

# A Note on Conservativity

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The goal of this note is to exhibit a principled weakening of the Conservativity Constraint on natural language determiners (Dets) so that it allows a few non-conservative expressions like *only* but still blocks the great range of functions noted as non-conservative in the literature. We review classical conservativity in (I), then in (II) adduce evidence that *only* and its confreres are plausibly treated as Dets, interpreted as non-conservative. In (III) we present our weak-conservativity constraint, and in (IV) calculate its strength, showing that it still excludes almost all the functions excluded by classical conservativity while admitting the *only* family. So the “cost” of admitting *only* as a Det if we choose to do so is not severe.

I. *Conservativity*<sup>1</sup> is standardly defined below treating Dets extensionally as functions  $D$  from subsets of an arbitrary universe  $E$ , to generalized quantifiers – functions from subsets of  $E$  into the boolean set  $\{0,1\}$  of truth values:

Def  $D$  is *conservative* (*cons*) iff for all  $p, q \subseteq E$ ,  $D(p)(q) = D(p)(p \cap q)$

The idea is that the value  $D$  assigns to a pair  $p, q$  of (extensional) properties depends at most on which individuals have  $p$  and which of those have  $q$ . Individuals with  $q$  but not  $p$  are irrelevant. So to evaluate *Det poets daydream* we may have to know who the poets are and which of them daydream, but knowing about daydreamers who are not poets is

1 *Conservativity* is the *lives on* relation in Barwise & Cooper 1981 and the *intersectivity* one in Higginbotham and May 1981. The term *conservativity* is from Keenan 1981.

irrelevant. Review of the copious lists of Dets in Keenan and Stavi 1986 (or Keenan & Moss 1985)<sup>2</sup> shows that the functions they may denote are indeed conservative. Syntactically, Dets are expressions which combine with common noun phrases (CNPs) to form argument expressions of predicates. Some samples are underlined in (1); the functions they denote are in bold in (2). Verification that these functions are conservative is straightforward.

- (1) a. No poets daydream  
 b. Every student but John got an A on the exam  
 c. Some but not all students are liberal  
 d. Seven out of ten students signed the petition
- (2) a. **no(p)(q) = 1 iff  $p \cap q = \emptyset$**   
 b. **(every...but john)(p)(q) = 1 iff  $p - q = \{\text{John}\}$**   
 c. **(some but not all)(p)(q) = 1 iff  $p \cap q \neq \emptyset$  &  $p - q \neq \emptyset$**   
 d. **(more than seven out of ten)(p)(q) = 1 iff  $10 \cdot |p \cap q| > 7 \cdot |p|$**

In contrast, the functions in (3) below are easily seen to fail conservativity and indeed we find no English Dets of type  $(et, (et, t))$  which denote them:

- (3) a.  $F(p)(q) = 1$  iff  $|p| = |q|$                       b.  $G(p)(q) = 1$  iff  $p \times q \neq \emptyset$   
 c.  $H(p)(q) = 1$  iff  $p^* \subseteq q$                         d.  $I(p)(q) = 1$  iff  $(p \cup q) = E$

(Here and later we write  $p^*$  for the complement of  $p$ , that is,  $p^* =_{def} E - p$ .)

The equicardinality function  $F$  is not cons as  $p$  and  $q$  may have the same size with  $p \cap q$  of lesser cardinality: set  $p = \{a, b, c\}$  and  $q = \{b, c, d\}$ . In general cardinal comparison functions,  $|p| = |q|$ ,  $|p| < |q|$ ,  $|p| = 3 \cdot |q|$ , etc., are naturally denotable by (conservative) two place Dets of type  $((et, et), (et, t))$ , as in *exactly as many students as teachers*, *fewer students than teachers*, etc. See Keenan & Moss 1985, Keenan & Westerstahl 1997.  $G$  in (3b) just says that neither  $p$  nor  $q$  is empty.  $H$  in (3c) is the **allnon** function in Chierchia and McConnell-Ginet 1990:427.  $I$  in (3d) just says everything is either a  $p$  or a  $q$  (some possibly both).

