



A Study of Neutrosophic Differential Equations By Using the One-Dimensional Geometric AH-Isometry Of Neutrosophic Laplace Transformation

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Abstract

In this paper, we study the neutrosophic differential equation by using the one-dimensional geometric AH-Isometry of Neutrosophic Laplace Transformation. Where we use this AH-isometry to find the algebraic image of this transformation, and then to apply this image directly on the problem of finding the solutions of differential equations.

Keywords: One-Dimensional Geometric AH-Isometry; Neutrosophic Differential Equation; Neutrosophic Laplace Transformation.

1. Introduction

Neutrosophic logic. Neutrosophy, Neutrosophic set, Neutrosophic probability and alike, are recently creations of Smarandache, being characterized by having the indeterminacies component of their framework, and a notable feature of neutrosophic logic is that can be considered a generalization of fuzzy logic, encompassing the classical logic as well [1]. Also, F. Smarandache, has defined the concept of continuation of a neutrosophic function in year 2015 in [1], and neutrosophic mereo-limit [1], mereo-continuity. Moreover, in 2014, he has defined the concept of a neutro-oscillator differential in [3], and mereo-derivative. Finally in 2013 he introduced neutrosophic integration in [2], and mereo-integral, besides the classical definitions of limit, continuity, derivative, and integral respectively. Among the recent applications there are: neutrosophic crisp set theory in image processing [4][5], neutrosophic sets medical field [6-10], in information geographic systems [11] and possible applications to database [12]. Also, neutrosophic triplet group application to physics [13]. Moreover several researches have made multiple contributions to neutrosophic topological [14 – 20, 23 – 33, 36 – 50], Also more researches have made multiple contributions to neutrosophic analysis [21]. Finally the neutrosophic integration may have application in calculus the areas between two neutrosophic functions.

2. Preliminaries

Definition 2.1. Neutrosophic Real Number: [1]

Suppose that w is a neutrosophic number, then it takes the following standard form: $w = a + bI$ where a, b are real coefficients, and I represents the indeterminacy, where $0 \cdot I = 0$ and $I^n = I$ for all positive integers n .

For example:

$$w = 1 + 2I, w = 3 = 3 + 0I.$$

Definition 2.2. Division of neutrosophic real numbers: [2]

Suppose that w_1, w_2 are two neutrosophic number, where

$$w_1 = a_1 + b_1I, w_2 = a_2 + b_2I$$

Then:

$$\frac{w_1}{w_2} = \frac{a_1 + b_1I}{a_2 + b_2I} = \frac{a_1}{a_2} + \frac{a_2b_1 - a_1b_2}{a_2(a_2 + b_2)}I$$

Definition 2.3 [21]

Let $R(I) = \{a + bI; a, b \in R\}$ where $I^2 = I$ be the neutrosophic field of reals. The one-dimensional isometry (AH-Isometry) is defined as follows: [21]

$$\begin{aligned} T: R(I) &\rightarrow R \times R \\ T(a + bI) &= (a, a + b) \end{aligned}$$

Remark 2.4. [21]

T is an algebraic isomorphism between two rings, it has the following properties:

- 1) T is bijective.
- 2) T preserves addition and multiplication, i.e.:

$$T[(a + bI) + (c + dI)] = T(a + bI) + T(c + dI)$$

And

$$T[(a + bI) \cdot (c + dI)] = T(a + bI) \cdot T(c + dI)$$

- 3) Since T is bijective, then it is invertible by:

$$\begin{aligned} T^{-1}: R \times R &\rightarrow R(I) \\ T^{-1}(a, b) &= a + (b - a)I \end{aligned}$$

- 4) T preserves distances, i.e.:

The distance on $R(I)$ can be defined as follows:

$$\begin{aligned} \text{Let } A = a + bI, B = c + dI \text{ be two neutrosophic real numbers, then } L = \|\overline{AB}\| &= d[(a + bI), (c + dI)] = \\ |a + bI - (c + dI)| &= |(a - c) + I(b - d)| = |a - c| + I[|a + b - c - d| - |a - c|]. \end{aligned}$$

