A Model of the Mechanisms of Language Extinction and Revitalization Strategies to Save Endangered Languages

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Abstract Why and how have languages died out? We have devised a mathematical model to help us understand how languages go extinct. We use the model to ask whether language extinction can be prevented in the future and why it may have occurred in the past. A growing number of mathematical models of language dynamics have been developed to study the conditions for language coexistence and death, yet their phenomenological approach compromises their ability to influence language revitalization policy. In contrast, here we model the mechanisms underlying language competition and look at how these mechanisms are influenced by specific language revitalization interventions, namely, private interventions to raise the status of the language and thus promote language learning at home, public interventions to increase the use of the minority language, and explicit teaching of the minority language in schools. Our model reveals that it is possible to preserve a minority language but that continued long-term interventions will likely be necessary. We identify the parameters that determine which interventions work best under certain linguistic and societal circumstances. In this way the efficacy of interventions of various types can be identified and predicted. Although there are qualitative arguments for these parameter values (e.g., the responsiveness of children to learning a language as a function of the proportion of conversations heard in that language, the relative importance of conversations heard in the family and elsewhere, and the amplification of spoken to heard conversations of the high-status language because of the media), extensive quantitative data are lacking in this field. We propose a way to measure these parameters, allowing our model, as well as others models in the field, to be validated.

Languages are culturally transmitted symbolic communication systems that are unique to humans (Jablonka and Lamb 2005). Only human language is productive; that is, in human language small phonological units can be combined to form

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new utterances in unlimited ways (Pinker 1994). The origin of language has not been solved yet, nor is it clear whether there was one protolanguage or whether language was developed independently on several continents (Christiansen and Kirby 2003). It is well known, however, that new languages generally arise through geographic isolation when former dialects become mutually unintelligible to the speakers and when languages get combined, that is, when creolization occurs (Thomason 2001; Thomason and Kaufman 1992).

In this paper, however, we deal with language extinction and various measures that could be taken to prevent it from happening. Thus we present an anthropological study that can help us to understand the problems of language endangerment and language extinction both in the past and in modern times. One can assume that languages have always been going extinct as the consequence of environmental catastrophes (e.g., earthquakes, droughts, or floods), intervention of new tools or weapons, development of agriculture, population movement to new territories, or religious or secular imperialism (Dixon 1998). Yet these changes have, according to Dixon (1998), historically had little impact on the relative equilibrium of languages and their overall number because constant expansion and splitting of peoples and languages have compensated for the loss of languages.

Now languages are going extinct at an increasing rate largely as a result of colonization and globalization, where the language of the economically powerful takes over (Mufwene 2001, 2004). In other words, the main reasons for language endangerment today are socioeconomic, political, and cultural (Campbell and Muntzel 1989; Fishman 1991; Nettle and Romaine 2000). Speakers of minority languages adopt the majority language so that their children will have better job prospects or because the minority language is simply not promoted in the society. Some minority groups choose not to speak their language for fear of persecution. Members of other minority groups see the invading dominant culture as more appealing and modern and abandon their traditional culture and language. These trends have accelerated with the rise of the nation-state and the one language–one state ideology (Dorian 1998) and with the introduction of the Western education system and economies (Aikhenvald 2002). The globalization of culture that accompanies economic integration has led to English competing with national languages and endangering minority languages (Grenoble and Whaley 2006). Preventing the loss of linguistic diversity is therefore a socioeconomic problem that involves changing the attitudes of speakers.

Over the past two decades concern has been growing over the loss of linguistic diversity (Crystal 2000; Dalby 2003; Grenoble and Whaley 1998, 2006; Harrison 2007; Krauss 1992; Nettle and Romaine 2000). It has been estimated that of the 6,000 languages spoken in the world, as much as 50% will become extinct in the near future (Grenoble and Whaley 2006; Romaine 2006b).

Despite some contrary opinions (e.g., Malik 2000), the reasons for maintaining linguistic diversity are numerous. Hale (1992) notes the importance of linguistic diversity to human intellectual life not only in providing subject matter for linguists but also as forms of artistic expression and cultural heritage. Different languages provide us with botanical, biological, and geographic information and insight into human cognition (Harrison 2007; Nettle and Romaine 2000). Most important, however, linguistic diversity can be considered a human right from the speakers' point of view (Hill 2002). When a language becomes endangered, it loses not only speakers but also domains (i.e., the social contexts where the language is spoken) and becomes impoverished and structurally simplified (Dal Negro 2004; UNESCO Ad Hoc Expert Group on Endangered Languages 2003), with heavy influence from the locally dominant language (Clyne 2003). Intergenerational transmission is lost, and younger speakers speak the more dominant language.

Linguists, members of endangered-language communities, governments, nongovernmental organizations, and international organizations such as UNESCO and the European Union are actively working to save and stabilize endangered languages. This is done, for example, by developing linguistic documentation, creating orthographies, producing dictionaries and language-learning materials, promoting positive attitudes toward an endangered language both outside and within the community, planning linguistic programs, and introducing and enforcing linguistic policies. In all this activity, a clear theoretical distinction is made between what Fishman calls "reversing language shift" (now generally referred to as language revitalization) and "language maintenance" (Fishman 1991, 2001). Language revitalization efforts aim to increase the number of speakers of an endangered language and to extend the use of the language to different domains, which requires a change in the attitudes of the speakers themselves; language maintenance, on the other hand, refers to the support given to languages that are still vital but that need to be protected from outsiders' attitudes (Grenoble and Whaley 2006). In practice both language revitalization and language maintenance are needed for the survival of a language. This is a complex and emotionally charged field, with disagreements about how best to intervene to save low-status languages from extinction.

The dynamics of human communication, cooperation, and competition has been modeled using social evolution models (Boyd and Richerson 2005; McElreath and Boyd 2007). Although modeling in the social sciences is increasingly influential (Gilbert and Troitzsch 2005), the real influence of mathematical modeling has been severely limited in the field of language revitalization. This is because models have not sought to engage with the intellectual framework used by linguists; rather, they impose an ecological or even a chemical kinetics metaphor that is not always helpful to linguists. Also, modelers lack the kind of statistical and formal data that would provide them with the measurements they require for their models to be validated and therefore for the dynamics of language revitalization to be properly understood and for the models to have predictive power. This lack of data itself reflects the paucity of mechanistic mathematical models—models that can identify the important parameters that need to be measured.

Our purpose in this paper is, first, to develop a more sophisticated mathematical model of language competition and death by introducing more realistic dynamics and more plausible parameters. Second, by investigating the mechanisms underlying language revitalization efforts, we can identify the parameters that are crucial for predicting the success of a language revitalization program. We aim for our model to be useful in language planning by showing the effects of public and private intervention strategies on an endangered minority language. We discuss some of the factors than can influence this success and thus highlight the most relevant statistics that need to be measured.

Modeling Language Competition, Death, and Survival

The modeling of language death and competition was made popular by Abrams and Strogatz (2003). In their model, speakers can speak either language X or language Y, with the interconversion of X and Y speakers modeled as a reversible reaction with a simple inevitable long-term behavior. Abrams and Strogatz found that the language with the lower status declined to 0 with a sigmoidal decay curve. Because the model is purely phenomenological rather than mechanistic, it cannot address the underlying dynamics of language extinction, that is, why language extinction is occurring and, more important, how the dynamics could be changed. Furthermore, Abrams and Strogatz say that the model "may be useful in the design and evaluation of language-preservation programs" (2003: 900), yet because of their lack of mechanistic considerations, they are unable to suggest anything more explicit than the obvious advantage of increasing the status of the low-status language.

