AGM Meets Abstract Argumentation: Contraction for Dung Frameworks

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Abstract. The aim of the paper is to combine two of the most important areas of knowledge representation, namely belief revision and argumentation. We present a first study of AGM-style contraction for abstract argumentation frameworks (AFs). Contraction deals with removing former beliefs from a given knowledge base. Our presented approach is based on a reformulation of the original AGM postulates. In contrast to the AGM setup, where propositional logic is used, we build upon the recently developed Dung-logics. These logics have been introduced to tackle the somehow inverse problem, namely adding new beliefs. Importantly, they satisfy the characterization property that ordinary equivalence in Dung logics coincides with strong equivalence for the respective argumentation semantics. Although using the same setup we prove a negative result regarding the unrestricted existence of contraction operators. This means, an analog to the Harper Identity, which allows to construct a contraction operator from a given revision operator, is not available. However, dropping the somewhat controversial recovery postulate leads to the existence of reasonable operators.

Keywords: Abstract argumentation · Argumentation frameworks · Belief contraction · Belief revision · Knowledge representation

Introduction

Argumentation theory has become a vibrant research area in Artificial Intelligence, covering aspects of knowledge representation, multi-agent systems, and also philosophical questions (cf. [30, 2] for excellent overviews). The simplicity of Dung's argumentation frameworks (AFs) [18], which are set-theoretically just directed graphs, has considerably contributed to the dominant role of them in the field of abstract argumentation. The latter is primarily concerned with the evaluation of arguments, viewed as abstract entities. The evaluation, i.e. the definition of acceptable sets of arguments, is performed by so-called argumentation semantics which are most commonly based on the attack relation among arguments [3].

Belief revision is concerned with changing the current beliefs of an agent, represented in a suitable representation language, in the light of new information (cf. [22, 24] for an overview). One central paradigm is that of *minimal change*, i.e.

the modification of the current knowledge base has to been done economically. Two main types of belief change are intensively studied, namely *revision* and *contraction*.

Whereas revision potentially replaces information with new knowledge, contraction removes information from a given knowledge base. Both revision and contraction have been studied in depth in the context of propositional logic, with AGM theory [1] certainly being the most influential account. Although, as just mentioned, AFs are widely used, and dynamic aspects obviously play a major role in argumentation, the dynamics of AFs have received an increasing interest over the last few years only. Confer [19] for historical relations between belief revision and argumentation. There are a few works which are dealing with revising AFs [14, 15, 5, 17]. All mentioned works are guided by an axiomatic approach inspired by the AGM postulates. However, several conceptional differences can be observed. For instance, they differ in their underlying equivalence notions, namely strong or ordinary equivalence, respectively as well as in their allowed types of manipulations, e.g. modifying the attack relation only vs. no restrictions at all. To the best of our knowledge the study of AF contraction has been neglected so far. The aim of the paper is to close this gap.

The presented approach builds upon the recently developed Dung-logics which have been introduced to tackle revision [5]. Although here we use the same setup as for revision, we can show a negative result regarding the unrestricted existence of contraction operators. This basically means that an analog to the Harper Identity [25], which allows to construct a contraction operator from a given revision operator, is not available. The main reason for this impossibility is simply that we can not rely on the same expressive power as in propositional logic. More precisely, we do not have an analog to disjunction nor negation in Dung-logics which is essential for the mentioned Harper Identity. It turns out, however, that dropping the somewhat controversial recovery postulate leads to the existence of reasonable operators.

The paper is organized as follows. In Section 2 we provide the background relevant for this paper covering abstract argumentation, ordinary and strong equivalence, kernels, Dung logics and AGM-style contraction. Section 3 presents contraction postulates for AFs, proves the non-existence of contraction operators in general and shows possible ways out. Section 4 summarizes the results of the paper and concludes.

Background

Abstract Argumentation

An *argumentation framework (AF)* is set-theoretically just a directed graph $F = (A, R)$ [18]. In the context of abstract argumentation we call an element $a \in A$ an *argument* and in case of $(a, b) \in R$ we say that *a attacks b* or *a* is *an attacker of b*. Moreover, an argument b is *defended by* a set A if each attacker of b is counter-attacked by some $a \in A$. For a set E we use $E^+ = \{b \mid (a, b) \in R, a \in E\}$ and define

$E^{\oplus} = E \cup E^{+}$. Throughout the paper we will write $F \sqcup G = (A_F \cup A_G, R_F \cup R_G)$ for the union of two AFs $F = (A_F, R_F)$, $G = (A_G, R_G)$ and $F \sqsubseteq G$ if $A_F \subseteq A_G$ and $R_F \subseteq R_G$. Moreover, for a set S we define the restriction of F to S as $F|_S = (S, R_F \cap (S \times S))$. In this paper we consider finite AFs only (cf. [7, 8] for a consideration of infinite AFs). We use $\mathcal{F}$ for the set of all finite AFs.

An *extension-based semantics* σ is a function which assigns to any AF $F = (A, R)$ a set of reasonable positions, so-called σ-*extension*, i.e. $\sigma(F) \subseteq 2^A$. Beside the most basic conflict-free and admissible sets (abbr. *cf* and *ad*) we consider the following mature semantics, namely stable, stage, semi-stable, complete, preferred, grounded, ideal and eager semantics (abbr. *stb*, *stg*, *ss*, *co*, *pr*, *gr*, *id* and *eg* respectively). A very good overview can be found in [3].

