Risk Propagation through Payment Distortion in Supply Chains

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The supply chain literature has devoted much attention to studying how the variability of orders propagates upstream. We focus, instead, on how the variability of payments to suppliers propagates upstream, which has a major impact on risk. To do so, we build a supply chain model based on empirical findings from the finance literature. We show that payment variability may occur even if orders are constant, and that this variability may propagate and even increase at upper echelons. We show that this phenomenon is caused by the limited access to debt and identify the factors that drive the propagation of variability—the industry risk, the firm’s operational leverage, the existence of a financial leverage target, and the cost of debt. By studying the limiting distribution of the corresponding Markov chain, we numerically illustrate the impact of these drivers on the risk of upper echelons as well as the interactions between order and payment variability. We provide a number of insights and propose measures for risk management.

Key words: risk; bullwhip effect; credit contagion; supply chain-finance link

1. Introduction

Supply Chain Management (SCM) is concerned with three flows—inventory, information, and money. The SC literature has traditionally devoted much attention to inventory and information flows, and not as much to financial flows. This does not mean that this literature has been oblivious to the importance of funds in operational settings. Cash, however, is usually seen as a “stock” more than a “flow” (e.g., financial constraints imposed when investing in capacity or inventory) and, since cash is what ultimately constrains the activity of most firms, not paying attention to financial flows may lead companies to financial distress and, possibly, bankruptcy. Paying attention to financial flows is even more relevant in the current economic context, when firms are more leveraged and have more difficulties raising additional funds. Furthermore, supply chains are becoming longer and broader, making it more difficult to assess the overall effect of decisions made on both realms, operational and financial.

A key observation that motivated this work, and that may well be driven by some of the facts just mentioned, is the existence of “financial contagion,” an effect that arises when firms facing customers’ defaults on trade credit (i.e., customers paying later than agreed) are more likely to default themselves to their suppliers (Boissay and Gropp 2007). This phenomenon has been addressed in the financial literature (e.g., Allen and Gale 2000, Egloff et al. 2007). The existence of financial contagion via trade credit defaults suggests not only that payments to suppliers are

1 For instance, our calculations show that retailers’ financial leverage in the US has increased by 40% in the last 40 years. Source: COMPUSTAT, US retailers (SCI Code 5331) 1969-2008.
subject to variability, but that that variability is somehow transmitted upstream. In the particular case of a firm holding inventory, the relationship between payments from customers and payments to suppliers may be intricate. In fact, material and financial flows are intimately related. Acquiring inventory today entails paying suppliers now or in the future, and decisions on how much inventory to buy in a period may well depend on when financial inflows from customers are estimated to occur.

If cash followed the order flow patterns, the variability of payments would exactly replicate the variability of orders and any of the causes of the bullwhip effect (Lee et al. 1997) would create payment variability amplification. However, in reality, cash flows may deviate from order flows, and payment variability may occur even if there is no order variability.

In this work, we are interested in exploring the causes and conditions of payment variability and amplification beyond the underlying order variability. Understanding the variability of payments is key to quantifying the risk of a firm, which is an issue of major importance, as the impact of financial contagion may spread from a single dyad to an entire industry, and even to the whole economy (Bardos and Stili 2007). The crisis that began in 2007 is a good example of widespread contagion because of “massive illiquidity” (Tirole 2010). Right after the Lehman Brothers episode in September 2008, the credit crisis worsened among financial institutions precisely because of the fear of financial contagion (Jorion and Zhang 2009).

Three recent trends make financial contagion through trade credit particularly relevant. First, firms heavily rely on trade credit. During 2001 in France, “trade payables stood at 103% of manufacturing firms’ financial debt and 219% of their bank borrowing” (Bardos and Stili 2007). For US retailers, accounts payable represent 53% of long-term debt and have increased by 38% in 40 years with respect to total assets. Second, firms default on trade credit agreements. According to the National Survey of Small Business Finance, as much as “46% of the firms declared that they had made some payments after the due date during the last year” (Cuñat 2007). Similar claims are found in Boissay and Gropp (2007). The consequences of trade credit defaults can be so strong on a supplier that they may push the company to bankruptcy. In fact, bankruptcy is caused by customers’ bankruptcy or default on trade credit in 10% to 20% of the cases (Blazy and Colombier 1997). Finally, firms who rely more on trade credit are more likely to go bankrupt themselves. This phenomenon increases the probability of starting or passing the financial “disease” (Boissay and Gropp 2007).

As our goal is to understand reality by observing it, our work is descriptive in nature, not normative. By better understanding the mechanisms that create and propagate payment variability, we hope to shed light on how to improve the ability of firms and regulators to prevent the potential undesired effects of this variability, such as financial distress and bankruptcy.

To address our research questions, we define a simple supply chain in which the first echelon faces stationary random demand. Every echelon pays its only supplier following simple rules, which are mainly grounded in the financial empirical literature (Boissay and Gropp 2007, Bardos and Stili 2007, Cuñat 2007). On analyzing this model, we study the limiting distributions of each echelon’s payments to its supplier, observe which portion of the variability of payments gets propagated for various sets of parameters, and identify the drivers of such a propagation.

To facilitate the analysis, our model is parameterized so that orders are constant for all echelons, i.e., we eliminated the causes of the order bullwhip effect (Lee et al. 1997). Despite this, we still find that payment variability and even amplification may occur. We show that these effects are caused by the inability of buyers to unlimitedly access financial debt and are modified by the drivers that impact industry, operational, and financial risks. We find that managers’ decisions on these drivers

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affect not only their firms’ returns, but also those of its higher echelons in the supply chain. The impact on upstream suppliers may be significant unless at least one of the firms in the supply chain exhibits “solid” financial statements (for instance, low financial leverage), which would mitigate the propagation of payment variability, a result in line with Boissay and Gropp (2007). However, when the echelons in the chain have “weak” financial statements, payment variability spreads and can even be amplified in upper echelons. We illustrate the interaction between order and financial flow variabilities by exploring up-to-level policies and by allowing managers to consider the firm’s financial status to inform their operational decisions. We find the interaction between order and payment variability intricate, as, for instance, our results qualitatively change depending on the demand distribution, or a financially-aware order policy might increase order variability but decrease overall payment variability. Finally, we explore the case of a supplier serving several retailers, and study the impact of the number of retailers and the correlation of retailers’ demands on the supplier’s payment variability.

The rest of the paper is organized as follows: §2 relates our work to the extant literature. In §3 we present the model and derive basic propositions about risk propagation in the supply chain. Then, we explore the impact of the firm’s industry, operational, and financial risk on payment variability as well as its interaction with order variability and the number of downstream players (§4). We propose measurements for assessing bankruptcy risk in §5 and conclude by discussing some managerial and regulatory implications of our work in §6.

2. Literature Review

Some of the mechanisms of risk transmission have already been studied. Within the financial literature, the empirical work by Bardos and Stili (2007) summarizes the work in Stili (2003), who studies defaults on trade credit in France and the “risk contagion” phenomenon, i.e., how borrowers’ risk is transmitted to lenders. They identify patterns necessary for risk contagion and bankruptcy to occur and find that risk transmission occurs when receivables represent a significant portion of total assets (Bardos and Stili 2007). Interestingly, they state that payment defaults are mainly provoked by retailers and wholesalers (43%), and most likely absorbed by wholesalers (80%). Boissay and Gropp (2007) extend the latter work and focus on the propagation of risk in long chains. They argue that trade credit default chains exist, and that firms that have difficulties accessing new funds pass the liquidity shocks they face to their suppliers. They identify the existence of “deep pockets”, i.e., firms with robust balance sheets who stop the chain of defaults by not passing the liquidity shocks to the suppliers, and who inject liquidity in the industry where is needed the most. They also state that, even when firms have suffered trade credit defaults, they continue to give trade credit, providing some sort of insurance to their customers.

This does not mean that firms continuously default on their suppliers. On the contrary, according to Cuñat (2007), trade credit is used only when other forms of credit (debt holders, shareholders) are not available. Cuñat (2007) presents a theory on the role of trade credit agreements, arguing that suppliers may have a comparative advantage over banks when lending money to firms because they can stop the supply of goods. In this paper, we do not look at trade credit agreements per se, but rather assume them and look at trade credit defaults while taking these agreements as given. The model in Cuñat (2007) focuses on under which conditions suppliers will become liquidity providers for their customers, i.e., will lend them money through trade credit. The empirical part is interesting for us in that it sheds light on how suppliers behave in the presence of customers’ liquidity shocks. Specifically, two assumptions we make are grounded in findings in Cuñat (2007), namely that firms affected by financial shocks rely on their suppliers, and that suppliers keep funding their customers even when they default on trade credit agreements.

