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Abstract—This manuscript provides a general analytical framework to accurately predict the performance of several slotted Random Access (RA) schemes such as Slotted Aloha and Diversity Slotted Aloha. Compared to past literature the proposed analytical framework allows to perform accurate performance estimation of these RA schemes exploiting state-of-the-art forward error correction schemes in the presence of packet power unbalance as it is the case in many practical systems. Furthermore, the analytical framework is also capable to accurately approximate the performance of more recent RA schemes particularly suitable for satellite communications such as Contention Resolution Diversity Slotted Aloha exploiting iterative interference cancellation techniques on top of packet repetition within the frame to further improve the RA scheme performance. The analytical results have been successfully verified against a comprehensive set of simulations covering cases of practical interest.

Index Terms—Satellite communication, SCADA systems, multiple access, time division multiple access, interference suppression.

I. INTRODUCTION

OVER the past years there has been a growing demand for low-cost interactive satellite networks supporting both fixed broadband consumer market and mobile applications, including machine-to-machine communications. These services call for the development of efficient multiple access protocols able to cope with large population of terminals sharing the available resources under very dynamic traffic conditions. In particular, in the return link of commercial satellite broadband access networks, residential users are likely to generate a large amount of low duty cycle bursty traffic with extended inactivity periods. A similar situation is occurring in satellite mobile as well for machine-to-machine communications, position reporting or other messaging and telematics applications networks whereby a large number of terminals typically generate infrequent packets for signalling transmission. As it is shown in [1], under these operating conditions the traditional Combined Free and Demand Assignment Multiple Access (CF-DAMA) satellite protocol [2] will not perform optimally. Random Access (RA) techniques are by nature, well matched to this type of traffic and capable to support large population of terminals sharing the same capacity. However, classical RA schemes have been widely investigated in the literature and are known not to perform efficiently in the satellite environment [3], [4]. RA techniques based on channel sensing [5], [6], commonly used in terrestrial networks, cannot be exploited in satellite networks because of the large channel propagation delay. Open loop RA protocols such as Slotted Aloha (S-ALOHA) for Time Division Multiple Access (TDMA) [7] are characterized by the fact that low packet collision probabilities (e.g. \( < 10^{-3} \)) are achieved at very low loads i.e. \( < 10^{-3} \) Erlangs. The Diversity Slotted Aloha (DSA) [8] is slightly improving the S-ALOHA performance by sending twice the same packet at random locations within the same frame to increase the time diversity thus reducing the Packet Loss Ratio (PLR) in the low load region. Operation in the high packet collision probability region is not practical in a satellite environment due to the high number of retransmissions needed yielding very high latencies. As a consequence, current RA's use in satellite networks is mainly limited to network login and the transmission of control packets. In some cases, RA is also used for the transmission of very small volumes of data. Example of systems where RA is used for control packets transmission and short data transmissions are the Digital Video Broadcasting Return Channel via Satellite standard [9] and the IP over Satellite standard [10].

As a result, it has become necessary to develop more efficient, highly reliable and low-complexity RA protocols better suited to the type of scenarios described above. Recently an enhanced version of DSA dubbed Contention Resolution Diversity Slotted Aloha (CRDSA) has been introduced [11]. The CRDSA key idea is to transmit two replicas per frame of the same packet at random locations within the same frame as in DSA but with a little extra signalling to point to the “twin” packet location. The incoming baseband frame samples are stored in the gateway demodulator memory to enable iterative signal processing. By scanning the memory, decodable packets are identified and canceled jointly with their replicas. The memory scan is repeated few times to solve the maximum number of packet collisions [12]. More recently an enhanced version of the CRDSA protocol dubbed CRDSA++ has been presented in [13]. CRDSA++ provides two main enhancements compared to the original version of the protocol: a) increased (optimized) number of packet repetitions (2 in CRDSA, 3-5 in CRDSA++); b) exploitation of the received packets power unbalance to further boost the RA performance\(^1\). Liva in [14] has been extending the concept

\(^1\)This was not the case in [11] whereby in case of packet collision no decoding attempt was performed.
of CRDSA to encompass an irregular repetition CRDSA scheme dubbed Irregular Repetition Slotted Aloha (IRSA).

To design the optimized irregular packet repetition scheme the author has been exploiting the bipartite graphs techniques typically used in the design and analysis of forward error correcting schemes. Although the proposed scheme shows some potential advantage compared to two-replicas CRDSA, for PLR $< 10^{-3}$ its throughput is lower than CRDSA with 3-4 replicas. Furthermore, the randomization of the number packet replicas in each frame is complicating the scheme implementation and the associated signalling mechanism.

RA analytical results for conventional RA schemes presented so far were assuming uncoded systems and equipped interfering packets [7], [15] which is not what is required by modern (satellite) communication systems. For CRDSA the analytical results reported in [11] were derived under even more restrictive assumptions i.e. negligible Additive White Gaussian Noise (AWGN) impact, only two replicas, no possibility to recover a packet in the presence of collision(s). Furthermore, only throughput simulation results were provided in [11] because the probability of successful detection derivation (eqn. (10)) was not accurate enough for PLR estimation. The CRDSA++ results reported in [13] were solely based on computer simulations with no analytical support. Some analysis of the power capture effect is reported in [16], [17] and [18], but existing models are limited to a finite population of terminals, discrete power levels, do not provide an accurate modeling of the RA Forward Error Correction (FEC) behavior and more importantly, do not include interference cancellation (IC) as implemented in most recent RA protocols.

In this contribution we define a generic analytical framework that allows calculating the performance of slotted random access techniques (e.g. S-ALOHA, DSA, CRDSA) in terms of PLR and throughput. The proposed approach assumes arbitrary power and traffic distributions, and accurately models the RA FEC behavior. Furthermore, the analytical framework is also capable to derive the performance of more recent RA schemes such as CRDSA exploiting IC techniques. Analytical results have been successfully compared to a wide set of simulation results showing the capability of the proposed analytical framework to predict the RA scheme performance in the vast majority of application cases.

The organization of the paper is as follows. In Section II we define the general model that is applicable to slotted RA techniques not employing IC. In Section III we customize the model to the well-known S-ALOHA and DSA techniques and verify the validity of the model. In Section IV we extend the framework to model two techniques for IC: iterative IC (IIC) and packet replicas IC due to successful reception of one of the packet replicas. In Section V we customize the model to the CRDSA technique and assess its performance by means of the analytical model and simulations. Section VI concludes the paper.