Definition 1. *Let $F = (A, R)$ be an AF and $E \subseteq A$.*

1. $E \in cf(F)$ *iff for no* $a, b \in E$, $(a, b) \in R$,
2. $E \in ad(F)$ *iff* $E \in cf(F)$ *and* E *defends all its elements,*
3. $E \in stb(F)$ *iff* $E \in cf(F)$ *and* $E^{\oplus} = A$,
4. $E \in stg(F)$ *iff* $E \in cf(F)$ *and for no* $I \in cf(F)$, $E^{\oplus} \subset I^{\oplus}$,
5. $E \in ss(F)$ *iff* $E \in ad(F)$ *and for no* $I \in ad(F), E^{\oplus} \subset I^{\oplus}$,
6. $E \in co(F)$ *iff* $E \in ad(F)$ *and for any* $a \in A$ *defended by* E, $a \in E$,
7. $E \in pr(F)$ *iff* $E \in co(F)$ *and for no* $I \in co(F)$, $E \subset I$,
8. $E \in gr(F)$ *iff* $E \in co(F)$ *and for any* $I \in co(F)$, $E \subseteq I$,
9. $E \in id(F)$ *iff* $E \in co(F)$, $E \subseteq \bigcap pr(F)$ *and for no* $I \in co(F)$ *satisfying* $I \subseteq \bigcap pr(F)$ *we have:* $E \subset I$,
10. $E \in eg(F)$ *iff* $E \in co(F)$, $E \subseteq \bigcap ss(F)$ *and for no* $I \in co(F)$ *satisfying* $I \subseteq \bigcap ss(F)$ *we have:* $E \subset I$.

Two AFs can be equivalent in many different ways (cf. [6] for an overview). The simplest form of equivalence is possessing the same extensions known as *ordinary* or *standard equivalence*. A further one is *strong equivalence* which requires semantical indistinguishability even in the light of further information. The latter plays an important role for nonmononotic formalisms. Consider the following definitions.

Definition 2. *Given a semantics σ. Two AFs F and G are*

1. *ordinarily σ-equivalent if* $\sigma(F) = \sigma(G)$ *and* $\qquad (F \equiv^{\sigma} G)$
2. *strongly σ-equivalent if* $\sigma(F \sqcup H) = \sigma(G \sqcup H)$ *for any* $H \in \mathcal{F}$. $\qquad (F \equiv^{\sigma}_{s} G)$

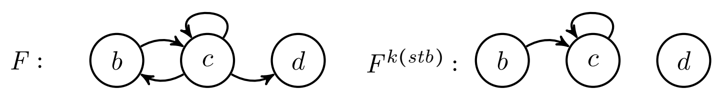
Clearly, both concepts are semantically defined. Surprisingly, in case of strong equivalence it turned out that deciding this notion is deeply linked to the syntax of AFs [27]. In general, any attack being part of an AF may contribute towards future extensions. However, for each semantics, there are patterns of redundant attacks captured by so-called *kernels*. Formally, a kernel is a function $k : \mathcal{F} \to \mathcal{F}$ where $k(F) = F^k$ is obtained from F by deleting certain redundant attacks. We call an AF F *k-r-free* iff $F = F^k$. The following kernels will be considered.

Definition 3. *Given an AF $F = (A, R)$. The σ-kernel $F^{k(\sigma)} = \left(A, R^{k(\sigma)}\right)$ is defined as follows:*

1. $R^{k(stb)} = R \setminus \{(a,b) \mid a \neq b \wedge (a,a) \in R\}$,
2. $R^{k(ad)} = R \setminus \{(a,b) \mid a \neq b \wedge (a,a) \in R \wedge \{(b,a),(b,b)\} \cap R \neq \emptyset\}$,
3. $R^{k(gr)} = R \setminus \{(a,b) \mid a \neq b \wedge (b,b) \in R \wedge \{(a,a),(b,a)\} \cap R \neq \emptyset\}$ *and*
4. $R^{k(co)} = R \setminus \{(a,b) \mid a \neq b \wedge (a,a),(b,b) \in R\}$.

Please note that for any considered kernel the decision whether an attack (a,b) has to be deleted does not depend on further arguments than a and b. Put differently, the reason of being redundant is *context-free*, i.e. it stems from the arguments themselves [4]. This property will play an essential role in several proofs.

Example 1. Consider the AF F and its associated stable kernel.



The only stable extension in our example is $\{b,d\}$. In order to compute a stable kernel, all attacks that come from a self-attacking argument must be deleted. In this example, we can see why this does not change extensions. c will have its self-attack remaining therefore it still can't be part of any extension as stable extensions must be conflict-free. Since stable extension must attack c, it does not matter whether c is attacking other arguments as well.

Kernels allow to efficiently decide on strong equivalence since one just needs to compute the respective kernels and compare them for equality. Hence, strong equivalence regarding AFs is a syntactical feature, i.e. it can be decided just be inspecting the syntax of two AFs.

Theorem 1 ([27],[10]). *For two AFs F and G we have,*

1. $F \equiv_s^\sigma G \Leftrightarrow F^{k(\sigma)} = G^{k(\sigma)}$ *for any semantics* $\sigma \in \{stb, ad, co, gr\}$,
2. $F \equiv_s^\tau G \Leftrightarrow F^{k(ad)} = G^{k(ad)}$ *for any semantics* $\tau \in \{pr, id, ss, eg\}$ *and*
3. $F \equiv_s^{stg} G \Leftrightarrow F^{k(stb)} = G^{k(stb)}$.

