Symbol Rate Estimation for DVB-S2 Broadcasting

Carlos Mosquera†, Sandro Scalise‡ and Roberto López-Valcarce†

† ETSE Telecomunicación, Universidad de Vigo
36310 Vigo, Spain
‡ DLR (German Aerospace Center)
Institute for Communications and Navigation
P.O. BOX 1116, 82230 Wessling, Germany
Email: {mosquera,valcarce}@gts.tsc.uvigo.es,sandro.scalise@dlr.de

1 INTRODUCTION

In this paper we present a Maximum Likelihood (ML) approach to the symbol rate estimation of a linearly modulated signal, as those found in satellite broadcasting with DVB-S [1] or DVB-S2 [2]. We focus in the general case for which all the sync parameters, including the mentioned baud rate, are unknown at the receive processing unit. This blind approach finds its main application on the passive analysis of signals, including automatic classification of modulations and quality monitoring. As a motivating starting point, note that the symbol rate can differ among different broadcasters for the same type of service. Let us consider, for example, the signals used in digital TV satellite broadcasting employing DVB-S or the new upcoming standard DVB-S2. It is the purpose of this paper to show how to exploit the structure of the signal to anticipate the underlying symbol rate without resorting to the trial and error decoding for all possible symbol rates.

Blind rate estimation has been considered in the literature, at great detail in some references, and from different points of view. Basically, the use of a bank of filters matched to the different signalling pulse shapes has been derived in an ad-hoc approach [3], or from a more formal point of view [4], assuming a known number of symbols in this latter case. A less computational demanding scheme, which avoids the use of matched filters, uses cyclic second order statistics; see [5], where an asymptotic analysis of this scheme was performed. Its immunity against frequency offsets poses an important advantage for practical scenarios, where frequency errors due to oscillators mismatch and relative movement are expected. The bandwidth excess is an issue for those methods based on second order cyclic statistics; in fact, the method analyzed in [5] fails for raised-cosine based signals of zero roll-off factor. This fact motivated the use of higher order statistics, such as the Godard’s contrast used in [6]: the performance does not degrade for low roll-off factors, although low SNRs (7 dB is considered the threshold in [6]) make the method fail. Our interest is focused to satellite scenarios and, in particular, to the emerging standard DVB-S2, for which signals with very low SNR (in the order of 0 dB) must be decoded. On the other hand, an adequate performance must be achieved for the lowest roll-off factor included in the DVB-S2 standard; this minimum value is 0.2, and it can be used when very tight bandwidth restrictions are in order.

In this contribution we offer a formal perspective of the estimation of the baud rate for a linearly modulated signal. The analysis will be guided by the ML criterion, which will allow to obtain the optimal estimator for low SNR scenarios from a given number of input samples. As detailed in the following sections, the ML approach will be proved to exploit the dependence of both the shaping pulse and its repetition rate with the symbol period, by means of a cost function that can be neatly interpreted. Thus all the previous results contained in the literature will be integrated in a common and formal framework, which will allow to obtain known results as particular cases and grasp some subtleties involved in the problem and unnoticed so far in the known literature. Performance results
will be presented for low SNRs and low roll-off factors, showing how an attractive performance can be achieved if some computational complexity can be allocated for the task of blind symbol rate detection\(^1\).

## 2 ANALYTICAL DERIVATIONS

Let us consider the following received signal model:

\[
r(t) = e^{j(2\pi f_0 t + \theta_0)} \sum_{k=-K/2}^{K/2-1} a_k g_T(t - kT - \epsilon_0 T) + w(t)
\]

(1)

corresponding to \( K \) transmitted symbols \( \{a_k\} \) which are zero-mean independent and identically distributed. The signaling pulse \( g_T(t) \) is a square-root raised cosine pulse, such that \( G_T(f) = 0, |f| > (1 + \alpha)/2T \), for a roll-off factor \( 0 \leq \alpha \leq 1 \) and baud rate \( 1/T \). The noise \( w(t) \) is circularly symmetric Additive White Gaussian Noise of Power Spectral Density \( N_0 \). Our focus is in the general case for which all the sync parameters \( \{T, \theta_0, f_0, \epsilon_0\} \) are unknown at the receive processing unit, as well as the symbol sequence \( \{a_k\} \). A simple scan of the channels currently in service under the DVB-S standard shows that the number of possible values of the symbol period \( T \) is very large\(^2\), and this will be possibly the case when DVB-S2 commercial services are well into service. In addition, the broadcasters will have an additional degree of freedom when synthesizing \( r(t) \), namely, the roll-off factor \( \alpha \) of the pulses. The 0.35 value used in DVB-S can be optionally substituted by 0.2 or 0.25 in DVB-S2, if tight bandwidth restrictions make it necessary.