We note (for purposes of one point to be made later) that the type notation  $(et, (et, t))$  is not quite as revealing as it might be (Keenan 2015), as argument expressions combine with predicates of any valence, not just one place ones: *Tim admires all poets*, *Some student gave every teacher an apple*, etc. Writing  $p$  to abbreviate  $(e, t)$  we might note the type for argument expressions as  $(p^{n+1}, p^n)$ , indicating that they combine with  $n+1$ -place predicates to form  $n$ -place ones. So generalized quantifiers are argument reducers. And a one-place Det maps a property to expressions of that type. Their semantics is straightforward (assumed and used in Keenan & Westerstahl 1997 and discussed in Keenan 2015). We illustrate the case where  $D(p)$  takes a binary relation as argument (so  $D(p)$  is its object, not its subject): *likes all poets* will denote  $(\mathbf{all}(\mathbf{poet}))(\mathbf{like})$ , which is the set of objects  $x$  which are such that  $(\mathbf{all}(\mathbf{poet}))$  holds of the set of individuals that  $x$  likes,  $\{a \in E \mid (\mathbf{all}(\mathbf{poet}))(\mathbf{like}_a) = 1\}$ . (For  $R$  a binary relation we write  $R_a$  for  $\{b \mid (a, b) \in R\}$ .) Note then that the value that a  $D(p)$  assigns to

2 The relative dates are misleading. Early versions of Keenan & Stavi which included the conservativity constraint circulated in 1981 and have been cited with that date (Thysse 1985, Westerstahl 1985). Keenan & Moss cites Keenan & Stavi even though the latter appeared in print later. The former has the greater abundance of complex Dets, especially ones of type  $((et, et), (et, t))$ .

an  $n+1$ -ary relation is determined by the values it assigns to the unary relations, which is what is given when it is defined as of type  $(et, (et, t))$ , so that type notation suffices, once we understand what it means.

Returning now to conservativity, Zuber 2004 presents several candidates for Dets whose denotations are not cons. One is the *relative proportional many* of Westerståhl 1985: *Many Scandinavians have won the Nobel Prize*, on the reading on which it says that the proportion of Nobel Prize winners who are Scandinavian is large. Such an extensional reading would indeed be non-conservative. However, Herburger 1997, Bastiaanse 2014 and Romero 2015 provide richer, more structured interpretations of *many* which, in our judgment, significantly improve our understanding of *many* and which, in different ways, rescue a conservative interpretation for it. The account in Bastiaanse is pleasingly general as it models the properly intensional character of *many* and so extends to many other value judgment Dets, such as *too many*, *not enough*, *surprisingly many*, etc. In view of these analyses we shall put *many* aside.

But to our knowledge *only*, as in (4), has not been convincingly reanalyzed (See von Fintel 1997 and von Fintel and Keenan 2018).

- (4) a. Only poets daydream      b.  $\text{only}(p)(q) = 1 \text{ iff}_{\text{def}} q \subseteq p$

(4a) is true iff anyone who daydreams is a poet. The truth of  $\text{only}(p)(q)$  varies with the choice of  $p$  and  $q$ , but  $\text{only}(p)(p \cap q)$  is always true, since  $p \cap q \subseteq p$  all  $p, q$ . So *only* is not cons.

But just how strong is the case that *only* ought to be treated as a Det? That is, does it combine with CNPs to form argument expressions of predicates, analogous to *no*, *every*, *some but not all*, etc.? Below we provide some support for treating *only* as a Det. We are not claiming that our observations are decisive, only that they constitute a prima facie case for *only* (and a few other non-conservative expressions) for being Dets, whence it is of interest to see by how little we can weaken the Conservativity Universal to accept them as such.