Abrams and Strogatz's model has also been criticized for phenomenological orientation, its lack of realism, and its neglect of bilingualism and different social spheres. A number of different studies have attempted to address some of these deficiencies. For instance, bilingualism is common in multilingual societies and can have an important influence on language use. In contrast to the assumptions of Abrams and Strogatz, if more speakers speak the high-status language, it does not automatically follow that the low-status language is lost, because speakers can become speakers in one language without loss of proficiency in the other. In an extension of Abrams and Strogatz's (2003) paper, Minett and Wang (2008) explored the effect of bilingualism (see also Wang and Minett 2005). Although the inclusion of bilingualism is an important addition, Minett and Wang's model retains the phenomenological notion of attractiveness, which is a function of status, and "peak attractiveness" (a minor modification of the Abrams-Strogatz model)—parameters that cannot be directly measured and whose values cannot be justified. In the Minett-Wang model, attractiveness is a zero-sum game so that it is impossible to make one language more attractive without making the other less so. Minett and Wang also considered attractiveness as depending solely on its presence in the community, disallowing many forms of community intervention such as schooling. Given these constraints, Minett and Wang concurred with

Abrams and Strogatz that one of two competing languages will eventually acquire all the speakers, irrespective of the initial conditions or the existence of bilingualism. It is not clear whether this result is due to constraints on their model (such as the zero-sum nature of language attractiveness or the limited forms of possible interventions) or because of the particular parameter values they used. In the absence of a stable multilingual equilibrium, they considered the intervention of simply swapping the status of the two languages whenever the frequency of the lower-status language declined below a certain threshold (30%) in the population of speakers. How such a dramatic intervention could be achieved is not explained.

In his paper on minority language survival, Grin (1992) considered the measurable individual quantity: the amount of time a bilingual speaker speaks one language or another. This work emphasized how individual motivations and decisions influence language survival and how these decisions can be set to maximize utility. Grin observed the possible presence of stable multilingual equilibria, with a critical threshold of minority language use below which the lower-status language is lost. He showed how interventions can be effective by modifying speakers' calculations of utility by, for instance, providing conditions in which the language can be used. Interestingly, Grin also modeled the role of *expectations* of language decline and found this to be a significant factor in intergenerational transmission, a perception that can be remedied by outspoken support for the minority language by authorities.

Castelló et al. (2007) and others have complemented the Abrams-Strogatz model by taking into consideration both the role of bilingual individuals and social structure in language competition. In Castelló et al.'s model, the two languages have equal status. Neither bilingualism nor social structure, which was modeled as a small-world network (Watts 1999), was sufficient to allow coexistence of two languages. Bilingual agents could not form stable communities but disintegrated into one of the monolingual domains, and in a small-world system they actually accelerated language death. This relates to the importance of social networks in minority and endangered language maintenance: Without strong social links with the original language community or without contact with the speakers of the same language, language shift is likelier (De Bot and Stoessel 2002; Sallabank 2007; Stoessel 2002).

Patriarca and Leppänen (2004) extended the Abrams-Strogatz model by using population dynamics and reaction-diffusion equations. Their model predicts that two competing languages can coexist if the main concentrations of their speakers are in two separate geographic areas, with diffusion in the border zones and higher status in one geographic area. In other words, the model predicts the survival of an endangered language with mating segregation. It correctly predicts the survival of a minority language of a particular region if that minority language is spoken as the majority language in some other region, such as the persistence of Swedish in Finland and of Finnish in Sweden. Although such geographic models can provide important insights, few regions are completely isolated anymore and most endangered minority languages are spoken within nation-states where they do not have an official status (Romaine 2006a). In these situations, children are educated in the majority language, the language of the media is the majority language, and the minority language is threatened despite the geographic center.

Most models of bilingualism do not make any assumptions about the nature of the two languages. Mira and Paredes (2005) showed that a strong tendency to bilingualism caused, for example, by language similarity (e.g., Galician and Castilian) can result in a stable bilingual situation. Stauffer and Schultze (2005) used a quasispecies-style stochastic bit-string model of multiple equally fit languages and examined the requirements for coexistence, with language mutations p (children speaking a slightly different language to their parents) and language transfer q resulting from imitation. They observed a phase transition in which one language dominates in the space of p and q parameters. This process occurs on time scales much slower than that responsible for the kinds of socioeconomic language decline we consider here.

A conceptually similar analysis to ours was performed by Wickström (2005). His theory is based on a separation of the utility of various languages compared with the degree of the speakers' emotional attachments. Wickström demonstrated that it is possible for bilingualism to be dynamically stable (either with or without monolingual speakers of both languages) so long as neither language has much greater status than the other. Various combinations of social status and emotional attachment (manifested in the rate at which children of monolingual-bilingual and bilingual-bilingual partnerships are raised bilingually, respectively) can result in either stable universal bilingualism, monolingualism-bilingualism coexistence, or coexistence of both languages with bilingualism. In these last two cases, it is important that children of mixed monolingual-bilingual parents have a sufficiently high probability of becoming bilingual. Wickström's analysis was theoretical rather than mechanistic, inasmuch as he did not consider the time evolution of language use, nor did he model the factors that would influence language choice. As a result, Wickström could not consider how interventions might change the dynamics and the resulting stable situations.

Our model is motivated by the need to create a mechanistic model characterized by parameters that are measurable in the field. We include bilingualism as an integral part of the model, replacing general functions that describe the probability of a child being monolingual or bilingual with more specific functions of the child's family type and speaker frequency that allow us to simulate the dynamics of the system. We include a factor that arises from explicit (language-frequency-independent) language teaching at school, and we distinguish between language heard in the public domain and language heard in the private domain (language-frequency-dependent effects). We also explicitly consider the kinds of intervention that might be capable of preserving the endangered minority language and explore a range of parameters. In addition, we present a phase diagram showing how stability depends on the parameters of our model. In agreement with Wickström (2005), we also observe that a strong tendency of individuals to learn both languages can result in stable bilingualism. We also observe monolingualism-bilingualism coexistence for the correct choice of parameters, although the possible range of these parameters is narrow. We do not observe any situation in which coexistence depends on the initial values of the various languages; there is no "threshold." However, we do find a situation in which the choice of which language dominates in monolingualism depends on the initial ratio of high-status and low-status language use.

Scope of Our Model

Our model is a social evolution model based on differential equations. It includes speakers of a high-status language, speakers of a low-status language, and bilingual speakers of both these languages. For the purposes of our model, we define bilingualism as the ability to function confidently in two languages, that is, the ability to have communicative competence in two languages. In practice, bilingualism is not as simple as this. For example, an individual may be able to read, write, or speak about certain things only in one of the languages, use one of the languages more, have a only passing knowledge of one of the languages, or have learned one of the languages as an adult and the other as a child (Butler and Hakuta 2006; Edwards 2006). Our bilingual speakers cover both successive and simultaneous bilingual speakers, that is, those who learned one language later or learned both languages simultaneously. They do not need to be idealized fully balanced bilingual speakers. In other words, they can be more proficient in one of the languages, although they may function well in both. Note that this definition differs from the model of Wickström (2005: 83), where it was explicitly stated that monolinguals "may be able to communicate very well in the other language, but not with the ease or comfort of a native speaker."

Our model takes into consideration not only intergenerational transmission [i.e., parents teaching their language(s) to their children] but also horizontal transmission (i.e., language acquired outside the home and learned formally in school). Thus, unlike previous models, for example, Wickström's (2005) model, we have included two domains in our model: a public domain and a private domain. This enables us to take into consideration the linguistic reality in any given bilingual society or community: One language is used in the private domain (e.g., at home, with friends, in personal letters), and the other is used in the public domain (e.g., in the city, at work, in education, in government, in media). This societal bilingualism involving a low-status and a high-status language, or a low and high register, is referred to as diglossia (Ferguson 1959; Romaine 2006b). The use of the endangered low-status language is often restricted to the private domain, whereas a high-status language is spoken in the public domains.

Because our goal is to show the effects of language revitalization efforts on a low-status endangered language, we have included three different types of intervention measures in our model. The three strategies are (1) increasing the perceived status of the low-status language so that bilingual families will choose to teach the low-status language to their children and children will be more motivated to use it; (2) increasing the amount of the low-status language heard in society, thus increasing exposure to the language and facilitating the learning of the language as well as raising its status; and (3) formal language teaching of the low-status language to children who would otherwise speak only the high-status language.

In practice, increasing the perceived status of the low-status language refers to awareness-raising programs such as providing promotional materials to parents highlighting the numerous benefits of speaking the low-status language or of being bilingual. In Wales leaflets given to bilingual parents have been successful in highlighting the benefits of being bilingual in Welsh and English (Edwards and Pritchard-Newcombe 2005). Often the interest linguists have shown in and the work they have conducted on the indigenous language have led to a linguistic and cultural revival (Bradley 2002). Local language activists can also achieve a change in attitudes (Florey 2008), as can a change in national attitudes and policies (Grenoble and Whaley 2006; Wurm 2002). This type of measure aims to secure intergenerational mother tongue transmission, which Fishman (1991) identifies as the key to reversing language shift.

