Correspondence

Comments on "Enhancement of Mammographic Features by Optimal Adaptive Neighborhood Image Processing"

STIG STEENSTRUP

Abstract—**In the paper by Dhawan *et al.*[1] a promising contrast enhancement procedure is developed. As a quantitative means of characterizing the picture, an entropy measure is introduced. It is the purpose of the present comment to clarify the use of the entropy measure and to argue that the "gray level" entropy introduced by Dhawan *et al.* is not a very useful concept.**

The entropy of an image has been much discussed (references are given in the above paper,[1] but see also [1]), and some brief remarks follows.

The information theoretic entropy measures an uncertainty. For definiteness, assume that there are n possible cases but we do not know for sure which one is the actual one; only a probability assignment is possible such that we can only say that there is a probability P_i that case i is the actual. Our uncertainty is then given by the entropy $H[P]$:

$$H[P] = -\Sigma P_i \ln P_i. \tag{1}$$

When using the entropy concept in image analysis, it is useful to distinguish between the picture and the scene. The picture is the actual hardware we are looking at, be it a photograph or numbers in an array, while the scene is the thing the picture is intended to represent. Strictly speaking, there is no uncertainty about the picture—we have it in front of us—the uncertainty lies in what scene the picture actually represents. Given the picture, it is only possible to make a probability statement as to which of n possible scenes is the actual. The probability that the picture represents scene i is P_i, and our uncertainty is given by the entropy expression (1). For brevity, the entropy of this probability distribution is often called the entropy of the image. In this sense, the entropy of an image seems to be a useful quantity as normally the interest is in the scene, while the picture is just a means of obtaining information about the scene.

To evaluate the entropy (1) in practice, some more definite models are required. The outcome of such models [1], [2] is that the entropy can be written in terms of the measured pixel values q_i in the picture with the result, using the notation $p_i = q_i / \Sigma q$, $i = 1, \cdots, N$, with N the number of pixels:

$$H = -\Sigma p_i \ln p_i. \tag{2}$$

The similarity between (1) and (2) is striking, but it is emphasized that (1) is quite general and P_i in (1) is the probabilities of scene i being the "true" given measured pixel values and the sum is over all possible scenes. Equation (2) only applies within definite models, and p_i in (2) is the normalized pixel value in pixel i, the sum in (2) being over all pixels.

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The author is with the Physics Laboratory, Universiteitsparken 5, 2100 Copenhagen, Denmark.
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The picture with the maximal entropy as given by (2) is the uniform picture with all pixel values equal, while pixtures with zero entropy are characterized by one pixel being different from zero, with all the others being equal to zero.

The "gray level" entropy, let it be denoted H_{qe}, is defined by Dhawan *et al.* as

$$H_{qe} = -\Sigma f(j) \ln f(j) \tag{3}$$

with $f(j)$ being the fraction of pixels with gray level j and the sum is over the possible gray levels. There does not seem to be any obvious model which relates (3) to (1), and (2) and (3) are conflicting. For instance, the picture with maximal H_{qe} is characterized by an equal number of pixels having each of the possible gray levels, e.g., a completely noisy picture—all gray levels present in equal proportions. On the other hand, H_{qe} is equal to zero for the uniform gray (or white or black) picture since then $f(j)$ is equal to 1 for $j = j0$ and $f(j) = 0$ for $j \neq j0$.

Loosely speaking, H as given by (2) measures the lack of structure in the picture, while H_{qe} measures the lack of structure in the histogram of gray levels.

The fact that the contrast enhancement procedure leads to larger H_{qe} simply means that the histogram of "gray level" frequencies becomes flatter, but it does not really tell anything about the structure of the picture itself.

It would be interesting to see the entropy of the image as evaluated by (2).

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Authors' Reply

ATAM P. DHAWAN AND RICHARD GORDON

Abstract—**The comment of Stig Steenstrup on the use of gray level entropy function in our paper[1] has been addressed. The entropy measure has not been used in the procedure of enhancement, but it has only been used to show statistically that the enhanced images have larger gray level entropy, and therefore better utilize the gray levels.**

We thank Stig Steenstrup for his comment on our paper[1] about the use of entropy measure. It is to be noted that the gray level entropy function has not been used in our enhancement procedure.

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A. P. Dhawan is with the Department of Electrical Engineering, University of Houston, Houston, TX 77004.
R. Gordon is with the Departments of Botany and Radiology, University of Manitoba, Winnipeg, Man., Canada R3T 2N2.
IEEE Log Number 8612792.

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It has only been used as an after-the-fact statistical measure of utilization of gray levels in the original and enhanced images. The increase in gray level entropy only shows that the enhanced image has a better distribution of gray levels, and therefore has more visible structure. Steenstrup's comment that our gray level entropy function measures the lack of structure in the histogram of gray levels is correct. Thus, larger gray level entropy leads to a flatter histogram of gray levels which is quite reasonable with regard to image processing contrast enhancement procedures. Histogram equalization is one of the standard techniques in image processing to enhance the overall contrast of an image [1], [2]. It works reasonably well on images such as outdoor scenes where the histogram of the original image has some well-defined structure. On the contrary, a mammographic image utilizes a narrow range of gray levels without a well-defined histogram structure. The application of histogram equalization to a mammographic image causes saturation in a number of regions. The saturated regions lose the detailed structure of mammographic features. Therefore, we need to apply neighborhood-based image processing techniques to enhance mammographic features. It is to be noted that for a flat gray level histogram utilizing 256 gray levels, our gray level entropy function gives the maximum value which is 8. Table I of our paper[1] (the correct table is published in the IEEE TRANSACTIONS ON MEDICAL IMAGING, vol. MI-5, p. 120, June 1986) shows that the gray level entropy of the enhanced image is larger than that of the original image, but still significantly less than 8, the maximum entropy corresponding to the perfect flat histogram. Thus, we avoid the problems of getting saturated regions in our enhanced images.

The remaining question is whether the enhanced structure is noise or information about mammographic features. In our enhancement procedure, we define various contrast enhancement functions showing different slopes. We have defined three segments of the required contrast enhancement curve' [3] where the first segment consisting of very small values of the neighborhood contrast (near to zero) relates to the noise variations. The visual inspection of enhanced images confirms that the enhanced structure is largely due to noise enhancement when the square root is taken as a contrast enhancement function (see Figs. 5(a) and 6(a) of our paper[1]). This is simply because the slope of the square root function near zero is infinite, which causes greater enhancement of noise variations. Thus, to restrict the noise enhancement, the first segment of the contrast enhancement curve should have a small slope. It is the second segment of the contrast enhancement curve that should have a larger slope to provide enhancement of mammographic features[1] [3]. The third segment may be required to keep input as well as output contrast values within the range of 0.0–1.0. Thus, knowledge about the noise and features, incorporated in the enhancement procedure, keeps us away from the noisy solutions.

We have come up with more efficient methods of defining the size and shape of the neighborhood on the basis of detection of feature or noisy background. This detection is based on the contrast histogram analysis [6].

The alternative definition of picture entropy, Steenstrup's equation (2), was introduced in [5] and [6] to measure convergence of computed tomography algorithms. However, we find the gray level entropy more relevant as a measure of contrast enhancement.

In conclusion, we would like to say that the use of gray level entropy measure as an overall statistical measure of better contrast is reasonable as long as one incorporates some knowledge in the enhancement procedure to be sure that the enhanced structure is not merely the enhancement of noise.

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