## II. Three types of support for the Det status of *only*.

1. (Metalinguistic) Several scholars have treated *only* as a Det, or more accurately, just assumed it was: Herburger 1997 (at “LF”), Higginbotham & May 1981, Johnsen 1987 and de Mey 1991 (who assumes *only* is syntactically a Det but dismisses it on grounds of its non-conservativity). Also von Fintel & Keenan 2018 accept the plausibility of treating *only* as a Det, the point of their article being to show that a certain proposed weakening of conservativity, motivated solely by *only*, is in fact equivalent to the classical notion.
2. If *only* combines with property denoting expressions to form argument expressions then it satisfies compositionality when interpreted as *only* above. So *only* appears to occur in the position where other Dets occur and is naturally interpreted as a function of type  $(et, (et, t))$ , as are established Dets.
3. Nominal expressions built from *only* share significantly the distribution of nominal argument expressions more generally. More explicitly, we argue that if expressions like *only students* are analyzed in such a way that *students* is a CNP and *only* is a Det then the expression *only students* should be an argument expression and thus occur in syntactic contexts where expressions already recognized as argument expressions occur. Support for this is (3.1) – (3.6) below.

3.1 *only*+CNP occurs as subject, (4a), direct object (5a), and possessor, (5b):

- (5) a. This school admits only women / no women / some excellent musicians  
 b. Only students' bicycles were stolen / Most students' bicycles were stolen

3.2 *only*+CNP Raises to Object, (6b), and Passivizes to subject, (6c):

- (6) a. We believe that only students / all students like jazz  
 b. We believe only students / all students to like jazz  
 c. Only students / All students are believed to like jazz

3.3 *only*+CNP occurs as pivot in Existential There constructions:

- (7) There were some/several/no/only undergraduates at the lecture

3.4 *only*+CNP interpreted as in (4b) is decreasing on its second argument and so licenses negative polarity items (in English) as expected, (8a) vs (8b):

- (8) a. Only/No linguists have *ever* visited Panini's birthplace  
 b. #Some/Several linguists have *ever* visited Panini's birthplace

3.5 Multiple argument positions of a given predicate can be simultaneously filled by *only*+CNP:

- (9) a. Only linguists admire only linguists  
 b. All linguists admire all linguists

3.6 *only*+CNP forms boolean compounds with [Det+CNP]s:

- (10) Most linguists but not only linguists speak several languages  
 We treat not only children but also their parents

We note that different classes of argument expressions have different privileges of occurrence. E.g. *some/several/only*+CNP occur in Existential There contexts, as in (7) but ones like *Most/All*+CNP are not natural there. Similarly the post *of* position in plural partitives seems limited to definite arguments *two of the/John's students* but *\*two of no/only students*. So again *only*+CNP behaves like other argument expressions.

These observations support the plausibility of treating *only* as a Det, but our data so far are limited. If we accept *only* as a Det is it an isolated counterexample to conservativity or are there others of a similar sort? If *only* is isolated we might just acknowledge it as an idiosyncratic lexical item of English and give it the ad hoc analysis in (11), where  $p, q, r$  range over subsets of  $E$  and *one* (more usually, *individual*) denotes  $E$ .

- (11) a. *Only*+CNP is a surface realization of *no individuals except* CNP,  
 E.g. *Only women objected*  $\equiv$  *No one except women objected*  
 b.  $(\text{no...except } r)(p)(q) = 1 \text{ iff}_{\text{def}} p \cap q \subseteq r$   
 c.  $(\text{no...except } r)$  is cons since  $p \cap q \subseteq r \text{ iff } p \cap (p \cap q) \subseteq r$

So on this analysis we could claim that *only* is indirectly cons, since *no...except*  $r$  is. But we reject this analysis of *only* for two reasons.

First, it lacks generality. Keenan and Paperno 2017 report on an extensive cross language study of quantifiers drawing on 36 languages from diverse families and areas. They report that all have a natural lexical expression translating *only*. So its presence in English is not just an idiosyncratic fact about an English vocabulary item – we can expect to find equivalents of *only* cross-linguistically.

Second there are other expressions in English with meanings similar to *only*:

- (12) a. Only liberals will object to that  
 b. Just liberals will object to that  
 c. At most liberals will object to that

(12a,b) seem to us complete paraphrases. (12c) somehow more definitively allows that there might be no liberals at all who object to that. We could enrich the definition in (4) to say  $\text{only}(p)(q) = 1$  iff  $\emptyset \subset q \subseteq p$ , and similarly for *just* but not for *at-most*. Then *at most* would be a non-conservative Det different from *only/just*. But even unenriched the non-conservativity of *only* is not an idiosyncrasy of a single word, *only*, at least two other expressions share it.