Increasing the amount of the low-status language heard in society targets the public sphere, that is, the higher domains. This type of intervention can include radio and television broadcasts, newspapers and other publications being printed in the low-status language, development of specialized terminologies in fields such as technology or commerce for that language, standardization, and orthography and literacy development (Grenoble and Whaley 2006).

The third intervention involves teaching in a formal setting (such as a school) the low-status language to children who would otherwise speak only the high-status language. Previous teaching efforts have concentrated on teaching the minority language in schools as a foreign or second language to semispeakers or nonnative speakers, or, more frequently, teaching it to bilingual speakers of the minority language as a mother tongue. According to Ó Riagáin (2001), the stable rates of bilingualism in Ireland are caused by successful language teaching in schools, not by intergenerational transmission among the original Irish-speaking community. Naturally, the fluency of these new bilingual speakers varies and there is bound to be interference from their dominant language in the formally learned language. Total immersion programs, in which all teaching is conducted in the local language, can lead to a high level of fluency and subsequent intergenerational transmission. The most famous immersion programs are the Maori language nest, Te Kōhanga Reo (King 2001), and the Hawaiian language nests, ʻAha Pūnana Leo (Warner 2001). Educational programs are costly, because they require skilled teachers and the development of teaching materials, and they often present serious logistical problems (e.g., Aikhenvald 2003). A perhaps less costly but similar intervention is the Master-Apprentice Program developed in California in 1992; this program involves an older, more fluent speaker of the endangered language interacting with a younger semifluent speaker or nonspeaker of the language as they engage in everyday activities (Hinton 2001; Hinton et al. 2002).

Here we model specifically the teaching of the low-status language to those children who previously spoke only the high-status language, that is, conversion to bilingualism from the community of high-status language speakers. We do not model semispeakers and so cannot render the full scope of the interventions discussed in the literature. Given this limitation, teaching of the low-status language to bilingual speakers would be included as interventions of the first two types to the extent that it increases both the status of the low-status language and the exposure to low-status language conversations. A component of teaching indeed works by this effect. The reason we have a distinct third intervention is that we wish to also describe those mathematical effects of teaching that do not depend on the frequency of low-status language conversations heard.

The model is used to study the conditions under which the low-status language and/or bilingualism is stable (i.e., has a nonzero steady-state equilibrium frequency) and how this stability depends on the nature and extent of any intervention.

We have chosen these three particular measures because they relate to the use of the endangered language in the private and public spheres, which has not been modeled extensively before. Furthermore, the three measures cover other, more detailed measures suggested in the literature (Crystal 2000; Fishman 1991, 2001; Grenoble and Whaley 2006); for example, literacy, use of technology, and use of the language in government are included in intervention type 2. Other measures, such as increasing the wealth of the low-status language speakers, could have been highlighted or chosen, but this would have led to a more complicated model that is out of the scope of this paper. Some degree of simplification is necessary in any mathematical model. Additional revitalization measures and their relationship to our model and the factors that we have chosen are discussed in the "Conclusion and Discussion" section.

The Model

We assume that there are three types of speakers: monolingual speakers of the high-status language (*H*), monolingual speakers of a low-status language (*L*), and bilingual (*B*) speakers. Speakers mate with each other and have children who speak at least one language. Speakers tend to mate with speakers of the same type, although bilingual speakers can mate with anyone, resulting in five types of families: *HH*, *HB*, *LL*, *LB*, and *BB*. Children who grow up in one of these families learn to speak one or both of the languages depending on the languages of the parents and the frequency of conversations that the child hears in each of the languages, both inside and outside the home. The probability that a child of *HH*, *HB*, or *BB* parents learns *L* depends on the amount of *L* they are exposed to and their susceptibility to *L*, parameterized by α_L (see Table 1). A similar parameter α_H represents the susceptibility of children from *LL*, *LB*, and *BB* parents to learn *H* as a function of the amount of *H* in their environment (see Table 1). Differences in social status may make α_H significantly larger than α_L.

Table 1. Assumptions of the Model: Choice of Parameter Values and the Form of the Equations[a]

Parameter	*Plausible Range*	*Justification*	*Data to Be Acquired*
Initial fractions of speakers of *L* and *H*	0–1	The outcome of the model depends on the initial concentration of speakers. In an endangered language situation, the initial number of *L* speakers is often low.	Proportions of speakers of *L* and *H*.
α_L: Responsiveness of a child born to *HH* or *HB* parents to speak *L* as a function of *L* conversations heard	1–1.5 (see Figure 1)	This figure is low because a child born to *HH* or *HB* parents is unlikely to speak *L* in a typical endangered language situation.	The probability that a child born to *HH* or *HB* parents becomes an *L* speaker in various environments with different *L* frequency.
α_H: Responsiveness of a child born to *LL* or *LB* parents to speak *H* as a function of *H* conversations heard	1–3 (see Figure 1)	This figure is high in our model, representing the enhanced desirability of speaking *H*.	The probability that a child born to *LL* or *LB* parents becomes an *H* speaker in various environments with different *H* frequency.
η: Ratio of the effect of family and nonfamily conversations heard by child on the language spoken by the child	1	We are assuming that a child receives 50% of her linguistic input at home and 50% outside the home.	The average percentage of *L* and *H* conversations heard by the child both at home and outside the home, combined with the fraction of children that are *L*, *H*, or *B*.
ω: Amplification factor of *H* conversations spoken in the community to *H* conversations heard from all public sources	1–3	Although we would expect *H* to dominate public sources, we would expect that the language exposure due to these sources is not much greater than that due to spoken conversations.	The relative amount of *L* and *H* heard in the public sphere, excluding and including public announcements, media, etc.
λ: Fraction of all conversations heard by a child as a result of government intervention	0–0.1	This figure is considered low in our model, but it could be higher in the presence of successful literacy or media projects.	Longitudinal data on the increased exposure to the minority language in the public sphere and its influence on number of speakers
m_{HB}: Rate at which teaching converts *H* children to *B* relative to the replacement rate of the population	0–0.1	This figure is assumed to be low in our model, but it could be higher with successful teaching programs.	Longitudinal data on the success rate for different types of programs where the second language learned is a minority language.

H, High-status language; *L*, low-status language; *B*, bilingual.

a. In this table we summarize the arguments for our choice of parameter values and suggest further experiments for field linguists to use to confirm these parameter values.

The amount of each language heard by the child has two components because of the influence of the family and the influence of the community, where η is a parameter in our model that represents the strength of the family contribution relative to the contribution from the community (see Table 1). This factor also includes the effectiveness of the families' discourse in influencing the language of the child. The amount of *H* and *L* spoken at home depends on the languages spoken by the parents. The amount of *H* and *L* spoken in society depends on the number of *H*, *L*, and *B* speakers. We also consider that, because of a difference in status, *H* might be given more prominence. As an example, *H* might be overrepresented in the mass media. We introduce another parameter, ω, which represents the extra prominence of *H* over *L* (see Table 1). ω amplifies the conversations of the *H* type so that for the same number of *H* conversations spoken, relatively more are actually heard by the child.

We can now consider three different strategies for public intervention in the interest of maintaining the presence of *L* in the society, either in the form of *L* or *B* speakers. The three strategies, introduced in the previous section, are (1) promoting the learning of *L* by raising its perceived status, thus encouraging its learning by children in *HH*, *HB*, and *BB* families; (2) using government programs to increase the amount of *L* heard in society; and (3) formally teaching *L* to children who would otherwise speak only *H*. Promoting the learning of *L* in the home is represented in our model by increasing the value of α_L so that children exposed to a given amount of *L* would have an increased motivation for learning *L*. We model governmental intervention programs to increase public discourse in *L* with an additional parameter λ, which represents the fraction of all discourse heard by the child as a result of this government intervention (see Table 1).

The third approach, direct teaching of *L* to schoolchildren, involves "converting" *H* children to *B* children. m_{HB} is the rate at which teaching converts *H* children to *B* relative to the replacement rate of the population (see Table 1).

The population dynamics are represented with a standard population genetics model of three-allele one-locus selection, with nonrandom mating (*H* speakers cannot mate with *L* speakers) and a non-Mendelian method of determining offspring types based on existing population frequencies, a Lamarckian process that is inappropriate for genetic inheritance but legitimate for language inheritance (McElreath and Boyd 2007). The dynamic variables in the model are the relative populations of *H*, *L*, and *B* speakers.