Dung-Logics

In propositional logic ordinary and strong equivalence coincide. Consequently, converting AGM postulates to a certain non-monotonic formalism $\mathcal{L}$ might be studied under two different guidelines, namely respecting ordinary or strong equivalence in $\mathcal{L}$. For Dung-style AFs the latter was firstly done in [5] for belief expansion and revision. In order to do so the authors introduced so-called *Dung-logics* which perform reasoning purely on the level of AFs. The heart of these logics are so-called *k-models*. A k-model of an AF F is again an AF which satisfies at least the information of F minus redundancy, but may have more information than encoded by F. Analogously to the relation between the logic of here and there and logic programs we have that Dung-logics are characterization logics for AFs [26, 9]. This means, ordinary equivalence in Dung-logics is necessary and sufficient for strong equivalence regarding argumentation semantics.

Definition 4. *Given a kernel k, two AFs F and G as well as a set of AFs $\mathcal{M}$.*

1. *The set of k-models is defined as: $Mod^k(F) = \{G \in \mathcal{F} \mid F^k \sqsubseteq G^k\}$ and $Mod^k(\mathcal{M}) = \bigcap_{F \in \mathcal{M}} Mod^k(F)$*
2. *The k-consequence relation is given as: $\mathcal{M} \vDash^k F \Leftrightarrow Mod^k(\mathcal{M}) \subseteq Mod^k(F)$*
3. *The ordinary k-equivalence is defined as: $F \equiv^k G \Leftrightarrow Mod^k(F) = Mod^k(G)$*

In the rest of the paper we will consider AGM-style contraction for single AFs. Therefore, as usual, we will drop braces and write $F \vDash^k G$ instead of $\{F\} \vDash^k G$. The following property is not explicitly mentioned in [5] and will be frequently used throughout the paper. It relates consequence relations with subgraph relations.

Lemma 1. *Given a kernel k and two AFs F, G,*

$$F \vDash^k G \Leftrightarrow G^k \sqsubseteq F^k.$$

Proof.

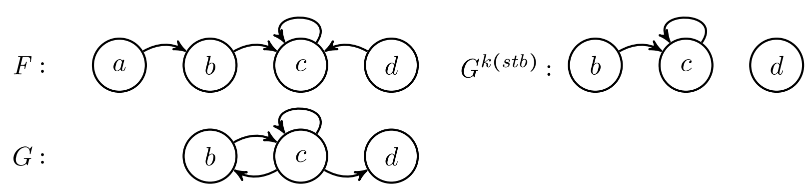
$$(\Rightarrow) \quad F \vDash^k G \Leftrightarrow Mod^k(F) \subseteq Mod^k(G) \qquad \text{(Def. 4.2)}$$
$$\Rightarrow F \in Mod^k(G) \Rightarrow G^k \sqsubseteq F^k \qquad \text{(Def. 4.1)}$$

$$(\Leftarrow) \quad G^k \sqsubseteq F^k \Rightarrow \left(\forall H \in \mathcal{F} : F^k \sqsubseteq H^k \Rightarrow \; G^k \sqsubseteq H^k\right) \qquad \text{(Def. } \sqsubseteq\text{)}$$
$$\Rightarrow \left(\forall H \in \mathcal{F} : H \in Mod^k(F) \Rightarrow H \in Mod^k(G)\right) \qquad \text{(Def.4.1)}$$
$$\Rightarrow Mod^k(F) \subseteq Mod^k(G) \Leftrightarrow F \vDash^k G \qquad \text{(Def.4.2)}$$

□

In the following we illustrate several definitions (cf. [5] for more details).

Example 2. Consider the AFs F, G and their associated stable kernels.



In contrast to G we observe that F is $k(stb)$-r-free since $F = F^{k(stb)}$, i.e. F does not possess any redundant attack w.r.t. stable semantics. Moreover, $G^{k(stb)} \sqsubseteq F^{k(stb)}$ verifies that F is a $k(stb)$-model of G. Loosely speaking, this means that F is a possible future scenario of G. Finally, according to Lemma 1 we deduce $F \vDash^{k(stb)} G$, i.e. believing in the information encoded by F justifies assertions encoded by G.

Figure 1 depicts this example and can be interpreted as a Hasse-diagram for the partial order $(\mathcal{F}, \sqsubseteq)$. Remember that this order possesses a least element, namely the uniquely defined tautology $(\varnothing,\varnothing)$ since $(\varnothing,\varnothing) \subseteq H$ for any $H \in \mathcal{F}$. Each of the cones stands for a set of k-models of a specific (redundant-free) AF which is located at the origin of the respective cone. This figure can also be interpreted as an upside down Hasse-diagram for the partial order $(\mathcal{F}, \models^k)$ as we know by Lemma 1 that the $\models^k$ relation is characterized by the $\subseteq$ relation. The diagram then possesses a *greatest* element, namely $(\varnothing,\varnothing)$. Although this figure successfully illustrates the $\subseteq$-relation of AFs and their models, it still is just an illustration and therefore inappropriate in a way. The figure conveys the impression that for any AFs $H, H' \in \mathcal{F}$ there is a non-empty intersection of their models, which is not the case. There are indeed such AFs H and H' that $Mod^k(H) \cap Mod^k(H') = \varnothing$ (cf. Example 3).