Another candidate for a non-conservative Det, similar to *only* in some ways but different in others, is *even*, as in (13a).

- (13) a. Even undergraduates came to the lecture  
 b. At least one undergraduate came to the lecture  
 c. At least one non-undergraduate came to the lecture

Def  $\text{even}(p)(q) = 1$  iff  $p \cap q \neq \emptyset$  and  $p^* \cap q \neq \emptyset$  (recall that  $p^* =_{\text{def}} E - p$ )

Interpreting *even* as above ensures that (13a) entails both (13b) and (13c). We leave up to pragmatics the implicature that attendance by undergraduates was unexpected. And *even* is easily seen to be non-conservative. For  $E = \{a,b,c\}$  with  $p = \{a,b\}$  and  $q = \{b,c\}$  we have that  $\text{even}(p)(q) = 1$  since  $p \cap q = \{b\} \neq \emptyset$  and  $\{a,b\}^* \cap \{b,c\} = \{c\}$ , also non-empty. But  $\text{even}(p)(p \cap q) = \text{even}(\{a,b\})(\{b\}) = 0$  since  $\{a,b\}^* \cap \{b\} = \emptyset$ . Note further that on the limited logical analysis of *even* above, when it occurs as a Det, we would assign the same logical analysis as *also*, as in *Also undergraduates came to the lecture*. The meaning difference between this sentence and (13a) would be purely pragmatic, with *also* lacking the unexpectedness implicature.

We note that *only*, *even* and *also* are generalized focus operators serving to focus many categories of expression other than nominals. In (14) for example they all can focus transitive verbs.

- (14) a. John only insulted Jim, he didn't threaten him  
 b. Eric even threatened Sam, in addition to insulting him  
 c. Eric also threatened Sam, in addition to insulting him

So *only* et al should be interpreted as functions taking denotations of a variety of types as arguments. But if one of those types are properties then we should still expect them to behave conservatively in that case. *All* for example combines with many expressions besides CNPs: *He is all alone now*, *He sobbed all night long*, *He's got mud all over him*, etc. Should that excuse it from having a conservative interpretation in *All teenagers love jazz*? Surely not, on pain of robbing the Conservativity claim of its empirical content.

Another candidate for a non-conservative Det is *mostly*, as in (15a), mentioned first to our knowledge in [Johnsen \(1987\)](#).

- (15) a. Mostly women voted for Reagan  
 b. Most people who voted for Reagan were women  
 c.  $\text{mostly}(p)(q) =_{\text{def}} \text{most}(q)(p), = 1 \text{ iff } 2 \cdot |q \cap p| > |q|$

Now *mostly* does occur as an S-level adverbial, as in (16).

- (16) Mostly / For the most part it is men who fight wars

And one might claim that the *mostly* in (15a) is a syntactically “lowered” S-level *mostly*. But this claim is hard to reconcile with the grammaticality of (17a-e).

- (17) a. Jake admires mostly linguists  
 b. Mostly linguists admire mostly linguists  
 c. A few men but mostly women voted for Reagan  
 d. Except for / Apart from John mostly women signed up for that course  
 e. Mostly, members of the Green Party support mostly liberals

Lastly, [Ahn and Sauerland 2017](#), henceforth A&S, discuss an intriguing non-conservative interpretation of a certain type of proportional Det:

- (18) a. That company hired fifty percent women (last year)  
 b. The company hired fifty percent of the women (last year)  
 c. The company hired all women (last year)

Now the object argument in (18b) is unproblematic and its Det behaves conservatively: *the women* in (18b) identifies a set of women in the domain and the Det says that fully half of them have the property expressed by  $\lambda x.the\ company\ hired\ x$ . But (18a) is understood differently. It says that half the people that the company hired were women. This interpretation is triggered (in English, see A&S for several other languages) by the presence of a proportional Det (e.g. percentage and fractional expressions, see below) directly adjacent to a CNP. We shall refer to these as *bare proportionals*. In English bare proportionals do not occur as subjects. ??*Twenty per cent women applied for the job*. We see in the next section that bare proportionals are non-conservative, as A&S claim.