The parameters that determine the dynamics in our model are shown in Table 1. We show the parameters along with their values, justification for their values, and numerical data that need to be acquired from minority and endangered language contexts to confirm their values where necessary.

We briefly discuss the justification for the parameter values. Understanding α_L and α_H requires looking at Figure 1. The higher these values are compared to 1, the more likely it is for a child to become bilingual as a function of the proportion of conversations heard in the language that is not shared by both parents. α_H is larger than α_L because we assume that children do not need to hear as many

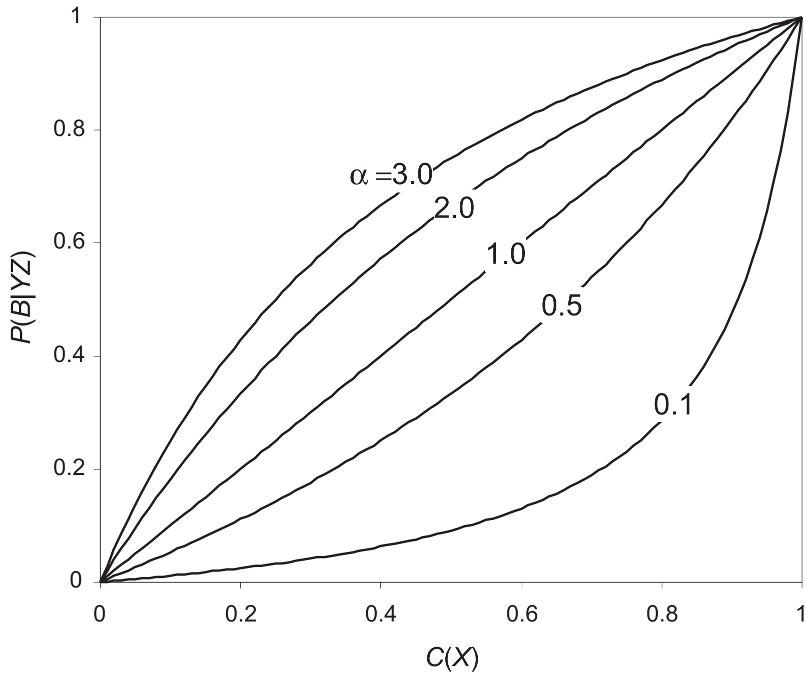

Figure 1. Equations (6) and (8) for various values of α, including α = 0.1, 0.5, 1.0, 2.0, and 3.0.

conversations in the high-status language in order to learn it as they do to learn a low-status language. Currently no experimental data show how these parameters should be set, but our results are robust for a wide range of α values. The η value of 0.5 assumes that half the child's conversations are heard in the public domain and half in the private domain. This is a neutral assumption, and if empirical estimation of this value is made, our model can be appropriately further constrained. ω is an amplification factor that converts conversations spoken into conversations heard. We assume that for the high-status language this amplification factor lies between 1 and 3. The results of the model are robust within at least this range. λ further modifies the ratios of conversations described by η. It is the ratio of conversations that the child is exposed to as a result of government intervention versus the conversation frequency heard without government intervention in the community. m_{HB} is the conversation-frequency-independent component of second-language learning resulting from teaching. Both λ and m_{HB} are assumed to be less than 10% of the proportion of speakers produced by vertical transmission.

We now show how the language competition situation just described is represented mathematically. The model is used to study the conditions in which the low-status language and/or bilingualism is stable (i.e., has a nonzero steady-state frequency in the population of speakers). ("Steady state" is a technical term that means the final values obtained if the model is run for a sufficiently long time for the values to stop changing.)

As stated earlier, our model assumes that there are three types of speakers: monolingual speakers of the high-status language (*H*), monolingual speakers of a low-status language (*L*), and bilingual (*B*) speakers. The proportions of speakers in the population are given by the variables p_H, p_L, and p_B, respectively.

Next we wish to obtain equations for the proportion of different couple types (i.e., parents) in the population as a function of the proportion of speakers in the population. We assume that speakers mate with each other and have children who speak at least one language. Speakers tend to mate with speakers of the same type, although bilingual speakers can mate with anyone. The fraction $p(X_iX_j)$ of all couples that are type X_iX_j, where $X \in \{H, L, B\}$ (i.e., where *X* can stand for *H*, *L*, or *B*), is given by

$$p(HH) = p_H^2 + p_H p_L, \tag{1}$$

$$p(HB) = 2p_H p_B, \tag{2}$$

$$p(LL) = p_L^2 + p_H p_L, \tag{3}$$

$$p(LB) = 2p_L p_B, \tag{4}$$

$$p(BB) = p_B^2. \tag{5}$$

These equations fulfill the appropriate conditions that (1) the total number of speakers of each language in each couple represents the overall fraction of total speakers [e.g., $p(HH) + \frac{1}{2}p(HB) = p_H$] and (2) in the absence of bilingualism [$p(B) = 0$], the fraction of monolingual speakers speaking *X* is the same as the fraction of *X* speakers.

Next we wish to obtain equations for the probability that a child will speak *H* or *L* or be *B*, as a function of the type of parents. We assume that children who grow up in one of these families learn to speak one or both of these languages depending on the languages of the parents (X_iX_j) and the frequencies of conversations $C(Y \mid X_iX_j)$ that the child hears in each of the languages ($Y \in \{H, L\}$), which also depends on the languages spoken by the parents and the frequencies of these languages in the surrounding society. [$C(Y \mid X_iX_j)$ is standard probability notation for a conditional probability (i.e., the probability of something happening, given that something else has already happened). It should be read as the probability of conversations the child hears spoken in language type *Y* given that the child is the offspring of couple type X_iX_j.] We will describe equations to determine the frequencies of conversations later. We assume that the children can always speak to both parents, so that a child in an *HH* or *HB* family can speak *H* or be *B*, but not only speak *L*. Consider a child of *HH* or *HB* parents. She will definitely speak *H* but will also speak *L* (i.e., will be *B*) with a probability $p(B|HX)$ ($X \in \{H, B\}$) that increases from 0 to 1 as the fraction of languages that she hears spoken in *L* increases from 0 to 1. A flexible function that describes these assumptions mathematically is

$$P(B|HX) = \frac{\alpha_L C(L|HX)}{1+(\alpha_L - 1)C(L|HX)}. \tag{6}$$

α_L measures the effectiveness of hearing language *L* in motivating its learning (i.e., the receptiveness of the child to *L*); if α_L is small, the child must be exposed to a significant amount of *L* before she is likely to learn it, whereas a large value of α_L means that a child will be motivated to learn *L* even if conversations in this language represent only a small fraction of those that she hears. α_L represents, among other things, the "status" of *L*, where status is used to mean the entire constellation of societal factors that motivate the learning of a given language.

The corresponding probability that the child will speak only *H* is then given by 1 minus this amount of *L*, because these are the only two probabilities; therefore they must add up to 1.

$$P(H|HX) = \frac{1 - C(L|HX)}{1+(\alpha_L - 1)C(L|HX)}. \tag{7}$$

We have similar functions for *LX* couples:

$$P(B \mid LX) = \frac{\alpha_H C(H|LX)}{1+(\alpha_H - 1)C(H|LX)}, \tag{8}$$

$$P(L \mid LX) = \frac{1 - C(H|LX)}{1+(\alpha_H - 1)C(H|LX)}, \tag{9}$$

where we do not assume that $\alpha_H = \alpha_L$. In particular, differences in social status may make α_H significantly larger than α_L. We will describe the effects of social status later.

The best way to understand these functions is to visualize them, as in Figure 1, which shows the proportion of children who become bilingual as a function of the proportion of conversations heard in the language that is not shared by both parents.