On the other hand, we have:

$$\begin{aligned} T(\|\overline{AB}\|) &= (|a - c|, |(a + b) - (c + d)|) = (d(a, c), d(a + b, c + d)) = d[(a, a + b), (c, c + d)] = d(T(a + \\ bI), T(c + dI)) \\ &= \|\overline{T(AB)}\|. \end{aligned}$$

This implies that the distance is preserved up to isometry. i.e. $\|T(AB)\| = T(\|AB\|)$

Definition 2.5. []

Let $f: R(I) \rightarrow R(I)$; $f = f(X)$ and $X = x + yI \in R(I)$ the f is called a neutrosophic real function with one neutrosophic variable.

a neutrosophic real function $f(X)$ written as follows:

$$f(X) = f(x + yI) = f(x) + I[f(x + y) - f(x)]$$

Theorem 2.6. any neutrosophic real function into two classical real functions, i.e., to the classical Euclidean plane $R \times R$.

Proof.

Let $f(X) = f(x + yI) = f(x) + I[f(x + y) - f(x)]$ a neutrosophic real function.

Now, Using the one-dimensional AH-isometry, we have.

$T(f(X)) = T(f(x) + I[f(x + y) - f(x)])$, then.

$(f_1, f_2) = (f(x), f(x + y))$, then, we have.

$$\begin{cases} f_1 = f(x) \\ f_2 = f(x + y) \end{cases}$$

the functions $f(x), f(x + y)$ are a real functions.

3. Neutrosophic Laplace Transformation.

In this section we define Neutrosophic Transformation of Laplace for a neutrosophic function $f(X)$.

Definition3.1.

Let $Y = y_1 + y_2I, X = x_1 + x_2I$, and $f(X) = f(x_1 + x_2I) = f(x_1) + I[f(x_1 + x_2) - f(x_1)]$ a neutrosophic function on $R(I)$, then we define a Neutrosophic Transformation Laplace of $f(X)$ as forms:

$$F(P) = \int_0^{-\infty} e^{-PX} f(X) dX$$

Where $P = p_1 + p_2I \in R(I)$.

Now, $F(P)$ can be written as follow:

$$F(p_1 + p_2I) = \int_0^{-\infty} e^{-(p_1+p_2I)(x_1+x_2I)} (f(x_1) + I[f(x_1 + x_2) - f(x_1)]) d(x_1 + x_2I)$$

$$F(p_1 + p_2I) = \int_0^{-\infty} e^{-[p_1x_1+(p_1x_2+p_2x_1+p_2x_2)I]} (f(x_1) + I[f(x_1 + x_2) - f(x_1)]) d(x_1 + x_2I)$$

$$F(p_1 + p_2I) = \int_0^{-\infty} [e^{-p_1x_1} + (e^{-(p_1x_1+p_1x_2+p_2x_1+p_2x_2)} - e^{-p_1x_1})I] (f(x_1) + I[f(x_1 + x_2) - f(x_1)]) d(x_1 + x_2I)$$

$$\begin{aligned}
 F(p_1 + p_2 I) &= \int_0^{-\infty} [e^{-p_1 x_1} + (e^{-(p_1+p_2)(x_1+x_2)} - e^{-p_1 x_1})I](f(x_1) + I[f(x_1 + x_2) - f(x_1)])d(x_1 + x_2 I) \\
 F(p_1 + p_2 I) &= \int_0^{-\infty} \left[e^{-p_1 x_1} f(x_1) \right. \\
 &\quad + I \left(e^{-p_1 x_1} f(x_1 + x_2) - e^{-p_1 x_1} f(x_1) + e^{-(p_1+p_2)(x_1+x_2)} f(x_1) + e^{-(p_1+p_2)(x_1+x_2)} f(x_1 + x_2) \right. \\
 &\quad \left. \left. - e^{-(p_1+p_2)(x_1+x_2)} f(x_1) - e^{-p_1 x_1} f(x_1 + x_2) + e^{-p_1 x_1} f(x_1) \right) \right] d(x_1 + x_2 I) \\
 F(p_1 + p_2 I) &= \int_0^{-\infty} \left[e^{-p_1 x_1} f(x_1) + I \left(e^{-(p_1+p_2)(x_1+x_2)} f(x_1 + x_2) - e^{-p_1 x_1} f(x_1) \right) \right] d(x_1 + x_2 I)
 \end{aligned}$$