II. GENERALIZED RANDOM ACCESS MODEL WITHOUT INTERFERENCE CANCELLATION

To define the generic slotted random access model we introduce the concept of packet of interest (PoI) and slot of interest (SoI) (see Fig. 1). The PoI is the packet for which we want to compute the packet loss probability and the SoI is the slot where the PoI is contained. In general, we assume that the traffic is bursty thus the number of received packets/slot is time variant and distributed according to a given arbitrary distribution. The average medium access control (MAC) load expressed in information b/s/Hz, in order to avoid any dependence with the modulation cardinality or coding rate used. The relation between $G$ and $\lambda$ is provided below. We assume that the packets belonging to the same frame and originated from the same transmitter are received with the same energy per bit to noise spectral density ratio $E_b/N_0$. For simplicity analysis we assume that packets belonging to different frames are distributed according to a given power distribution common to all incoming packets. $N_{rep}$ is the total number of packet replicas transmitted in each frame. It follows that $N_{rep} > 1$ for diversity RA techniques such as DSA or CRDSA while $N_{rep} = 1$ for S-ALOHA. The MAC protocol throughput $T$ is simply related to the MAC load and $N_{rep}$ through the following equation:

$$T(G, N_{rep}) = G \cdot [1 - PLR(G, N_{rep})],$$

where the packet loss error rate $PLR$ is given by:

$$PLR(G, N_{rep}) = \int_0^\infty \{P_{loss}(\omega; \lambda(G, N_{rep}))\}^{N_{rep}} \cdot f_\Omega(\omega) \cdot d\omega,$$

where $f_\Omega(\omega)$ is the probability density function (PDF) for the energy per bit to noise power spectral density $[E_b/N_0]_{PAI} = \omega$. The PoI, $P_{loss}(\omega; \lambda(G, N_{rep}))$ is the probability of loss for the PoI when its $[E_b/N_0]_{PAI}$ is equal to $\omega$ and $\lambda(G, N_{rep}) = N_{rep} \cdot G \cdot G_p$ is the average traffic load measured in packets/slot. In the following for notation simplicity we drop the $\lambda$ dependence on the variables $G$ and $N_{rep}$. The processing gain $G_p$ is defined as $G_p = \frac{B}{R_c} = \frac{1}{\rho \log_2(M)}$, where $R_c$ is the channel baud rate, $B$ is information bit rate, $\rho$ is the channel coding rate and $M$ is the modulation cardinality. In the derivation of $\lambda$ we have disregarded the overhead of packet headers, preambles, pilot symbols and the transmit square-root raised-cosine (SRRC) roll-off factor extra bandwidth. The reason for these assumptions is to facilitate the comparison of the results with other research work which typically does not include system specific parameters. The packet loss probability is given by:

$$P_{loss}(\omega; \lambda) = \sum_{k=0}^{\infty} \{P^K_{loss}(\omega|k)\} \cdot f_k(k; \lambda),$$

where $f_k(k; \lambda)$ is the probability mass function for the packet arrivals in a slot and $P^K_{loss}(\omega|k)$ is the probability of loss of the PoI when its $[E_b/N_0]_{PAI}$ is equal to $\omega$ provided that there are $k$ colliding packets. The probability of loss of the PoI in the presence of $k$ colliding packets is approximated by:

$$P^K_{loss}(\omega|k) \approx \int_0^\infty \Gamma \left[10 \log_{10} \left(\frac{\omega}{1 + \chi}\right)\right] \cdot f_\Omega(\omega; k) \cdot d\chi,$$

where $f_\Omega(\omega; k)$ is the PDF for the interference to noise Power Spectral Density (PSD) ratio $\chi = I_0/N_0$ when there are
When collisions occur. By comparing Figs. 2 and 3 it can be observed that the advantage in terms of throughput compared to S-ALOHA.

To derive the PDF \( f_{\Xi}(\chi; k) \) of the sum of \( k \) lognormal random variables we can either perform the convolutions of the interfering packets PDFs as specified in eqn. (5), or approximate the sum of rvs by a new lognormal random variable with mean \( \mu_{\Xi} \) and standard deviation \( \sigma_{\Xi} \). The derivation of \( \mu_{\Xi} \) and \( \sigma_{\Xi} \) as a function of \( \mu_{\chi_n} \) and \( \sigma_{\chi_n} \) is a special case of [22] and it is reported in Appendix C. We will consider that the probability mass function for the packet arrivals in a slot \( f_{K}(k; \lambda) \) follows a Poisson distribution:

\[
 f_{K}(k; \lambda) = \frac{\lambda^k \exp(-\lambda)}{k!}, \quad (7)
\]

For the FEC PLR characteristics the reader can refer to Appendix A.

In Fig. 2 we compare the results obtained by a detailed simulation and the analytical model derived in this paper for S-ALOHA. The match of the analytical model is very good and the small deviations at high load values are small errors in the analytical PLR derivations that are magnified in the throughput curve at high loads as \( T = G(1 - PLR) \). The results without power unbalance (\( \mu = \sigma = 0 \) dB) also match the throughput results presented in [7]. In Fig. 3 we present the DSA performance results using both the analytical model derived in this paper and a detailed simulation for different values of the power unbalance standard deviation. The accuracy of the analytical model is also very good for DSA. As we can see, in both schemes the throughput improves with larger power unbalance as collisions become easier to be resolved (packet capture effect). However, the packet loss ratio is not significantly improved as the weaker packets are lost when collisions occur. By comparing Figs. 2 and 3 it can be remarked that for \( PLR < 10^{-2} \) DSA provides a considerable advantage in terms of throughput compared to S-ALOHA.

IV. GENERALIZED RANDOM ACCESS MODEL WITH INTERFERENCE CANCELLATION

In this section we extend the model defined in Section II to include the IIC process applied in more recent Random Access techniques (e.g. CRDSA). In the IIC process, packets interfering with the PoI may be cancelled in the SoI for two reasons. First, when the FEC is powerful enough to allow the interfering packet to be correctly decoded also in the presence of one or more colliding packets (power capture effect). Second, when several replicas are present in other slots than the SoI within the same frame (time diversity), one of the replicas is successfully decoded and the interfering packet can be cancelled from the SoI by using the information of the recovered data [12]. In the remaining of this Section, we will extend the model to cope with both situations.

A. Model for Iterative Interference Cancellation within the Slot of Interest

In the first case, we will start with a new definition of the probability of loss of the PoI when its \( E_b/N_0 \) PoI is equal to
defined in eqn. (4). The exact computation of $P$ \( \rho \)ponentially with the number of interfering packets $k$ and with the interferers' power distribution spread (infinite in general). As it is common practice when implementing successive IC, the demodulator will rank the interferers' power distribution \( \omega \) and when there are $k$ colliding packets, i.e. the $P_{\text{loss}}(\omega|k)$ defined in eqn. (4). The exact computation of $P_{\text{loss}}(\omega|k)$ is not simple when we want to take into account the effects of the IC process. The complexity of the calculation grows exponentially with the number of interfering packets $k$ and with the interferers' power distribution spread (infinite in general). As it is common practice when implementing successive IC, the demodulator will rank the $k$ colliding packets and the PoI based on their actual $E_b/N_0$ from stronger to weaker. Furthermore, we will assume that the demodulator will only attempt to decode the PoI when the preceding packets with higher $E_b/N_0$ have been successfully decoded and removed from memory. The assumption on the conditional detection of weaker packets subject to the successful detection of the stronger ones represents a tight upper bound for the probability of loss\(^2\). We will now derive the expression of $P_{\text{loss}}^K(\omega|k)$ taking into account the above assumption. Assuming we have $k$ interfering packets, the PoI with an \( [E_b/N_0]_{\text{PoI}} = \omega \) will occupy a ranked position $r$ from 0 to $k$. The case $r = 0$ will correspond to the case where the PoI has a higher or equal $E_b/N_0$ than any of the $k$ colliding packets in the SoI. And the case for $r = k$ will correspond to the case where the PoI has the lowest $E_b/N_0$. If we consider that the received packets $E_b/N_0$ are i.i.d. with a PDF $f_{E_0}(\cdot)$, the probability mass function $f_R(r|\omega,k)$ of the ranked position of the PoI follows a binomial distribution $B(n,p)$ with parameters $n = k$ and $p = 1 - F_{E_0}(\omega)$. Thus the probability of getting exactly $r$