(18c), not drawn from A&S, seems to us ambiguous. The, improbable, conservative reading says that the company hired all the women in the domain. The more natural reading is that all the people the company hired were women (last year), and this reading is non-conservative just as (18a) is.

### III. Weak Conservativity

Various (unconvincing) proposals for weakening conservativity have been suggested. [Von Stechow & Keenan 2018](#) is a recent detailed discussion of one such case. Here we propose *weak conservativity*:

#### DEFINITION:

- a. D is  $\text{cons}_2$ , *conservative on its 2<sup>nd</sup> argument*, iff  $D(p)(q) = D(p)(p \cap q)$ , all  $p, q$   
 b. D is  $\text{cons}_1$ , *conservative on its 1<sup>st</sup> argument*, iff  $D(p)(q) = D(p \cap q)(q)$ , all  $p, q$   
 c. D is *weakly conservative (weak-cons)* iff D is  $\text{cons}_1$  or D is  $\text{cons}_2$

**THEOREM:** For all  $D$  of type  $((e,t)(et,t))$ , all  $i \neq j \in \{1,2\}$ ,  $D \in \text{cons}_i$  iff  $D^{-1} \in \text{cons}_j$  where  $D^{-1}(p)(q) =_{\text{def}} D(q)(p)$

pf:  $D \in \text{cons}_2$  iff  $Dp,q = Dp,p \cap q = 1$  iff  $D^{-1}p \cap q,p = 1$  iff  $D^{-1}q,p = 1$  iff  $D^{-1} \in \text{cons}_1$   $\square$

**PROPOSITION:** The converse function  $^{-1}$  from  $\text{cons}_i$  to  $\text{cons}_j$ ,  $i \neq j$ , is a boolean isomorphism in each case. Also  $D^{-1-1} = D$  and  $H = D^{-1}$  iff  $H^{-1} = D$ .  $\square$

**COR:** **only** is  $\text{cons}_1$  since **only** = **all** $^{-1}$  and **all** is  $\text{cons}_2$   
**mostly** is  $\text{cons}_1$  since **mostly** = **most** $^{-1}$  and **most** is  $\text{cons}_2$

**FACT 1:** **even** is  $\text{cons}_1$ , as

**even** $(p \cap q)(q) = 1$  iff  $(p \cap q) \cap q \neq \emptyset$  &  $(p \cap q) * \cap q \neq \emptyset$  Def even  
iff  $(p \cap q) \neq \emptyset$  &  $(p * \cup q *) \cap q \neq \emptyset$   
iff  $(p \cap q) \neq \emptyset$  &  $(p * \cap q) \cup (q * \cap q) \neq \emptyset$   
iff  $(p \cap q) \neq \emptyset$  &  $p * \cap q \neq \emptyset$   
iff **even** $(p)(q) = 1$   $\square$

Thus **only/just, at most, mostly** and **even/also** are weakly conservative but not conservative ( $\text{cons}_2$ ) simpliciter.

We note that the corollary regarding *only* appears to be language general. Keenan and Paperno 2017 support that essentially all the languages in their sample have a natural translation of *only* and a different natural translation (often many) for *all*. Exactly how bare proportionals are identified varies cross linguistically as A&S show (case marking and definiteness are factors). Exactly which Dets allow converse interpretation needs further investigation.

Let us note now that the bare proportional Det in (18a) is weak-cons, as it is  $\text{cons}_1$ . We begin by defining an illustrative function **k%**, for  $k$  a natural number with  $0 \leq k \leq 100$ , assuming an arbitrary non-empty domain  $E$ . (We take **k%** in the exactly sense, but **at least k%** and **more than k%** behave comparably).

**DEFINITION:** **k%** $(p)(q) = 1$  iff  $|p \cap q|/|p| = k/100$  ( $p$  finite, non-empty)

**DEFINITION:**  $D$  of type  $(et,(et,t))$  is *proportional* iff for all  $p,q,p'q' \subseteq E$ ,

If  $|p \cap q|/|p| = |p' \cap q'|/|p'|$  then  $D(p)(q) = D(p')(q')$   $\square$

The idea is that if the proportion of  $p$ 's that are  $q$ 's is the same as the proportion of  $p$ 's that are  $q$ 's then  $Dp,q$  and  $Dp',q'$  have the same truth value – both 1 or both 0.

