Apparent unsettled value of the recently measured muon magnetic moment

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In this paper, we point out that the anomalous value of the muon magnetic moment recently measured at FERMILAB appears to be unsettled due to the experimentally unresolved behavior of the mean life of muons with speed caused by non-local internal effects as well as the tirreversibility of the decay process.

1 The Einstein-Podolsky-Rosen argument

Recent, accurate measurements conducted at FERMILAB [1] have indicated the following difference between the experimental value of the *muon* g-factor, g_μ^{EXP}, and its prediction via quantum electrodynamics, g_μ^{QED},

$$\begin{aligned} g_\mu^{EXP} - g_\mu^{QED} &= \\ = 2.00233184122 - 2.00233183620 &= \\ = 0.00000000502 &> 0. \end{aligned} \tag{1}$$

Additional accurate measurements have shown deviations from quantum mechanical predictions in various fields, such as experiments [2] for *atoms* in condensed matter and measurements [3] for *heavy ion* showing an apparent increase of the deviations with the increase of the dimension of the considered particle.

The above experiments supports the 1935 view by A. Einstein, B. Podolsky and N. Rosen that: *Quantum mechanics is not a complete theory* (EPR Argument) [4].

Some historical confirmations of the EPR argument are given by the non-linear completion of quantum mechanics by W. Heisenberg [5], the non-local completion by L. de Broglie and D. Bohm [6], the completion via *hidden variables* by D. Bohm [7], and others.

R. M. Santilli initiated theoretical studies on the completion of quantum into *hadronic mechanics* for the representation of *extended* particles under Hamiltonian and non-Hamiltonian interactions due to mutual penetration of their wavepackets [8], [9] [10] (see Refs. [11] [12] [13] for recent reviews of the basic methods and Ref. [14] for an independent outline). The studies are based on the axiom-preserving isotopy of the universal enveloping associative algebra of quantum mechanics in terms of the *isoproduct*

$$A \star B = A\hat{T}B, \quad \hat{T} > 0, \tag{2}$$

where the quantity $\hat{T} = \hat{T}(t, r, p, E, d, \psi, \pi, \tau, ...)$ called the *isotopic element* is positive-definite but possesses otherwise an unrestricted dependence on all needed local variables, such as time t, coordinates r, momenta p, energy E, density d, wave functions ψ, pressure π, temperature τ, etc. (herein tacitly assumed).

For the case of systems reversible over time, the isotopic element has realizations of the type [10]

$$\begin{gathered} \hat{T} = \Pi_{\alpha=1,2,...,N} Diag.(\tfrac{1}{n_{1,\alpha}^2}, \tfrac{1}{n_{2,\alpha}^2}, \tfrac{1}{n_{3,\alpha}^2}, \tfrac{1}{n_{4,\alpha}^2})e^{-\Gamma}, \\ n_{\mu,\alpha} > 0, \quad \Gamma > 0,, \quad \mu = 1,2,3,4, \quad \alpha = 1,2,...,N, \end{gathered} \tag{3}$$

by therefore characterizing:

1) The dimension and shape of particles via semi-axes $n_{k,\alpha}^2, k = 1,2,3$ (with n_3 parallel to the spin);

2) The density $n_{4,\alpha}^2$, with all n-characteristic quantities normalized to the value $n_{\mu,\alpha}^2 = 1$ for the vacuum;

3) Non-Hamiltonian interactions caused by the mutual penetration of the charge distribution of hadrons via the term e^Γ, where $\Gamma(r, p, E, d, \psi, \pi, \tau, ...)$ is a positive-definite quantity with an unrestricted functional dependence on local variables.

For the case of systems irreversible over time, the isotopic element $\hat{T}$ has time-dependent realizations more general than that of Eq.s (3) (see Vol.II, Chapter 8 of Ref. [9]).