\(^2\)Only in very unlikely circumstances (e.g. very low FEC code rates and robust modulation formats) it may be possible to decode a packet even when one or few of the stronger packets have not been successfully decoded and cancelled from memory.
successes in $k$ trials is given by the probability mass function:

$$f_R(r|\omega, k) = \binom{k}{r} \left[ 1 - F_1(\omega) \right]^r \left[ F_1(\omega) \right]^{k-r},$$

(8)

where $F_1(\omega)$ is the cumulative distribution function for the $E_0/N_0$ computed at the level of the PoI. Therefore, by making $p = 1 - F_1(\omega)$ in the binomial distribution we are computing the probability that $r$ out of the $k$ interfering packets have an $E_b/N_0$ greater than $\omega$, which corresponds to the $[E_b/N_0]_{PoI}$ of the PoI. The $P^K_{loss}(\omega|k)$ can now be computed as:

$$P^K_{loss}(\omega|k) = \sum_{r=0}^{k} P^R_{loss}(\omega|k, r) f_R(r|\omega, k),$$

(9)

where $P^R_{loss}(\omega|k, r)$ is the probability of loss of the PoI when its $E_b/N_0$ is equal to $\omega$, there are $k$ colliding packets and it occupies $E_b/N_0$ ranking position $r$.

The upper bound to $P^K_{loss}(\omega|k, r)$ can be obtained as 1 minus the lower bound of the probability of success of the PoI. The lower bound to the probability of success of the PoI can be derived as the product of two terms. The first term $\left[ 1 - P^K_{loss}(\omega, k) \right]^r$ represents the probability that the $r$ stronger packets than the PoI are successfully decoded, and the second term $\left[ 1 - P^K_{loss}(\omega|k-r) \right]$ represents the probability that the PoI with $E_b/N_0 = \omega$ is successfully decoded provided that there are $k - r$ weaker interfering packets. $P^K_{loss}(\omega|k, r)$ represents the probability that an interfering packet stronger than the PoI in terms of $E_b/N_0$ is lost when $k$ interfering packets are present in the SoI. $P^K_{loss}(\omega|k-r)$ represents the probability that the PoI is lost when there are $k - r$ weaker interfering packets (i.e. for all $k-r$ interfering packets $\omega_{W_n} \leq \omega$ being $\omega_W$ the $E_b/N_0$ associated to the weak packets). Thus:

$$P^K_{loss}(\omega|k, r) \leq 1 - \left[ 1 - P^K_{loss}(\omega, k) \right]^r \cdot \left[ 1 - P^K_{loss}(\omega|k-r) \right].$$

(10)

In the remaining of this section we derive the probabilities $P^K_{loss}(\omega, k)$ and $P^K_{loss}(\omega|k-r)$.

The $P^K_{loss}(\omega, k)$ can be computed as an average for all possible values of $\omega_S > \omega$, being $\omega_S$ the $E_b/N_0$ associated to the stronger packets:

$$P^K_{loss}(\omega, k) = \frac{1}{\Delta \omega} \int_{\omega + \Delta \omega}^{\omega} P^K_{loss}(\omega_S|k) f_{\Omega}(\omega_S|\omega) d\omega_S,$$

(11)

where $f_{\Omega}(\omega_S|\omega)$ is PDF for $E_b/N_0$ of an interfering packet, provided that it is stronger than the $[E_b/N_0]_{PoI} = \omega$ of the PoI. Therefore, the variable $\omega_S$ is defined in the interval ($\omega$, $+\infty$). $P^K_{loss}(\omega_S|k)$ is the probability of loss of the interfering packet when its $E_b/N_0 = \omega_S$ provided that there are $k$ interfering packets. As we have already stated, the PoI and the $k$ interfering packets $E_b/N_0$ are assumed to be i.i.d random variables. Therefore, $P^K_{loss}(\omega|k)$ corresponds to eqn. (9), but for values $\omega_S > \omega$. Consequently, we have introduced here a recursive function as $P^K_{loss}(\omega|k)$ is a function of $P^K_{loss}(\omega|k)$ for all $\omega_S > \omega$. This does not represent a problem for the computation of $P^K_{loss}(\omega|k)$, as in the IIC model the probability of loss is derived from stronger to weaker packets, and all values of $P^K_{loss}(\omega|k)$ are known. Finally, the PDF $f_{\Omega_{S}}(\omega_S|\omega)$ can be computed as:

$$f_{\Omega_{S}}(\omega_S|\omega) = \frac{f_{\Omega}(\omega_S) u(\omega_S - \omega)}{1 - F_1(\omega)}$$

(12)

where $u(x)$ is the unit step function i.e. $u(x) = 1$ for $x \geq 0$ and $u(x) = 0$ for $x < 0$. $F_1(\omega)$ is the cumulative distribution function for the $E_b/N_0$ computed at the level of the PoI introduced in eqn. (8) and $f_{\Omega}(\omega)$ is the PDF for the $E_b/N_0$ of the interfering packet, which was introduced after eqn. (2).

We now derive the probability $P^K_{loss}(\omega|k-r)$ introduced in eqn. (10). The equation is obtained in a similar way as eqn. (4), but with the constraint that the $k-r$ interfering packets have an $E_b/N_0$ weaker than the PoI (i.e. $\omega_{W_n} \leq \omega$).

The noise power spectral density $N_0$ is constant, but the interference power spectral density $I_0$ is a random variable, as it is the result of the sum of $k-r$ colliding packets weaker than the PoI, but with a random packet power level. The $I_0/N_0$ ratio for the PoI can be derived as $\chi_{PoI} = [I_0/N_0]_{PoI} = \omega/G_p$. Since $f_{\Omega_{W_n}}(\cdot)$ represents the PDF of the $I_0/N_0$ ratio resulting from the sum of $k-r$ random variables, it can be derived by performing the convolution of the PDF of the $k-r$ interfering packets:

$$f_{\Omega_{W_n}}(\chi_{PoI} | k-r, \chi_{PoI}) = f_{\Omega_{W_n}}(\chi_{W_n} | \chi_{PoI}) \cdot f_{\Omega_{W_n}}(\chi_{W_{n-r}} | \chi_{PoI}),$$

(13)

where $f_{\Omega_{W_n}}(\chi_{W_n} | \chi_{PoI})$ are the PDFs of the $I_0/N_0$ of the $k-r$ interfering packets provided that $\chi_{W_n} \leq \chi_{PoI}$. Considering that they are independent and identically distributed (i.i.d.) and that $G_p > 0$, they can be computed in a similar way as it has been done for eqn. (12):

$$f_{\Omega_{W_n}}(\chi_{W_n} | \chi_{PoI}) = G_p \frac{f_{\Omega}(\chi_{W_n} G_p) u(\chi_{PoI} + \chi_{W_n} - \chi_{PoI})}{F_1(\chi_{PoI} G_p)}$$

(14)

for $n = 1 \ldots (k-r)$.

