CONTROL ALLOCATION FOR AIRSHIPS

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Abstract

Article presents application of two approaches to airship control allocation – sequential quadratic programming and model predictive control. Airship actuators produced forces and moments unified mathematical model is given, making possible of application for mostly any actuators configuration with any positioning and orientation. Simulation results of given forces and moments vector allocation on 4 rotatable actuators for spherical airship is given.

Keywords: airship, blimp, control, actuator, mathematical model, control allocation, unified control system, predictive control, quadratic programming, ACADO, resource management, dynamic scheduling, optimization.

Introduction

At the present time, the requirements for airship control systems are increasing with regard to the requirement of their wide configuration, which is related to the market demand in a unified control system applicable to airships of various designs. Separate control systems for each design of airships development appears to be economically inefficient, since airships are produced by various companies in small series. Thus, developing of methods for designing control systems for airships challenge is connected with unification to technical characteristics of airships and design of their actuators and control allocation problem consequently.

Airship can be equipped with a number of actuators (about ten), which significantly increases the dimensionality of the control problem being solved. In this connection, it is required to apply effective algorithms for calculating the required thrusts and angles of rotation of the engines according to the specified control forces and moments. In addition, the reliability requirements for aviation technology lead to the need to work in emergency mode in the event of failure of any executive mechanism, which also requires the use of new technologies for automatic reconfiguration of the control system.

Limited number of publications were devoted to this topic of control allocation for airships. The redistributed pseudoinverse method has been widely used to solve the unconstrained control allocation problem. Static control allocation for airship is used in [1]. Work [2] show a pseudo-inverse matrix approach for explicit control allocation problem for airships with no actuators limitation applied and note, that actuator limits introduction requires further development. Works [3, 4] shows possible approaches for solving this nonlinear non-convex optimization problem, including its linearization and using QP and MPC approaches, as far as explicit solution using Lagrange Multipliers for simplified task without actuators limitation applied for boat actuators model. Similar approaches could be applied for airship. A predictive control allocation application has also been proposed in [5].

Actuation mathematical model

Airship control system acts by means of control forces and moments F_u, M_u, formed by individual actuators forces F_{ui} and moments M_{ui}.

$$F_u = \sum_{i=1}^{n} F_{ui},\ M_u = \sum_{i=1}^{n} M_{ui},$$

where n – is number of actuators.

Actuators scheme configuration is one of parametrization of unified airships control system (in addition to airship body parameters).

Equations of the dynamics of actuators have the following form:

$$T\frac{d\delta}{dt} + \delta = K\delta_{cmd}, \tag{1}$$

where δ – the vector of actuators state (rotational speed of the propellers, the angles of rotations of the thrust vectors, etc.); *T*, *K* – diagonal matrices of time constants and gains of actuators; δ_{cmd} – vector of actuators controls. In the case where the actuator time constants are much less than the time constant of an airship, the matrix of time constants T in equation (1) is usually neglected.

Let's write down the forces and moments created by some actuator with thrust f_i located at the position determined by the radius vector r_i in the body coordinate system and fixed with the orientation determined by the rotation matrix with respect to the associated coordinate system R_i. We also assume that the actuator has a variable thrust vector for this model. Let, the *i*-th actuator thrust vector could rotate by pitch angle α_i from a horizontal axis.

Let's denote

$$\delta = \begin{bmatrix}\alpha \\ f\end{bmatrix},\ \delta_{cmd} = \begin{bmatrix}\alpha_{cmd} \\ f_{cmd}\end{bmatrix}, \tag{2}$$

where f is a *nx1* vector of actuators' thrusts f_i, α – *nx1* vector of actuators' rotation angles α_i, f_{cmd} and α_{cmd} are thrusts and angles controls respectively.