By Definition 2.9 , we have.

$$\begin{aligned}
 F(p_1 + p_2 I) &= \int_0^{-\infty} e^{-p_1 x_1} f(x_1) d(x_1) + I \left[\int_0^{-\infty} e^{-(p_1+p_2)(x_1+x_2)} f(x_1 + x_2) d(x_1 + x_2) - \right. \\
 &\quad \left. \int_0^{-\infty} e^{-p_1 x_1} f(x_1) d(x_1) \right] \dots \dots (1)
 \end{aligned}$$

Method of solution.

1. Take AH-Isometry for equation(1), we have.

$$\begin{aligned}
 T(F(p_1 + p_2 I)) &= T \left(\int_0^{-\infty} e^{-p_1 x_1} f(x_1) d(x_1) \right. \\
 &\quad \left. + I \left[\int_0^{-\infty} e^{-(p_1+p_2)(x_1+x_2)} f(x_1 + x_2) d(x_1 + x_2) - \int_0^{-\infty} e^{-p_1 x_1} f(x_1) d(x_1) \right] \right) \\
 [F(p_1), F(p_1 + p_2)] &= \left[\int_0^{-\infty} e^{-p_1 x_1} f(x_1) d(x_1), \int_0^{-\infty} e^{-(p_1+p_2)(x_1+x_2)} f(x_1 + x_2) d(x_1 + x_2) \right]
 \end{aligned}$$

Then.

$$\begin{cases}
 F(p_1) = \int_0^{-\infty} e^{-p_1 x_1} f(x_1) d(x_1) \\
 F(p_1 + p_2) = \int_0^{-\infty} e^{-(p_1+p_2)(x_1+x_2)} f(x_1 + x_2) d(x_1 + x_2)
 \end{cases}$$

$F(p_1)$ and $F(p_1 + p_2)$ are two Laplace transformation classical.

2. We find $F(p_1)$ and $F(p_1 + p_2)$.

3. We Take invertible AH-Isometry, then, we have a Neutrosophic Laplace transformation.

$$F(p_1 + p_2 I) = T^{-1}(F(p_1), F(p_1 + p_2)) = F(p_1) + (F(p_1 + p_2) - F(p_1))I$$

Now, We show the laplace transform table for some analytical functions classical.

$f(x)$	$F(p) = L[f(x)]$
a	$\frac{a}{p}$
1	$\frac{1}{p}$
x^n	$\frac{n!}{p^{n+1}}; n = 1, 2, 3, \dots$
\sqrt{x}	$\frac{\sqrt{\pi}}{2p^{\frac{3}{2}}}$
$\sin ax$	$\frac{a}{p^2 + a^2}$
$\cos ax$	$\frac{p}{p^2 + a^2}$
$x \sin ax$	$\frac{2ap}{(p^2 + a^2)^2}$
$x \cos ax$	$\frac{p^2 - a^2}{(p^2 + a^2)^2}$
e^{ax}	$\frac{1}{p - a}$
$\sin(ax + b)$	$\frac{p \sin b + a \cos b}{p^2 + a^2}$
$\cos(ax + b)$	$\frac{p \cos b - a \sin b}{p^2 + a^2}$
$e^{ax} \sin bx$	$\frac{b}{(p - a)^2 + b^2}$
$e^{ax} \cos bx$	$\frac{p - a}{(p - a)^2 - b^2}$
$\sinh ax$	$\frac{a}{p^2 - a^2}$
$\cosh ax$	$\frac{p}{p^2 - a^2}$

Properties laplace transform classical.