B. Model for Iterative Interference Cancellation due to successful detection of replicas

In this section we model the effect of the IC process across slots. Following [11], we assume that the demodulator stores in memory the signal samples corresponding to the current frame slots. Furthermore, the fact that all packet replicas within the same frame have the same power is also accounted for. Whenever a packet is successfully decoded, the decoded packet and all its replica(s) will be cancelled in their corresponding memory location, thanks to the signaling information providing the location of all replicas within the frame [11]. Therefore, after each frame processing iteration,
some of the interfering packets to the PoI may be cancelled because one of their replicas has been successfully decoded. In the example shown in Fig. 1, we can see that interfering packet \( I_2(2) \) is alone in the slot and most likely will be successfully decoded, thus both copies will be cancelled from the frame, including the replica \( I_2(1) \) in the SoI. It is worth noting, when we perform the IC process across slots, the probability of detection for the interfering packets replicas is not always independent of each other. In the example provided in Fig. 1, we have seen that the decoding of \( I_2(2) \) was completely independent of the other replica \( I_2(1) \) in the SoI, but if we look at \( I_1(2) \), which is a replica of \( I_1(1) \), its probability of decoding is dependent to the probability of decoding of PoI(2) which at the same time is dependent to the probability of decoding of PoI(1) in the SoI where the interfering packet \( I_1(1) \) is present. Therefore, the probabilities of decoding of the two replicas from packet \( I_1 \) are not independent from each other. We will refer to this situation as a "loop" and their probability of occurrence is linked to the length of the frame, the MAC load and more importantly the number of replicas from each packet. Longer frames and larger number of replicas will significantly reduce the occurrence of loops.

To simplify the model, in a first instance it is assumed that the detection of the different interfering packets replicas are independent of each other, i.e. no "loops" can take place among them. This approach provides an upper bound to the throughput and a lower bound to the PLR, as the occurrence of "loops" is always limiting the performance of the IIC process across the slots in a frame. An extension of the theoretical framework to include the loop occurrence probability impact on the CRDSA performance is provided in Appendix D. In this Appendix it is shown that for practical frame sizes, the approximation to neglect the loop probability holds well when the number of CRDSA replicas is larger than two. In [13] it was shown that 3 or 4 replicas are required to get the best CRDSA performance, thus the approximation is well justified in practice. The optimum number of replicas for CRDSA results from a compromise between the loop and the irresolvable packet collision events. As shown in Fig. 1 of the Appendix D, by increasing the number of replicas the loop probability is reduced. The latter represents the dominating factor affecting the PLR performance with two replicas. At the same time for a given MAC load \( G \), a larger number of replicas is increasing the probability of packet collision thus eventually reducing the RA throughput. As we have seen in Section II, at the end of the reception process within the SoI we derive the probability of loss of the PoI as a function of its \( E_b/N_0 \) value (see eqn. (3)). Considering that all packets \( E_b/N_0 \) are i.i.d., and assuming that the detection of the different replicas of a packet are independent of each other (i.e. no loops can take place) then all replicas of the interfering packets present in other slots than the SoI will follow the same \( P_{loss}(\omega; \lambda) \) distribution as for the PoI generated by eqn. (3). Therefore, once all slots of the frame have been processed as described in Section II, some interfering packets in the SoI will have been recovered due to the IC process across slots previously described (e.g. \( I_2(1) \) in the frame example provided in Fig. 1).

We introduce here an iterative model where \( N_{iter} \) represents the frame processing iteration and we consider that the IC process across slots takes place at the end of each frame processing iteration. The interfering packets in the SoI with \( E_b/N_0 \) equal to \( \omega_n \) with \( n = 1, 2 \ldots k \) has the following probability distribution:

\[
f_{\Omega}^{N_{iter}}(\omega_n) = \begin{cases} 
  f_{\Omega}(\omega_n) & \text{for } N_{iter} = 1 \\
  \alpha^{N_{iter}-1}\delta(\omega_n) + f_{\Omega}(\omega_n) & \text{for } N_{iter} = 2 \ldots N_{iter}^{\max} 
  \end{cases}
\]  (16)

where \( f_{\Omega}(\omega_n) \) is the \( E_b/N_0 \) PDF distribution of the interfering packets. The latter is assumed to be same as for the PoI at \( N_{iter} = 1 \), \( P_{loss}^{N_{iter}-1}(\omega_n; \lambda) \) is the packet loss probability defined in eqn. (3) at iteration \( N_{iter} - 1 \) and \( \alpha^{N_{iter}-1} = 1 - \int_0^\infty f_{\Omega}(\omega_n) P_{loss}^{N_{iter}-1}(\omega_n; \lambda) \ d\omega_n \) represents the probability that an interfering packet has been successfully decoded at the iteration \( N_{iter} - 1 \) and its interference cancelled (i.e. the probability that \( \omega_n = [E_b/N_0]_n = 0 \), represented by the Delta Dirac function).

We can now derive in an iterative manner the interfering packets \( \Xi_n = [I_0/N_0]_n \) distribution, previously defined in eqn. (6) as follows:

\[
f_{\Xi}^{N_{iter}}(\chi_n) = G_p \cdot f_{N_{iter}}(\chi_n; G_p) \quad \text{for } n = 1 \ldots k
\]

and for \( N_{iter} = 1 \ldots N_{iter}^{\max} \). (17)

Equations (3) to (5) from the model defined in Section II can now be computed iteratively for \( N_{iter} = 1 \ldots N_{iter}^{\max} \) updating the interfering packets \( \Xi_n \) according to eqn. (17) at the end of each iteration:

\[
P_{loss}^{N_{iter}}(\omega; \lambda) \geq \sum_{k=0}^{\infty} \left[ P_{loss}^{K,N_{iter}}(\omega|k) \right] \cdot f_K(k; \lambda), \quad (18)
\]

\[
P_{loss}^{K,N_{iter}}(\omega|k) \geq \int_0^{\infty} \Gamma \left( \log_{10} \left( \frac{\omega}{1 + \chi} \right) \right) \cdot f_{\Xi}^{N_{iter}}(\chi; k) \ d\chi, \quad (19)
\]

\[
f_{\Xi}^{N_{iter}}(\chi; k) = f_{\Xi}^{N_{iter}}(\chi_1) \otimes f_{\Xi}^{N_{iter}}(\chi_2) \ldots \otimes f_{\Xi}^{N_{iter}}(\chi_k). \quad (20)
\]

As previously explained, this model provides a lower bound to the packet loss ratio as we have assumed the reception of the different replicas of a packet to be an independent process (i.e. no occurrence of loops).