Despite its simplicity, iso-product (2) requires, for consistency, a compatible axiom-preserving isotopy of 20th century applied mathematics into *iso-mathematics* [15] with corresponding completion of quantum mechanics into the *isotopic branch of hadronic mechanics* also called *iso-mechanics,* including:

A) The compatible, completion of the basic unit $\hbar = 1$ of quantum mechanics into the integro-differential *iso-unit*

$$\hat{I} = 1/\hat{T} > 0, \quad \hat{I} \star A = A \star \hat{I} = A, \tag{4}$$

with ensuing completion of numeric fields $F(n, \times, 1)$ of real $\mathcal{R}$, complex C and quaternionic Q numbers n into the *iso-fields* $\hat{F}(\hat{n}, \star, \hat{I})$ of *iso-real* $\hat{\mathcal{R}}$, îso-complex $\hat{C}$ and *iso-quaternionic* $\hat{Q}$ *iso-numbers* $\hat{n} = n\hat{I}$ and related isotopic operations [16] (see Ref. [17] for an independent study).

B) The completion of conventional Euclidean (and other) coordinates r into *iso-coordinates* and functions $f(r)$ into *iso functions* [15] (see Ref. [18] for an independent study)

$$\hat{r} = r\hat{I}, \quad \hat{f}(\hat{r}) = [f(r\hat{I})]\hat{I}, \tag{5}$$

as well as the completion of the Newton-Leibnitz differential calculus into the *iso-differential calculus* verifying the basic conditions [15] (see Refs. [19] for comprehensive studies and ref. [20] for a general review)

$$\hat{d}\hat{r} = \hat{T} d\hat{r}, \quad \frac{\hat{\partial}\hat{f}(\hat{r})}{\hat{\partial}\hat{r}} = \hat{I}\frac{\partial \hat{f}(\hat{r})}{\partial \hat{r}}. \tag{6}$$

C) The completion of conventional spaces S over F into *iso-spaces* $\hat{S}$ over iso-fields $\hat{F}$ [15]. In particular, the conventional Minkowski space $M(x, \eta, I)$ over $\mathcal{R}$ with spacetime coordinates $x \in \mathcal{R}$, $x^4 = ct$, metric $\eta = Diag(1, 1, 1, -1)$ and unit $I = Diag(1, 1, 1, 1)$, is mapped into the *iso-Minkowski iso-space* $\hat{M}(\hat{x}, \hat{\Omega}, \hat{I})$ [21] [22] (see also Ref. [12] for a recent account and Ref. [23] for notes on R. M. Santilli's lectures delivered at the ICRP, Trieste, Italy) over the iso-real iso-numbers $\hat{\mathcal{R}}$ with iso-coordinates $\hat{x} = x\hat{I} \in \hat{\mathcal{R}}$, iso-metric $\hat{\Omega} = (\hat{\eta})\hat{I} = (\hat{T}\eta)\hat{I}$, and iso-interval

$$\begin{aligned}(\hat{x}^\rho - \hat{y}^\rho)^{\hat{2}} &= (\hat{x}^\rho - \hat{y}^\rho) \star \hat{\Omega}_{\rho\nu} \star (\hat{x}^\nu - \hat{y}^\nu) = \\ &= [\tfrac{(x_1-y_1)^2}{n_1^2} + \tfrac{(x_2-y_2)^2}{n_2^2} + \tfrac{(x_3-y_3)^2}{n_3^2} - \tfrac{(t_x-t_y)^2 c^2}{n_4^2}]\hat{I},\end{aligned} \tag{7}$$

where the exponential term $exp\{-\Gamma\}$ is imbedded into the n-characteristic quantities.

D) The compatible completion of all branches of Lie's theory first studied in Vol. II of Ref. [8] (see also Vol. II of Refs. [9] for a general treatment, Ref. [24] for an independent study, and Ref. [25] for a recent update). For instance, an N-dimensional Lie algebra L with Hermitean generators X_k, $k = 1, 2, ...N$ is completed into the infinite family of Lie-Santilli iso-algebras $\hat{L}$ with iso-commutation rules

$$[X_i, X_j]^* = X_i \star X_j - X_j \star X_i = C_{ij}^k X_k. \tag{8}$$

E) The completion of all known space-time symmetries into the *iso-symmetries* of iso-space-time (6) [9] [12], including: the completion of the Lorentz symmetry $SO(3.1)$ into the *iso-Lorentz symmetry* $\hat{SO}(3.1)$ [21] with iso-transformations

$$x^{1'} = x^1, \ \ , x^{2'} = x^2, \tag{9}$$

$$x^{3'} = \hat{\gamma}(x^3 - \hat{\beta}\frac{n_3}{n_4}x^4), \quad x^{4'} = \hat{\gamma}(x^4 - \hat{\beta}\frac{n_4}{n_3}x^3), \tag{10}$$