Then *i*-th actuator's thrust vector in body frame is calculated by:

$$F_{ui} = R_i \begin{bmatrix}\cos\alpha_i & 0 & \sin\alpha_i \\ 0 & 1 & 0 \\ -\sin\alpha_i & 0 & \cos\alpha_i\end{bmatrix}\begin{bmatrix}T_i \\ 0 \\ 0\end{bmatrix} = R_i \begin{bmatrix}\cos\alpha_i \\ 0 \\ -\sin\alpha_i\end{bmatrix} f_i. \tag{3}$$

Actuator's torque (in body frame) is calculated as follows:

$$M_{ui} = r_i \times F_i = \begin{bmatrix}0 & -r_{xi} & r_{yi} \\ r_{zi} & 0 & -r_{xi} \\ -r_{yi} & r_{xi} & 0\end{bmatrix} R_i \begin{bmatrix}\cos\alpha_i \\ 0 \\ -\sin\alpha_i\end{bmatrix} f_i. \tag{4}$$

For further use, we also introduce controls constraints for the *i*-th actuator:

$$f_i^{min} \le f_i \le f_i^{max} \tag{5}$$

$$\alpha_i^{min} \le \alpha_i \le \alpha_i^{max} \tag{6}$$

The total force and moment created by all actuators are determined by the expressions:

$$F_u = \sum_{i=1}^{n} F_{ui} = R_1 \begin{bmatrix}\cos\alpha_1 \\ 0 \\ -\sin\alpha_1\end{bmatrix} f_1 + R_2 \begin{bmatrix}\cos\alpha_2 \\ 0 \\ -\sin\alpha_2\end{bmatrix} f_2 + \cdots + R_n \begin{bmatrix}\cos\alpha_n \\ 0 \\ -\sin\alpha_n\end{bmatrix} f_n \tag{7}$$

$$M_u = \sum_{i=1}^{n} M_{ui} = \begin{bmatrix} 0 & -r_{x1} & r_{y1} \\ r_{z1} & 0 & -r_{x1} \\ -r_{y1} & r_{x1} & 0 \end{bmatrix} R_i \begin{bmatrix} \cos\alpha_1 \\ 0 \\ -\sin\alpha_1 \end{bmatrix} f_1 + \cdots + \begin{bmatrix} 0 & -r_{xn} & r_{yn} \\ r_{zn} & 0 & -r_{xn} \\ -r_{yn} & r_{xn} & 0 \end{bmatrix} R_n \begin{bmatrix} \cos\alpha_n \\ 0 \\ -\sin\alpha_n \end{bmatrix} f_n \quad (8)$$

We denote the combined vector of forces and moments of the actuators by τ:

$$\tau = \begin{bmatrix} F_u \\ M_u \end{bmatrix}. \quad (9)$$

Let us also designate the vector of desired (obtained from the automatic control system or operator) forces and moments through τ_{ref}.

We could reformulate (9) according to (7) and (8) as follows:

$$\tau = B(\alpha) f, \quad (10)$$

where $B(\alpha)$ is a 6xn control matrix, *i*-th 6x1 column of $B(\alpha)$ has following form:

$$B_i(\alpha_i) = \begin{bmatrix} R_i \begin{bmatrix} \cos\alpha_i \\ 0 \\ -\sin\alpha_i \end{bmatrix} \\ \begin{bmatrix} 0 & -r_{xi} & r_{yi} \\ r_{zi} & 0 & -r_{xi} \\ -r_{yi} & r_{xi} & 0 \end{bmatrix} R_i \begin{bmatrix} \cos\alpha_i \\ 0 \\ -\sin\alpha_i \end{bmatrix} \end{bmatrix}. \quad (11)$$

Control allocation

The corresponding control allocation problem is posed as follows: For the constrained system (1), (10) with constraints (5), (6) find $\delta_{cmd} = [\alpha_{cmd} \quad f_{cmd}]^T$ such that τ is as closely to τ_{ref} as possible.

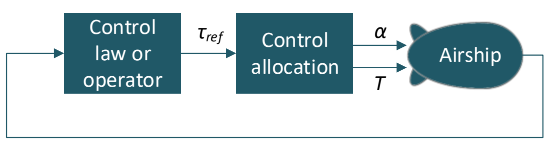

Figure 1 - The task of control allocation

Sequential quadratic programming control allocation

We form the problem of control allocation as a minimization problem:

$$\min_{\alpha_{cmd}, f_{cmd}} \{J = f^T W f + s^T Q s + (\alpha - \alpha_0)^T \Omega (\alpha - \alpha_0)\} \quad (12)$$

subject to

$$s = \tau - \tau_{ref} \quad (13)$$

$$f_{min} \preccurlyeq f \preccurlyeq f_{max} \quad (14)$$

$$\alpha_{min} \preccurlyeq \alpha \preccurlyeq \alpha_{max} \quad (15)$$

$$\Delta\alpha_{min} \preccurlyeq \Delta\alpha \preccurlyeq \Delta\alpha_{max} \quad (16)$$