V. Model Customization for CRDSA and Performance Analysis

The CRDSA scheme combines the two IC techniques described in Sections IV-A and IV-B. Considering that the IIC process within the SoI described in Section IV-A represents an upper bound to the packet loss probability and the IC process across slots defined in Section IV-B represents a lower bound to the packet loss probability, the combination of the two techniques will result in an approximation to the PLR.

At the beginning of each frame iteration we compute the \( n \)-th interfering packet \( \omega_n = [E_b/N_0]_n \) PDF \( f_{\Omega}^{N_{iter}}(\omega_n) \), as described by eqn. (16) in Section IV-B. The corresponding cumulative distribution function for the interfering packets is \( P_{loss}^{N_{iter}}(\omega_n) \). It shall be noted that the \( E_b/N_0 \) distribution of the PoI \( f_{\Omega}(\omega) \) remains constant and the only packets that
experiment changes in their $E_b/N_0$ distribution due to IIC process are the interfering ones.

We then compute an approximation to the packet loss probability following the model defined in Section IV-A, but replacing in eqn. (2), (3) and (9) the interfering packets $E_b/N_0$ distribution $f_{\Omega}(\omega_n)$ by the iterative version $f_{\Omega}^{N_{iter}}(\omega_n)$:

$$PLR_{N_{iter}}^G(N, N_{rep}) \approx \int_0^\infty \left[ P_{loss}^{N_{iter}}(\omega; \lambda) \right]^{N_{rep}} f_{\Omega}(\omega) \cdot d\omega,$$

Finally, from eqns. (13)-(14) and (15) the iterative version of $P_{loss}$ can be computed as:

$$P_{loss}(\omega; \lambda) \approx \sum_{k=0}^\infty \left[ P_{K, N_{iter}}^{loss}(\omega|k) \cdot f_K(k; \lambda),
$$

$$P_{loss}(\omega|k) = \sum_{r=0}^k P_{R, N_{iter}}^{loss}(\omega|k, r) \cdot f_{\Omega}^{N_{iter}}(\omega|k) \cdot f_{\Omega}^{N_{iter}}(\omega|k-r),
$$

where $f_{\Omega}^{N_{iter}}(\omega|k) = \left( \frac{\lambda}{\Omega} \right) [1-P_{\Omega}^{N_{iter}}(\omega)]^r [F_{\Omega}^{N_{iter}}(\omega)]^k-r$ as defined in eqn. (8), but this time using the iterative cumulative distribution function $F_{\Omega}^{N_{iter}}(\cdot)$ for the interfering packets defined above. Equation (10) can now be rewritten as:

$$P_{loss}(\omega, r) \leq 1 - \left\{ 1 - P_{loss}^{S,N_{iter}}(\omega, k) \right\}^r \cdot \left[ 1 - P_{loss}^{W,N_{iter}}(\omega|k-r) \right].$$

From eqns. (11)-(12) the iterative version of $P_{loss}$ can be computed as:

$$P_{loss}(\omega) = \int_{\omega+}^{\infty} P_{K, loss}(\omega|k)f_{\Omega}^{N_{iter}}(\omega|k)d\omega,$$

$$f_{\Omega}^{N_{iter}}(\omega|k) = f_{\Omega}^{N_{iter}}(\omega|k)u(\omega - \omega) / 1 - P_{\Omega}^{N_{iter}}(\omega).$$

Finally, from eqns. (13)-(14) and (15) the iterative version of $P_{loss}$ is derived as follows:

$$P_{loss}(\omega|k-r) = \int_0^\infty \Gamma \left[ 10 \log_{10} \left( \frac{\omega}{1 + \chi W} \right) \right] f_{\Omega}^{N_{iter}}(\omega|k-r, \chi_{P|of})d\chi W,$$

$$f_{\Omega}^{N_{iter}}(\omega|k-r, \chi_{P|of}) = f_{\Omega}^{N_{iter}}(\omega_1|\chi_{of}) \otimes f_{\Omega}^{N_{iter}}(\omega_2|\chi_{of}) \ldots \otimes f_{\Omega}^{N_{iter}}(\omega_{k-r}|\chi_{of}),$$

$$f_{\Omega}^{N_{iter}}(\omega_n|\chi_{P|of}) = G_p f_{\Omega}^{N_{iter}}(\chi_{P|of} - \chi_{W_n}) / f_{\Omega}^{N_{iter}}(\chi_{P|of} G_p)$$

for $n = 1 \ldots (k-r)$.

In Fig. 4 we present the results for CRDSA with 3 replicas and we compare the analytical results with those derived from a comprehensive system simulation. The simulator is encompassing independent generation of coded and modulated $N_{rep}$ replica packets within each frame with Poisson traffic packets distribution with normalized MAC average load $G$ (see Sect. II), lognormal power distribution$^3$, random carrier phase for each packet. The different packets generated in each slot are summed together with the AWGN. The demodulator is implementing the CRDSA frame memory-based I-SIC cancellation algorithm detailed in [12] inclusive of real FEC decoding for the detected packets. The only simplification compared to a real CRDSA demodulator is the assumption of ideal channel estimation. This is justified by the fact that channel estimation performance impact has been shown to be negligible in [12]. As shown in Appendix D, in this case the probability of loops is quite small and the analytical model matches quite well the simulation results also when the lognormal packets power standard deviation is as large as 3 dB. In this case the throughput is increased by 50 % compared to the equi-powered packets case. This result is due to the fact that IIC performance is enhanced when the incoming signal power is not constant [23]. The analytical model accuracy will be even higher for 4 replicas but less good for the 2 replicas case which is investigated in Appendix D. Looking at Fig. 4 we can observe that at high loads the throughput curves from the analytical model are a bit lower than the simulated ones. As it has been explained above, the IC process within the SoI implemented in the CRDSA analytical model and described in detail in Section IV-A, represents an upper bound to the packet loss probability hence a lower bound to the throughput. However, the difference between the two results is quite marginal. By comparing Figs. 2, 3 and 4 it can be remarked that CRDSA provides an impressive advantage in terms of throughput for $PLR \leq 10^{-3}$ compared to S-ALOHA and DSA. This is a key improvement for a satellite environment where packet retransmissions shall be minimized due to the long propagation delays.