We drew G with dashed lines as its place in the $(\mathcal{F}, \models^k)$ order coincides with the place of $G^{k(stb)}$. Remember, that the $\models^k$-relation is determined only by looking at the kernels of AFs. This partial order therefore could also be defined on $\equiv^k$-equivlance classes.

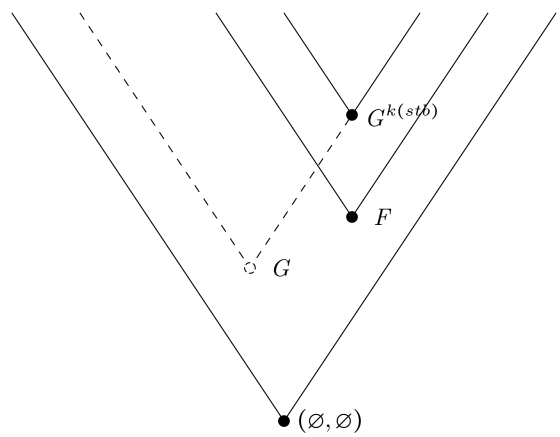


Fig. 1. Dung-logic example

AGM-style Contraction

Let us now take a closer look at AGM-style belief contraction [1]. In the AGM paradigm the underlying logic is assumed to be classical logic and the beliefs are modeled by a theory, i.e. a deductively closed set of sentences (a so-called *belief set*). The provided postulates address the problem of how a current belief set K should be changed in the light of removing a former belief p. In the following we list the basic postulates. We use $\models$ for the classical consequence relation, $K \div p$ for the results of contracting a belief p from K and $K + p$ for $\subseteq$-least deductively closed set of formulas containing both K and p. The latter operator is called *expansion* and it simply adds new beliefs without restoring consistency.

$C1$ $K \div p$ is a belief set *(closure)*
$C2$ $K \div p \subseteq K$ *(inclusion)*
$C3$ $p \notin K \Rightarrow K \div p = K$ *(vacuity)*
$C4$ $\nvDash p \Rightarrow p \notin K \div p$ *(success)*
$C5$ $K \subseteq (K \div p) + p$ *(recovery)*
$C6$ $\vDash p \leftrightarrow q \Rightarrow K \div p = K \div q$ *(extensionality)*

The *closure* axiom $C1$ states that the result of a contraction is a deductively closed theory. The postulate of *inclusion* $C2$ stipulates that when contracting, nothing should be added. The so-called *vacuity* postulate $C3$ complements the former axiom since it determines the precise result of contraction whenever p is not included in the current beliefs. $C4$ encodes the *success* of contraction, i.e. p is no longer believed given that p is not tautological. Postulate $C5$, axiom of *recovery*, states a relation between contraction and expansion. It stipulates that no information is lost if we contract and afterwards expand by the same belief. This axiom is often subject to criticism and is intensively discussed in the literature (see [20, Section 3.1] for more details). The *extensionality* axioms $C6$ encode that the results of belief change are independent of the syntactic form, i.e. results of contraction do not differ if contracting with semantical equivalent formulae.

Dung-style Contraction

Strong equivalence can be seen as the non-monotonic analog of ordinary equivalence in classical logic since it respects the so-called substitution principle (cf. [33] for more details), i.e. for two equivalent sets of formulas Σ and Δ and any set of formulas Γ, $\Gamma \cup \Sigma$ is equivalent to $\Gamma \cup \Delta$. In [5] the authors tackled belief revision for AFs in a way which respects exactly this strong notion of equivalence. In this section we will continue this line of research and consider AGM-style contraction with regard to Dung-logics. We recap the definitions of k-tautology and k-expansion firstly introduced in [5].

Definition 5. *Given a kernel k. An AF F is a k-tautology if $Mod^k(F) = \mathcal{F}$.*

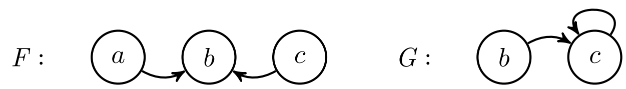
It turns out that the empty framework $F_\varnothing = (\varnothing, \varnothing)$ is the uniquely determined k-tautology. Analogously to classical logic we define expansion semantically, namely as the intersection of the initial models.

Definition 6. *Given a kernel k. For two AFs F, G we define the result of k-expansion as, $Mod^k(F +_k G) = Mod^k(F) \cap Mod^k(G)$.*

In classical logic, the realization of expansion is straightforward from a technical point of view since the intersection of the models can be simply encoded by using conjunctions. It was one main result that even the intersection of k-models is always realizable if considering sets of AFs (cf. [5, Theorem 5, Lemma 6] for more details). In this paper we will require the following result only.

Lemma 2. *Given $k \in \{k(stb), k(ad), k(gr), k(co)\}$. For any two AFs F, G, s.t. $Mod^k(F) \cap Mod^k(G) \neq \varnothing$ we have: $F +_k G = F^k \sqcup G^k$.*

Example 3. Consider the AFs F, G and $k = k(stb)$.



Both AFs are k-r-free, as argument c in G does not have any attacks outgoing. In this case, what does hold regarding $Mod^k(F) \cap Mod^k(G)$? At first glance, one might think that the intersection is not empty, as one would surely find an AF that is an extension of both F and G. However, remember that k-models are not required to include (w.r.t. subgraph relation) the respective AF of which they are a model but to do so modulo redundancy. A k-model of both AFs must include the attacks (b, c), (c, b) and (c, c) which is not possible as (c, b) is a k-redundant attack when (c, c) is given. Therefore we have $Mod^k(F) \cap Mod^k(G) = \varnothing$ leading to $F +_k G$ being undefined.