**Proposition** **k%** is proportional and  $\text{cons}_2$ . Also the proportional functions are closed under the pointwise boolean operations:  $(F \wedge G)(p)(q) = Fp,q \wedge Gp,q$  and  $F * p,q = (Fp,q) *$ .  $\square$

We now stipulate as a condition on the interpretation of English, which uses the following definition:

**DEFINITION:**  $D$  is *intersective* iff for all  $p,q,p',q'$  if  $p \cap q = p' \cap q'$  then  $Dp,q = Dp',q'$ .

Dets such as *some*, *no*, *no...but Tim*, *more than/less than five*, ... are intersective (many meeting stronger conditions).

### Condition on Interpretation

If a Det  $d$  is interpreted as a proportional but non-intersective function  $\mathbf{d}$  in a partitive  $[d \text{ of } DP_{def}]$  then in a bare proportional  $[d + \text{CNP}]$   $d$  is interpreted as  $\mathbf{d}^{-1}$

Since the converses of  $\text{cons}_2$  functions are  $\text{cons}_1$  they are weakly conservative. We illustrate with the interpretation of (19) below, where *John* denotes  $j$ :

- (19)  $\llbracket \text{John hires } 50\% \text{ women} \rrbracket = 1$   
     iff  $j \in \{ \text{al}(50\%)^{-1}(\text{woman})(\text{hire}_a) = 1 \}$                       def interpretation  
     iff  $j \in \{ \text{al}(50\%)(\text{hire}_a)(\text{woman}) = 1 \}$                       def converse  
     iff  $(50\%)(\text{hire}_j)(\text{woman}) = 1$                                       set theory  
     iff  $|\text{hire}_j \cap \text{woman}| / |\text{hire}_j| = 50/100$                       def  $k\%$

Thus in (19) we are comparing the number of women John hired to the number of individuals he hired, as desired. So bare proportional Dets are interpreted in  $\text{cons}_1$  because they are the converses of elements of  $\text{cons}_2$ . This also holds for **only** as it is the converse of **all**. Note that **all** satisfies the defining condition for being proportional; **some** does not, it is proportional but also intersective.

It is worth noting that the Condition on Interpretation that we are proposing is logically natural. It maps  $\mathbf{d}$  to an isomorphic image  $\mathbf{d}^{-1}$ .  $\mathbf{d}^{-1}$  bears the same logical relations to the elements of  $\text{cons}_1$  as  $\mathbf{d}$  itself bears to the elements of  $\text{cons}_2$ .

The above analysis is obviously in need of extensive empirical study, but it does show how the basic examples of bare proportionals can be formally classified as weakly conservative. And it does predict acceptable complex cases like *Our company hires at least 10% and not more than 50% women every year*. None of these were among the cases that motivated our original definition, so we count this as satisfying the criterion of applying to new cases.

### IV. How strong a constraint is weak conservativity?

We answer this explicitly as follows: the number of properties (subsets of  $E$ , where  $|E| = n$ ) is  $2^n$ , so the number of pairs of properties is  $2^n \cdot 2^n = 4^n$ , whence the number of functions from pairs of properties into  $\{0,1\}$  is 2 raised to the power  $4^n$ . (The cardinality of the set of functions from a set  $A$  into a set  $B$  is  $|B|^{|A|}$ ). So even in a small domain,  $|E| = 2$ , there are  $2^{16} = 65,536$  functions from pairs of properties into truth values!