- 1- $L[e^{ax} f(x)] = F(p - a)$
- 2- $L[x^n f(x)] = (-1)^n \frac{d}{dp} F(p)$
- 3- $L\left[\frac{f(x)}{x}\right] = \int_p^{-\infty} F(p) dp$
- 4- $L[y'] = pL[y] - y(0)$
- 5- $L[y''] = p^2L[y] - py(0) - y'(0)$
- 6- $L[y'''] = p^3L[y] - p^2y(0) - y''(0) + py'(0)$
- 7- $L[y^{(n)}] = p^nL[y] - p^{n-1}y(0) - p^{n-2}y'(0) - \dots - py^{(n-2)}(0) - y^{(n-1)}(0)$

Example3.2. Find a Neutrosophic Laplace transformation:

$$f(X) = 2\sin 4(X) + X$$

Solution.

Let $X = x_1 + x_2I$. Then.

$$f(X) = 2\sin 4(x_1 + x_2I) + (x_1 + x_2I) = 2\sin 4(x_1) + I[2\sin 4(x_1 + x_2) - 2\sin 4(x_1)] + x_1 + I[(x_1 + x_2) - (x_1)]$$

$$f(X) = 2\sin 4(x_1) + x_1 + I[2\sin 4(x_1 + x_2) + (x_1 + x_2) - 2\sin 4(x_1) - (x_1)]$$

Now, we have.

$$F(p_1 + p_2I) = \int_0^{-\infty} e^{-p_1x_1}(2\sin 4(x_1) + x_1)d(x_1) \\ + I \left[\int_0^{-\infty} e^{-(p_1+p_2)(x_1+x_2)}(2\sin 4(x_1 + x_2) + (x_1 + x_2))d(x_1 + x_2) \right. \\ \left. - \int_0^{-\infty} e^{-p_1x_1}(2\sin 4(x_1) + x_1)d(x_1) \right]$$

Now, Take AH-Isometry, we have.

$$\begin{cases} F(p_1) = \int_0^{-\infty} e^{-p_1x_1}(2\sin 4(x_1) + x_1)d(x_1) \\ F(p_1 + p_2) = \int_0^{-\infty} e^{-(p_1+p_2)(x_1+x_2)}(2\sin 4(x_1 + x_2) + (x_1 + x_2))d(x_1 + x_2) \end{cases}$$

Then.

$$F(p_1) = L(2\sin 4(x_1) + x_1) = \frac{8}{p_1^2 + 16} + \frac{1}{p_1^2}$$

$$F(p_1 + p_2) = L(2\sin 4(x_1 + x_2) + (x_1 + x_2)) = \frac{8}{(p_1 + p_2)^2 + 16} + \frac{1}{(p_1 + p_2)^2}$$

Now, Take invertibleAH-Isometry, we have.

$$F(p_1 + p_2I) = T^{-1}(F(p_1), F(p_1 + p_2)) = F(p_1) + (F(p_1 + p_2) - F(p_1))I$$

$$F(p_1 + p_2I) = \frac{8}{p_1^2 + 16} + \frac{1}{p_1^2} + \left(\frac{8}{(p_1 + p_2)^2 + 16} + \frac{1}{(p_1 + p_2)^2} - \frac{8}{p_1^2 + 16} - \frac{1}{p_1^2} \right) I$$

$$F(p_1 + p_2I) = \frac{8}{(p_1 + p_2I)^2 + 16} + \frac{1}{(p_1 + p_2I)^2}$$

Example 3.3. Find a NeutrosophicLapalce transformation:

$$f(X) = e^{-X}\sin X$$

Solution.