where

$$\hat{\beta}_k = \frac{v_k/n_k}{c_o/n_4}, \ \hat{\gamma}_k = \frac{1}{\sqrt{1 - \hat{\beta}_k^2}}; \tag{11}$$

the completion of the Lorentz-Poincaré symmetry $P(3.1)$ into the *iso-Lorentz-Poincaré symmetry* $\hat{P}(3.1)$ [26]; and the completion of the spinorial covering of the Lorentz-Poincaré symmetry $\mathcal{P}(3.1)$ into the *iso-spinorial covering of the Lorentz-Poincaré-Santilli iso-symmetry* $\hat{\mathcal{P}}(3.1)$ [27].

We should also recall from Ref. [28] that all aspects of regular iso-mathematics and iso-mechanics can be derived via simple *non-unitary transforms*

$$UU^\dagger = \hat{I} \neq I, \tag{12}$$

of *all* conventional mathematical or physical aspects, under which: the unit of quantum mechanics is mapped into the iso-unit of iso-mechanics

$$\hbar = 1 \rightarrow U1U^\dagger = \hat{I}; \tag{13}$$

the conventional associative product AB is mapped into the iso-product

$$AB \rightarrow U(AB)U^\dagger = (UAU^\dagger)(UU^\dagger)^{-1}(UBU^\dagger) = \hat{A}\hat{T}\hat{B}; \tag{14}$$

and the same holds for the construction of all remaining regular iso-theories.

As indicated, the isotopic element $\hat{T}$ represents physical characteristics of particles. Hence, the invariance of its numeric value is essential for the consistency and experimental verification of any iso-theory. Such an invariance does indeed occur under the *infinite class of iso-equivalence* of isotopic methods which is given by the isotopic reformulation of non-unitary transforms called *iso-unitary iso-transforms* [28]

$$UU^\dagger = \hat{I} \neq I, \quad U = \hat{U}\hat{T}^{1/2}, \quad \hat{U} \star \hat{U}^\dagger = \hat{U}^\dagger \star \hat{U} = \hat{I}, \tag{15}$$

under which we have the numeric invariance of the iso-unit

$$\hat{I} \rightarrow \hat{U} \star \hat{I} \star \hat{U}^\dagger = \hat{I}' \equiv \hat{I}, \tag{16}$$

and of the isotopic element

$$\hat{A} \star \hat{B} \rightarrow \hat{U} \star (\hat{A} \star \hat{B}) \star \hat{U}^\dagger = \hat{A}'\hat{T}'\hat{B}', \quad \hat{T}' \equiv \hat{T}. \tag{17}$$

Thanks to the use of iso-mathematics and iso-mechanics, R. M. Santilli provided the following verifications and application of the EPR Argument:

I) The inapplicability (rather than the violation) of Bell's inequality [29] with ensuing existence of classical counterparts for a system of extended spin $1/2$ particles under Hamiltonian as well as non-Hamiltonian interactions due to their mutual penetration as occurring in a nuclear structure [30];

II) The progressive recovering of Einstein's determinism [4] with the increase of the density for extended particles within hadrons, nuclei and stars, and the full recovering of Einstein's determinism at the limit of Schwartzschild's horizon [31];

III) The conversion of quantum mechanical strongly divergent perturbative series into strongly convergent isotopic series, evidently in view of the very small value of the isotopic element (9) sandwiched in between all products, by therefore setting up the foundations for the elimination of divergencies in physics (see Chapters 11 and 12, Vol. II of Ref. [9] and upgrade [10]). As an application, *isochemistry* achieved the first known *attractive force* between the *identical* electrons of valence bonds (see Chapter 4 on of [32]) whose resulting *strong valance bond* achieved the first known exact representation of experimental data for the hydrogen [33] and water [34] molecules without divergent perturbative series.

2 Apparent unsettled aspects in the muon magnetic moment

To implement due scientific process on values (1), we should recall P. A. M. Dirac's [35] and other authoritative doubts on the final character of the numeric values obtained from quantum electrodynamics due to the divergence of Feynman's and other series (see Ref. [36] for a recent account on QED divergencies).