VI. SUMMARY AND CONCLUSIONS

In this paper a generalized analytical framework capable to accurately predict the performance of the most common slotted random access techniques such as S-ALOHA, DSA and CRDSA exploited in satellite communication systems have been developed. The proposed analytical model allows to predict the performance of the RA schemes in the presence of arbitrary FEC schemes, packet size, traffic and received packets power distributions. The model takes into account the fact that thanks to powerful physical layer coding, packets may be decoded with non zero probability also in the presence of collisions. The model also considers the effect of slot time diversity in case more than one packet replica is transmitted in the same frame as it is the case for DSA. Two types of IC strategies have been modeled (IIC over the SoI and IC across the slots due to successful reception of packet replicas in other slots than the SoI) and used characterize the performance of more recent RA schemes such as CRDSA. The analytical findings are shown to be very close to Monte Carlo simulation results for S-ALOHA and DSA. For the more complex CRDSA scheme results are accurate when the

$^3$It is assumed that the lognormally distributed power $r v$ is the same for all packet replicas within the frame.
match with the simulated PER is provided by the following best fit law:

$$
\Gamma(x) = \begin{cases} 
10^{\left(\sum_{m=0}^{10} F_m x^m\right)} & \text{if } x \geq T_h, \\
1 & \text{if } x < T_h 
\end{cases}
$$

(30)

with $x = 10 \log_{10} \left(\frac{E_b}{N_0}\right)$ and $F_0 = -0.29846$, $F_1 = -0.53778$, $F_2 = -0.23827$, $F_3 = 0.02605$, $F_4 = -0.004$, $F_5 = -0.01752$, $F_6 = 3.45 \times 10^{-3}$, $F_7 = 2.02 \times 10^{-3}$, $F_8 = -3.52 \times 10^{-4}$, $F_9 = -3.46 \times 10^{-5}$, $F_{10} = 0$ and $T_h = -2$ dB.

The FEC code with rate $\rho = 1/2$ has been derived from the previous 3GPP Turbo FEC code with rate $\rho = 1/3$ by changing the puncturing pattern from $\text{pun \_ pattern}_{\text{13}} = [110; 00; 11]$ into $\text{pun \_ pattern}_{\text{12}} = [110; 00; 01]$. Equation (30) has been used for the interpolation with $F_0 = -0.1096$, $F_1 = -0.2432$, $F_2 = -0.1854$, $F_3 = 0.0434$, $F_4 = 0.0226$, $F_5 = -0.1373$, $F_6 = 0.0444$, $F_7 = 0.0561$, $F_8 = -0.0445$, $F_9 = 0.0116$, $F_{10} = -1.0611 \times 10^{-3}$ and $T_h = -1$ dB.

B. On the MAI Gaussian Approximation for S-ALOHA

In this Appendix we are providing some justification for the Gaussian MAI interference approximation used in the RA analysis. In case of S-ALOHA, SA or CRDSA very few (down to one for S-ALOHA or CRDSA after some IC iterations) interfering packets/slot are typically present even in loaded conditions thus weakening the conditions for the Central Limit Theorem (CLT) applicability. The issue of QPSK co-channel interference approximation by a white Gaussian process has already been investigated in the literature for the case of time asynchronous interference [25]. In our case packets are aligned at symbol level thus the hypothesis of uniform distribution of the relative time offset is not applicable. First we investigate the packet error performance in the presence of few interferers at physical layer level. The worst case is represented by a single interfering packet. The equivalent $E_b/(N_0 + I_0) = E_b/N_t$ considering the interference as a Gaussian process is given by:

$$
\frac{E_b}{N_t} \left(\frac{C}{T}\right) \approx \frac{E_b}{N_0} \left[1 + \frac{E_b}{N_0} \left(\frac{C}{T}\right)^{-1}\right].
$$

(31)

In the following example we assume that the S-ALOHA transmitted packets are using the 3GPP turbo code with rates $\rho = 1/3$, information packet size of 100 bits and QPSK modulation. The energy per symbol to noise PSD $E_s/N_0$ value used in the RA simulations in the absence of lognormal power fluctuation is 10 dB. The two packets are time aligned and assumed to be continuously transmitted with the carrier-to-interference power ratio $C/I$ ranging from $-3.10$ to $0.72$ dB for the PoI. For the particular case of a single interfering packet with the same $E_s/N_0$ value as the PoI (i.e. $C/I = 0$ dB), the corresponding $E_b/N_t$ values will be $+1.34$ dB. For what concerns the interferer’s carrier phase we have been simulating two different scenarios: a) completely coherent carrier phase, b) random phase at symbol level. Simulated results reported in Fig. 5 shows the simulated PER vs the $C/I$ in the presence of real co-channel interference or with AWGN emulating the co-channel interference characterized by...
an equivalent noise PSD $N_i$ such that $E_b/N_i = E_b/(N_o+I_0)$. As it is apparent from Fig. 5 the Gaussian approximation holds quite well for $10^{-1} \leq \text{PER} \leq 1$ when the carrier phase is randomized symbol-by-symbol. The approximation is definitely looser when the interferer has the same carrier phase of the useful signal. The symbol-by-symbol random carrier phase error is quite realistic considering that in practice different transmitters will have a slight carrier frequency offset, thus the received packets are offset in frequency and the relative phase will be time variant. The physical layer interleaver will further randomize the carrier phase offset across the received packet symbol samples thus justifying the hypothesis of interfering packet(s) symbol-by-symbol random carrier phase.

We can infer that the Gaussian approximation is more accurate at packet than symbol level assuming an Ergodic type of behavior for the PER calculation. The fact that the good match between simulations and $E_b/N_i$ Gaussian approximation holds true in a limited PER range is not of major concern as the region where the approximation accuracy is weakening corresponds to rather large values of $C/I$ i.e. $C/I = 0$ dB or higher. In this region the $E_b/N_i$ is high enough (i.e. 1.5 dB or higher) so that there is an high probability that the useful packet is decoded. Instead the Gaussian approximation holds well in a region whereby the colliding packet has an impact on the useful packet detection capability and its accurate modeling is important. To better understand the importance of the random phase assumption on the MAI interference approximation we have developed an analytical model of the MAI for the case of interest. The model is assuming that packets are aligned in time at symbol level so that the signal at the receive side for the $k$-th packet symbol after symbol matched filtering, perfect useful packet carrier recovery and in the absence of AWGN\(^5\). The resulting signal can be expressed as:

$$z_0(k) = d_0(k) + \sum_{n=1}^{N_{int}} A_n d_n(k) \exp[j \theta_n(k)], \quad (32)$$

where $N_{int}$ represents the number of interfering packets, subscript $d_0(k)$ refers to the useful packet symbol $k$, subscript $n$ represents the $n$-th interfering packet, $A_n$ is the real-valued amplitude of the $n$-th interfering terminal, $d_n(k) = d_n^R(k) + j d_n^I(k)$ is the complex QPSK symbol for the $n$-th signal with $d_n^R(k) = \pm 1$ and $d_n^I(k) = \pm 1$ i.i.d. random variables, $\theta_n(k)$ is the phase offset of the $n$-th interfering packet. We assume that the amplitude of each interferer $A_n$ is an i.i.d. random variable with PDF $p_{A_n}(a)$. Concerning the interfering packets carrier phase offset $\theta_n(k)$ we make two different assumptions about its PDF $p_{\theta_n}(\theta)$:

$$p_{\theta_n}(\theta) = \begin{cases} \frac{1}{2\pi} & \text{for } -\pi \leq \theta \leq \pi \\ \delta(\theta) & \text{for uniform phase offset} \\ \delta(\theta) & \text{for zero phase offset.} \end{cases}$$