Now consider the more complicated case of *BB* parental pairs. Here the children may be one of three types: speakers of *H* or of *L*, or *B*. We assume that parents may omit to teach one of the languages. We assume that the relative fraction of *H*-speaking children (i.e., *H* and *B*) who also speak *L* is given by an expression analogous to Eq. (6):

$$\frac{p(B \mid BB)}{p(B \mid BB) + (H \mid BB)} = \frac{\alpha_L C(L \mid BB)}{1 + (\alpha_L - 1)C(L \mid BB)}. \tag{10}$$

The relative number of *L* speakers who also speak *H* is given by an expression analogous to Eq. (8). This results in

$$p(H \mid BB) = K \frac{1 - C(L \mid BB)}{\alpha_L C(L \mid BB)}, \tag{11}$$

$$p(L \mid BB) = K \frac{1 - C(H \mid BB)}{\alpha_H C(H \mid BB)}, \tag{12}$$

$$p(B \mid BB) = K = \left[\frac{1 - C(H \mid BB)}{\alpha_H C(H \mid BB)} + \frac{1 - C(L \mid BB)}{\alpha_L C(L \mid BB)} + 1 \right]^{-1}. \tag{13}$$

Figure 2 shows these equations. The probability of a child being of type *H*, *L*, or *B*, if born to *BB* parents depends on the probability of hearing *L* conversations in the environment *C*(*L*) and on the values of α_H and α_L. If receptiveness is equal to both languages with $\{\alpha_H, \alpha_L\} = \{0.5, 0.5\}$, then one obtains the solid curves. If $\{\alpha_H, \alpha_L\} = \{3.0, 1.0\}$ so that the responsiveness to *H* is three times the responsiveness of *L*, one obtains the dashed curves. The difference can be seen if one considers the situation in which *C*(*L*) = 0.5, that is, where 50% of the conversations heard are in *L*. With $\{\alpha_H, \alpha_L\} = \{0.5, 0.5\}$, a child born to *BB* parents has a 40% chance of speaking *H*, a 40% chance of speaking *L*, and a 20% chance of being *B*. With $\{\alpha_H, \alpha_L\} = \{3.0, 1.0\}$, at the same value of *C*(*L*) the child has a 43% chance of speaking *H*, a 14% chance of speaking *L*, and a 43% chance of being *B*.

Now we wish to obtain equations for the frequency of conversations heard by the child in the environment as a function of the frequency of conversations actually spoken. The amount of each language heard by the child $P(Y| X_i X_j)$ has two components because of the influence of the family and the influence of the community:

$$C(Y) = \eta C_{\text{family}}(Y \mid X_i X_j) + (1 - \eta) C_{\text{community}}(Y), \tag{14}$$

where η represents the strength of the family contribution relative to the contribution from the community, $(1 - \eta)$. This factor also includes the effectiveness of the families' discourse in influencing the language of the child. The speech in the family, $C_{\text{family}}(Y|X_i X_j)$, is an explicit function of the language category of the parents $(X_i X_j)$.

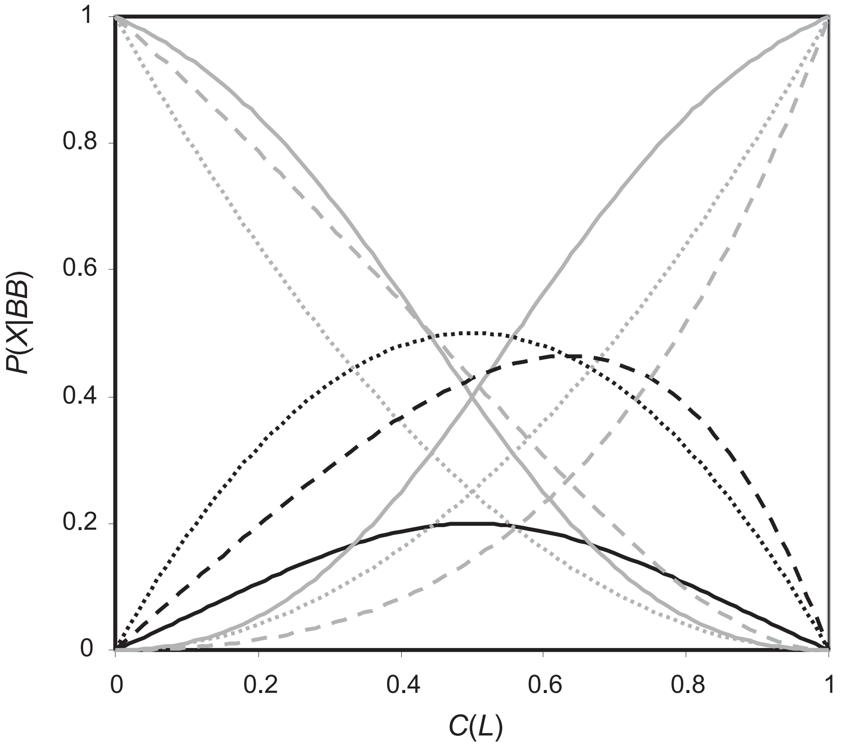

Figure 2. Plot of Eqs. (11)–(13) showing $P(H|BB)$ (gray curves, top-left to bottom-right), $P(L|BB)$ (gray curves, bottom-left to top-right), and $P(B|BB)$ (black curves), as a function of $C(L)$ for $\{\alpha_H, \alpha_L\} = \{0.5, 0.5\}$ (solid curves), $\{2.0, 2.0\}$ (dotted curves), and $\{3.0, 1.0\}$ (dashed curves).

We first consider the community distribution of languages. We assume that conversations are dyadic and that the probabilities of a conversation between two people are given by expressions identical to those of Eqs. (1)–(7) (i.e., that conversations between pairs is of equal ratio to mating between pairs). Two interacting people have only one language in common, which they speak, with the exception of conversations between two bilingual speakers, who, we assume, speak equally frequently in either language. That is,

$$C_{\text{community}}(H) = \left(p_H^2 + p_H p_L\right) + \left(2 p_H p_B\right) + \tfrac{1}{2} p_B^2 \qquad (15)$$

with a similar expression for $C_{\text{community}}(L)$.

We make another small modification to these equations to consider that, because of a difference in status, H might be given more prominence. (As an example, H might be overrepresented in the mass media.) We introduce another parameter, ω, representing the extra prominence of H over L.

$$C_{\text{community}}(H) = \frac{\omega}{\beta}\left(p_H^2 + p_H p_L + 2p_H p_B + \tfrac{1}{2}p_B^2\right), \tag{16}$$

$$C_{\text{community}}(L) = \frac{1}{\beta}\left(p_L^2 + p_H p_L + 2p_L p_B + \tfrac{1}{2}p_B^2\right), \tag{17}$$

where β is a normalization term:

$$\beta = 1 + (\omega - 1)C_{\text{community}}(H). \tag{18}$$

ω amplifies the conversations of *H* type so that for the same number of *H* conversations spoken, relatively more are actually heard by a child.

For the family contribution, we consider that *HH* families speak only *H* at home, whereas *LL* families speak only *L*. Compared with *HH* families, we would expect *HB* families to expose their children to more *L*, especially when the bilingual parent speaks *H* as a second language. The presence of *L* in such families might depend on the relative status of the two languages and on their frequency in the surrounding culture. In this model we make the simple approximation that *HB* families speak *H* five-sixths of the time, representing that conversations could involve one or both parents and that only a fraction of those conversations involving only the *B* parent could occur in *L*. We might expect that the fraction of *H* spoken in *LB* families might be larger because of the higher status of *H*, but we use the simple approximation that *L* will be used five-sixths of the time. Again, somewhat simplistically, we assume that *BB* families speak both languages equally frequently. This results from the assumption that half of *B* speakers are primarily *L* speakers and that half of *B* speakers are primarily *H* speakers. Modifications of these fractions affect the specific results of the model but not the general conclusions.

Effectively, what we have done is to modify a standard population genetics model of three-allele one-locus selection with nonrandom mating (bilingual speakers can mate with everyone) and a non-Mendelian method of determining offspring types based on existing population frequencies (McElreath and Boyd 2007). Assuming that the number of children does not depend on the language spoken by the parents, we arrive at the following set of differential equations for the change of language (allele) frequencies over time. $p_K{}'$ is the rate of change of p_K over time:

$$p_K{}' = \gamma\left[\sum_{\langle ij\rangle} p(X_i X_j)P(K \mid X_i X_{ij})\right] - \gamma p_K \qquad \text{for } K = \{L, B, H\}, \tag{19}$$

where the sum is over all types of families with X_i and X_j taking values from the range *K* and γ is the birth rate. To maintain $p_H + p_B + p_L = 1$, so that *p* can properly be interpreted as the proportion of each speaker type in the community, we set the death rate equal to the birth rate, resulting in a loss of population of the form γp_K.

This term effectively imposes the selection pressure on speaker types; growth of one type of speaker means the decline of the other two types of speaker. The equilibrium conditions obtained in the "Results" section are by numerical simulation of the given equations.