How many of these functions are conservative ( $\text{cons}_2$ )? *Thyssen 1983* and *Keenan & Stavi 1986* give an answer: 2 raised to the power  $3^n$ . Here we reiterate the intuition behind this figure. The “minimal” (atomic) conservative functions are those of the form  $F_{p,q}$ , where  $p$  and  $q$  are properties with  $q \subseteq p$ . By definition  $F_{p,q}(s)(t) = 1$  iff  $s = p$  and  $s \cap t = q$ . The number of such functions corresponds to the number of pairs of sets  $(p,q)$  with  $q \subseteq p$ . Such pairs can be thought of as partitioning  $E$  into three subsets: any  $b \in E$  is in exactly one of:  $q$ ,  $p \cap q^*$ , or  $p^* \cap q^*$  as  $q \subseteq p$ , so  $q \cap p^*$  is empty. Thus there are  $3^n$  such pairs and so  $3^n$  such atomic functions. Each conservative function  $D$  corresponds to a set of  $F_{p,q}$ , those for which  $D(p)(q) = 1$ . So there are 2 raised to the power  $3^n$  such functions, that is,  $|\text{cons}_2| = 2$  to the  $3^n$ . So in a domain of just 2 entities, there are  $2$  to  $3^2 = 2^9 = 512$  such functions. This is



a massive reduction from 65,536, the total number of functions of type  $(et, (et, t))$  in a two element model.

By symmetry  $|cons_1| = |cons_2|$ . The converse function  $^{-1}$  mapping each  $D$  to  $D^{-1}$  maps each  $cons_2$  function to a  $cons_1$  one;  $^{-1}$  is one to one and onto, so  $|cons_1| = |cons_2| = 2$  to the power  $3^n$ .

However the number of weakly conservative functions is not just double the number of  $cons_2$  ones, 2 to the power  $3^n + 1$ , as that would count functions in  $cons_1 \cap cons_2$  twice. Provably  $cons_1 \cap cons_2$  is just the set of intersective functions.

**THEOREM:**  $D$  of type  $(et, (et, t))$  is in  $cons_1 \cap cons_2$  iff  $D$  is intersective

pf:  $\Rightarrow$  Let  $D \in cons_1 \cap cons_2$ . Show  $D$  intersective. Let  $p \cap q = p' \cap q'$ . Then  $Dp q = Dp, p \cap q$  as  $D$  is  $cons_2$ ,  $= D(p \cap p \cap q)(p \cap q)$  as  $D$  is  $cons_1$ ,  $= Dp \cap q, p \cap q = Dp' \cap q', p' \cap q' = Dp' \cap p' \cap q', p' \cap q' = Dp', p' \cap q'$ , as  $D$  is  $cons_1$ ,  $= Dp' q'$  as  $D$  is  $cons_2$ .

$\Leftarrow$  Let  $D$  intersective. Then  $Dp q = Dp, p \cap q$  since  $p \cap q = p \cap (p \cap q)$  and  $D$  is intersective, so  $D$  is  $cons_2$ . Mutatis mutandis  $D$  is  $cons_1$ , completing the proof.

Zuber 2005 provides an alternate proof. □

How many intersective functions are there? Well, the value of each such function is decided by a single property of which there are  $2^n$ , so in effect we are looking at the set of functions from a set of size  $2^n$  into  $\{0, 1\}$ , so it has size 2 to the power  $2^n$ . Formally: the function  $h$  which maps each intersective  $D$  to the map  $h(D)$  from properties into  $\{0, 1\}$  given by  $h(D)(p) = D(E)(p)$  is bijective, hence the set of intersective functions has cardinality 2 raised to the power  $2^n$ . Thus:

**PROPOSITION:** For  $|E| = n$ ,  $|weak-cons| = 2^m - 2^k$ , where  $m = 3^n + 1$  and  $k = 2^n$ .

So in a domain with just 2 objects, there are 65,536 functions of type  $(et, (et, t))$ , of which only  $2^{10} - 2^4 = 1024 - 16 = 1008$  are weakly conservative. So weak-cons is still a very strong constraint: of the logically possible Det denotations in a world with just two individuals only 1 in 65 satisfies weak conservativity.

Thus, while we may not want to consider as Dets *only/just, at most, mostly* and *even/also*, if we do, and interpret them extensionally as above, and we also allow bare proportionals, an open class, we still maintain a very strong constraint on Det denotations. We leave it as an exercise for the reader to show that the four non-conservative (non- $cons_2$ ) functions in (3) are also not weakly conservative.

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