The real and imaginary parts of $z_0$ from eqn. (32) are thus:

$$\Re \{z_0(k)\} = d_0^R(k) + \sum_{n=1}^{N_{int}} A_n [d_n^R(k)X_n(k) - d_n^I(k)Y_n(k)],$$

$$\Im \{z_0(k)\} = d_0^I(k) + \sum_{n=1}^{N_{int}} A_n [d_n^I(k)X_n(k) + d_n^R(k)Y_n(k)], \quad (33)$$

having named with $X_n(k) = \cos[\theta_n(k)]$ and $Y_n(k) = \sin[\theta_n(k)]$. The PDF $p_{X_n} = p_{Y_n}$ of the two Random Variables (RVs) $X_n$ and $Y_n$ can be easily derived as:

$$p_{X_n}(x) = \begin{cases} \frac{1}{\pi} \frac{1}{\sqrt{1-x^2}} & \text{for } -1 \leq x \leq 1 \\ \delta(x-1) + \delta(x+1) & \text{for uniform phase} \\ \delta(x-1) & \text{for zero phase.} \end{cases}$$

Since $X_n(k)$ and $Y_n(k)$ are independent RVs with values $\{-1, +1\}$ by naming:

$$\overline{X}_n(k) = d_n^R(k)X_n(k), \quad \overline{Y}_n(k) = -d_n^I(k)Y_n(k),$$

$$\overline{X}_n(k) = d_n^I(k)X_n(k), \quad \overline{Y}_n(k) = d_n^R(k)Y_n(k), \quad (34)$$

we can rewrite eqn. (33) as:

$$\Re \{z_0(k)\} = d_0^R(k) + \sum_{n=1}^{N_{int}} A_n \overline{X}_n(k) + \sum_{n=1}^{N_{int}} A_n \overline{Y}_n(k) = d_0^R(k) + J^R(k),$$

$$\Im \{z_0(k)\} = d_0^I(k) + \sum_{n=1}^{N_{int}} A_n \overline{X}_n(k) + \sum_{n=1}^{N_{int}} A_n \overline{Y}_n(k) = d_0^I(k) + J^I(k),$$

---

\(^5\)This represents a worst-case condition for assessing the Gaussian MAI approximation.
Looking at eqn. (34) it is easy to see that for the RVs $X_n$, $\overline{X}_n$, $\overline{Y}_n$, $\overline{Y}_n$ PDFs the following equality holds:

$$p_{X_n}(x) = p_{\overline{X}_n}(\overline{x}) = p_{\overline{Y}_n}(\overline{y}) = p_{\overline{Y}_n}(\overline{y}) = p_{X_n}(x).$$ (36)

Now defining $U_n$ as $U_n = A_nX_n$ and observing that due to the independence of the rvs $A_n$ and $X_n$ we get [26]:

$$p_{U_n}(u) = \int_{-\infty}^{\infty} \frac{1}{|x|} f_{X_n}(x) f_{A_n}\left(\frac{u}{x}\right) dx.$$ (37)

The interfering terms affecting the real and imaginary part of $z_0(k)$ are thus $J^R(k)$ and $J^I(k)$ whose PDF is given by:

$$p_{J^R}(x) = p_{J^I}(z) = p_{U_1}(x) \otimes p_{U_2}(x) \otimes \cdots \otimes p_{U_n}(x).$$ (38)

To be remarked that there are $2N_{\text{int}}$ convolution components in the calculation of $p_{J^R}(x)$ which helps the convergence to a Gaussian PDF in particular for the random carrier phase case. The statistics of the co-channel interference are also dependent on the type of the transmission shaping filter $h(t)$ adopted through the factor $\beta = R_n \int_{-\infty}^{+\infty} |H(f)|^2 df$ [25] where $H(f)$ is the Fourier transform of $h(t)$. Assuming a SRRC pulse shaping it can be found that for a typical roll-off factor of 0.2 $\beta = 0.95$. For the zero roll-off factor case considered in this paper $\beta = 1$. The calculation of the MAI amplitude distribution following eqn. (38) for the case of zero phase gives a discrete PDF with non-zero values at $\{0, \pm 2, \pm 4\}$ for $N_{\text{int}} = 1$ and $\{0, \pm 2, \pm 4, \pm 6\}$ for $N_{\text{int}} = 3$. A different situation is observed in case of $N_{\text{int}} = 1, 2, 3$ and random carrier phase as shown in Fig. 6-a for equal power interferers. In this case for two interferers the Gaussian amplitude PDF approximation starts to be fairly good even in the absence of AWGN. For the case of lognormally distributed interferers shown in Fig. 6-b the Gaussian approximation is tighter for the single interferer.

Having remarked the goodness of the Gaussian approximation in the single interferer no AWGN worst-case condition, we now investigate the impact of this assumption on the PLR calculation for S-ALOHA according to eqn. (4) with FEC coding rate $1/3$ and $\sigma = 0$ dB. Simulation versus theoretical findings under various interferer carrier phase hypothesis are shown in Fig. 7. As expected, there is a good match with simulation findings when interferers random carrier phase is applied. On the other hand, analytical results are distant from simulation findings when the unrealistic case of a completely coherent carrier phase is considered.

### C. First Order Statistics of the Sum of Lognormal rvs

It is know from literature that the sum of $N$ lognormal rvs can be approximated with a lognormal rv. Particularizing the result of the Wilkinson’s method reported in [22] to the case of $N$ independent lognormal rvs each with mean $\mu$ and standard deviation $\sigma$ (referred to the corresponding normal distribution) it can be found that the sum of these $N$ rvs is well approximated by a lognormal rv having mean $\mu_\Sigma$ and standard deviation $\sigma_\Sigma$ provided by:

$$\mu_\Sigma(N, \mu, \sigma) = \frac{1}{\gamma} \left( 2 \ln u_1(N, \mu, \sigma) - \frac{1}{2} \ln u_2(N, \mu, \sigma) \right),$$

$$\sigma_\Sigma(N, \mu, \sigma) = \frac{1}{\gamma} \sqrt{\ln u_2(N, \mu, \sigma) - 2 \ln u_1(N, \mu, \sigma)},$$

$$u_1(N, \mu, \sigma) = N \exp \left( \gamma \mu + \frac{\gamma^2}{2} \sigma^2 \right),$$

$$u_2(N, \mu, \sigma) = N \exp \left( 2 \gamma \mu + 2 \gamma^2 \sigma^2 \right) + 2 \sum_{i=1}^{N-1} (N-i) \exp(2\gamma \mu + \gamma^2 \sigma^2),$$