To reiterate, the dynamic variables in the model are the relative populations of *H*, *L*, and *B* speakers, given by p_H, p_L, and p_B, respectively. The parameters in the model are (1) the effectiveness of each language in motivating speakers, given as α_H and α_L; (2) the relative contributions of family and society to the languages spoken by the children, parameterized by η; (3) the bias in the prominence of *H* compared to *L*, given by ω; and (4) the replication rate γ, which, so long as it is sufficiently small, will not affect the steady-state concentrations.

Status is represented as follows in our model. The fact that *H* is the higher-status language is represented by α_H being greater than α_L (i.e., children are more motivated to learn *H*) and by ω being greater than 1 (i.e., a greater proportion of sentences spoken in *H* are heard by children than sentences spoken in *L*).

Modeling Intervention

We can now consider three different strategies for public intervention in the interest of maintaining the presence of *L* in the society, either in the form of *L* speakers or *B* speakers. The three strategies are (1) promoting the speaking of *L* by raising its perceived status, (2) using government programs to increase the amount of *L* heard in society, and (3) formal teaching of *L* to children who speak *H*.

How do we describe intervention 1 in our model? Promoting the learning of *L* in the home is represented in our model by increasing the value of α_L so that children exposed to a given amount of *L* will have an increased motivation for learning *L*.

How do we describe intervention 2 in our model? We assume that government programs act to increase the amount of *L* perceived by language learners, thus increasing $C_{\text{community}}(L)$. To model this effect, we consider

$$C(Y) = \eta C_{\text{family}}(Y \mid X_i X_j) + \lambda C_{\text{government}}(Y) + (1-\eta-\lambda) C_{\text{community}}(Y), \tag{20}$$

where, for a governmental program to enhance *L*, $C_{\text{government}}(L) = 1$ and $C_{\text{government}}(H) = 0$. That is, we introduce a government-mediated term into the equation that describes the proportion of language conversations heard. λ represents the strength of the intervention and is the fraction of all discourse heard by the child that is due to this government intervention.

How do we describe intervention 3 in our model? The third approach would "convert" *H* children to *B* children. This can be modeled as a modification of Eq. (19) to

$$p_H{}' = \gamma \left[\sum_{\langle ij \rangle} p(X_i X_j) P(H \mid X_i X_j) \right] - \gamma p_H - \gamma m_{HB} p_H, \tag{21}$$

$$p_L{}' = \gamma \left[\sum_{\langle ij \rangle} p(X_i X_j) P(L \mid X_i X_j) \right] - \gamma p_L, \tag{22}$$

$$p_B{}' = \gamma \left[\sum_{\langle ij \rangle} p(X_i X_j) P(B \mid X_i X_j) \right] - \gamma p_B - \gamma m_{HB} p_H, \tag{23}$$

where m_{HB} is the rate at which teaching converts *H* children to *B* relative to the replacement rate of the population. This effect is independent of the frequency of language speakers in the community.

Intervention 3 is specifically targeted at nonnative speakers of *L*. A more typical intervention, systematic teaching of *L* to existing bilingual and semispeakers of *L*, cannot be explicitly included because we do not model degrees of language fluency. This intervention, however, can be modeled as both an increase in the amount these children are exposed to *L* [intervention 2, increasing $C_{\text{community}}(L)$] and an increase in the perceived status of *L* in these communities (intervention 1, modeled as an increase in α_L).

Results

Intergenerational Transmission. We first consider the situation without intervention in the public sphere, shown in Figures 3A and 3B. Typical temporal dynamics are shown in Figures 3A and 3B, starting from a population of 50% *L* speakers and 50% *H* speakers, where there is a status differential favoring language *H* (i.e., $\alpha_L = 1$, $\alpha_H = 3$, and $\omega = 2$). As can be seen, *L* goes extinct over a relatively short period of time. The mechanism for this change is the tendency of children of *L* speakers to be *B*. This allows them to mate with other *B* speakers as well as with *H* speakers, with a high likelihood of having offspring who can speak only *H*.

Figures 3C and 3D demonstrate the result of public intervention at year 100 during the period of language decay. The intervention we model is a mixed one that encourages children to be more receptive to *L* at home by setting α_L = 1.5, where *L* is heard more in public ($\lambda = 0.1$) and where government programs encourage teaching ($m_{HB} = 0.1$), representing that approximately 10% of the *H*-speaking children learn *L*. At year 300 the intervention is removed and the various parameters return to their initial values. As can be seen, *L* is rapidly eliminated from the population, indicating that its preservation depends on continuous intervention.

Figure 4 explores how the finding that *L* is lost (without intervention) depends on the initial conditions (i.e., the initial proportions of speakers of *H*, *L*, and *B* in the community). A ternary plot is used, where each point inside the triangle corresponds to a triplet of numbers that gives the initial proportions of *H*, *L*, and

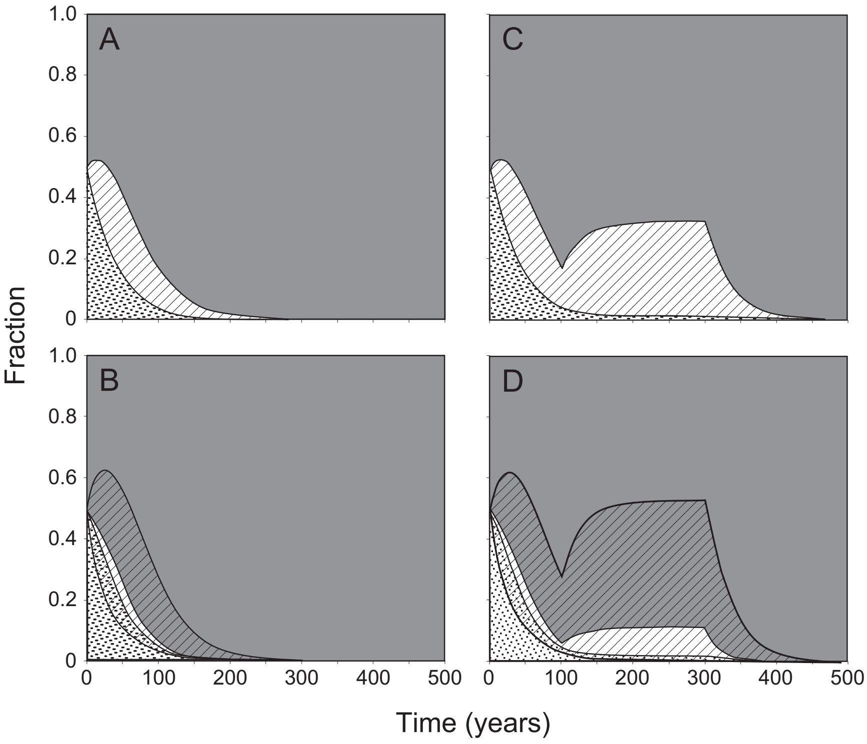

Figure 3. Language dynamics for a low-status language *L* competing with a higher-status language *H*. This difference of status is represented by $\alpha_H = 3$, $\alpha_L = 1$, and $\omega = 2$. The initial population starts with 50% *H* speakers and 50% *L* speakers. Parts A and C show changes in the fraction of the population speaking *H* (gray area), *L* (stippled area), and both (hatched area). Parts B and D show the relative fraction of couples that are *HH* (solid gray area), *HB* (gray-hatched area), *BB* (white-hatched areas), *LB* (white stippled and hatched area), and *LL* (white stippled area). Parts C and D show the effect of a mixed governmental intervention program starting at year 100 consisting of a government intervention to encourage more spoken *L* (λ = 0.1), with formal teaching (m_{HB} = 0.1) and encouraging learning at home (α_L increased to 1.5). We have used a value of γ = 0.01, resulting in a biological "generation time" of 100 simulation steps. If we assume that a human generation is approximately 25 years, then this means that each simulation step corresponds to approximately one-fourth year. The result is the stable maintenance of *L*, primarily among bilingual speakers, so long as the intervention continues. When the intervention ends at year 300, *L* is quickly lost in the population.

B speakers in the community. We see that only if the community starts with a high proportion of *L* does *H* not invade, dominate, and eliminate *L*.