The financial literature also provides models on credit contagion. Kiyotaki and Moore (1997) define a network of firms to study how shocks propagate and why firms do not insure against
accounts receivable shocks. They also study the relationship between credit limits and collateral prices, and find that this relationship plays a key role in the transmission of shocks. Kiyotaki and Moore (2002) study why contagion seems to be country-dependent. More recent financial models resort to using Markov chains (Giesecke and Weber 2006, Frey and Backhaus 2004, Egloff et al. 2007). All these financial models are high-level, parsimonious models, in which the local interactions between firms are not always entirely captured. For example, they tend to ignore the effect of inventory decisions on credit chains. This is rather surprising since a large proportion of trade credit defaults occurs among wholesalers and retailers (Bardos and Stili 2007), who hold large levels of inventory (Gaur et al. 2005), and usually have the ability to decide on their inventory level target.

In the operations management literature, the models on risk propagation are more specific about the local interaction between firms. Battiston et al. (2007) study bankruptcy propagation, either upstream or downstream, in production networks connected by credit ties, where financially constrained firms have to adapt when one of the firms in the vicinity of the network goes bankrupt. The focus is not on cash flow variability, but on bankruptcies. Using simulation, they measure the impact of the cost of debt and the ability to adapt on production levels and growth. Tsai (2008) studies the impact of reducing working capital, a common practice in many industries, on the risk of a manufacturer, as measured by cash flows variability. He argues that the usual mechanisms aimed at reducing the cash-to-cash cycle have a negative impact on risk due to the uncertainty about the time when payments are made. Xu et al. (2010) also focus on firms bankruptcies, and propose the use of collaborative formulas between firms, such as information sharing and vendor-managed inventory, to reduce the probability of bankruptcy. They propose modifications of these mechanisms (e.g., VMI) to encourage cooperation between firms.

Our model borrows the findings from the financial literature mentioned above to state its assumptions, and makes relevant contributions in developing an operational understanding of the mechanisms of financial contagion. Specifically, it explains how payment variability gets created, propagated, and amplified as one moves upstream of the supply chain. In this regard, we depart from Tsai (2008), who measures risk creation for inflows and outflows, but is not concerned with how that risk gets propagated and/or amplified, and from Battiston et al. (2007) and Xu et al. (2010), whose focus is not on risk. Secondly, it describes the interactions between the echelons in the supply chain in more detail, considering the impact on the different elements of the balance sheets of firms. Thirdly, it does not focus on the effects of variability, such as bankruptcy as in Battiston et al. (2007) or Xu et al. (2010), but on the mechanisms that drive such effects. Finally, it considers the role of inventory decisions that managers make in response to the financial or operational status of the firms they manage, and measures the impact of such inventory decisions on risk propagation.

3. Framework
As noted, our main concern is to study the propagation of risk in supply chains due to the distortion of payments received. Therefore, we will focus on the variability of payments received, which is a key element of risk, and its upstream propagation. By focusing on payments received, we ignore some other aspects of risk, such as the impact of the financial and operational leverages of the firm on its own risk, but still consider the impact of those on the risk of upper echelons, since the payments made to suppliers depend on the own financial and operational leverages.

In the next sub-section we describe our model. Our key financial assumptions—such as those related to capital structure, dividends policy, or payment priorities—are grounded on findings from the empirical financial literature. We provide the relevant references as we present our model.
3.1. Model

Consider a serial supply chain consisting of \( n \) echelons, a retailer \((i=1)\) and its subsequent suppliers \((i = 2, 3, \ldots, n)\), where each echelon serves only one customer (an assumption relaxed in §4.6) and is served by only one supplier (see Figure 1). The retailer is a newsvendor who faces i.i.d. demand, \( \bar{\xi}_0 \), and sells at constant price, \( p \). To simplify physical flows, we assume that leftovers are lost and lead-time is zero. When computing order quantities, all echelons maximize their expected profit as if bankruptcy did not exist—this assumption is relaxed in §4.4. Echelons buy the demand they face at constant cost, \( c_i \), thus upper echelons sell at constant price, \( c_{i-1} \).

All echelons may make use of financial debt \((d_i)\) and trade credit \((y_i)\) as sources of funds. Financial debt includes cash, so it can be negative. According to trade credit agreements, buyers should pay suppliers \( \tau_i \) periods after receiving the goods, while the market pays cash. The portion of accounts payable not agreed with the supplier through \( \tau_i \) is denoted \( \bar{y}_i \). Financial leverage is represented by the parameter \( \theta_i \), defined as the ratio of relevant debt, \( d_i + \bar{y}_i \), to equity level, \( e_i \), at book value. Observe that relevant debt is total debt (from banks and suppliers) less the portion of accounts payable agreed with the suppliers. All echelons define a leverage target, \( \theta^*_i \), such that if leverage falls below that target, excess cash is paid to shareholders as dividends (e.g., Lin and Drekic 2003). The financial debt provider (e.g., a bank) imposes a limit, \( \theta^f_i \), on the financial leverage of firm \( i \), above which additional financial debt cannot be obtained. Also, as observed by Cuñat (2007), sellers accept that buyers default on trade credit up to a certain limit, \( \theta^b_i \), i.e., as long as \( \theta_i \leq \theta^b_i \). We assume herein that \( \theta^*_i \leq \theta^f_i \leq \theta^b_i \). The cost of debt and equity level at each echelon, \( r_i \) and \( e_i \), are constant and exogenous. Table 1 summarizes the notation used.

The order of events is as follows:

1. All echelons, in increasing order, place orders, i.e., lower echelons order first.
2. All echelons, in decreasing order, receive inventory.
3. Market demand is realized.
4. All echelons, in increasing order, make payments, i.e., the market pays first. We assume some “pecking order” when paying, since firms may default on trade credit only when other forms of credit are not available (Cuñat 2007). Thus, payments are made according to the following list of decreasing priorities:
   1. Fixed costs and interest expenses.
   2. Purchases due payments. If an echelon has not enough cash to pay its supplier, debt is raised from a bank to complete the payment, up to a limit, \( \theta^f_i \). If the payment cannot be completely satisfied, the buyer defaults on trade credit, and the buyer has an obligation to pay for the unsatisfied portion the following period (Boissay and Gropp 2007). If accounts payable grow beyond a threshold, such that \( \theta_i > \theta^b_i \), the firm cannot resort to additional sources of funds and goes bankrupt. In this case, the portion of payables that make leverage exceed the limit \( \theta^b_i \) is released.
   3. Loan repayment. If possible, a portion of the principal of the debt is repaid so as to reduce the financial leverage of the firm down to the leverage target, \( \theta^*_i \).
### Table 1  Notation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
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<tbody>
<tr>
<td>( p )</td>
<td>Market price</td>
</tr>
<tr>
<td>( c_i )</td>
<td>Purchasing cost at echelon ( i )</td>
</tr>
<tr>
<td>( \tau_i )</td>
<td>Payment delay agreed between buyer ( i ) and its supplier</td>
</tr>
<tr>
<td>( r_i )</td>
<td>Cost of debt at echelon ( i )</td>
</tr>
<tr>
<td>( f_i )</td>
<td>Fixed cost incurred at echelon ( i )</td>
</tr>
<tr>
<td>( e_i )</td>
<td>Constant equity level at echelon ( i )</td>
</tr>
<tr>
<td>( \theta_i^* )</td>
<td>Financial leverage target at echelon ( i ) (exogenously defined)</td>
</tr>
<tr>
<td>( \theta_i^f )</td>
<td>Financial limit imposed by echelon ( i )'s bank</td>
</tr>
<tr>
<td>( \theta_i^b )</td>
<td>Bankruptcy limit above which echelon ( i ) goes bankrupt</td>
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<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
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<tbody>
<tr>
<td>( \tilde{\xi}_0 )</td>
<td>Random market demand</td>
</tr>
<tr>
<td>( \xi_{0,t} )</td>
<td>Market demand realization at time ( t )</td>
</tr>
<tr>
<td>( \varphi_{0,t} )</td>
<td>Payment made by the market at time ( t )</td>
</tr>
<tr>
<td>( \varphi_{i,t} )</td>
<td>Payment made by echelon ( i ) at time ( t )</td>
</tr>
<tr>
<td>( q_{i,t} )</td>
<td>Quantity ordered by echelon ( i ) at time ( t )</td>
</tr>
<tr>
<td>( d_{i,t} )</td>
<td>Financial debt level at echelon ( i ) and time ( t )</td>
</tr>
<tr>
<td>( y_{i,t} )</td>
<td>Accounts payable at echelon ( i ) and time ( t )</td>
</tr>
<tr>
<td>( y_{i,t}^\ast )</td>
<td>Portion of ( y_{i,t} ) above agreement with supplier ( i + 1 )</td>
</tr>
<tr>
<td>( x_{i,t} )</td>
<td>Relevant debt at echelon ( i ) and time ( t ), ( x_{i,t} = d_{i,t} + y_{i,t} )</td>
</tr>
<tr>
<td>( m_{i,t} )</td>
<td>Buyer’s payment less fixed costs and interest expenses at echelon ( i ) and time ( t )</td>
</tr>
<tr>
<td>( l_{i,t} )</td>
<td>Money available to pay the supplier at echelon ( i ) and time ( t )</td>
</tr>
<tr>
<td>( \theta_{i,t} )</td>
<td>Financial leverage at echelon ( i ) and time ( t ), ( \theta_{i,t} = x_{i,t}/e_i )</td>
</tr>
</tbody>
</table>

4. Dividends. All remaining cash, if any (i.e., that available after reducing leverage down to the target, \( \theta_i^* \)), is paid as dividends.