Contraction Postulates for Kernel k

The axiom translation is relatively straightforward because we can utilize a full-fledged logic which allows us to translate all axioms in a direct fashion. Note that since belief sets are closed sets we can rephrase any axioms using subset/element relations into postulates using consequence relations. More precisely, for two deductively closed sets of propositional formulas Γ and Δ we find, $\Gamma \subseteq \Delta \Leftrightarrow \Delta \vDash \Gamma$. For instance, the inclusion axiom $K \div p \subseteq K$ translates to $K \vDash K \div p$. Now, converting these from classical logic to Dung-logics, i.e. replacing $\vDash$ with $\vDash^k$ results in $F \vDash^k F \div_k G$ which is equivalent to $(F \div_k G)^k \sqsubseteq F^k$ according to Lemma 1. All other axioms can be translated in the same fashion.

$C1^k$ $F \div_k G$ is an AF *(closure)*
$C2^k$ $(F \div_k G)^k \sqsubseteq F^k$ *(inclusion)*
$C3^k$ $G^k \not\sqsubseteq F^k \Rightarrow (F \div_k G) \equiv^k F$ *(vacuity)*
$C4^k$ $G^k \neq (\varnothing, \varnothing) \Rightarrow G^k \not\sqsubseteq (F \div_k G)^k$ *(success)*
$C5^k$ $F^k \sqsubseteq \big((F \div_k G) +_k G\big)^k$ *(recovery)*
$C6^k$ $G \equiv^k H \Rightarrow F \div_k G \equiv^k F \div_k H$ *(extensionality)*

Definition 7. *An operator $\div_k : \mathcal{F} \times \mathcal{F} \to \mathcal{F}$ where $(F, G) \mapsto F \div_k G$ is called a k-contraction iff axioms **$C1^k$**-**$C6^k$** are satisfied.*

Let us start with some reflections on what is required for an operator in order to be a k-contraction. The redundancy-free version of the result $F \div_k G$ is by postulate **$C2^k$** upwards bounded by F^k, i.e. $(F \div_k G)^k \sqsubseteq F^k$. Since postulate **$C1^k$** ensures that we have to end up with an AF we deduce that any contraction for one of the kernels considered in this paper has to delete single arguments together with their corresponding attacks or attacks only. Note that nothing has to be deleted if F is no k-model of G (axiom $\boldsymbol{C3}^k$). Now, given that G is non-empty the success postulate $\boldsymbol{C4}^k$ requires that at least one argument or one non-redundant attack in G is not contained in $(F \div_k G)^k$. Consider Figure 2 for illustration purposes. The depicted situation illustrates that $G^k \sqsubseteq F^k$ or equivalently $F \models^k G$.

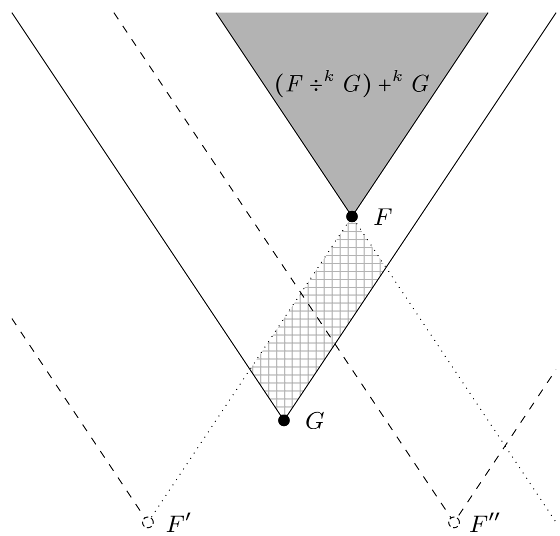


Fig. 2. Dung-logic contraction

What are possible places so far for $F \div_k G$? The AFs F' and F'' show two options. It is possible to move F' and F'' around inside the dotted cone originating from F but it is crucial for them to not lie in the patterned area between F and G since this would lead to $G^k \sqsubseteq (F \div_k G)^k$ and an unsuccessful contraction ($\boldsymbol{C4}^k$). It would however not be possible to move them out of the dotted cone since this would conflict with the inclusion postulate $\boldsymbol{C2}^k$. The postulate $\boldsymbol{C5}^k$ demands the possibility of a recovery of a contracted AF. This means the result of expanding $F \div_k G$ by G has to be located in the grey-drawn space. Unfortunately, we will see that in general this is impossible due to formal reasons.

Non-Existence of Contraction Operators

We now formally prove the non-existence of contraction operators for Dung-logics. Roughly speaking, the reason why there is no suitable operator is that arguments and attacks do not possess the same independency status regarding their own existence. More precisely, in order to reobtain an AF we observe that the deletion of certain arguments necessarily causes the deletion of attacks. In contrast, deleting attacks from an AF results in an AF too. As already discussed, on the one hand, postulates $\boldsymbol{C2}^k$ and $\boldsymbol{C4}^k$ enforce the deletion and prohibit the addition of information and on the other hand, axiom $\boldsymbol{C5}^k$ postulates the possibility of restoring all information. The proof of the following theorem shows that removing arguments that have attacks dependent on them which are not part of the contracted AF cause an irreversible loss of information.