The signal \( r(t) \) is sampled at \( f_s = 1/T_s \), after being filtered by a low-pass filter of cutoff frequency \( 1/2T_s \). The oversampling factor \( T/T_s \) will be denoted as \( N_s \). Thus, the oversampled signal is expressed as:

\[
r(nT_s) = e^{j(2\pi \gamma_0 + \theta_0)} \sum_{k=-K/2}^{K/2-1} a_k g_T(nT_s - kT - \epsilon_0 T) + w(nT_s)
\]

(2)

with \( w(nT_s) \) i.i.d. Gaussian circularly complex noise samples of variance \( \sigma_w^2 = N_0/T_s \). The normalized timing and frequency offset are denoted respectively by \( \epsilon_0 \) and \( \gamma_0 = f_0 T_s \). For the sake of a compact and clear presentation we will group the samples in vectors and matrices, which will be denoted by boldface symbols. Based on the discretized signal, the core problem of the paper can be posed in the following terms.

**Problem statement.**

Let us consider the vector \( r \in \mathbb{C}^N \) of received samples \( r \doteq \left[ r\left(-\frac{N}{2}T_s\right), \ldots, 0, \ldots, r\left(\frac{N}{2} - 1\right)T_s\right]^T \)
as a noisy observation of a linear modulation characterized by the model \( r = G(\psi)a + w \). The matrix \( G(\psi) \in \mathbb{C}^{N \times K} \) is parameterized by the unknown parameters contained in \( \psi = [N_s, \theta, \gamma, \epsilon] \), and it is given by

\[
[G(\psi)]_{nk} = e^{j2\pi \gamma_0 + \theta} \cdot g_T(nT_s - kN_sT_s - \epsilon N_sT_s), \quad -N/2 \leq n < N/2, -K/2 \leq k < K/2
\]

(3)

The symbols contained in the vector \( a \doteq \left[ a_{-K/2}, \ldots, a_0, \ldots, a_{K/2-1}\right]^T \in \mathbb{C}^K \) are i.i.d. with average energy \( \sigma_a^2 \), and the noise vector \( w \) is Gaussian distributed with covariance matrix \( \sigma_w^2 I \). The

\(^1\)The word *detection* could also apply to the problem under study, as in practical cases we have a limited number of symbol rates to choose from. The context should determine if *detection* is more appropriate than *estimation*.

\(^2\)Some specific symbol rates are encountered very often in practice, e.g., 27.5 or 22 Mbaud. On the other side, a few of them are used in exceptional cases.
number $K$ of symbols included in the observation interval is considered as unknown\(^3\). The objective of this paper is the derivation of the ML estimator of the symbol period $T$ based on the $N$ received values contained in $r$.

The likelihood function, given by the probability density function of the received samples, follows a Gaussian distribution which depends on the different unknown parameters:

$$\Lambda(r|\psi, K, a) = \frac{1}{(2\pi\sigma_w^2)^N} \exp\left\{ -\frac{1}{\sigma_w^2} ||r - G(\psi)a||^2 \right\}$$  \hspace{1cm} (4)

and must be maximized to obtain the ML estimates of the different parameters, in particular, the ML estimate of $T$ or, equivalently, $N_s$. This ML function is well-known and has been widely studied for synchronization purposes. The unknown symbols $a$ are usually dealt with resorting to different assumptions, given that the expectation of $\Lambda$ with respect to $a$ cannot be obtained in closed form except for very particular cases. A good overview is presented in [7]: low-SNR, high-SNR and Gaussian symbols approximations provide solutions applicable in different scenarios. Interestingly, it is shown that for low SNR the ML cost function happens to be independent of the actual symbol distribution. In the Appendix we have included the algebraic manipulations of the above probability density function that, for low SNR, lead to the following log-likelihood function:

$$\ell(r|\psi) = \sum_{k=-K/2}^{K/2-1} |y_T(kN_sTs + \epsilon N_sTs)|^2 - \frac{1}{N_s} \sum_{n=-N/2}^{N/2-1} |r(nTs)|^2$$ \hspace{1cm} (5)

where $y_T(nTs)$ denotes the output of the receive filter matched to the pulse corresponding to the symbol period $T$:

$$y_T(nTs) = r(nTs)e^{-j(2\pi n\gamma + \theta)} * g_T(-(nTs)) = \sum_{m=-N/2}^{N/2-1} r(mTs)e^{-j(2\pi m\gamma + \theta)} g_T((m - n)Ts)$$ \hspace{1cm} (6)