To write the implied dynamics mathematically, let a second subscript denote time (e.g., \( \varphi_{i,t} \) is the payment made by echelon \( i \) at time \( t \)). Unless otherwise stated, \( i \in \{1, 2, \ldots, n\} \) and \( t \in \{1, 2, \ldots\} \) herein.

Since \( \tilde{\xi}_0 \) is i.i.d. and leftovers are lost, the retailer is a repetitive newsvendor who orders the usual critical fractile

\[
q_{1,t} = \arg\max_{q_{1,t} \geq 0} \{ pE\min(q_{1,t}, \tilde{\xi}_0) - c_1 q_{1,t} \} \tag{1}
\]

Upper echelons buy the demand they observe

\[
q_{i,t} = q_{i-1,t}, \quad i = 2, 3, \ldots, n \tag{2}
\]

Right after market demand is realized the market pays the retailer (cash)

\[
\varphi_{0,t} = p \min(q_{1,t}, \xi_{0,t}) \tag{3}
\]

Let \( m_i \) be the buyer’s payment less fixed costs (\( f_i \)) and interest expenses. For tractability, demand is such that \( m_i \) is always non-negative.

\[
m_{i,t} = \varphi_{i-1,t} - f_i - r_i d_{i,t-1}, \quad \text{with} \quad m_{i,t} = 0 \quad \text{for} \quad t \leq \tau_i \tag{4}
\]

The money available to pay the seller, \( l_i \), depends on the current disposable amount of financial debt plus the buyer’s payment surplus of the current period, \( m_{i,t} \)

\[
l_{i,t} = \theta_i^f e_i - d_{i,t-1} + m_{i,t} \tag{5}
\]
The actual payment to the seller is the minimum of two quantities: the money available to pay and the due payment to the seller, which is the sum of the purchasing cost \( \tau_i \) periods ago plus previous unsatisfied payments due \( (\bar{y}_{i,t-1}) \), if any

\[
\varphi_{i,t} = \min(l_{i,t}, c_i q_{i,t-\tau_i} + \bar{y}_{i,t-1}), \quad \text{with } \varphi_{i,t} = 0 \text{ for } t \leq \tau_i \tag{6}
\]

The financial debt is limited by two bounds, the bank’s imposed limit, above which the firm cannot resort to additional funds from banks, and the leverage target, below which the firm pays dividends

\[
d_{i,t} = \min(\theta_i^f e_i, \max(\theta_i^* e_i, d_{i,t-1} + \varphi_{i,t} - m_{i,t})) \tag{7}
\]

As noted, the financial debt may be negative if the leverage target is negative.

As for accounts payable, if the firm has gone bankrupt in the current period, some payables are released such that the firm is left on the edge of bankruptcy; otherwise, payables are normally updated. The term \((\theta_i^b - \theta_i^f) e_i\) in equation (8) limits the amount of accounts payable beyond the agreement with the supplier, which is captured by the term \(\sum_{j=t-\tau_i+1}^t c_i q_{i,j}\).

\[
y_{i,t} = \min\left((\theta_i^b - \theta_i^f) e_i + \sum_{j=t-\tau_i+1}^t c_i q_{i,j}, y_{i,t-1} + c_i q_{i,t} - \varphi_{i,t}\right) \tag{8}
\]

\[
\bar{y}_{i,t} = \left(y_{i,t} - \sum_{j=t-\tau_i+1}^t c_i q_{i,j}\right)^+, \quad \text{with } a^+ = \max(0, a) \tag{9}
\]

Relevant debt, \(x_{i,t}\), is the sum of financial debt and the portion of accounts payable not agreed with the seller

\[
x_{i,t} = d_{i,t} + \bar{y}_{i,t} \tag{10}
\]

Finally, the initial state is defined by \(d_{i,0} = y_{i,0} = \bar{y}_{i,0} = 0\).

Given the structure of equations (1) to (10), the process can be described as a Markov chain, where the state is defined by the triplet \((x_{i,t}, y_{i,t}, \bar{y}_{i,t})\) at each echelon. The relevant debt at each echelon is limited by two barriers, one due to the payment of dividends \((\theta_i^* )\), another due to the trigger of the bankruptcy procedure \((\theta_i^b)\). Figure 2 pictorially represents the Markov chain through the relevant debt position. It consists of \(n\) special random walks, where the magnitude of the transitions depends not only on the parameters, but also on the own and downstream states.
We will confine ourselves to the interesting case where for low (high) values of $\phi_{i-1,t}$, $m_{i,t}$ is lower (higher) than $c_{i}q_{i,t}$. These conditions guarantee that both barriers at each echelon are hit with positive probability.

### 3.2. Payment variability creation

In this simple model, if firms had unlimited access to financial debt, then buyers would always be able to pay sellers on time and payments would follow the same pattern as orders and inventory (lagged if lead-time is not zero or trade credit is allowed), and so the variability of payments would exactly replicate the variability of orders. However, if the access to financial debt is limited, firms may not be able to pay sellers on time, thus increasing variability of payments as sometimes the payment will be lower than the corresponding order and, later, the payment will be higher than the corresponding order to reduce the buyers’ payables. Therefore, it should be apparent that the limited access to financial debt may create variability of payments beyond the variability of orders, as the following proposition formalizes.

**Proposition 1.** If $\theta_{f}^{i} < \infty$, then $CV\phi_{i} > CVq_{i} = 0$. Furthermore, if $\theta_{f}^{i} = \infty$, $CV\phi_{i}$ decreases with $\theta_{f}^{i}$.

In words, the first result states that, if credit is limited for the buyer, the variability of payments to the supplier, as measured by the coefficient of variation, is larger than the variability of orders to the supplier. This is expected, since a borrowing limit prevents the buyer from paying always the seller on time, as $\text{Prob}(l < c_{i}q_{i})$ becomes positive—see equation (6). This will occur after a number of periods with relatively low demand during which the buyer should face her obligations to pay the seller, but cannot resort to a bank to raise additional funds. The proposition does not hold if the access to financial debt is not restricted, since the bank could always provide the buyer with the funds necessary to pay the seller on time. In the latter case, the variability of payments and orders would be zero. Only when financial debt is restricted, the variability of payments is greater than zero, i.e., some variability is created.

The second result states that the payment variability increases with financial credit restrictions (increasing/decreasing are used here in a weak sense), which is a non-trivial result, as some of the payments made under restricted credit conditions will be closer to the mean payment than those made when credit is not restricted. A direct consequence is that restricting the buyer’s financial credit leads to increasing the seller’s risk. We note that the condition $\theta_{b}^{i} = \infty$ is sufficient, and we conjecture that is not needed for the result to hold, although we were not able to prove it.

Finally, it should be noted that, in the order space, the use of order variance as a measure of comparable variability (Lee et al. 1997) requires the assumption of a single unit of input for unit of output. In our case, because of the different prices among echelons, we use the coefficient of variation of payments as a comparable metric of variability. This adjustment of metric would also be necessary to compare order variability if multiple units of input were required per unit of output, e.g., tires for a vehicle.

### 3.3. Payment variability propagation

If created, variability may be propagated upstream, since it may have an impact on the sellers’ ability to pay their own suppliers on time.