Let $X = x_1 + x_2I$. Then.

$$f(X) = e^{-(x_1+x_2I)} \sin(x_1 + x_2I) = [e^{-(x_1)} + I(e^{-(x_1+x_2)} - e^{-(x_1)})]. [\sin(x_1) + I(\sin((x_1 + x_2)) - \sin(x_1))]$$

$$f(X) = e^{-(x_1)} \sin(x_1) + I[e^{-(x_1+x_2)} \sin(x_1 + x_2) - e^{-(x_1)} \sin(x_1)]$$

Now, we have.

$$F(p_1 + p_2I) = \int_0^{-\infty} e^{-(p_1)(x_1)} e^{-(x_1)} \sin(x_1) d(x_1) + I \left[\int_0^{-\infty} e^{-(p_1+p_2)(x_1+x_2)} e^{-(x_1+x_2)} \sin(x_1 + x_2) d(x_1 + x_2) - \int_0^{-\infty} e^{-(p_1)(x_1)} e^{-(x_1)} \sin(x_1) d(x_1) \right]$$

Now, Take AH-Isometry, we have.

$$\begin{cases} F(p_1) = \int_0^{-\infty} e^{-(p_1)(x_1)} e^{-(x_1)} \sin(x_1) d(x_1) \\ F(p_1 + p_2) = \int_0^{-\infty} e^{-(p_1+p_2)(x_1+x_2)} e^{-(x_1+x_2)} \sin(x_1 + x_2) d(x_1 + x_2) \end{cases}$$

Then.

$$F(p_1) = L(e^{-(x_1)} \sin(x_1)) = \frac{1}{(p_1 + 1)^2 + 1}$$

$$F(p_1 + p_2) = L(e^{-(x_1+x_2)} \sin(x_1 + x_2)) = \frac{1}{(p_1 + p_2)^2 + 1}$$

Now, Take invertibleAH-Isometry, we have.

$$F(p_1 + p_2I) = T^{-1}(F(p_1), F(p_1 + p_2)) = F(p_1) + (F(p_1 + p_2) - F(p_1))I$$

$$F(p_1 + p_2I) = \frac{1}{(p_1 + 1)^2 + 1} + \left(\frac{1}{(p_1 + p_2)^2 + 1} - \frac{1}{(p_1 + 1)^2 + 1} \right) I$$

$$F(p_1 + p_2I) = \frac{1}{(p_1 + p_2I)^2 + 1}$$

4 Neutrosophic differential equation by using Laplace transformtion.

In this section is defined a Neutrosophic differentialequationby Using the One-Dimensional Geometric AH-Isometryof Laplace transformtionand solutions are found for this equation.

Definition 4.1 .

Let $Y = y_1 + y_2I$, $X = x_1 + x_2I$ We define the Neutrosophic differential equations form:

$$Y^{(n)} + (a_1 + a_2I)Y^{(n-1)} + \dots + (b_1 + b_2I)Y' + (c_1 + c_2I)Y = f(X) \dots \dots (2)$$

And takes a initial conditions.

Method of solution.

- 1- We take the laplace transform of both sides of the equation (2).
- 2- We substitute the initial conditions.
- 3- We take the inverse laplace transform.

Example 4.2.find the solution of equation.

$$Y'' + Y = X \dots \dots (3)$$

Where.

$$Y'(0) = 2, Y(0) = 1 \dots \dots (4)$$

Solution.

Let $Y = y_1 + y_2I, X = x_1 + x_2I$. Then.

$$y_1'' + y_1 + I[(y_1 + y_2)'' + (y_1 + y_2) - (y_1'' + y_1)] = x_1 + I[(x_1 + x_2) - x_1]$$

And:

$$y_1'(0) + I[(y_1 + y_2)'(0) - y_1'(0)] = 2, y_1(0) + I[(y_1 + y_2)(0) - y_1(0)] = 1$$

Now, Take AH-Isometry for the differential equation, and Take AH-Isometry for initial conditions, we have..

$$\begin{cases} y_1'' + y_1 = x_1 \dots \dots (5) \\ (y_1 + y_2)'' + (y_1 + y_2) = (x_1 + x_2) \dots \dots (6) \end{cases}$$