Figure 5 demonstrates that, in general, bilingualism is unstable without intervention for low values of α_L. The shadings show which kind of language persists at equilibrium for different parameter values. Stippling shows that *L* persists, gray shading that *H* persists, and hatching that *B* persists. It is possible to have

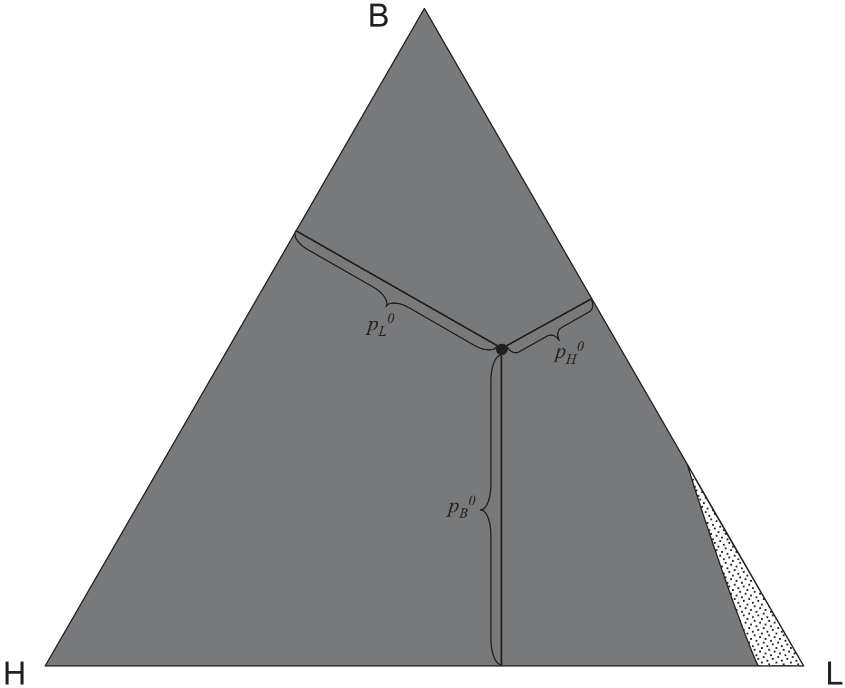

Figure 4. Ternary plot showing how the resulting final population depends on the initial population. The location in the graph represents the initial proportion of *H*, *L*, and *B* speakers. Consider an initial population, represented by the black dot. The relative proportions of *H*, *L*, and *B* speakers are given by the lengths of the lines notated as p_{H0}, p_{L0}, and p_{B0}, respectively. A point in the middle, for example, represents the situation in which all three types are equally numerous. A point in the bottom right represents a system starting with only *L* speakers. The shading of the region surrounding every point represents the final population. Dotted regions represent the area where only *H* speakers exist at equilibrium, and dark regions represent the area where only *L* speakers exist at equilibrium. In all cases the steady-state solution is the existence of a single language, generally *H* unless the initial population contains few *H* speakers. The model is the same as in Figure 3: $\alpha_H = 3$, $\alpha_L = 1$, and $\omega = 2$.

stable bilingualism so long as the propensity to learn a language (i.e., the child's receptiveness) is sufficiently strong; that is, α_H and α_L are sufficiently large. This is shown in Figure 5, which shows steady-state population as a function of α_H and α_L for $\omega = 1$ (i.e., equal status; Figure 5A) and $\omega = 2$ (i.e., *H* status higher than *L* status; Figure 5B). The hatched areas show where there is stable bilingualism for high values of α_H and α_L. To the left of the hatched region, *H*, *L*, and *B* speakers coexist in a steady state.

Intervention in the Public Sphere. In the previous section we examined the dynamics of a model in which *H* and *L* started at equal frequency but *L* eventually

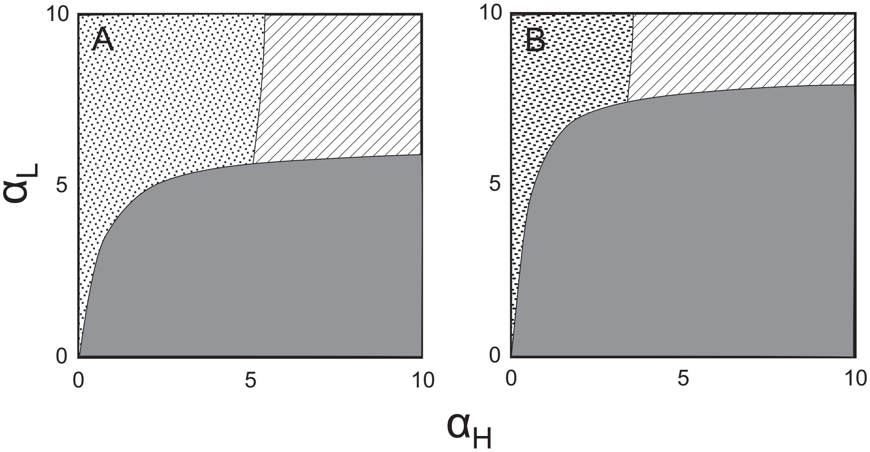

Figure 5. Equilibrium concentration of population when the initial conditions are 90% *H* speakers and 10% *L* speakers for various values of α_H and α_L for (A) $\omega = 1$ and (B) $\omega = 2$. Where the final population is of *L* speakers, the region is stippled. Where the final population is of *H* speakers, the region is solid gray. Where the final population is bilingual, the parameter region is shaded with lines. The lack of symmetry in part A is due to the difference in initial population. Large values of α_H and α_L result in stable bilingualism. Even a moderate change in ω increases the required value of α_L necessary for *L* to dominate or for stable bilingualism.

went extinct. What manipulations to the system could save *L* when *L* has already been decaying so that its proportion is low (10%)?

Dynamics of a mixed intervention in Figures 3C and 3D show that an intervention strategy can promote language stability. What types of intervention work best? Figure 6 shows the results of exploring the parameter space of possible interventions. In general, increasing the receptiveness of children to *L* (i.e., changes in α_L) are not sufficient unless drastic. Moderate values of λ and m_{HB} can be effective, especially when used together along with an increase in α_L. In general, an increase in λ is about twice as effective as a similarly sized increase in m_{HB} in increasing the proportion of *L* speakers in a community (see Figure 6). Once the parameter settings of the model have been confirmed, governments could decide to invest in altering λ, m_{HB}, or α_L, to achieve maximum effect.

Conclusion and Discussion

Our results are relevant to the field of language policy and language revitalization because they give mathematical validation to the supportive policies whose goal is language revitalization and maintenance. The main features of the previous policy recommendations are guaranteeing intergenerational transmission, increasing the status of the endangered language, changing the attitudes of the speakers, improving the economic situation of the people, teaching the