**Proposition 2.** If $\theta_{b}^{i} = \infty$, then $CV\phi_{i}$ and $CV\phi_{i+1}$ decrease with $\theta_{f}^{i}$.

Put differently, as credit to the buyer gets tighter, the variability of payments to the seller and its supplier (weakly) increase. This results follows from the first part of Proposition 1, as facing a higher variation coefficient of payments implies that some payments are lower, which is equivalent to have tighter financial credit restrictions as far as the ability to pay the supplier is concerned.
Again, note that if the access to financial debt were not restricted, the proposition would not hold, since the seller could always pay his supplier on time, and the variability of the payments to the supplier would not increase with the variability of the payments to the seller, but would be identical to the variability of the orders placed to his supplier (i.e., zero). Only when financial debt is restricted, may some variability be transmitted from the seller to his supplier. In the latter case, note that not all variability is transmitted, since the buffer of cash provided by the seller’s access to financial debt (plus the period’s $m_i$ and the cash reserve, if they are positive) may absorb a portion of that variability. Finally, similar to our discussion after Proposition 1, restricting the buyer’s financial credit leads to increasing not only the seller’s risk, but also that of the seller’s supplier.

3.4. Payment variability amplification

If variability is propagated, a relevant question is also whether it increases or not upstream, i.e., whether there is variability amplification à la Lee et al. (1997) in the payment space. To isolate the conditions under which there is amplification and get a better intuition of why that is the case, we explore an extreme case with special conditions by focusing on a retailer facing the simplest demand distribution (low-high).

**Proposition 3.** Consider a retailer facing demand $\tilde{q}_0 = q_1 B(0.5)$, where $B(\cdot)$ is a Bernoulli distribution. Let $p = kc_1$, with $k > 2$—to avoid trivial solutions—and integer. If $f_1 = \tau_1 = \theta_1^U = \theta_1^I = 0$, and $\theta_1^b = \infty$, then $CV(\varphi_1) > CV(\varphi_0)$.

In words, the variability of the payments to the wholesaler is always larger than the variability of the payments to the retailer.

Note that the assumptions of the proposition are sufficient to observe amplification. Less restrictive assumptions may also lead to amplification, as shown in the next section. However, relaxing some of the assumptions would make the analytical proof intractable. We are making the point that amplification in the financial space is possible under restrictive conditions even if there is no order variability in the information space. In the real world, while variability can be introduced in the chain by a weak player, and it can be further propagated, the conditions for that variability to be amplified as we move up the supply chain may be unusual. While tempting to refer to the contagion effect as the “financial bullwhip”, we found that the conditions to create payment variability amplification (beyond the variability of orders) are so restrictive that it would only happen in the most cash-restricted supply chains, e.g., during a credit crisis. Thus, the main focus of the paper is on propagation and not on amplification.

Note also that the result could apply to any dyad buyer-seller as long as the buyer faced a similar demand distribution. Again, the assumption that financial debt is restricted is key for the proposition to hold. Furthermore, the conditions of the proposition ensure that the cash buffers are absent both in the firm (through an aggressive dividend policy) and in banks (since raising financial debt is forbidden). Under these conditions, the variability of payments is amplified as it propagates through the supply chain.

In sum, the three propositions above state that, even if there is no order variability, variability of financial flows may be created, propagated upstream, and amplified if access to the financial markets is restricted. These findings resonate with work previously done on order amplification in supply chains (Sterman 1987, Lee et al. 1997). While Lee et al. find four necessary and sufficient conditions for order amplification, we identify one necessary condition for payment variance and amplification (limited access to debt). Furthermore, just like order amplification creates operational distress in upstream echelons through higher production costs, payment amplification creates financial distress in upstream echelons through higher financial costs.

In the following section, we explore the effects of different risk sources on payment variability, how they impact its propagation in the supply chain, and how payment variability interacts with order variability.
4. Numerical study
In this section, we first characterize the Markov chain that results from the model defined so as to calculate the payment limiting distributions, and then numerically study the impact of various drivers on variability creation, propagation, and amplification.

4.1. Limiting distributions
Our concern is to find how the limiting distribution of payments made by the retailer (respectively, wholesaler) is related to that of the market (respectively, retailer) depending on the parameters. In particular, we focus on the variability of the payment distributions, as they are related to the risk faced at each echelon. We find the payment distributions for both retailer and wholesaler through the calculation of the limiting distribution of the Markov chain, whose transition-probability matrix, $P$, is derived according to equations (1) to (10). Unless otherwise stated, we limit ourselves to settings where the chain is finite, so that $P$ is also finite.

Denoting by $\pi$ the column vector of probabilities for the limiting distribution, $\pi$ satisfies $\pi = P\pi$, with $\sum_{i=1}^{N} \pi_i = 1$, where $N$ is the number of states. Let $\hat{P}$ be the transpose of the matrix that results from substituting the last row of $P - I_N$ by ones, where $I_N$ is the $N$-dimension diagonal matrix. It can be shown that $\pi = \hat{P}^{-1}e_N$, where $e_N$ is the $N$-dimension column vector $(0, 0, \ldots, 0, 1)$.

To find the distribution of payments, note that the payments to the sellers, $\varphi_1$ and $\varphi_2$, are unambiguously defined by the state and the demand realization. Given $\pi$, the limiting distribution of the payments, $\varphi$, can be calculated as

$$\varphi = C\pi = C\hat{P}^{-1}e_N$$

where $C$ is an $M \times N$ matrix, whose elements $c_{ij}$ contain the probability of payments $(\varphi_{2i}, \varphi_{mi})$ given the state $(x_{1j}, x_{2j})$, with $M$ being the cardinality of the state space of payments.

4.2. Impact of drivers on payment variability
As pointed out in the literature—e.g., Penza and Bansal (2001)—the drivers that modify firms’ risk may fall into the following three main categories of risk: industry, operational, and financial. Industry risk depends on the volatility of the industry demand and can be measured by demand variability, operational risk depends on the technology chosen by the firm and can be measured by the proportion of fixed costs with respect to total costs, and financial risk depends both on the firm’s capital structure and the cost of debt.

In this section, we observe payment variability and its impact on risk as some of the parameters of the model change. Unless otherwise noted, parameters used are as shown in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market price</td>
<td>$p = $10$</td>
</tr>
<tr>
<td>Purchasing cost</td>
<td>$c_1 = $4, c_2 = $2$</td>
</tr>
<tr>
<td>Agreed payment delay</td>
<td>$\tau_1 = \tau_2 = 0$</td>
</tr>
<tr>
<td>Cost of debt</td>
<td>$r_1 = r_2 = 0$</td>
</tr>
<tr>
<td>Fixed cost</td>
<td>$f_1 = $300, f_2 = $150$</td>
</tr>
<tr>
<td>Equity level</td>
<td>$e_1 = e_2 = $1,000$</td>
</tr>
<tr>
<td>Leverage target (desired financial debt/equity)</td>
<td>$\theta_{f1} = \theta_{f2} = 0.3$</td>
</tr>
<tr>
<td>Financial limit (max. financial debt/equity)</td>
<td>$\theta_{f1} = \theta_{f2} = 0.6$</td>
</tr>
<tr>
<td>Bankruptcy limit (max. debt position/equity)</td>
<td>$\theta_{b1} = \theta_{b2} = 0.9$</td>
</tr>
<tr>
<td>Demand realizations with probability 0.5 each</td>
<td>$\xi_{0,t} \in {50, 100}$</td>
</tr>
</tbody>
</table>
Figure 3  Payment variability as a function of industry, operational, and financial risks.

The left panel of Figure 3 shows the coefficient of variation of the payments made by the retailer as a function of the demand coefficient of variation and the retailer’s level of fixed costs, i.e., industry and operational risks. The right panel shows the payments coefficient of variation as a function of the retailer’s leverage target and cost of debt; indicators of financial risk.

Note that $CV_{q_t} = 0$, thus all the variability in Figure 3 has been created due to lack of access to financial debt. As expected, payment variability increases with the usual drivers of risk (e.g., demand variability, operational leverage, leverage target level, and cost of debt). Our model, however, allows us to explain the mechanisms through which these risk factors impact the payment variability. Specifically, the payment variability increases with the leverage target. When this target increases, the dividends barrier is moved to the right in Figure 2 and dividends are paid more frequently, diminishing the buffers of cash available to pay the supplier. Therefore, the probability of both not paying on time and paying larger quantities in subsequent periods increases, leading to higher payment variability. Furthermore, increasing the buyer’s operational or financial risk may trigger a harmful positive loop for the seller, as they may lead to the buyer’s bank reducing credit, which further increases the sellers risk, according to proposition 1. Also, if price increases, ceteris paribus, it is interesting to note that the coefficient of variation of the demand does not change, but the variability of payments decreases. The reason is that, as price increases, more money is injected on average into the firm, which acts as a cash buffer reducing the variability of payments.