The total power consumption is represented by the first term $T^T WT$ in the criterion, combining the power consumptions of the individual actuators. The second term $s^T Qs$ penalizes the error s between the commanded and achieved generalized force. This slack variable s formulation is necessary in order to guarantee that the optimization problem always has a feasible solution. The diagonal weights in the matrix Q are chosen so large that the constraint $s = \tau - \tau_{ref}$ is satisfied with $s \approx 0$ whenever possible. The maximum and minimum thrusts produced by the actuators f and angles of their rotation α are specified through the constraints (13), (14), where $\preccurlyeq$ means that the inequality is taken element-wise over the vectors left and right. Moreover, the rate-of-change in actuators rotation angles $\Delta\alpha$ is constrained and minimized such that a large change is only allowed if this is necessary, represented by the third term $(\alpha - \alpha_0)^T \Omega(\alpha - \alpha_0)$ in the criterion and the constraints (15). The matrix Ω is used to tune this objective, and the vector α_0 contains the azimuths at the previous sample.

Work [4] also introduces additional singularity avoidance term $\frac{\varrho}{\varepsilon + \det\left(B(\alpha)B^T(\alpha)\right)}$, where $\varepsilon > 0$ is required to avoid numerical problems and $\varrho > 0$ is a weighting parameter. Please refer [4] for details.

Dynamic reconfigurability and fault handling can be achieved by dynamically changing the constraint limits or weighting matrices.

The problem (12) – (16) can be locally approximated with a convex QP problem [4] by assuming that the power consumption can be approximated by a quadratic term in f, near the last force f_0 such that $f = f_0 + \Delta f$, $B(\alpha)$ could be approximated by a linear term linearized about the last azimuth angle α_0 such that $\alpha = \alpha_0 + \Delta\alpha$. The resulting QP criterion is [4]:

$$\min_{T,\alpha}\{J = (f_0 + \Delta f)^T W(f_0 + \Delta f) + s^T Qs + \Delta\alpha^T \Omega\Delta\alpha\} \tag{17}$$

subject to

$$s + B(\alpha_0)\Delta f + \frac{\partial}{\partial\alpha}(B(\alpha_0)f)\Big|_{\alpha_0, f_0} \Delta\alpha = \tau - B(\alpha_0)f_0 \tag{18}$$

$$f_{min} - f_0 \preccurlyeq f \preccurlyeq f_{max} - f_0 \tag{19}$$

$$\alpha_{min} - \alpha_0 \preccurlyeq \alpha \preccurlyeq \alpha_{max} - \alpha_0 \tag{20}$$

$$\Delta\alpha_{min} \preccurlyeq \Delta\alpha \preccurlyeq \Delta\alpha_{max} \tag{21}$$

The convex QP problem (17) – (21) can be solved by using standard software for numerical optimization. Authors used MATALB quadprog for simulation presented below.

Convergence toward a local minimum of J may happen, but simulation shows this approach is practical.

Model predictive control allocation

It is assumed that all control actuators have dynamics, which can be modelled by (1). The corresponding MPC control allocation problem is posed as follows: For the constrained system (1), (10) with constraints (5), (6) find $\delta_{cmd} = [\alpha_{cmd} \quad f_{cmd}]^T$ such that τ tracks τ_{ref} as closely as possible.

The system (1) is used to predict the commanded control inputs δ_{cmd}, the control states δ and outputs throughout the prediction horizon. The MPC algorithm finds the optimal set of $\hat{\delta}_{cmd}$ by minimizing a cost function on the form:

$$\min_{\alpha_{cmd}, f_{cmd}} J = \int_{t=0}^{T} \left(\tau(t) - \tau_{ref}(t)\right)^T W_\tau \left(\tau(t) - \tau_{ref}(t)\right) + \int_{t=0}^{T} f_{cmd}^T(t) W_P f_{cmd} \quad (22)$$

subject to

$$T\frac{d\delta}{dt} + \delta = K\delta_{cmd},$$

$$\tau = B(\alpha)f$$

$$f_i^{min} \le f_i \le f_i^{max}$$

$$\alpha_i^{min} \le \alpha_i \le \alpha_i^{max}.$$

In the cost function (22), W_τ is a weight matrix weighing the importance of tracking. W_P weighs the relative cost of use of effector.