Additionally, measurements [1] have been done via the assumption that the mean life of muons behaves with speed according to the time dilation law of special relativity

$$t = t_o \sqrt{1 - \frac{v^2}{c^2}}. \tag{18}$$

The exact validity of the above law for electrons and other *point-like* particles can be considered, nowadays, to be beyond scientific doubt.

However, at this writing there exist unresolved aspects in regard to the experimental behavior of law (18) for the behavior of the mean life with speed (or, equivalently, with energy) of *unstable, thus composite particles.*

In fact, in 1965, D. I. Blokhintsev [37] pointed out the expected inapplicability (rather than the violation) of special relativity for the interior of hadrons due to non-local effects in their hyperdense structure and suggested that deviations due to internal effects could be measured in the outside via deviations from time dilation law (18).

Independently from Blokhintsev's studies, R. M. Santilli pointed out in his Ph. D. thesis in the mid 1960's [38] the strict time-reversibility of special relativity and relativistic. quantum mechanics, with the ensuing lack of exact validity for irreversible physical and chemical processes, such as nuclear fusions or fuel combustion. Santilli identified the origin of the time reversibility in the invariance of space-time Lie algebras under anti-Hermiticity, and suggested the completion/covering of isotopic methods into the broader *genotopic methods* with Lie-admissible structure [39].

Subsequently, when he was at Harvard University under DOE support, Santilli pointed out the inapplicability (rather than the violation) of special relativity and relativistic quantum mechanics for irreversible particle events such as spontaneous decays, and suggested their treatment via the Lie-admissible/gentopic branch of hadronic mechanics [40] [41] [42] (see Refs. [9] for detailed treatments and Ref. [10] for a recent update).

These studies triggered a number of generalizations of time dilation law (18), such as those by L. B. Redei [43], D. Y. Kim [44] and others.

In 1983, R. M. Santilli [21] (see also Refs. [22] [23] and Section 8 of the recent update [10]) showed that the axioms of special relativity remain valid for the interior of hyperdense particles when realized on the iso-Minkowskian isospace over an iso-field, and then projected in our spacetime

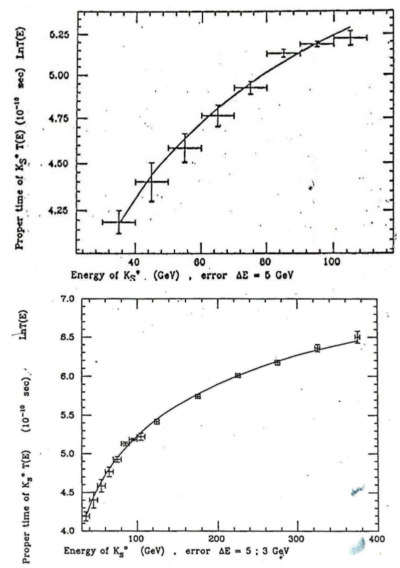

Fig. 1: *In this figure, we reproduce the exact fits of isotopic time dilation law (19) obtained by F. Cardone et al. [48] of: 1) Deviations [46] from the time dilation law (18) for the behavior of the* $K^0 - \bar{K}^0$*-system from* 0 *to* 100 *GeV (top view); 2) The exact fit of both deviations* 0 *to* 100 *GeV [46] and apparent verification in the range from* 100 *to* 350 *GeV [47] (bottom view).*

over a conventional field in which case they yield the isotopic completion of law (18)

$$t = t_o \sqrt{1 - \frac{v^2/n_3^2}{c^2/n_4^2}}, \tag{19}$$

where the characteristic n-quantities are those of the isotopic element (3) for time reversible processes, with broader time non-invariant realizations for irreversible processes.

In 1989, A. K. Aringazin [45] proved that all preceding generalizations of law (18) are particular cases of the isotopic law (19) because they can be obtained via different expansions of the latter law in terms of different parameters and with different truncation, thus restricting the experiments to the test of law (19).

In 1983, S. H. Aronson *et al.* [46] reported the outcome of experiments conducted at FERMILAB showing apparent *deviations* from law (18) for the $K^0 - \bar{K}^0$ system in the energy range from 0 to 100 GeV.

In 1987, N. Grossman *et al.* [47] reported counter-experiments also conducted at FERMILAB showing an apparent *confirmation* of law (45), but in the *different energy range* from 100 to 250 GeV.