The most significant implication of this analysis, however, is the fact that risk drivers for the buyer have an impact on the sellers’ risk. For instance, the type of technology chosen by the retailer has an impact on the supplier, who sees that his risk profile worsens due to a decision exogenous to him.

4.3. Variability Propagation

We next explore the impact of one of the risk drivers, specifically, the capital structure, on the propagation of risk through the supply chain. We focus on this financial driver of variability for two reasons: First, while we have shown that increasing the industry or operational risk drivers

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4 Because, in the assessment of the financial risk we allowed $r_i > 0$ and the number of states is infinite, it was not possible to find the limiting distribution as per equation (11). Instead, we relied in the observed distribution of a simulation with 10,000,000 periods captured after 1,000,000 periods of simulation to eliminate possible initialization transients—see Law and Kelton (2000, section 9.5.1) for a discussion on the warm-up periods for simulation models of limiting distributions. This simulation horizon and warm-up periods were enough to ensure that simulation errors of the output were almost always below 0.1% for the extensive tests we made using both simulation and equation (11).
also affect the payment variability, we want to focus on the fact that this phenomenon could be entirely generated and sustained within the financial domain of the organization. Second, financial leverage is the easiest to visualize in Figure 2, as it limits the state space from the left. All the other drivers have to do with the cost structure as opposed to limiting the viable space. While the specific response to the changes in other drivers might differ from the results reported there, the analysis of one driver allows us to develop qualitative statements on the sign and intensity of these responses.

Figure 4 shows the impact of the retailer’s leverage target on the payments made by both the retailer and the wholesaler. As the retailer’s financial leverage increases, not only the variability created at the retailer is passed to the wholesaler, but a significant portion of that variability is in turn passed to the wholesaler’s supplier. The same propagation mechanism may be replicated for higher echelons in the supply chain, and also for other drivers. In fact, the propagation phenomenon just described may well lead to the appearance of credit chains and financial contagion, in line with the observations in the financial literature (e.g., Boissay and Gropp 2007).

To explore the propagation of payment variability upstream, we fix the retailer’s leverage target at 0.3, and let the wholesaler’s leverage target vary from −0.6 to 0.6. Figure 5 reports the ratio of \( CV \varphi_2 \) to \( CV \varphi_1 \) as the wholesaler’s leverage target varies. Observe first that variability is greatly attenuated if the wholesaler’s leverage target is low. The wholesaler is injecting liquidity into the supply chain, and does it where is needed the most, a result in line with Boissay and Gropp (2007). Thus, in terms of financial contagion, the supply chain is de-coupled by the presence of firms with deep pockets. Note also that variability is propagated when the wholesaler’s leverage target is relatively large, since he cannot access additional funds to buffer against the payment variability he faces. Also observe that, if his leverage target is above approximately 0.4, then variability is not only propagated, but also amplified. Note that amplification may not be as rare as the strong assumptions in Proposition 3—which gives a sufficient condition for amplification to occur—might suggest. On the contrary, it can occur for realistic parameters. Finally, if not only the retailer increased her leverage target, but also the wholesaler simultaneously did, the consequences would be more harmful for the supply chain in terms of additional risk.

Consider the implications of these examples for the economy, as they can be extended to other combinations of drivers, various players in the supply chain, and even various supply chains in the economy. The crisis that started in 2007 may be an example of the consequences of “massive illiquidity” (Tirole 2010) (partially perhaps caused by high leverage targets) at a global scale.
In this section, we have analyzed the impact of several drivers on the creation and propagation of payment variability. The four drivers identified (industry risk, operational risk, leverage target, and cost of debt) may be seen as the counterparts of lead-time in the order space (Lee et al. 1997), in the sense that all these drivers do not create variability per se, but, once variability exists, they aggravate or attenuate its effects.

4.4. Variability interactions

So far, we have assumed that all the causes of the bullwhip effect for orders were absent. Therefore, only the “financial” drivers were accountable for the increase in payment variability. In this subsection, we relax this assumption to observe the interactions between order and payment variability. We explore how the financial status of a firm impacts the order behavior by making the retailer’s manager sensitive to the financial situation of her firm so that she reduces her orders whenever the firm is close to bankruptcy, thus reducing her exposure to bankruptcy distress. We find this driver relevant for two reasons: first, it is a phenomenon that may appear only if the access to financial debt is limited; second, since it impacts the order variability, it has consequences in both spaces, information and financial.

Let the retailer’s manager have bankruptcy aversion level $b$ and assume that, rather than buying the profit-maximizing quantity according to equation (1) ($q_{1,t} = q_1^* = 100$ units $\forall t$), she now buys:

$$q_{1,t} = \left[1 - b \left(1 - \frac{\theta_1 - \theta_{1,t-1}}{\theta_1 - \theta_1^*}\right)\right]q_1^* \text{ units, with } b \geq 0,$$

i.e., the amount she orders decreases with the distance from the current financial leverage to the bankruptcy barrier. Figure 6 shows the coefficient of variation of orders and payments made by the retailer and wholesaler’s payments as the bankruptcy aversion level varies.

The order variability increases with the bankruptcy aversion level, as the orders placed further depart from the profit-maximizing order quantity. However, the variability of payments to the wholesaler monotonically decreases even if order variability (which, other things being equal, creates payment variability) increases and financial debt is restricted. The retailer’s behavior is beneficial for the wholesaler, who faces lower risk as $b$ increases. Interestingly, this benefit is not usually transferred to the wholesaler’s supplier. For relatively large values of $b$, the impact of order variability created by the retailer’s behavior outweighs the smoother payment function due to the firm moving away from bankruptcy. The intuition is that the retailer’s manager, by keeping the retailer, on average, far from the bankruptcy barrier, reduces the probability of late payments and subsequent larger payments to the wholesaler. However, the retailer’s manager’s behavior does not prevent the wholesaler from being far from the bankruptcy barrier, and then the payment
variability to his supplier is mainly driven by the order variability he faces. The consequence is that a conservative response by the retailer does have the desired effect on her performance, but additional risk and variance are introduced into the upper echelons.

4.5. Trade credit
As mentioned, trade credit agreements play a major role as sources of funds for firms. A plausible question is what is the role of trade credit agreements on payment variability propagation. We performed a number of numerical studies (not reported) and found that trade credit agreements are irrelevant as far as payment variability propagation is concerned when order quantities are constant over time. Even though trade credit has an impact on payments during the transient period, in the long run, if orders are constant, delaying a constant payment at every period is equivalent to not delaying it at every period. Therefore, trade credit just introduces a delay on when payments are made, and limiting distributions for payments remain unaffected.

Payment variability to suppliers at each time result in general from the interaction of two realms, information through orders and financial through financial limits—$\theta_i^\alpha, \theta_i^\beta, \theta_i^\gamma$, which distort natural payments, $c_i d_i - \tau_i$, by either delaying or reducing them. Only if orders are constant, the impact of the information realm becomes trivial and trade credit, as described, is irrelevant.

When orders over time are not constant, however, the impact of the information realm is relevant, and the interaction between both realms changes as payments are delayed. For instance, payments made by the wholesaler in Figure 6 slightly change when the wholesaler pays one period later to his supplier ($\tau_1 = 0, \tau_2 = 1$).

These observations (§4.4 and §4.5) should open the door to broader decision models that transcend the operational realm and give opportunity to study more complex interactions between inventory, cash reserves, orders, and cash flows (e.g., Yang and Birge 2011).

4.6. The case of several retailers
In this sub-section, we briefly explore the case of a wholesaler that serves several identical retailers, and study the impact of the number of retailers and the correlation among retailers’ demands on the variability of payments made by the wholesaler. Given the size of the space state, we resort again to simulation, and build the retailers’ demand series such that any pair of retailers $(i,j)$ has the same coefficient of correlation, $\rho = \rho_{i,j}$. To obtain comparable results, we increase the relevant parameters of the wholesaler, namely equity level, order quantity, and fixed cost, proportionally to
Figure 7  Payment variability as a function of the number of retailers and the coefficient of correlation of retailers’ demands

the number of retailers (for instance, equity becomes $3,000, order quantity 300 units, and fixed cost $450 when the wholesaler serves 3 retailers.)