Theorem 2. *Given $k \in \{k(stb), k(ad), k(gr), k(co)\}$. There is no operator $\div_k : \mathcal{F} \times \mathcal{F} \to \mathcal{F}$ such that $\div_k$ satisfies $\boldsymbol{C1}^k$-$\boldsymbol{C6}^k$.*

Proof. Striving for a contradiction let us assume the existence of an operator $\div_k$ satisfying $\boldsymbol{C1}^k$-$\boldsymbol{C6}^k$. Consider the AFs $F = (A, R_F) = (A, \{(a,a) \mid a \in A\})$ and $G = (A, R_G) = (A, \varnothing)$ with $A \neq \varnothing$. Obviously $F \models^k G$ for any kernel k considered. Due to $\boldsymbol{C2}^k$ we have $(F \div_k G)^k \sqsubseteq F^k$. Since $G \neq (\varnothing, \varnothing)$ and therefore $\not\models^k G$ we also have $G^k \not\sqsubseteq (F \div_k G)^k \sqsubseteq F^k$ because of $\boldsymbol{C4}^k$. Note that F and G are k-r-free, i.e. $F^k = F$ as well as $G^k = G$ for all kernels k considered, as self-loops never are removed. Hence, $G \not\sqsubseteq (F \div_k G)^k \sqsubseteq F$. Since $R_G = \varnothing$ and therefore for any $H = (A_H, R_H) \in \mathcal{F}$, we have $R_G \subseteq R_H$, our only chance to satisfy the claimed relation lies in removing some set of arguments A' with $\varnothing \subset A' \subseteq A$ in F. Thus, a respective AF F' not entailing G can be identified by $F' = (A_{F'}, R_{F'}) = \big(A \setminus A', R_{F'}\big)$ with $R_{F'} = R_F \cap (A_{F'} \times A_{F'})$ due to postulate $\boldsymbol{C1}^k$. Please observe that $R_{F'} \subset R_F$. Since $\div_k$ satisfies $\boldsymbol{C5}^k$ we must end up with $F^k \sqsubseteq (F' +^k G)^k$. Note that F' and G are both k-r-free. Moreover, $G \sqsubseteq F'$ and $G \sqsubseteq G$ witnesses $Mod(F') \cap Mod(G) \neq \varnothing$. Consequently, we may conclude as follows:

$$
\begin{aligned}
F' +^k G &= F'^k \sqcup G^k = F' \sqcup G && \text{(Lem. 2, } k\text{-}r\text{-freeness)}\\
&= (A \setminus A', R_{F'}) \sqcup G && \text{(Def. } F'\text{)}\\
&= \big((A \setminus A') \cup A, R_{F'} \cup \varnothing\big)\\
&= (A, R_{F'})\\
&\neq F = (A, R_F) && (R_{F'} \subset R_F).
\end{aligned}
$$

Contradiction! □

Example 4. Given $F, G \in \mathcal{F}$ as considered in the proof of Theorem 2. We illustrate the counterexample in Figure 3. The result of $(F \div_k G) +^k G$ should be located in the grey shaded area above F as $\boldsymbol{C5}^k$ demands that $F^k \sqsubseteq (F \div_k G) +^k G$ which means that F should be a $\sqsubseteq$-smaller AF than the result of the expansion.

Expanding $F \div_k G$ with G results in an AF that is $\sqsubseteq$-greater than G. But, as we have shown, the contraction of F with G results in an irreversible loss of information, namely any argument removed from F, as G does not posses all information formerly removed from F and therefore $(F \div_k G) +^k G$ will be truly $\sqsubseteq$-smaller than F which conflicts with $\boldsymbol{C5}^k$. Intuitively speaking, when expanding $F \div_k G$ with G, G is not able to "push" the contraction result "above" F. This means that the result of $(F \div_k G) +^k G$ is located somewhere in the area "between" F and G, highlighted by dotted lines. One imaginable result for $(F \div_k G) +^k G$ is depicted by F'.

Remember that one single kernel may serve for different semantics (Theorem 1). This means, the non-existence of a syntactically-based version of contraction applies to any semantics characterizable through one of the considered kernels. Moreover, the proof reveals that only very common properties of kernels are used. This indicates that further semantics might be affected by this negative result. A study of this issue will be part of future work.

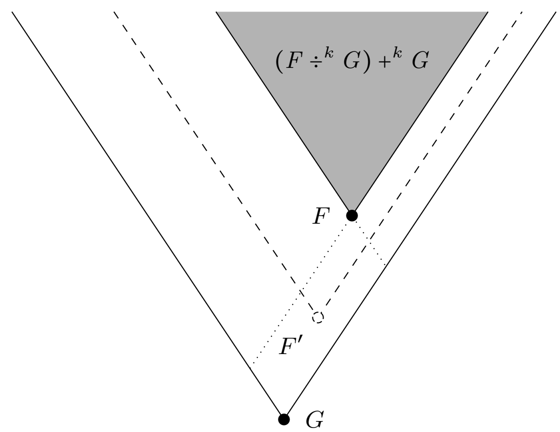


Fig. 3. Counterexample (Theorem 2)

Brute Contraction

In this section we will show that dropping the somewhat controversial recovery postulate leads to the existence of reasonable operators by introducing the brute contraction operator. In [20, Section 3.1] a good example for why the recovery postulate might lead to counter-intuitive results is given. Suppose you know that Cleopatra has a son but later you find out that Cleopatra doesn't have a child at all so you contract your belief set with the belief "Cleopatra has a child". Obviously, this would lead to forgetting that Cleopatra has a son as well. If you were then to learn that Cleopatra has indeed a child thereby expanding your belief set by "Cleopatra has a child", the recovery postulate would demand the belief "Cleopatra has a son" to be part of your new belief set as well. This counter-example holds in general. Whenever you contract a belief set with a more general belief, all more specific beliefs that imply the contracted one must be removed as well. However, when then expanding the more general belief all more specific ones, although not implied by the more general belief, must be restored.