And:

$$\begin{cases} y_1'(0) = 2, (y_1 + y_2)'(0) = 2 \\ y_1(0) = 1, (y_1 + y_2)(0) = 1 \end{cases}$$

Now, we find solution of equation (5), then.

$$L[y_1''] + L[y_1] = L[x_1]$$

$$\Rightarrow (p_1^2)L[y_1] - p_1y_1(0) - y_1'(0) + L[y_1] = [x_1] \Rightarrow (p_1^2)L[y_1] - p_1y_1(0) - y_1'(0) + L[y_1] = \frac{1}{p_1^2}$$

$$\Rightarrow (p_1^2)L[y_1] - p_1[1] - [2] + L[y_1] = \frac{1}{p_1^2} \Rightarrow (p_1^2)L[y_1] - p_1 - 2 + L[y_1] = \frac{1}{p_1^2}$$

$$\Rightarrow (p_1^2 + 1)L[y_1] - 2 = \frac{1}{p_1^2} \Rightarrow (p_1^2 + 1)L[y_1] = \frac{1}{p_1^2} + p_1 + 2$$

$$\Rightarrow (p_1^2 + 1)L[y_1] = \frac{p_1^3 + 2p_1^2 + 1}{p_1^2} \Rightarrow L[y_1] = \frac{p_1^3 + 2p_1^2 + 1}{p_1^2(p_1^2 + 1)} \Rightarrow L[y_1] = \frac{p_1^3 + 2p_1^2 + 1}{p_1^2(p_1^2 + 1)}$$

$$\frac{p_1^3 + 2p_1^2 + 1}{p_1^2(p_1^2 + 1)} = \frac{A}{p_1} + \frac{B}{p_1^2} + \frac{Cp_1 + D}{p_1^2 + 16} \Rightarrow A = 0, B = C = D = 1$$

$$\Rightarrow \frac{p_1^3 + 2p_1^2 + 1}{p_1^2(p_1^2 + 1)} = \frac{1}{p_1^2} + \frac{p_1}{p_1^2 + 16} + \frac{1}{p_1^2 + 16}$$

$$\Rightarrow L[y_1] = \left(\frac{1}{p_1^2} + \frac{p_1}{p_1^2 + 16} + \frac{1}{p_1^2 + 16} \right)$$

$$L^{-1}[y_1] = L^{-1}\left(\frac{1}{p_1^2}\right) + L^{-1}\left(\frac{p_1}{p_1^2 + 16}\right) + L^{-1}\left(\frac{1}{p_1^2 + 16}\right)$$

$$\Rightarrow y_1 = x_1 + \cos x_1 + \sin x_1$$

By the method same, The solution of equation (6) written as follow:

$$(y_1 + y_2) = (x_1 + x_2) + \cos(x_1 + x_2) + \sin(x_1 + x_2).$$

Now, Take invertible AH-Isometry, then, we have the solution of a Neutrosophic differential equation .

$$Y = y_1 + y_2 I = T^{-1}(x_1 + \cos x_1 + \sin x_1, (x_1 + x_2) + \cos(x_1 + x_2) + \sin(x_1 + x_2))$$

$$Y = y_1 + y_2 I = x_1 + \cos x_1 + \sin x_1 + I[(x_1 + x_2) + \cos(x_1 + x_2) + \sin(x_1 + x_2) - (x_1 + \cos x_1 + \sin x_1)]$$

$$Y = y_1 + y_2 I = x_1 + I((x_1 + x_2) - x_1) + \cos x_1 + I[\cos(x_1 + x_2) - \cos x_1] + \sin x_1 + I[\sin(x_1 + x_2) - \sin x_1]$$

$$Y = y_1 + y_2 I = (x_1 + x_2 I) + \cos(x_1 + x_2 I) + \sin(x_1 + x_2 I)$$

$$Y = y_1 + y_2 I = X + \cos X + \sin X$$

Example 4.3. find the solution of equation.

$$Y'' + 2Y' + 5Y = e^{-X} \sin X \dots \dots (7)$$

Where.

$$Y'(0) = 1, Y(0) = 0 \dots \dots (8)$$

Solution.