Figure 7 shows the wholesaler’s payment variability as a function of the number of retailers, \( m \), and the coefficient of correlation of retailers’ demands, \( \rho \). Note that the plot for the \( m = 1 \) case is just the point \((1, 0.292)\), consistent with the results in Figures 4 \((CV_\varphi_2 \text{ when } \theta^*_1 = 0.3)\) and 6 \((CV_\varphi_2 \text{ when } b = 0)\). Two main results arise: first, the wholesaler’s payment variability decreases with the number of retailers, since the pool of retailers reduces the probability of not paying the wholesaler on time, which, in turn, reduces the probability of not paying the wholesaler’s supplier on time, reducing payment variability, in line with proposition 2; second, payment variability increases with the coefficient of variation of retailers’ demands, since demand correlation counteracts the pooling effect just described. This implies that payment variability becomes more relevant in a context of financial shocks affecting simultaneously most of firms in an industry, as in an economic downturn.

5. Application

A major concern for managers and regulators may not be how variability is propagated per se, but the fact that, as a consequence of that variability, firms in their supply chain may have a higher risk exposure. Specifically, as illustrated in Figure 2, the probability of “hitting” the bankruptcy barrier increases with the payment variability, as the magnitude of changes in the state also increase with payment variability. This is especially important for those companies whose performance strongly depends on their suppliers’ survival, such as the automotive industry. Major car manufacturers have programs to assess the probability of first-tier suppliers going bankrupt. In turbulent times, bankruptcies arise and suppliers have to be closely overseen. As an example, consider the crisis that started in 2007. During 2008, roughly 150 (of 1200) of a major German OEM’s first-tier suppliers went bankrupt. Having the ability to anticipate suppliers’ bankruptcies is key to avoiding major disruptions in the channel. To do so, buyers deploy control programs to try to anticipate suppliers’ bankruptcies. After interviewing some of these buyers, we were surprised to learn that tools such as Altman’s z-score (Altman 1968) are used in practice. A weak point of these tools, however, is that they rely only on accounting data (other than the market value of the firm), and do not capture potential future scenarios or operational aspects of the firm.

Given that, within a time horizon, OEM production rates are usually constant over time—which may induce constant orders, our model may help buyers in this industry (and possibly others)
to define additional or alternative control mechanisms to monitor their suppliers and anticipate possible bankruptcies. A key observation is that, for either echelon, the probability of a firm going bankrupt depends on the debt position with respect to the distance between the two barriers (see Figure 2). Anticipating how the debt position will change under various scenarios can help managers to proactively take action with troubled suppliers.

We propose using the following ratio as a heuristic to monitor suppliers:

$$STBI = \frac{\text{Prob}(\Delta x_i > 0) \cdot \mathbb{E}[(\Delta x_i | \Delta x_i > 0)]}{(\theta^b_i - \theta^* i) e_i},$$

which we name the short-term bankruptcy index (STBI). The first factor in the numerator measures the probability of a “bad event” occurring (the firm going towards bankruptcy), whereas the second factor measures the severity of such an event if it occurs. The denominator normalizes the measurement to allow for ratio comparison across firms or over time. Parameters in the denominator and the current state can be obtained from the firm financial statements, due diligence, and industry standards, whereas the two factors in the numerator may come from simulation tests performed using equations (1) to (10) under a number of potential scenarios. These tests would follow a similar approach to the stress tests conducted for banks by the Federal Reserve System in the US or the European Bank Authority in Europe. This index takes into consideration the current debt position, and so it contains information on how close the firm is to the bankruptcy barrier, which may be useful to make tactical decisions.

An alternative heuristic would be to ignore the current debt position and just focus on a long-term bankruptcy index (LTBI) by monitoring the ratio of the standard deviation of the debt position change to the distance between barriers. Formally,

$$LTBI = \frac{\sigma(\Delta x_i)}{(\theta^b_i - \theta^* i) e_i},$$

A good feature of these indexes is that all the main sources of risk (industry, operational, financial) are jointly considered in one measure. Also, they capture the future rather than relying only on historical data. By and large, these indexes may be useful to help buyers assess the probability of suppliers going bankrupt and categorize them when devising risk assessment programs, consider alternative suppliers to distressed ones, or select suppliers for new parts.

6. Conclusions and managerial implications

This paper studies how risk propagates upstream through payment distortion. We leverage on the existing financial literature to build a theory (as suggested by Boissay and Gropp 2007) on how firms behave when constrained by limited access to funds. When this factor is present, we show that payment variability may be created, propagated, and amplified. Using Markov chain tools, we conduct a series of numerical studies to assess the impact of various drivers on the variability of payments. We identify four factors that exacerbate the impact of limited access to debt on payment variability: industry risk, operational leverage, leverage target, and cost of capital. The impact of these drivers on payment variability is in line with the expected impact of industry, operational, and financial risk on the firm’s risk profile. The decisions concerning these drivers not only affect the decision maker firms, but also their suppliers and suppliers’ suppliers. Also we find that managers’ behavior to avoid bankruptcy reduces payment variability but increases order variability. While this has the desired stabilizing effect for the immediate echelons, this benefit is not transferred to upper echelons as the new order variability results in greater payment variability. Payment variability is a major driver of firms’ risk, hence our findings about variability can be extrapolated to how risk is created, propagated, and amplified.
Our results are consistent with the empirical results in Boissay and Gropp (2007) and Bardos and Stili (2007). In fact, our findings support the existence of credit chains and the beneficial impact of “deep pockets,” who reduce the propagation of variability. Our study improves on these works as we provide explanations for the firms’ observed behaviors based on operation and financial constraints. The phenomenon of propagation implies that the risk of a firm is not only driven by its customers, but also by its customers’ customers and so on. Although serving a broad base of customers reduces payment variability and risk in normal times, in turbulent times this reduction might not be as relevant due to cash flow shocks being highly correlated. Likewise, a firm decision has an impact on its cost of capital and that of its suppliers and suppliers’ suppliers. Furthermore, the combined effect of two or more exacerbating simultaneous decisions in two echelons of the chain may greatly impact the risk of the array of the upstream suppliers, thus the health of the whole supply chain and, eventually, the economy.

Our work may help managers judge how their decisions will impact bankruptcies in the supply chain. For instance, trade credit is sometimes seen as “free lunch” for buyers having a dominant position in the market, such as large retailers or manufacturers. These seem to be able to obtain agreements to pay their sellers later without expecting further negative consequences, such as subsequent price increases, lower quality, or late deliveries. However, other things being equal, reaching agreements to paying sellers later will move the buyer’s financial barrier (and hence the bankruptcy barrier) to the left, shortening the distance between barriers, therefore taking both buyer and seller closer to bankruptcy, and increasing de facto the risk of both players. When used extensively by all of the large players in the channel, trade credit (typically combined with high financial leverage) may set the conditions for a major disruption in payments, if one of the lower echelons triggers the financial contagion.

The same logic can be applied to other strategies, such as reverse factoring, typically used by strong buyers to get around the financial burden imposed to weak sellers that have to finance their trade credit agreements. Buyers claim that this is a win-win solution, since they benefit from paying later, the sellers get better conditions from the lenders, and the lenders increase their income plus have access to new customers. While this may well be true regarding cash flows, it is not clear that value increases for the chain as a whole. The reason is, again, that paying sellers later decreases the distance between barriers, rising buyer’s cost of capital which also increases the seller’s cost of capital. As a consequence, rational lenders should ask for higher returns.

On the other hand, the fact that risk may propagate implies paying attention to whatever occurs up and downstream. For instance, in a B2B environment, managers should be especially cautious when selecting customers, who may bring along an array of supply chain echelons subject to various, and maybe “contagious,” levels of risk. Likewise, investors, when deciding on the weight of the stock of a firm in their portfolios, should not be oblivious of the peculiarities of the various echelons related to that particular firm, since the shareholders’ cost of capital is affected by what occurs several echelons down the supply chain. By the same token, banks should calculate risk premia depending not only on the characteristics of a firm, but also on those of its customers’ customers.

From a regulatory perspective, policy makers may want to impose limits on trade credit agreements that may decrease variability. By doing so, they not only protect the small (usually weaker) players, but all the echelons in the supply chain, the industry, and, eventually, the whole economy. This might be why some regulators are imposing limits to trade credit in some industries, as was recently decided upon in some European countries (Sersiron and Dany 2008). Finally, financial authorities may have to decide where to inject liquidity (e.g., whom to bail out) if needed, since the risk profiles of the different echelons may well depend on where money is injected. For example, it might be more efficient to inject liquidity into a buyer than into a seller that may go bankrupt due to the inability of that buyer to pay on time.