In 1992, F. Cardone *et al.* [48] indicated that counter-measurements [47] from 100 to 350 GeV leave basically unresolved the deviations of law (18) from 0 to 100 GeV [46], and that the isotopic law (19) provides an exact fit for both measurements [46] [47] (Figure 1).

Finally, in 1998, Yu. Arestov *et al* [49] pointed out apparent flaws in the theoretical elaboration of the experimental data of measurements [47].

3 Concluding remarks

As it is well known, the standard model assumes that muons are *elementary particles,* under which assumption, the sole known possibility of representing deviation (1) is the search for new particles and/or new interactions.

In 1978, R. M. Santilli [50] noted that *muons are naturally unstable* with time-irreversible spontaneous decays

$$\mu^{\pm} \rightarrow e^{\pm} + \nu + \bar{\nu}, \ \% \ 10^{-11}, \tag{20}$$

$$\mu^{\pm} \rightarrow e^{\pm} + 2\gamma, \ \% \ 10^{-11}, \tag{21}$$

$$\mu^{\pm} \rightarrow e^{-} + e^{\pm} + e^{+} \ \% \ 10^{-12}. \tag{22}$$

Consequently, muons are expected to be *extended bound states* of elementary particles suitable to trigger their decay.

Therefore, Santilli [50] assumed that muons are bound states of particles produced free in the spontaneous decay with the lowest mode, Eq. (22), with structure according to hadronic mechanics (hm)

$$\mu^{\pm} = (e^{-}_{\downarrow}, e^{\pm}_{\uparrow}, e^{+}_{\downarrow})_{hm}. \tag{23}$$

By recalling that an electron-positron pair annihilates into 2γ, the presence in the muon structure of an electron-positron pair was confirmed by spontaneous decay (21), by allowing a quantitative representation of the unstable character of the muons as well as of their mean life.

Following the approximation of model (23) as a *two-body* bound state at short distance of a positronium and a central electron, Santilli worked out a detailed analytic solution of the iso-Schrödinger equation of iso-mechanics for structure (23), and achieved in 1978 a numerically exact and time invariant, non-relativistic representation of *all* characteristics of the muons, including [50]: rest energy 105.658 MeV, mean life $\tau = 2.19703 \times 10^{-6}\ s$, charge radius of about $R = d/2 = 10^{-13}\ cm$, spin 1/2, charge $\pm e$, parity; and a representation of the muon magnetic moment of the time, as being equal to that of the electron, with gyromagnetic factor $g = 2$.

Following the discovery of the anomalous value (1) of the muon magnetic moment, Santilli re-examined in the recent paper [51] (see Ref. [13] for a recent account) his 1978 model [50] at the relativistic level via the use of the iso-symmetry $\hat{\mathcal{R}}(3.1)$ of interval (7) on isofields $\hat{\mathcal{F}}(3.1)$ [27] with isotopic element (3) and achieved the values of the n-characteristic quantities of the muons

$$n_1^2 = n_2^2 \approx 0.4926, \ \ n_3^2 \approx 0.0149, \quad n_4^2 \approx 0.0149, \tag{24}$$

confirming the expectation that model (23) characterizes in the ground state an essentially flat structure in view of the point-like character of the constituents.

In particular, Ref. [51] identified the value of the muon gyromagnetic factor

$$\hat{g}^{EXP}_{\mu} == 2.00233184122 - 2.00233183620 = \frac{n_4}{n_3} g^{QED}_{\mu}, \tag{25}$$

from which

$$\frac{n_4}{n_3} = 1.00000000502. \tag{26}$$

The above results appear to confirm the lack of exact character of time evolution law (18) for the behavior of the mean life of unstable particles with speed. In fact, under the assumption in first approximation that the muon spontaneous decay is time-reversible, isotopic time dilation law (19) with values (24) predicts the *increase* of anomalous value (1)

$$t = t_o \sqrt{1 - 1.00000000502.\frac{v^2}{c^2}}, \tag{27}$$

with the expectation of bigger predictions for the full time irreversible treatment.

In conclusion, it seems plausible to expect that, in the event deviations [46] from time dilation (18) are confirmed, experimental value (1) of the muon magnetic moment should be correspondently revised.

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