We coined the name *brute contraction* and we will explain why. As indicated in the proof of Theorem 2, for two AFs F and G with $F \vDash^k G$ it suffices to remove some non-empty subset of the arguments in G from F when contracting G from F to accomplish success postulate $\boldsymbol{C4}^k$. This leaves us with the problem to decide *which* elements to remove whilst ensuring that there is a deterministic decision procedure to determine the result of a contraction. It seems that no heuristic selecting arguments to be removed can accomplish this without relying on a strict order on the set of arguments in some way. The brute contraction operator avoids this problem by being *brute* in the sense that it removes *any* argument and attack of G from F when contracting F with G. Consider therefore the following definition.

Definition 8. *Given a kernel k. We define the brute contraction operator $-_k$: $\mathcal{F} \times \mathcal{F} \to \mathcal{F}$ as:*

$$(A,R) -_k (A',R') = \begin{cases} \big((A,R)|_{A \smallsetminus A'}\big)^k & (A',R')^k \sqsubseteq (A,R)^k \\ (A,R)^k & \textit{otherwise.} \end{cases}$$

Example 5. Consider the AFs F and G discussed in Example 2 and let $k = k(stb)$. We already observed that $G^k \sqsubseteq F^k$. Hence, the first case of Definition 8 applies, i.e. $F -_k G = (\{a\}, \varnothing)$.

We proceed with the main theorem of this section stating that the brute contraction operator satisfies all contraction postulates apart from the recovery axiom.

Theorem 3. *Given $k \in \{k(stb), k(ad), k(gr), k(co)\}$. The brute contraction operator $-_k$ satisfies $\boldsymbol{C1}^k$-$\boldsymbol{C4}^k$ and $\boldsymbol{C6}^k$ but does not satisfy $\boldsymbol{C5}^k$.*

Proof. Given $k \in \{k(stb), k(ad), k(gr), k(co)\}$ and the brute contraction $-_k$. In the following we consider $F = (A_F, R_F)$, $G = (A_G, R_G)$ and $H = (A_H, R_H)$ as arbitrary but fixed AFs.

$\boldsymbol{C1}^k$ By definition we have that restrictions as well as kernels of AFs are AFs again. Hence, in both cases of Definition 8 we obtain that $F -_k G$ is an AF.

$\boldsymbol{C2}^k$ We have to prove that $(F -_k G)^k \sqsubseteq F^k$.
Consider the first case of Definition 8. Since any considered kernel k is *context-free* [4, Definition 7] we have that for any AF H and any set A, $\big(H|_A\big)^k \sqsubseteq H^k$. Consequently,

$$F -_k G =^{(\text{Def. } 8)} \big((A_F, R_F)|_{A_F \smallsetminus A_G}\big)^k \sqsubseteq \big(A_F, R_F\big)^k = F^k.$$

Since $F -_k G \sqsubseteq F^k$ we further deduce $(F -_k G)^k \sqsubseteq F^k$ since $\big(F^k\big)^k = F^k$ and thereby, $(F -_k G)^k = F -_k G$ for any considered kernel k.
In the second case we have $F -_k G = F^k$. Again, since $\big(F^k\big)^k = F^k$ we deduce $(F -_k G)^k \sqsubseteq F^k$.

$\boldsymbol{C3}^k$ We have to show $G^k \not\sqsubseteq F^k \Rightarrow (F -_k G) \equiv^k F$.
Assuming $G^k \not\sqsubseteq F^k$ pushes us to the second case, i.e. $F -_k G = F^k$. We deduce $(F -_k G)^k = \big(F^k\big)^k = F^k$ which means $(F -_k G) \equiv^k F$.

$\boldsymbol{C4}^k$ We have to prove $G^k \neq (\varnothing,\varnothing) \Rightarrow G^k \not\sqsubseteq (F -_k G)^k$.

For any considered kernel k we have, $G^k \neq (\varnothing,\varnothing)$ if and only if $G \neq (\varnothing,\varnothing)$. Hence, assume $G \neq (\varnothing,\varnothing)$. In the first case we have by definition, $F -_k G = \left((A_F, R_F)|_{A_F \smallsetminus A_G}\right)^k = \left(A_F \smallsetminus A_G, R_F \cap (A_F \smallsetminus A_G \times A_F \smallsetminus A_G)\right)^k$. Consequently, $G^k \not\sqsubseteq (F -_k G)^k$ since $A_G \not\subseteq A_F \smallsetminus A_G$ because A_G is assumed to be non-empty. In the second case, we may assume $G^k \not\sqsubseteq F^k$. Since $F -_k G = F^k$ by definition and furthermore, $\left(F^k\right)^k = F^k$ for any considered kernel k we immediately verify $G^k \not\sqsubseteq (F -_k G)^k$.

$\boldsymbol{C5}^k$ Confer Example 6.

$\boldsymbol{C6}^k$ We have to show $G \equiv^k H \Rightarrow F -_k G \equiv^k F -_k H$.

