. Further, as privacy cost paid by the consumers is sunk, firms correctly anticipate that old consumers with $\theta \in [0, \theta_A]$ and $\theta \in [\theta_B, 1]$ will purchase the product in stage 3 if

$$v - p^c_{AN2} - t\theta \geq 0 \text{ and } v - p^c_{BN2} - t(1-\theta) \geq 0,$$

respectively. This implies that the optimal collusive prices for the market for old consumers who opt for privacy are

$$p^c_{AN2} = v - t\theta_A, \; p^c_{BN2} = v - t(1-\theta_B) \tag{B.2}$$

Stage 1 - Consumers' privacy decisions (Revisited): Now, we consider the old consumers' expectation formation to fully characterize the equilibrium pricing. We solve the problem recursively and start with consumers with strategic sophistication level $k = 0$, who are referred to as 'naive' consumers. Consumers with sophistication $k = 0$ do not anticipate that their privacy decisions indicate their higher willingness to pay. Therefore, they expect the regular collusive price in the market for new consumers. That is, $p^e_{iN2} = v - \frac{t}{2}$. Firms having higher sophistication ($k+1 = 1$) anticipate this and set prices according to Eq. ($A.7$).

Therefore, from equations $(B.1)$ and $(B.2)$, we have

$$\theta_A(k=0) = \frac{1}{2} - \frac{c}{t},\ \theta_B(k=0) = \frac{1}{2} + \frac{c}{t},\ \ p^c_{AN2} = v - \frac{t}{2} + c$$

It can be seen that firms charge a higher price than their expected price to the old consumers who opt for privacy. To be more specific, the actual price charged is equal to the expected price plus the privacy cost. Next, suppose that consumers have sophistication level $k = 1$. Then, a consumer expects firms to have sophistication level equal to 1, and other consumers to have sophistication level equal to 0. Therefore, consumers expect the price $p^e_{AN2}(k = 1) = v - \frac{t}{2} + c \implies \theta_A(k = 1) = \frac{1}{2} - \frac{2c}{t}$ and $\theta_B(k = 1) = \frac{1}{2} + \frac{2c}{t}$. Firms having higher sophistication level $(k + 1) = 2$ infer θ_A and θ_B correctly, and set a price $p^c_{iN2} = v - \frac{t}{2} + 2c$. That is, with every additional sophistication level, consumers pay more than before. Solving recursively, we get the general expression for θ_A, θ_B and the collusive prices.

Suppose that consumers and firms have sophistication levels k and $(k+1)$, respectively. Then

$$\theta_A(k) = \frac{1}{2} - \frac{(k+1)c}{t},\ \theta_B(k) = \frac{1}{2} + \frac{(k+1)c}{t},\ p^c_{iN2} = v - \frac{t}{2} + (k+1)c \qquad (B.3)$$

A strictly positive fraction of old consumers opt for privacy if $k < \frac{t}{2c} - 1 \equiv \bar{k}$.

For $k < \bar{k}$, the total collusive profit of firm A is

$$\pi^c_A = \frac{v - t/2}{2} + \theta_A\, p^c_{AN2} + \int_{\theta_A}^{\frac{1}{2}} (v - t\theta)\, d\theta = \frac{(2v - t)t + ct(k+1) - c^2(k+1)^2}{2t}$$

As firms are symmetric, the total collusive profit of firm $i (i = A,\ B)$ is

$$\pi^c_i(k) = \frac{(2v - t)t + ct(k+1) - c^2(k+1)^2}{2t} \qquad (B.4)$$

Note that when $k \geq \bar{k}$ such that no consumer opts for privacy, the collusive profit is

$$\pi^c_i(k)|_{k=\bar{k}} = v - \frac{3t}{8}$$

same as the collusive profit as implied by corollary 1.

Optimal Deviation

Suppose that firm A, without loss of generality, deviates from the collusive agreement. As consumers can not predict if a firm is going to deviate, their privacy decisions are based on the expectations of collusive prices. Therefore, for sophistication levels k and (k +1) of consumers and firms, respectively, we have

$$\theta_A(k) = \frac{1}{2} - \frac{(k+1)c}{t},\ \theta_B(k) = \frac{1}{2} + \frac{(k+1)c}{t}$$

In the market for new consumers, given that firm B sets a collusive price, firm A sets a price p^d_{AN1} to maximize its own profit. We have, $p^d_{AN1} = \frac{v}{2} + \frac{t}{4}$, and the profit of firm A from the market for new consumers is $\pi^d_{AN1} = (\frac{v}{4t} + \frac{1}{8})(\frac{v}{2} + \frac{t}{4})$ (follows from the proof of Corollary 1). This is for interior solution when $v < \frac{7t}{2}$.