Note that $K$ samples must be taken at the rate given by $1/T$. Although not detailed here, the same simplified description of the problem results from the conventional approach, that is, after keeping the first terms of the Taylor series of the exponential term in (4), we only need to use the second order statistics of the symbols, without assuming any specific distribution for them. In consequence, it is necessary to compute the energy of the different matched filters sampled at the corresponding symbol rate. The normalized frequency offset $\gamma$ plays an important role, whereas the phase $\theta$ is irrelevant in the computation of (5). In addition, the likelihood function depends on the timing offset $\epsilon$, which can be extracted for a given symbol period using the received signal second-order cyclostationarity [8]. As noticed in [9], this is optimal for baseband transmission or perfectly known frequency offset $\gamma$. This ML estimator turns out to be the Oerder&Meyr estimator [10]:

$$\hat{\epsilon} = -\frac{1}{2\pi} \arg \left\{ \sum_{n=-N/2}^{N/2-1} |y_T(nTs)|^2 e^{j\frac{2\pi}{N_s}n} \right\}$$ \hspace{1cm} (7)

Finally, once the timing offset $\epsilon$ is substituted by its ML estimate, the likelihood function is given by

$$\ell(r|N_s, \gamma) = \sum_{n=-N/2}^{N/2-1} |y_T(nTs)|^2 + \frac{2}{N_s} \left| \sum_{n=-N/2}^{N/2-1} |y_T(nTs)|^2 e^{j\frac{2\pi}{N_s}n} \right| - \frac{1}{N_s} \sum_{n=-N/2}^{N/2-1} |r(nTs)|^2$$ \hspace{1cm} (8)

\(^3\)In [4] the number of symbols is fixed and known; for the sake of practical applications, we assume that such a number is not available.
The first term of (8) is measuring the energy at the output of the filters matched to the different signaling pulses, i.e., to the different baud rates. The second term exploits the cyclostationarity of the information signal; it can be considered as a matched filter followed by a bandpass filter at the cyclostationarity frequency: this resembles well-known results of classical synchronization [11]. For roll-off factors close to zero the second term would be very small, and the only second order information can be found in the shape of the received pulse.

3 FREQUENCY CORRECTION

A frequency mismatch which is not correspondingly compensated in the matched filters degrades the performance of the maximum likelihood estimator. We can consider the frequency offset as an unknown deterministic parameter, so that the normalized timing offset $\epsilon$ and frequency offset $\gamma$ can be jointly estimated. This joint estimation cannot be properly addressed in an explicit way using an ML criterion, and some kind of simplification is needed. For this matter we can use again the cyclostationarity of the received signal. Suboptimal estimates of $\gamma$ and $\tau \gamma \pm \epsilon$ can be derived from the estimates of the cyclic correlation of $r(nT_s)$ as follows\(^4\) [12]:

\[
e^{-j2\pi \hat{\gamma} / N_s} = \frac{1}{\sigma_a^2 c_0(N_s; \tau)} \sum_{n=-N/2}^{N/2-1} r(nT_s) r^*((n+\tau)T_s)\]

\[
e^{-j2\pi (\hat{\tau} \gamma \pm \hat{\epsilon}) / N_s} = \frac{1}{\sigma_a^2 c_\pm(N_s; \tau)} \sum_{n=-N/2}^{N/2-1} r(nT_s) r^*((n+\tau)T_s) e^{\mp j2\pi n / N_s} \]

where

\[
c_u(N_s; \tau) = \frac{1}{N_s} \sum_{n=-\infty}^{\infty} g_T(nT_s) g^*_T((n+\tau)T_s) e^{-j(2\pi/N_s)n} \]

If these estimates are inserted into expression (5), corresponding to the low SNR approximation of the likelihood function for a given number of symbols $K$, it can be proved that

\[
\ell(r|N_s) \doteq \ell(r|N_s, \hat{\gamma}, \hat{\epsilon}) = \sum_{\tau=-\gamma}^{\gamma} \left| \sum_{n} r(nT_s) r^*((n+\tau)T_s) e^{-j\frac{2\pi}{N_s} n} \right|^2
\]

the well-known cyclic-correlation based estimator analyzed in [5]. The use of this estimator is advocated by its performance regardless the frequency offset, which makes it especially suited for practical scenarios, although as expected, its performance does not achieve that corresponding to the use of matched filters, as the next simulations illustrate. As a bonus, we can conclude that this estimator can be improved if we use more accurate estimators for the carrier frequency and the timing offset, as those detailed in [12]. In other words, we could use the previously derived matched-filter based estimators after a more accurate frequency estimation has been performed, especially for low roll-off factors for which the cyclostationarity content reduces.