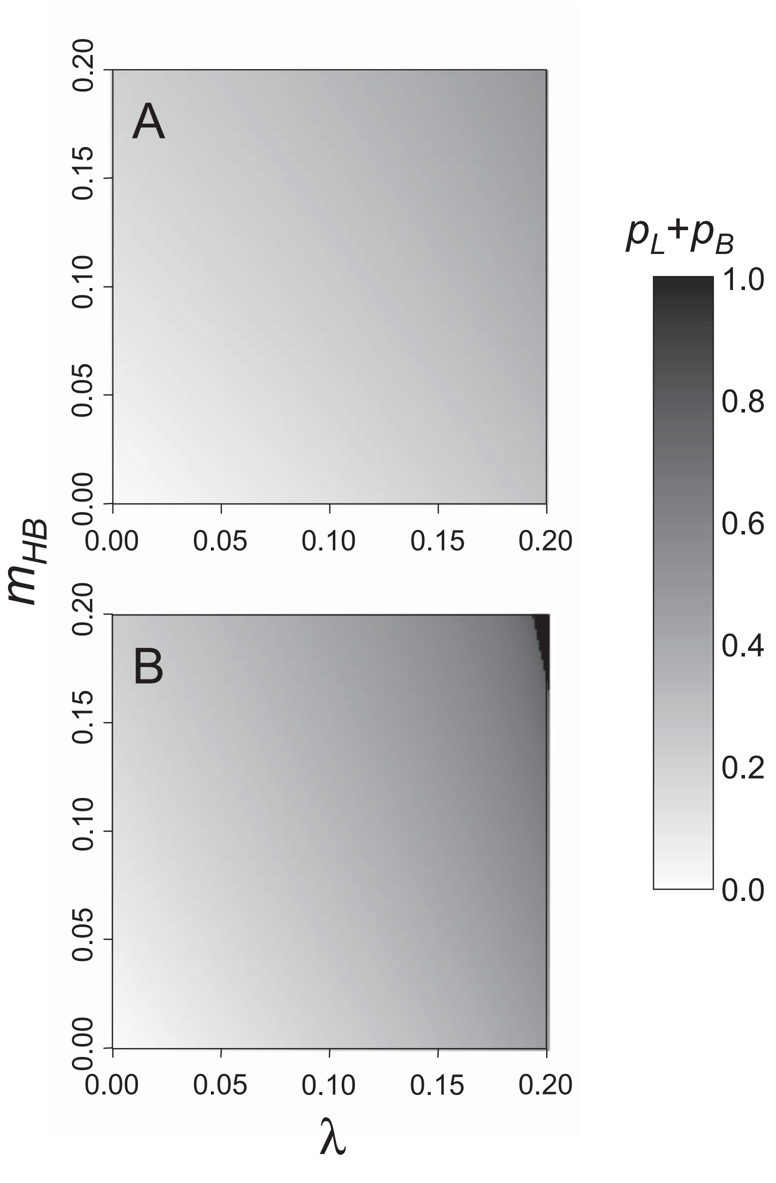

Figure 6. Effect of government intervention by increasing m_{HB} and λ, including when α_L is increased. Darkness of shading represents steady-state proportion of *L* and *B* speakers in the population. (A) $\alpha_H = 3$, $\alpha_L = 1$, $\omega = 2$, and starting population is 90% *H* and 10% *L*. (B) Additional effect of increasing α_L to 1.5. Note that strong intervention can result in *H* going extinct (i.e., the entire population speaks *L*). In general, increasing λ is about twice as effective as increasing m_{HB} by the same amount.

endangered language in schools, increasing its use in the public domain, and improving legislation (Cantoni 1996; Crystal 2000; Fishman 1991; Grenoble and Whaley 2006; Nettle and Romaine 2000; UNESCO Ad Hoc Expert Group on Endangered Languages 2003). Our results also emphasize the interconnectedness of various revitalization measures and show which measures are most effective. The following discussion relates our results to existing supportive policies and language planning solutions.

First, our main finding, consistent with previous models, is that languages are mutually exclusive unless children are highly responsive to learning both languages (large, roughly equal values of α_L and α_H). Existing evidence suggests that without intervention this is unlikely to be the case, because minority languages are in fact dying out. It is an empirical question whether the high values of α_L and α_H required for language coexistence can be achieved. Some effective policies and planning measures for increasing α_L include not only giving the low-status language an official legal status (Romaine 2002) but also taking active measures to make the language economically useful and attractive in the labor market, that is, giving it market value (Grin 1999), and conducting language awareness campaigns that highlight and explain the benefits of being bilingual (Edwards and Pritchard-Newcombe 2005). Greater representation of higher-status languages in the media, resulting in $\omega > 1$, increases the value of α_L necessary for maintenance of L. This suggests that actively reducing ω can assist in language revitalization efforts.

Second, our model shows that increasing the amount of the low-status language in the public sphere or providing formal language instruction can work; however, interventions that increase the proportion of the low-status language heard in the public sphere are particularly effective and synergistic with interventions that increase responsiveness α_L or provide formal teaching (m_{HB}) (see Figures 3 and 6). Assuming that the parameter values in Figure 6 can be supported with quantitative data, we find that continuous intervention that increases the use in public domains is approximately twice as effective as interventions that use formal teaching (Figure 6). Teaching (m_{HB}) can achieve an initial concentration of speakers of the low-status language, and this concentration can be maintained with other interventions.

The observed synergy between interventions highlights the interconnectedness of revitalization policies and measures. Strubell (2001) and Walsh and McLeod (2008) also suggested that increasing public services available in the low-status language makes it more attractive, which increases its attractiveness and leads to the likelihood of it being studied and used, and this in turn increases the need for public services and so forth. Note that the implications of our results differ from Fishman's (1991) policy recommendation of securing intergenerational transmission and achieving stable diglossia (i.e., increasing the language being spoken in low-status domains) before bringing it to public spheres or schools. It is difficult to achieve the suggested high initial population by targeting the private domain on its own, as our first result shows. A more integrated approach is required instead of the stages that Fishman suggests.

Third, our model shows that, as in Figure 4, the lower-status language can persist against an externally stable higher-status language if the initial proportion of speakers of the low-status language is extremely high, as in Quebec, where 80% of the population already spoke French when language revitalization was initiated (Bourhis 2001), or in England, when lower-status English was able to survive the Norman conquest. In both cases current bilingualism is due to contact and population mixing with external populations. The policy recommendation that follows this result is that language maintenance should be started when a low-status language has a high initial population in a certain area. This differs from the general practice in which communities and policy makers are generally not concerned about securing the future of large languages.

Fourth, our model shows that if the status of the two languages cannot be made high, then continuous government intervention is needed to preserve the low-status language. When government support measures are abandoned, the low-status language dies out unless one has managed to dramatically increase its number of speakers so that the low-status language is able to dominate the high-status language (see Figure 4). A robust finding (over a wide range of realistic parameter values) of our model is therefore that the low-status language can be stable (in coexistence with the high-status language) only if intervention is continuous (Figures 3 and 6). Continuous intervention (e.g., $\alpha_L = 1.5$, $\lambda = 0.1$, $m_{HB} = 0.1$) can result in stable bilingualism even if the low-status language has a negligible number of speakers to begin with. In practice this means that abandoning a revitalization project too early would reverse any gains made, which is what Grenoble and Whaley (2006) pointed out.

Fifth, our model shows that segregation can support language maintenance together with intervention in the public sphere (cf. Patriarca and Leppänen 2004). Our model initially assumed that all speakers of the languages can mate with each other, so long as they can communicate with each other, which leads to the low-status language going extinct as speakers of the low-status language mate with bilingual speakers and their bilingual children mate with speakers of the high-status language. Increasing the amount of segregated mating in our model slowed down the loss of the low-status language (given no intervention) but did not affect the ultimate outcome. It did, however, increase the steady-state concentration of the low-status language and of bilingualism when government intervention was increased. Thus segregation is synergistic with government intervention but is not a long-term solution on its own.

Mating segregation could in practice refer to a situation in which cultural segregation exists sympatrically (as is the case with the successfully revitalized Hebrew or with the Amish, who have managed to maintain their language) or where regionalism is strong (i.e., where speakers of endangered languages choose to stay in their traditional area). These are instances of self-imposed boundary maintenance (Paulston 1994), or what Bourhis (2001) calls a separatist orientation, which is rather rare. Policy makers should instead try to encourage the speakers of an endangered language to stay in a local area through development programs that would secure an income in that particular place, without the speakers having to move to urban centers [or to the vicinity of rubber plantations or mines, as has happened, for example, in the Amazon (Aikhenvald 2003)]. Similarly, Scots Gaelic speakers were forced to switch over to English because they lost their local livelihood as fishers and had to seek other employment elsewhere, which led to the disintegration of their society (Dorian 1981). This further emphasizes the need to develop language policies together with economic and regional policies (Nettle and Romaine 2000).

The remaining elements of model validation are to justify the choice of parameter values for η and ω. We have assumed that η is 0.5; that is, 50% of language learning and linguistic socialization takes place at home and 50% takes place outside the home. This value can be changed according to the type of situation in question based on the empirical data acquired. The value of ω is an important parameter for understanding the role of media and how government discourse dominates public discourse, in this case, in the use of language.

One should bear in mind that not all language revitalization projects have the same aims. The aim is not always to achieve full fluency by all community members and to use the endangered language in all domains. Few programs reach that goal. The revitalization of Cornish has been successful in teaching people some Cornish, not expecting fluency, and the same applies to Hawaiian, which is spoken mainly in schools but not in the private domain (Grenoble and Whaley 2006). So in these cases there has in essence been an increase of semispeakers using the language in limited domains. Efforts in cases where a language has few speakers should perhaps be focused on documentation work and cultural awareness programs, as it is rare that formal teaching will lead to intergenerational transmission. The small languages could instead survive as markers of identity, types of cultural and symbolic tokens that are used in ceremonies or when giving names or greeting, as is the case with the reclaimed Australian language Kaurna (Amery 2000). Modeling these alternative goals remains for future work. The modeling of multilingual systems also remains for further work. In practice, it is important to set realistic and practical goals in language revitalization projects (Dauenhauer and Dauenhauer 1998) and not to expect policy changes to give instant results without long-term implementation and a multifaceted approach, which is supported by our model.

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