Further, in the personalized market, firm A decreases its collusive price slightly at each $\theta \in (\frac{1}{2}, \theta_B]$ to capture the entire personalized market. The profit of firm A from the personalized market is $\pi^d_{AO} = \int_{\theta_A}^{\theta_B}(v - t\theta)d\,\theta = \frac{(2v-t)c(k+1)}{t}$.

From the market of old consumers who opt for privacy, firm A serves consumers with $\theta \in [0, \theta_A]$ at the collusive price as it is not optimal for firm to set a lower price to attract consumers with $\theta \in [\theta_B, 1]$. Therefore, profit of firm A from this segment is $(\frac{1}{2} - \frac{(k+1)c}{t})(v - \frac{t}{2} + (k+1)c)$. Adding the profits from three segments, we get the total deviation profit of firm A. As firms are symmetric, if firm i deviates from the collusive agreement, given that firm j sets collusive prices , its profit is

$$\pi^d_i(k) = \frac{-32c^2(k+1)^2 + 32c(k+1)v - 7t^2 + 20tv + 4v^2}{32t},\ i \neq j,\ i,j = A,\ B \qquad (B.5)$$

Non-cooperative equilibrium

When firms compete non-cooperatively, no consumer opts for privacy, and therefore limited strategic sophistication does not play a role in this case. To illustrate this point, consider an old consumer with $\theta \in [0, \frac{1}{2}]$. Consumers understand the nature of the personalized market. Therefore, their expectations about firms' prices in the personalized market is correct, i.e., $p^e_{AO} = p^n_{AO} = \max\{t(1-2\theta), 0\}$. Next, we start with consumers with sophistication $k = 0$. Consumers with sophistication $k = 0$ expect the regular non-cooperative price $p^e_{AN2} = t$ in the market for old consumers who opt for privacy. Therefore, an old consumer with $\theta \in [0, \frac{1}{2}]$ opts for privacy iff $v - t\theta - p^e_{AN2} - c \geq v - t\theta - p^e_{AO}(\theta) \implies t + c \leq t(1-2\theta)$, which is a contradiction. Therefore, no consumer opts for privacy. In turn, firms even with a higher sophistication level can not exploit consumers' naivety. Under non-cooperative equilibrium, firm i $(i = A, B)$ earns the profit $\pi^n_i = \frac{3t}{4}$ (Lemma 3). Thus, for any level of strategic sophistication k, we have

$$\pi^n_i(k) = \frac{3t}{4} \tag{B.6}$$

Now, suppose that $\delta^N(k)$ is the critical discount factor above which collusion is sustainable. Then, using equations (2), (B.4), (B.5), and (B.6), we have (for $v < \frac{7t}{2}$)

$$\delta^N(k) = \frac{16c^2(k+1)^2 + 16c(k+1)(t-2v) - (3t-2v)^2}{32c^2(k+1)^2 - 32c(k+1)v + 31t^2 - 20tv - 4v^2} \tag{B.7}$$

It can be checked that $0 < \delta^N(k) < 1$.

Now consider the limiting case when the deviating firm captures the entire market for new consumers, i,e., $v \geq \frac{7t}{2}$. Then, the deviation price in the market for new consumers will be $p^d_{AN1} = v - \frac{3t}{2}$.

The critical discount factor in this case is obtained as follows.

$$\delta^N(k) = \frac{2c^2(k+1)^2 + 2c(k+1)(t-2v) + t(5t-2v)}{4c^2(k+1)^2 - 4c(k+1)v + 2t(5t-3v)} \qquad (B.8)$$

Combining the above, we have the critical discount factor in the case of narrow scope of privacy regulation with limited cognitive abilities as follows.

$$\delta^N(k) = \begin{cases} \frac{16c^2(k+1)^2+16c(k+1)(t-2v)-(3t-2v)^2}{32c^2(k+1)^2-32c(k+1)v+31t^2-20tv-4v^2} & \text{when } v < \frac{7t}{2} \\ \frac{2c^2(k+1)^2+2c(k+1)(t-2v)+t(5t-2v)}{4c^2(k+1)^2-4c(k+1)v+2t(5t-3v)} & \text{when } v \geq \frac{7t}{2} \end{cases} \qquad (B.9)$$

Differentiating $\delta^N(k)$ with respect to c and k in both cases, we have $\frac{\partial \delta^N(k)}{\partial c} > 0$ and $\frac{\partial \delta^N(k)}{\partial k} > 0$, ignoring the integer constraint on k.

[QED]