From all the above considerations, we propose a simple frequency estimator before the application of the simplified ML cost function (8). Thus, in addition to the required robustness to frequency offsets in practical settings, we expect to improve the estimator (12) by exploiting the additional information

\(^4\)Individual estimates of the timing offset $\epsilon$ and the frequency offset $\gamma$ can also be obtained from the cyclic correlation in more accurate forms. See [12] for details.
contained in the shape of the received pulse. In order to avoid the increase of the computational complexity in a high extent, we propose the following simple frequency estimator:

$$\hat{\gamma} = \frac{1}{2\pi} \arg \left\{ \sum_{n=-N/2+1}^{N/2-1} r(nT_s)r^*((n-1)T_s) \right\}$$

(13)

analyzed in [8] in detail. As the results in the next section show, its use is enough to improve the performance of estimator (12).

4 SIMULATION RESULTS

Before detailing some numerical examples to illustrate the performance of the presented methods, it is important to understand some relevant subtleties of the estimation problem under study. Thus, Figure 1 plots the log-likelihood function $\ell(r|N_s)$, where the estimate (13) is being applied before the bank of matched filters. The second term in (8) is responsible for the narrow peak and all the other spurious peaks; in fact, as the number of samples goes to infinity, the width of the spectral line of interest goes to zero. This poses a problem in practical settings, and makes it necessary to have some cautions as anticipated in [5]. The smooth curve is the result of evaluating the sum of the first and last term in (8). As observed, the maximum of this pruned function is biased with respect to the true oversampling factor. It can be proved that this bias, which increases with the roll-off factor, is caused by the low SNR regime approximations which are detailed in the Appendix. In fact, we have checked that there is a linear relation between the locations of the biased and the true maxima. From all this, we can envision a two-step strategy for the estimation of the symbol rate, or equivalently, the oversampling factor:

1. An initial search using the energy at the output of the matched filters. The number of the filters can be quite low, given the smoothness of the cost function

$$\ell_1(r|N_s) = \frac{1}{N_s} \sum_{n=-N/2}^{N/2-1} |y_T(nT_s)|^2 - \frac{1}{N_s} \sum_{n=-N/2}^{N/2-1} |r(nT_s)|^2$$

(14)

The estimated position of the maximum must be modified to account for the existing bias. This correction depends on the roll-off factor; therefore, if the roll-off factor is unknown, a higher error must be expected in this first stage.

2. A refined search around the coarse estimate of the previous stage. If the possible symbol rates are known, a simple evaluation of the cost function at each of them within a predetermined range around the coarse estimated value would suffice:

$$\ell_2(r|N_s) = \frac{2}{N_s} \left| \sum_{n=-N/2}^{N/2-1} |y_T(nT_s)|^2 e^{j\frac{2\pi}{N_s} n} \right|^2$$

(15)

A two-step approach was already pointed out in [5], therein referred to as coarse-search and fine-search, to handle the problems derived from locating a very narrow spectral line. The ML approach allows to ground on formal roots this two-step procedure. We have applied this two-fold strategy to a QPSK signal with a random frequency offset, with the SNR ranging from -5 dB up to 5 dB. Figure 2 shows the coarse search performance, measured as the probability of making an initial estimate within the 10% range of the true oversampling factor, which in this case if $N_s = 5$. For a given setting, once established the desired probability and the tolerance of this initial acquisition, the number of necessary samples can be determined.
Figure 1: Normalized ML cost function and its corresponding coarse approximation, after a given realization of 2000 samples. \( E_s/N_0 = 0 \) dB, \( \alpha = 0.25 \). The maximum of the log-likelihood function is at the true \( N_s = 5 \).

Figure 2: Coarse search results. The probability of acquiring the correct symbol rate within an error margin of 10% is plotted against the SNR and for different sample sizes. The true oversampling factor is \( N_s = 5 \), and the roll-off factor \( \alpha = 0.25 \) is assumed to be known.

Figure 3: Comparison of the fine search performance of (15) and (12). The mean square error (mse) is plotted against the SNR, for an initial acquisition range of 10% around the true value \( N_s = 5 \).

For the purpose of assessing the fine search performance, we have evaluated both (15) and (12) on a fine grid of oversampling ratios \( N_s \in (4.5, 5.5) \) for a true oversampling factor 5. Figure 3 shows the results after 5000 realizations, in terms of the mean square error. The new proposed estimator outperforms the cyclic correlation-based estimator, at the cost of a higher complexity. For the evaluated cases, the gain ranges between 3.5 and 4 dB. For the latter estimator we have used a factor \( \Upsilon = 40 \) in (12), since no improvement was noticed in [5] for higher values.