Only the first commanded control sample is applied to the actuator. The whole algorithm is repeated.

The toolkit employed to generate the nonlinear solver is ACADO [6] which is able to generate very fast custom C code solvers for general optimal control problems.

Simulation results

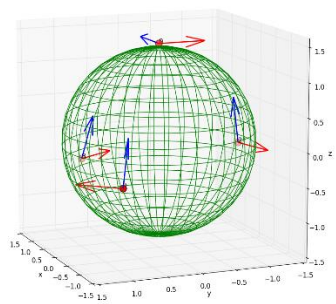

Figure 2 – Actuators position on sphere airship

Let's consider an example of distribution of controls for a spherical blimp with a diameter of 2.84 m (volume 12 m3) with 4 rotary propellers (Aerotain-like, [7]). The coordinates of the location of the actuators are determined from the condition of uniform distribution on the sphere:

$$r_1 = [0.000 \;\; 0.000 \;\; 1.420], \qquad r_2 = [-0.995 \;\; 0.929 \;-0.404],$$

$$r_3 = [0.325 \;-1.370 \;-0.187], \qquad r_4 = [1.133 \;\; 0.745 \;-0.421].$$

The orientation of the actuators is determined by the rotation matrices:

$$R_1 = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \qquad R_2 = \begin{bmatrix} 0.682 & -0.700 & 0.208 \\ 0.731 & 0.654 & -0.194 \\ 0 & 0.284 & 0.958 \end{bmatrix},$$

$$R_3 = \begin{bmatrix} -0.973 & 0.229 & -0.030 \\ -0.231 & -0.965 & 0.128 \\ 0 & 0.132 & 0.991 \end{bmatrix}, \qquad R_4 = \begin{bmatrix} 0.550 & 0.798 & -0.248 \\ -0.835 & 0.525 & -0.162 \\ 0 & 0.296 & 0.955 \end{bmatrix}$$

We first set τ_{ref} to simulating ascent, forward motion, yaw rotation, forward motion rotating around y axis as follows:

$$\tau_{ref}(t) = \begin{cases} [50;\ 0;\ -50;\ 0;\ 0;\ 0], & t < 250 \\ [100;\ 0;\ 0;\ 0;\ 0;\ 0], & 250 \le t < 500 \\ [0;\ 0;\ 0;\ 0;\ 0;\ 20], & 500 \le t < 750 \\ [100;\ 0;\ 0;\ 0;\ 20;\ 0], & 750 \le t < 1000 \end{cases}$$

SQP approach simulation results are show below.