Let $Y = y_1 + y_2 I, X = x_1 + x_2 I$. Then.

$$y_1'' + 2y_1' + 5y_1 + I[(y_1 + y_2)'' + 2(y_1 + y_2)'] + 5(y_1 + y_2) - (y_1'' + 2y_1' + 5y_1) \\ = e^{-x_1} \sin x_1 + I[e^{-(x_1 + x_2)} \sin(x_1 + x_2) - e^{-x_1} \sin x_1]$$

And:

$$y_1'(0) + I[(y_1 + y_2)'(0) - y_1'(0)] = 1, y_1(0) + I[(y_1 + y_2)(0) - y_1(0)] = 0$$

Now, Take AH-Isometry for the differential equation, and Take AH-Isometry for initial conditions, we have..

$$\begin{cases} y_1'' + 2y_1' + 5y_1 = e^{-x_1} \sin x_1 \dots \dots (9) \end{cases}$$

$$\begin{cases} (y_1 + y_2)'' + 2(y_1 + y_2)' + 5(y_1 + y_2) = e^{-(x_1 + x_2)} \sin(x_1 + x_2) \dots \dots (10) \end{cases}$$

And:

$$\begin{cases} y_1'(0) = 1, (y_1 + y_2)'(0) = 1 \\ y_1(0) = 0, (y_1 + y_2)(0) = 0 \end{cases}$$

Now, we find solution of equation (9), then.

$$L[y_1''] + 2L[y_1'] + 5L[y_1] = L[e^{-x_1} \sin x_1]$$

$$\Rightarrow (p_1^2)L[y_1] - p_1 y_1(0) - y_1'(0) + 2p_1 L[y_1] - 2y_1(0) + 5L[y_1] = \frac{1}{(p_1 + 1)^2 + 1}$$

$$\Rightarrow (p_1^2)L[y_1] - 1 + 2p_1 L[y_1] + 5L[y_1] = \frac{1}{(p_1 + 1)^2 + 1}$$

$$\Rightarrow (p_1^2 + 2p_1 + 5)L[y_1] = 1 + \frac{1}{p_1^2 + 2p_1 + 3} \Rightarrow (p_1^2 + 2p_1 + 5)L[y_1] = \frac{p_1^2 + 2p_1 + 4}{p_1^2 + 2p_1 + 3}$$

$$L[y_1] = \frac{p_1^2 + 2p_1 + 4}{(p_1^2 + 2p_1 + 3)(p_1^2 + 2p_1 + 5)}$$

$$\frac{p_1^2 + 2p_1 + 4}{(p_1^2 + 2p_1 + 3)(p_1^2 + 2p_1 + 5)} = \frac{Ap_1 + B}{p_1^2 + 2p_1 + 5} + \frac{Cp_1 + D}{p_1^2 + 2p_1 + 3}$$

$$\Rightarrow A = 0, B = \frac{1}{2}, C = 0, D = \frac{1}{2}$$

$$\Rightarrow \frac{p_1^2 + 2p_1 + 4}{(p_1^2 + 2p_1 + 3)(p_1^2 + 2p_1 + 5)} = \frac{1}{2} \left(\frac{1}{p_1^2 + 2p_1 + 5} \right) + \frac{1}{2} \left(\frac{1}{p_1^2 + 2p_1 + 3} \right)$$

$$\Rightarrow L[y_1] = \frac{1}{2} \left(\frac{1}{p_1^2 + 2p_1 + 5} \right) + \frac{1}{2} \left(\frac{1}{p_1^2 + 2p_1 + 3} \right)$$