$$\gamma = \ln 10/10.$$ (39)
the PoI. The above expression assumes that when no loops are present \( l = 0 \), then the PLR expression derived in eqn. (21) applies. However, when one or more loops are present \( l \geq 1 \), we compute the equivalent \( E_b/(N_0 + f_0) \) for the PoI (where \( E_i \{ \chi \} = l \cdot \chi \omega l / G_p l = l \cdot \omega / G_p l \)) assuming equal power conditions for all received packets and derive the corresponding PLR by using eqn. (30) from Appendix A. It is worth noting that for \( l \gg 1 \), the corresponding PLR \( \rightarrow 1 \). Therefore, eqn. (40) can be simplified as follows:

\[
\begin{align*}
\text{PLR}^{N_{\text{iter}}} (G, N_{\text{rep}}, N_{\text{slots}}) & \approx \text{PLR}^{N_{\text{iter}}} (G, N_{\text{rep}}) \\
& - P_{\text{loop}}^0 (G, N_{\text{rep}}, N_{\text{slots}}) \\
& + \sum_{l=1}^{\infty} \{ \Gamma (10 \log_{10} \left( \frac{\omega}{1 + E_i \{ \chi \} l} \right) \}^{N_{\text{rep}}/N_{\text{iter}}} \\
& \cdot P_{\text{loop}}^l (G, N_{\text{rep}}, N_{\text{slots}})
\end{align*}
\]

The PLR expression derived in eqn. (21) can be extended as follows when the probability of loops is not negligible (e.g. CRDSA with 2 replicas):

\[
\begin{align*}
\text{PLR}^{N_{\text{iter}}} (G, N_{\text{rep}}, N_{\text{slots}}) & \approx \text{PLR}^{N_{\text{iter}}} (G, N_{\text{rep}}) \\
& - P_{\text{loop}}^0 (G, N_{\text{rep}}, N_{\text{slots}}) \\
& + \sum_{l=1}^{\infty} \{ \Gamma (10 \log_{10} \left( \frac{\omega}{1 + E_i \{ \chi \} l} \right) \}^{N_{\text{rep}}/N_{\text{iter}}} \\
& \cdot P_{\text{loop}}^l (G, N_{\text{rep}}, N_{\text{slots}})
\end{align*}
\]

where \( P_{\text{loop}}^l \) represents the probability to have \( l \) loops affecting

\[
\begin{align*}
\text{PLR}^{N_{\text{iter}}} (G, N_{\text{rep}}, N_{\text{slots}}) & \approx \text{PLR}^{N_{\text{iter}}} (G, N_{\text{rep}}) \\
& - P_{\text{loop}}^0 (G, N_{\text{rep}}, N_{\text{slots}}) \\
& + \sum_{l=1}^{\infty} \{ \Gamma (10 \log_{10} \left( \frac{\omega}{1 + E_i \{ \chi \} l} \right) \}^{N_{\text{rep}}/N_{\text{iter}}} \\
& \cdot P_{\text{loop}}^l (G, N_{\text{rep}}, N_{\text{slots}})
\end{align*}
\]

We now derive the probabilities \( P_{\text{loop}}^l \) to have \( l \) loops with \( l \) integer and \( l \geq 0 \). The number of different combinations that occur in CRDSA when \( N_{\text{rep}} \) replicas are transmitted in a frame of \( N_{\text{slots}} \) can be simply computed as \( N_{\text{iter}} (N_{\text{slots}}, N_{\text{rep}}) = \binom{N_{\text{slots}}}{N_{\text{rep}}} \). Therefore, the probability that an interfering packet selects the same combination of \( N_{\text{rep}} \) slots than the PoI is \( p (N_{\text{slots}}, N_{\text{rep}}) = 1/N_{\text{iter}} (N_{\text{slots}}, N_{\text{rep}}) \). Consequently, the probability \( p \) will quickly decrease as we increase the number of replicas from 2 to 4. The average number of packet arrivals per frame is \( \lambda_f (N_{\text{slots}}) = G \cdot G_p \cdot N_{\text{slots}} \), and \( P_{\text{loop}}^0 (G, N_{\text{rep}}, N_{\text{slots}}) \) can be computed as:

\[
P_{\text{loop}}^0 (G, N_{\text{rep}}, N_{\text{slots}}) = [1 - p (N_{\text{slots}}, N_{\text{rep}})]^{\lambda_f (N_{\text{slots}})}.
\]

\( P_{\text{loop}}^0 \) follows a Binomial distribution with parameters \( B (\lambda_f, p) \) and corresponds to the zero successes case (i.e. no loops). In a similar way, we can derive \( P_{\text{loop}}^l \) as:

\[
P_{\text{loop}}^l (G, N_{\text{rep}}, N_{\text{slots}}) = \left( \frac{\lambda_f}{l} \right)^l \cdot p^l \cdot (1 - p)^{\lambda_f - l}
\]

In Fig. 8 the \( P_{\text{loop}} = 1 - P_{\text{loop}}^0 \) is reported for the case \( N_{\text{rep}} = 2, 3 \) and 4 replicas as a function of \( G \). As for the calculations performed in Section V, we assume an \( E_s/N_0 = 10 \) dB for the PoI, QPSK modulation and a FEC coding rate \( \rho = 1/3 \), with a number of slots per frame \( N_{\text{slots}} = 100 \). As anticipated, the probability of occurrence of loops quickly reduces with the number of replicas per packet. Observing eqn. (42) we remark that the \( P_{\text{loop}} \) decreases with the number of slots per frame and the effect of loops becomes negligible as the frame size increases towards infinite.

Finally, Fig. 9 reports the PLR derived adopting the extended analytical model defined in eqn. (41) and compared against the simulated results for CRDSA with 2 replicas. The analytical curve based on eqn. (41) better approximates the simulation results than the curve obtained using eqn. (21). Figure 9 confirms that the PLR performance of CRDSA with 2 replicas is limited by the higher probability of loops
Fig. 8. Probability of loop for \( N_{\text{rep}} = 2, 3 \) and \( N_{\text{slots}} = 100 \).

Fig. 9. CRDSA PLR approximation due to effects of loops vs. simulated results for \( N_{\text{rep}} = 2, N_{\text{iter}} = 15, N_{\text{slots}} = 100 \) for QPSK modulation, 3GPP FEC \( \rho = 1/3 \), packet block size 100 bits, \( E_s/N_0 = 10 \text{ dB} \) in the presence of lognormal packets power unbalance with mean \( \mu = 0 \text{ dB} \), standard deviation \( \sigma = 0 \text{ dB} \) and Poisson traffic.

occurrence. The match between the analytical and simulation results is not perfect because in the above analysis only the simpler case of loops has been characterized. We can conclude that increasing the number of replicas (e.g. \( N_{\text{rep}} = 3 \) or 4) seems the best approach to combat loops and to get the maximum CRDSA performance, while still using practical frame lengths (i.e. \( N_{\text{slots}} \approx 100 \) slots).

REFERENCES


[9] ETSI EN 301 700 v1.2.2, “Digital Video Broadcasting (DVB); interaction channel for satellite distribution systems.”


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