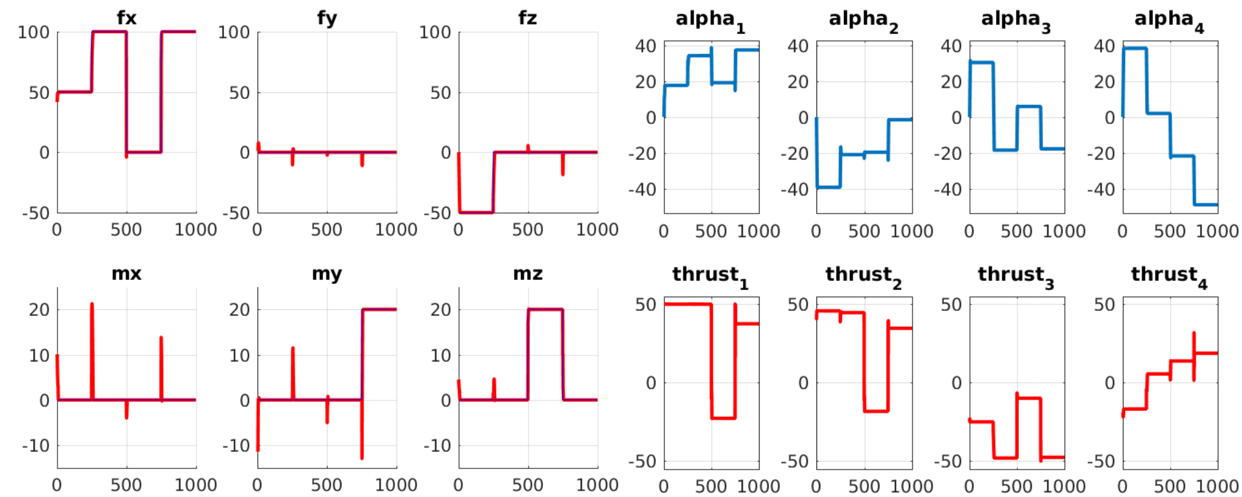

Figure 3 – Sequential quadratic programming control allocation simulation: left – given (blue) and resulting (red) forces and moments, right – controls

MPC approach simulation results are show below.

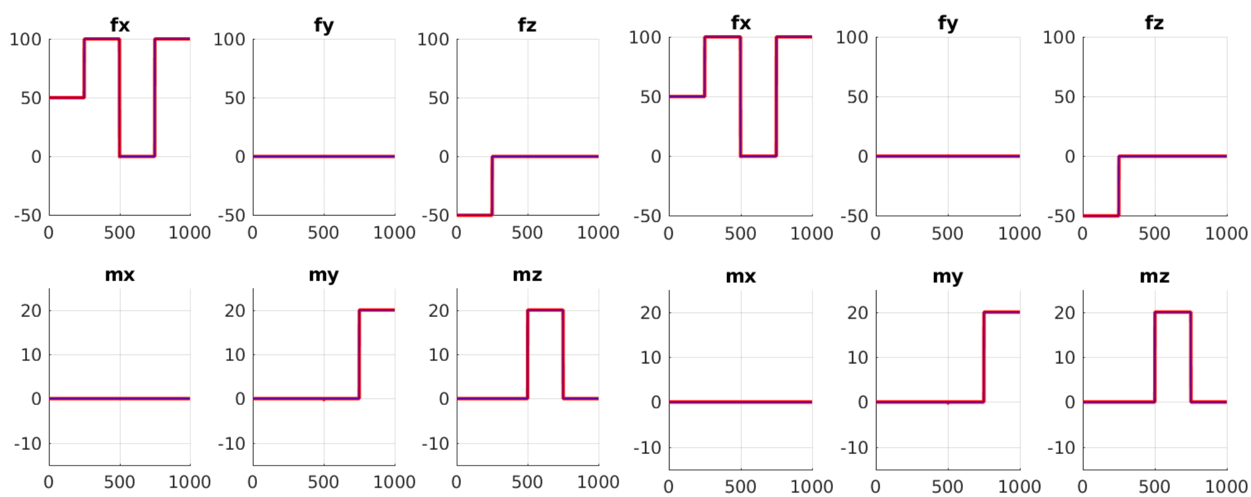

Figure 4 – MPC control allocation simulation: left – given (blue) and resulting (red) forces and moments, right – controls

At second experiment we use some continuous functions to set τ_{ref}:

$$\tau_{ref} = \begin{bmatrix} 20 * \sin(t/100) \\ 30 * \cos(t/100) + t/5 \\ 40 * \sin(t/200) \\ 10 * \sin(t/150) \\ t/50 \\ t^2/50000 \end{bmatrix}$$

SQP approach simulation for second τ_{ref} results are show below.