$$L^{-1}[y_1] = L^{-1} \left[\frac{1}{2} \left(\frac{1}{p_1^2 + 2p_1 + 5} \right) \right] + L^{-1} \left[\frac{1}{2} \left(\frac{1}{p_1^2 + 2p_1 + 3} \right) \right]$$

$$\Rightarrow L^{-1}[y_1] = \frac{1}{2} \left(\frac{1}{p_1^2 + 2p_1 + 5} \right) + \frac{1}{2} L^{-1} \left(\frac{1}{p_1^2 + 2p_1 + 3} \right)$$

$$\Rightarrow L^{-1}[y_1] = \frac{1}{2} L^{-1} \left\{ \frac{1}{(p_1 + 1)^2 + 4} \right\} + \frac{1}{2} L^{-1} \left\{ \frac{1}{(p_1 + 1)^2 + 2} \right\}$$

$$\Rightarrow y_1 = \frac{1}{4} e^{-x_1} \sin 2x_1 + \frac{1}{2\sqrt{2}} e^{-x_1} \sin \sqrt{2} x_1$$

By the method same, The solution of equation (10) written as follow:

$$(y_1 + y_2) = \frac{1}{4} e^{-(x_1 + x_2)} \sin 2(x_1 + x_2) + \frac{1}{2\sqrt{2}} e^{-(x_1 + x_2)} \sin \sqrt{2}(x_1 + x_2).$$

Now, Take the inverse AH-Isometry, then, we have the solution of the Neutrosophic differential equation .

$$Y = y_1 + y_2 I = T^{-1} \left(\frac{1}{4} e^{-x_1} \sin 2x_1 + \frac{1}{2\sqrt{2}} e^{-x_1} \sin \sqrt{2} x_1, \frac{1}{4} e^{-(x_1+x_2)} \sin 2(x_1 + x_2) + \frac{1}{2\sqrt{2}} e^{-(x_1+x_2)} \sin \sqrt{2}(x_1 + x_2) \right)$$

$$Y = y_1 + y_2 I = \frac{1}{4} e^{-x_1} \sin 2x_1 + \frac{1}{2\sqrt{2}} e^{-x_1} \sin \sqrt{2} x_1 + I \left[\frac{1}{4} e^{-(x_1+x_2)} \sin 2(x_1 + x_2) + \frac{1}{2\sqrt{2}} e^{-(x_1+x_2)} \sin \sqrt{2}(x_1 + x_2) - \left(\frac{1}{4} e^{-x_1} \sin 2x_1 + \frac{1}{2\sqrt{2}} e^{-x_1} \sin \sqrt{2} x_1 \right) \right]$$

$$Y = y_1 + y_2 I = \frac{1}{4} e^{-x_1} \sin 2x_1 + I \left(\frac{1}{4} e^{-(x_1+x_2)} \sin 2(x_1 + x_2) - \frac{1}{4} e^{-x_1} \sin 2x_1 \right) + \frac{1}{2\sqrt{2}} e^{-x_1} \sin \sqrt{2} x_1 + I \left[\frac{1}{2\sqrt{2}} e^{-(x_1+x_2)} \sin \sqrt{2}(x_1 + x_2) - \frac{1}{2\sqrt{2}} e^{-x_1} \sin \sqrt{2} x_1 \right]$$

$$Y = y_1 + y_2 I = \frac{1}{4} e^{-(x_1+x_2)I} \sin 2(x_1 + x_2) I + \frac{1}{2\sqrt{2}} e^{-(x_1+x_2)I} \sin \sqrt{2}(x_1 + x_2) I$$

$$Y = y_1 + y_2 I = \frac{1}{4} e^{-X} \sin 2X + \frac{1}{2\sqrt{2}} e^{-X} \sin \sqrt{2} X$$

5. Conclusion

In this paper, we have applied the one dimensional neutrosophic AH-isometry to study the algebraic direct image of Laplace transformation. Then, we have used this image to find solutions for some neutrosophic differential equations.

In the future, we aim to find more applications of AH-isometry in the study of neutrosophic structures.

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