We acknowledge the limitations of our model in several aspects that can be further explored in subsequent work.
For instance, alternative dividends policy, such as paying “sticky” dividends, which are common in some industries (Myers 1984), might have an impact on the way variability propagates. Additionally, studying the role of trade credit may unveil relevant insights, as our experiments reveal that trade credit has an impact on the variability of payments when orders change over time. Finally, looking at how risk propagates downstream, due to poor quality or service that may ultimately lead to lower and more variable buyer’s returns (e.g., Harrison et al. 2009), may nicely complement our work.

References


Serrano, Oliva, and Kraiselburd: Risk Propagation through Payment Distortion in Supply Chains


Appendix

Proof of Proposition 1 Since $\xi_0$ is i.i.d. and non-satisfied demand and leftovers are lost, it follows from equations (1) and (2) that $d_{i,t} = q^* v_i = 1,2,\ldots,n \forall t = 1,2,\ldots$. Therefore, $CV q_i = 0$, and it will suffice to show that $CV \varphi_i \geq 0$.

From (6), using (4) and (5), once the system has reached its steady state,

$$\varphi_{i,t} = \min(\theta_i e_i - d_{i,t-1} + \varphi_{i-1,t} - f_i - \bar{y}_{i,t-1})$$

If for low values of $\varphi_{i-1,t}$, $m_{i,t}$ is lower than $c_i q^*$, then $d_{i,t}$ will increase according to equation (7) as $\varphi_{i,t} > m_{i,t}$.

Therefore, unless $d_{i,t}$ can grow indefinitely (i.e., unless $\theta_i = \infty$), it will follow that

$$\theta_i e_i - d_{i,t-1} + \varphi_{i-1,t} - f_i - \bar{y}_{i,t-1} < c_i q^* + \bar{y}_{i,t-1}$$

with positive probability given the assumptions of the proposition, thus $CV \varphi_i > 0$.

To prove the second part of the proposition, note that, in the context of the process defined in Section 3.1, once the steady state is reached, the buyer ought to pay additional $c_i q_i$ to the seller at every period. If credit to the buyer is not limited, the buyer can always pay such an amount to the seller, and there will be no variability in her payments. However, as $\theta_i$ decreases, and given the conditions of the proposition, for low realizations of the buyer’s received payments, the buyer may not be able to satisfy the payments to the seller on time, but only a portion of it—possibly zero. However, as $\theta_i = \infty$, the non-satisfied portion will be eventually satisfied, thus $E \varphi_i$ will not change when $\theta_i$ decreases. Therefore, to show that $CV \varphi_i$ increase when $\theta_i$ decreases, it will be suffice to show that $E \varphi_i^2$ does, as $CV \varphi_i = \sqrt{E \varphi_i^2/\sqrt{\varphi_i}} = \sqrt{E \varphi_i^2}/E \varphi_i$.

Specifically, consider that a partial payment of magnitude $\alpha c_i q_i$ is made at time $t$, where $0 \leq \alpha < 1$. According to equation (6), the non-satisfied portion of the payment, $(1 - \alpha) c_i q_i$, will be satisfied in subsequent periods together with the period’s payment plus, possibly, previous non-satisfied amounts corresponding to periods other than $t$. Let $m$ be the number of periods necessary to pay for the unsatisfied amount $(1 - \alpha) c_i q_i$. Let $\delta_j$ be the $j$-th payment (out of $m$) paid on top of the period’s payment $\beta c_i q_i$, where $\beta \geq 1$.

Given these definitions, note that

$$\sum_{j=1}^m \delta_j = c_i q_i - \alpha c_i q_i = (1 - \alpha) c_i q_i \tag{12}$$

The incremental flows occurring when a partial payment is made are as follows: 1) $-c_i q_i$, which is the payment that cannot be afforded; 2) $\alpha c_i q_i$, which is the partial payment made; 3) $\beta_j c_i q_i + \delta_j$ with $j = 1,\ldots,m$, which include the incremental $m$ payments to compensate for the unsatisfied part of the payment in 2); and 4) $-\beta c_i q_i$, which are the $m$ payments not made because of the payments in 3).

The sum of the squares of these incremental payments is

$$-c_i^2 q_i^2 + \alpha^2 c_i^2 q_i^2 + \sum_{j=1}^m (\beta_j c_i q_i + \delta_j)^2 - \sum_{j=1}^m \beta_j^2 c_i^2 q_i^2 =$$
Using equation (12), the latter expression can be written as

\[(\alpha^2 - 1) c_i^2 q_i^2 + \sum_{j=1}^{m} \delta_j^2 + 2c_i q_i \sum_{j=1}^{m} \beta_j = \]

\[(\alpha^2 - 1) c_i^2 q_i^2 + \sum_{j=1}^{m} \delta_j^2 + 2c_i q_i \sum_{j=1}^{m} (\beta_j - 1) \delta_j > 0\]

As the sum of the squares of the incremental payments is positive and it contributes to increase \(E \varphi_i^2\), it follows that, as \(\theta_i^t\) decreases, partial payments will occur with (weakly) higher probability, leading to increasing \(E \varphi_i^2\), thus \(C V \varphi_i\). □

Proof of Proposition 2 A key observation is that, according to equation (5), the impact of a lower \(m_{i,t}\) on the money available to pay the seller, \(l_{i,t}\), is equivalent to the impact of a lower \(\theta_i^t\), as it lowers the additional disposable credit, \(\theta_i^t - d_{i,t}\). Increasing \(C V \varphi_i\) has two consequences—see equation (4): 1) the probability of a lower \(m_{i,t}\) occurring increases, and 2) the probability of a higher \(m_{i,t}\) occurring increases. Because of 1), partial payments (lower than \(c_i q_i\)) will occur with higher probability; because of 2), partial payments will occur with lower probability and more dividends will be paid, since the dividends barrier will be hit more frequently. As the money entering the seller from the buyer remains invariable when \(C V \varphi_i\) changes, the overall impact of increasing \(C V \varphi_i\) is that partial payments will occur with lower probability. Therefore, \(C V \varphi_{i+1}\) will increase as shown in the proof of Proposition 1. □

Proof of Proposition 3 We first show the result for \(k=3\), and then generalize for any \(k > 2\) integer.

Since \(\xi_0\) is i.i.d. and non-satisfied demand and leftovers are lost, it follows from equations \(1)\) and \(2)\) that \(q_{i,t} = q^* \forall t = 1, 2, \ldots\) Given that \(\xi_0\) can take two values, namely 0 and \(q_1\) with probability \(0.5\) each, \(q^* = q_1\), as long as \(k > 2\).

To find the coefficient of variation of the limiting distribution of payments to the supplier, we calculate the two first central moments, \(E \varphi_1\) and \(E \varphi_2^2\).

To find the first moment, we find \(E \varphi_{1,t}\) for \(t = 1, 2, \ldots\) under the conditions of the proposition. From equations \(3), (4),\) and \(5), we obtain

\[\varphi_{1,1} = \min(c_1 q_1, \varphi_{0,1})\]

\[E \varphi_{1,1} = \frac{1}{2} (c_1 q_1 + 0) = \frac{c_1 q_1}{2}\]

\[\varphi_{1,2} = \min(c_1 q_1 + (c_1 q_1 - \varphi_{0,0})^+, \varphi_{0,2})\]

\[E \varphi_{1,2} = \frac{1}{2^2} (\min(c_1 q_1, 0) + \min(2c_1 q_1, 3c_1 q_1) + \min(c_1 q_1, 3c_1 q_1)) = c_1 q_1 \left(\frac{1}{2} + \frac{1}{4}\right)\]

where \(\alpha^+ = \max(\alpha, 0)\).