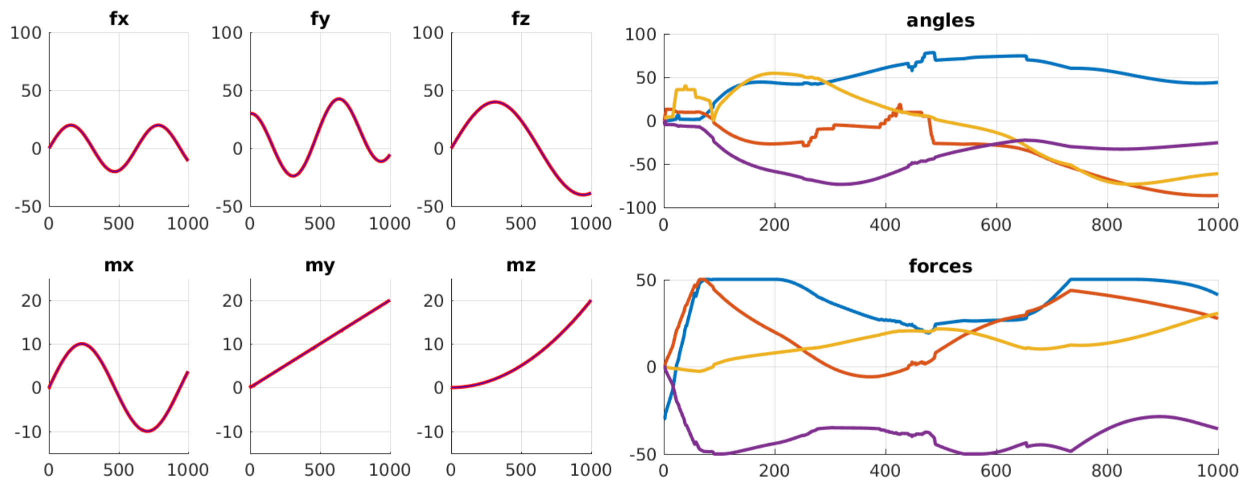

Figure 5 – Sequential quadratic programming control allocation simulation: left – given (blue) and resulting (red) forces and moments, right – controls

MPC approach simulation for second τ_{ref} results are show below.

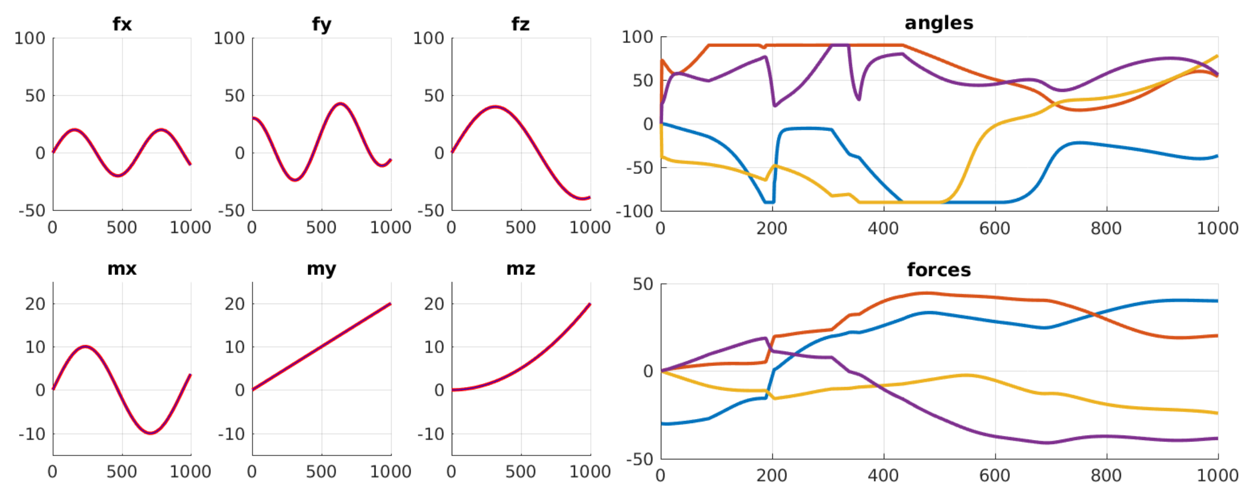

Figure 6 – MPC control allocation simulation: left – given (blue) and resulting (red) forces and moments, right – controls

Conclusion

Application of two approaches to airship control allocation – sequential quadratic programming and model predictive control presented. Unified mathematical model of actuators forces and moments is given, making possible of application for mostly any actuators configuration with any positioning and orientation. Simulation results of given forces and moments vector allocation on 4 rotatable actuators for spherical airship is given.

Simulation shows both approaches are applicable for control allocation task. Sequential programming showed some error at moments of large change of reference, but is similar to MPC result for continuous reference change, when there are not much alphas angles changes.

Further research should be done on closed loop system with airship dynamics and performance comparison would be interesting too.

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