5 CONCLUSIONS

A global Maximum Likelihood approach to the baud rate estimation problem has been addressed. With low SNR scenarios in mind, new results have been obtained which clearly show the information
to exploit for symbol rate estimation, namely, the shape of the received pulse together with the cyclostationarity of the information signal. If uncertainty in the carrier frequency is present, the Maximum Likelihood criterion has been proved to yield a well known suboptimal estimator if suboptimal estimates are used for the timing and frequency offset. This scheme estimates the cyclostationarity period of the signal, although does not exploit the shape of the received pulse, which makes it robust against frequency errors and frequency-selective fading, but offers a poor performance for low SNR and low roll-off factors. A new method, equally robust against frequency errors, has been presented improving the performance in the range of 0 dB, which makes it especially interesting for signals with a strong degree of coding protection, such as those corresponding to DVB-S2.

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A ML COST FUNCTION SIMPLIFICATION

Next we compute the average of the probability density function with respect to the symbol sequence $a$:

$$E_a \{ \Lambda(r|\psi, K, a) \} = \int \frac{1}{(\pi \sigma_a^2)^K} \exp \left\{ -\frac{1}{\sigma_a^2} ||r - G(\psi)a||^2 \right\} \frac{1}{(\pi \sigma_w^2)^K} \exp \left\{ -\frac{||a||^2}{\sigma_w^2} \right\} da$$  \hspace{1cm} (16)

Note that we are assuming a Gaussian distribution for the symbols. If we solve the integral, and after dropping some irrelevant constants, we get to

$$\Lambda(r|\psi, K) = \frac{1}{(\sigma_a^2)^K} \det \left( \frac{1}{\sigma_a^2} I + \frac{1}{\sigma_w^2} G^H(\psi)G(\psi) \right)^{-1} \exp \left\{ \frac{1}{\sigma_a^2} r^H G \left( \frac{1}{\sigma_a^2} I + \frac{1}{\sigma_w^2} G^H(\psi)G(\psi) \right)^{-1} G(\psi)r \right\} \hspace{1cm} (17)$$

Let us choose the shaping pulse $g_T(t)$ such that $G^H(\psi)G(\psi) = I$. This is the case, for example, with squared-root raised cosine pulses with $N$ sufficiently large and with normalized energy. With $\text{SNR} = \frac{\sigma_a^2}{\sigma_w^2}$, and taking the logarithm, we have

$$\ell(r|\psi, K) = -K \ln (1 + \text{SNR}) + \frac{\text{SNR}}{\sigma_a^2 + \sigma_w^2} r^H G(\psi)G^H(\psi)r$$  \hspace{1cm} (18)

As expected, the GML estimator depends on the SNR. For $\sigma_w^2 \to \infty$, we have $\ln(1 + \text{SNR}) \approx \text{SNR}$, and the above expression boils down to

$$\ell(r|\psi, K) = -K + \frac{1}{\sigma_w^2} r^H G(\psi)G^H(\psi)r$$  \hspace{1cm} (19)

which corresponds to the low SNR approximation. The approximation that we are going to apply is only valid for very low SNR, but allows to avoid the knowledge of the operation parameters:

$$||r||^2 \approx N \cdot \sigma_w^2 = K \cdot N_s \cdot \sigma_w^2$$  \hspace{1cm} (20)

since for a given oversampling factor $N_s$, the number of symbols $K$ contained in $N$ samples is $N/N_s$. If we make use of this last consideration, then the likelihood approximation for low SNR ends up being

$$\ell(r|\psi) \approx \ell(r|\psi, K) = r^H G(\psi)G^H(\psi)r - \frac{1}{N_s} ||r||^2$$  \hspace{1cm} (21)
which can be written as

$$\ell(r|\psi) = \sum_{k=-K/2}^{K/2-1} |y_T(kT_sN_s + \epsilon T_sN_s)|^2 - \frac{1}{N} \sum_{n=-N/2}^{N/2-1} |r(nT_s)|^2 ; \ K = N/N_s \quad (22)$$

with

$$y_T(nT_s) = r(nT_s)e^{-j(2\pi n \gamma + \theta)} + g_T(-nT_s) = \sum_{m=-N/2}^{N/2-1} r(mT_s)e^{-j(2\pi m \gamma + \theta)} g_T((m-n)T_s) \quad (23)$$

References

[1] EN 300 421 V1.1.2, Digital Video Broadcasting (DVB); Framing structure, channel coding and modulation for 11/12 GHz satellite services, ETSI, August 1997.