Likewise,

\[\varphi_{1,3} = \min(c_1 q_1 + (c_1 q_1 + (c_1 q_1 - \varphi_{0,0})^+ - \varphi_{0,1})^+, \varphi_{0,3})\]

\[E \varphi_{1,3} = \frac{1}{2^3} (3c_1 q_1 + c_1 q_1 + 2c_1 q_1 + c_1 q_1) = c_1 q_1 \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right)\]

\[\varphi_{1,4} = \min(c_1 q_1 + (c_1 q_1 + (c_1 q_1 - \varphi_{0,0})^+ - \varphi_{0,1})^+ - \varphi_{0,2})^+, \varphi_{0,4}\]

\[E \varphi_{1,4} = \frac{1}{2^4} (3c_1 q_1 + c_1 q_1 + 2c_1 q_1 + c_1 q_1 + 3c_1 q_1 + c_1 q_1 + 2c_1 q_1 + c_1 q_1) = c_1 q_1 \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{16}\right)\]

\[\varphi_{1,5} = \min(c_1 q_1 + (c_1 q_1 + (c_1 q_1 + (c_1 q_1 - \varphi_{0,0})^+ - \varphi_{0,1})^+ - \varphi_{0,2})^+ - \varphi_{0,3})^+, \varphi_{0,5}\]

\[E \varphi_{1,5} = \frac{1}{2^5} (3c_1 q_1 + 2c_1 q_1 + \cdots + c_1 q_1) = c_1 q_1 \left(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3}\right)\]

\[\varphi_{1,6} = \min(c_1 q_1 + \cdots + (c_1 q_1 + (c_1 q_1 - \varphi_{0,0})^+ - \varphi_{0,1})^+ - \cdots - \varphi_{0,4})^+, \varphi_{0,6}\]

\[E \varphi_{1,6} = \frac{1}{2^6} (3c_1 q_1 + \cdots) = c_1 q_1 \left(\frac{1}{2} + \frac{1}{2^2} + \frac{2}{2^3} + \frac{1}{2^4}\right)\]
From the latter equations, note that
\[
E\varphi_{1,2} - E\varphi_{1,1} = c_1 q_1 \frac{1}{2^2} = \frac{1}{2^2} \left( \frac{3}{3} \right) (0)
\]
\[
E\varphi_{1,3} - E\varphi_{1,2} = c_1 q_1 \frac{1}{2^3} = \frac{1}{2^3} \left( \frac{3}{3} \right) (1)
\]
\[
E\varphi_{1,4} - E\varphi_{1,3} = 0
\]
\[
E\varphi_{1,5} - E\varphi_{1,4} = c_1 q_1 \frac{1}{2^5} = \frac{1}{2^5} \left( \frac{3}{3} \right) (2)
\]
\[
E\varphi_{1,6} - E\varphi_{1,5} = c_1 q_1 \frac{2}{2^6} = \frac{1}{2^6} \left( \frac{3}{3} \right) (3)
\]

Working in this fashion, it can be shown that
\[
E\varphi_{1,t} - E\varphi_{1,t-1} = \begin{cases} \frac{1}{2^t} \left( \frac{t-2}{2t-1} \right), & \text{if } t = 3 \\ \frac{1}{2^t} \left( \frac{t-2}{2t-1} \right), & \text{if } t = 3 - 1 \\ 0, & \text{if } t = 3 - 2 \end{cases}
\]

Let \( \xi_{0,t} \) denote the market demand time series up to time \( t \) and \( \xi_{0,t} \) a vector of realizations. For any \( t = 3, \)
\[
\varphi_{1,t} = \min(c_1 q_1 + \cdots + (c_1 q_1 + (c_1 q_1 - \varphi_{0,0})^+ - \varphi_{0,1})^+ \cdots - \varphi_{0,(t-2)}^+ + \varphi_{0,t})
\]
\[
E\varphi_{1,t} = \sum_{\forall \xi_{0,t}} \min(c_1 q_1 + \cdots + (c_1 q_1 + (c_1 q_1 - \varphi_{0,0})^+ - \varphi_{0,1})^+ \cdots - \varphi_{0,(t-2)}^+ + \varphi_{0,t}) \text{ Prob}(\xi_{0,t} = \xi_{0,t})
\]
\[
= E\varphi_{1,1} + (E\varphi_{1,2} - E\varphi_{1,1}) + (E\varphi_{1,3} - E\varphi_{1,2}) + \cdots + (E\varphi_{1,t} - E\varphi_{1,t-1})
\]
\[
= c_1 q_1 \left( \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^{t-2}} \right) + \frac{3}{3} \left( \frac{t-3}{2^{t-3}} \right) + \frac{1}{2^{t-1}} \left( \frac{t-2}{2t-3} \right)
\]

As \( t \to \infty \)
\[
E\varphi_1 = c_1 q_1 \left[ \frac{1}{2} + \sum_{j=1}^{\infty} \frac{1}{2^{j-1}} \left( \frac{3j-3}{2j-3} \right) \left( \frac{3j-2}{2j-1} \right) \right]
\]
\[
= c_1 q_1 \left[ \frac{1}{2} + \sum_{j=1}^{\infty} \frac{1}{2^{j-1}} \left( \frac{3j-3}{2j-3} \right) \left( \frac{3j-2}{2j-1} \right) \right]
\]
\[
= c_1 q_1 \left[ \frac{1}{2} + \sqrt{\frac{3}{3}} \sin \left[ \frac{3}{4} \arcsin \left( \frac{3}{\sqrt{2}} \right) \right] + 2 - \frac{2}{3} \cos \left[ \frac{3}{4} \arcsin \left( \frac{3}{\sqrt{2}} \right) \right] \right]
\]
\[
= c_1 q_1
\]

Note that this result was expected, since, in the long run, the wholesaler receives all his money, as payables are never released.

The second moment for \( t = 1, 2, \ldots \) is
\[
E\varphi_{1,1}^2 = E\min(c_1 q_1, \varphi_{0,0})^2 = c_1 q_1^2 \frac{1}{2}
\]
\[
E\varphi_{1,2}^2 = \frac{1}{2^2} [(2 c_1 q_1)^2 + c_1 q_1^2] = c_1 q_1^2 \left( \frac{1}{2} + \frac{3}{4} \right)
\]

Following the same approach as with the first moment we obtain
\[
E\varphi_1^2 = c_1 q_1^2 \left[ \frac{1}{2} + 3 \left( \frac{1}{4} + \frac{1}{32} + \frac{3}{256} + \cdots \right) + 5 \left( \frac{1}{8} + \frac{2}{64} + \cdots \right) \right]
\]
\begin{align*}
&\frac{1}{2} + 3 \sum_{i=1}^{\infty} \frac{1}{2^{3i-1}} \frac{1}{2i-1} \left(3i-3\right) + 5 \sum_{i=1}^{\infty} \frac{1}{2^{3i}} \frac{1}{2i-1} \left(3i-2\right) \\
&= c_1^2 q_1^2 \left[1 + \frac{1}{2} \left(2^2 - 1^2\right) S_{3,1} + \left(3^2 - 2^2\right) S_{3,2}\right] = c_1^2 q_1^2 \left(2 + 2 S_{3,2}\right)
\end{align*}

where \( S_{3,1} = \sum_{i=1}^{\infty} \frac{1}{2^{3i-1}} \frac{1}{2i-1} \left(3i-3\right) \) and \( S_{3,2} = \sum_{i=1}^{\infty} \frac{1}{2^{3i}} \frac{1}{2i-1} \left(3i-2\right) \).

Given the expression of the two first moments, the calculation of the variance easily follows as

\[
\text{Var} = \mathbb{E}[\varphi_1^2] - \left(\mathbb{E}^2 \varphi_1\right) = c_1^2 q_1^2 (1 + 2 S_{3,2})
\]

And the coefficient of variation is

\[
\text{CV} \varphi_1 = \frac{\sqrt{\text{Var} \varphi_1}}{\mathbb{E} \varphi_1} = \sqrt{1 + 2 S_{3,2}}
\]

Since \( \text{CV} \varphi_0 = 1 \), the result for \( k = 3 \) follows.

For \( k = 4 \), given the relationship between the first two central moments, it can be shown that

\[
\mathbb{E} \varphi_1^2 = c_1^2 q_1^2 \left[1 + \frac{1}{2} \left(2^2 - 1^2\right) S_{4,1} + \left(3^2 - 2^2\right) S_{4,2} + \left(4^2 - 3^2\right) S_{4,3}\right]
\]

\[
= c_1^2 q_1^2 \left(2 + 2 S_{4,2} + 4 S_{4,3}\right)
\]

with \( S_{4,1} + S_{4,2} + S_{4,3} = 1/2 \), and \( S_{4,j} > 0, j = 1, 2, 3 \).

In general, for any \( k > 2 \) integer, it can be shown that

\[
\mathbb{E} \varphi_1^2 = c_1^2 q_1^2 \left[1 + \frac{1}{2} \left(2^2 - 1^2\right) S_{k,1} + \left(3^2 - 2^2\right) S_{k,2} + \left(k^2 - (k-1)^2\right) S_{k,k-1}\right]
\]

\[
= c_1^2 q_1^2 \left(2 + 2 S_{k,2} + 4 S_{k,3} + \cdots + 2(k - 2) S_{k,k-1}\right)
\]

with \( S_{k,1} + S_{k,2} + \cdots + S_{k,k-1} = 1/2 \), and \( S_{k,j} > 0 \forall j = 1, 2, \ldots\)

Therefore

\[
\text{CV} \varphi_1 = \sqrt{1 + 2 \sum_{j=2}^{k-1} (j-1) S_{k,j}} > 1
\]

and the result follows for any \( k > 3 \) integer. \( \square \)