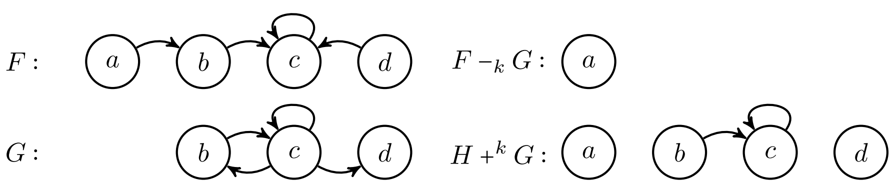
Assume $G^k = H^k$. Consequently, $G^k \sqsubseteq F^k$ if and only if $H^k \sqsubseteq F^k$. For the second case we have nothing to show since the result F^k neither depend on G, nor on H. Now, for the first case. It is essential to see that $G^k = H^k$ implies $A_G = A_H$ for any considered kernel k. Thus,

$$F -_k G = \left((A_F, R_F)|_{A_F \smallsetminus A_G}\right)^k = \left((A_F, R_F)|_{A_F \smallsetminus A_H}\right)^k = F -_k H$$

Therefore $F -_k G \equiv^k F -_k H$ since they are even identical. □

The following example shows that brute contraction indeed does not satisfy the recovery postulate $\boldsymbol{C5}^k$.

Example 6. Consider again the AFs F and G introduced in Example 2 and let $k = k(stb)$. For convenience, we depicted all relevant frameworks below including the running examples F and G.



According to Example 5 we have $H = F -_k G$ as depicted above. Since $G^k \sqsubseteq F^k$ as well as $H^k \sqsubseteq F^k$ we have $Mod^k(G) \cap Mod^k(H) \neq \varnothing$. Hence, in consideration of Lemma 2 we infer that $H +^k G = H^k \sqcup G^k$ as displayed above. One can clearly see that the recovery postulate $\boldsymbol{C5}^k$ is not fulfilled since $F^k \not\sqsubseteq \left((F \div_k G) +_k G\right)^k = H +_k G$. The attack (a, b) in F is irrecoverably contracted.

Discussion

In [25] it was shown that a contraction operator can be constructed from a given revision operator via the union of sets of models as well as the complement of a set of models which can be implemented with disjunction or negation, respectively, in case of belief bases (cf. [13, Definition 3] for excellent explanations). This raises the question why we cannot construct contraction operators although we are equipped with revision operators for Dung-logics [5, Theorem 9]. The reason for this is simply that we do not have an analog to disjunction or negation in Dung-logics. In other words, not every set of k-models is realizable. In [5, Section 3] a first study was presented and it will be an exciting project to exactly characterize the expressive power of Dung-logic.

A further possible view why contraction is not possible in Dung-logics is as follows: We already mentioned that the syntax of AFs is layered, i.e. one part depends on the other whilst the converse does not. In contrast, the classical AGM-postulates were phrased with propositional logic in mind where such a kind of dependence is not given. Consequently, one possible way out is to rephrase all axioms or single problematic postulates in such a way that they better match a logic whose syntax is layered.

Finally, we want to mention that the recovery postulate $\boldsymbol{C5}^k$ does not carry the sole fault regarding the non-existence of contraction operators. It can be shown that dropping the success postulate $\boldsymbol{C4}^k$ leads to the existence of operators but giving up this axiom would be in conflict with the very idea of contraction.

Related Work and Summary

In this paper we presented a first study of AGM-style contraction for abstract argumentation frameworks. Since ordinary and strong equivalence coincide in the underlying formalism of the AGM setup, i.e. propositional logic, one may pursue two in principle different options for converting AGM postulates to a certain non-monotonic logic. In this paper we focus on strong equivalence. Such an approach was applied to logic programs [16] and abstract argumentation [5] for AGM revision. The authors considered so-called *SE*-models and k-models which capture strong equivalence for logic programs under answer set semantics or certain argumentation semantics, respectively [28, 21, 5]. In order to translate the AGM postulates of contraction we used so-called Dung-logics constituted by k-models. This means, the paper complements the previous studies on AGM-style revision and expansion as presented in [5].

The general result is a negative one, that is, there are no contraction operators satisfying all 6 translated postulates. This means, an analog to the Harper Identity, which allows to construct a contraction operator from a given revision operator, is not available (cf. discussion part). Interestingly, a similar problem was discovered for contraction in the realm of logic programming [11]. Instead of the problematic recovery postulate the authors considered the alternative *relevance postulate* introduced in [23]. Such a kind of consideration, i.e. a study of further postulates given in the literature, will be part of future work.

As mentioned in the Introduction, we are not aware of any alternative approaches to contraction in the context of abstract argumentation. In [31] argument contraction (as well as revision) for structured argumentation was studied. The authors presented postulates also influenced by the AGM-postulates but adapted to the more involved ASPIC$^+$ system [29]. In contrast to the original setup contraction functions may return several alternative theories and moreover, the resulting theories might be inconsistent. Interestingly, although a straightforward analog to the Harper Identity is not given, the authors showed how to define a meaningful revision in terms of contraction.

There are several directions for future work. In particular, we plan to extend the analysis of revision and contraction to the more general abstract dialectical frameworks (ADFs) [12]. It will be interesting to compare these results with the ones already proposed in [16, 11] since there are standard translations such that the semantics of logic programs and ADFs coincide [32].

Acknowledgments

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