Antijamming Capacity and Performance Analysis of Multiple Access Spread Spectrum Systems in AWGN and Fading Environments

Reza Nikjah, Student Member, IEEE, and Norman C. Beaulieu, Fellow, IEEE
iCORE Wireless Communications Laboratory, University of Alberta

Abstract—A unified multiple access channel model based on the time-bandwidth dimensionality is considered for the user capacity analysis of a wide variety of multiuser spread spectrum systems contaminated by smart jamming. Different users are distinguished by signatures of either direct sequence or hopping type. It is found that the jammer should spread its energy evenly over all degrees of freedom in order to minimize the average capacity or maximize the outage probability. Also, for the communicator to best make use of the channel resources, it is necessary to avoid hopping type signatures. As a practical realization, we consider the performance analysis of a synchronous frequently cited multicarrier frequency hopping CDMA system in AWGN and Rayleigh fading jammed uplink channels. We show that the best counter-jamming performance is attained when the number of subcarriers is maximal. It is demonstrated that when the receiver does not know the jammer state, concentrating instead of spreading the jamming power over the channel degrees of freedom will give rise to the worst performance for the communicator. Optimal weights for the receiver soft outputs in different channels are also obtained, for practical purposes.

I. INTRODUCTION

The pervasive appeal of multicarrier modulation is indisputable owing to its resistance to channel frequency selectivity, thereby avoiding multipath, and its easy implementation using fast-Fourier-transform devices. On the other hand, code division multiple access (CDMA) systems are well known for the suppression of narrowband interference, graceful degradation with system loading, and in some cases, resilience against jamming signals. The literature is replete with different proposed multicarrier CDMA schemes and their performance analyses when hostile interference is present [1]-[6]. A general treatment, within a simple framework, of the antijamming properties of these systems as well as some other time-spread CDMA schemes, together with a well known multicarrier frequency hopping system is presented in this paper.

In the first part of this paper, we approach the problem of intentional interference in multiuser CDMA systems from an information theoretic viewpoint to derive some general results on the performance of a wide range of multiple access spread spectrum systems under partial-band or partial-time jamming. Generally speaking, a channel of bandwidth $W$ used for a time interval of $T$ has $N = 2WT$ dimensions or degrees of freedom for the transmission of data [7]. We split any block of $N$ bins (bins in frequency or in time) into $L$ segments, each having $h = N/L$ bins. In any block, each user according to its assigned bin hopping signature is given one bin of every segment for data transmission. Consequently, any user possesses $L$ bins in every block. For each user, selected bins hop within the segments from block to block. Independent bin hopping signatures dedicated to users in the system provide multiple access capability. In the special case of $N = L$ or $h = 1$, bin hopping signatures cannot be utilized, and direct sequence type signatures are exploited instead. The information to be transmitted in each bin is modulated by one bit of the direct sequence signature in a similar fashion to direct-sequence CDMA systems. For $h > 1$, we can also use direct sequence signatures with bin hopping signatures for better interference resilience in certain scenarios. Our model and analysis accommodates all of the preceding. The hostile interferer can be of partial-band or partial-time type, and impinges upon the bins by introducing additional signal corruption.

Two representative examples that fit into the above framework include time-hopping ultrawideband systems [8],[9], and multicarrier frequency hopping (MC-FH) schemes [10],[11]. Both may be affected by partial-band and/or partial-time jammers. The former exploit equal-length time frames with an identical number of time chips within each frame. Frames play the role of segments while chips take the role of bins. In the latter, a frequency counterpart of the former, the whole available frequency band is divided into $L$ frequency subbands (segments) each having $h$ frequency subcarriers (bins). In each signaling interval, $L$ subcarriers are chosen from $L$ frequency subbands according to the user’s hopping signature code. Encoded bits modulate the phase of the $L$ selected subcarriers. Modulated subcarriers are then transmitted simultaneously at the signaling interval. The user’s hopping signature code selects a different set of subcarriers in each signaling interval. Multicarrier CDMA (MC-CDMA) schemes are a special case of MC-FH systems when $h = 1$, and hence, direct sequence signatures are used in lieu of hopping type signatures.

MC-FH systems have received great attention because they take advantage of both multicarrier modulation and the fre-
quency hopping concept, and because they can be implemented coherently at the receiver side [10]. They have been analyzed in different non-jamming multiple access channels in [11]-[14], and in uncoded form in jamming environments in [6],[10]. The second part of this paper gives a detailed performance evaluation of synchronous coded MC-FH systems in additive white Gaussian noise (AWGN) and Rayleigh fading uplink channels in the presence of deliberate interference. New results and practical guidelines for the design of such systems are presented.

The paper is organized as follows. Capacity results are derived in Sec. II. Sec. III describes the MC-FH system model. Static results in analytical form and fading channels are carried out in Sec. IV. Sec. V presents some numerical results. Finally, conclusions are drawn in Sec. VI.

II. CAPACITY ANALYSIS AND DISCUSSION

Consider a CDMA channel with available blocks of $N$ bins, as described in the previous section. The bins are affected by a Gaussian jammer with average energy per bin, $N_j$, which is assumed to be sustainable in perpetuity by the jammer. If the jammer impinges on only a fraction $\rho$ of the bins, its average energy over each bin hit reaches $N_j/\rho$. Ambient noise is considered negligible compared with the jamming noise. In channels with Gaussian interference and noise, each bin has capacity $\log(1 + SNIR)$ bits per two-dimensional symbol, where $SNIR$ is the instantaneous signal-to-noise-plus-interference ratio at the receiver. Note that achieving capacity requires all transmitters to send Gaussian distributed information through the channel, and hence the multiple access interference is assumed to be Gaussian. This assumption is validated by a central limit theorem when the number of active users in the system is large, and when Rayleigh fading is present.

Assume that there are $K$ users trying to randomly access the channel. Any user, in each of its $L$ available segments, is assigned one bin among $h$ bins based on its signature which is perceived as random by the other users. Therefore, the chance of any bin being hit by another is $1/h$. We assume that the effects of path loss and large-scale fading are compensated at either the transmitter or the receiver side such that the average received energy per bin is $E_\gamma = E/L$, where $E$ is the total energy per block per user. We denote the channel small-scale fading coefficient on the $l$th selected bin from a total of $L$ for a typical user by $g_l$. In static channels, $g_l = 1$, whereas in fading channels $g_l$ is assumed to be a complex Gaussian random variable with zero mean and $E(|g_l|^2) = 1$. Under these assumptions, the capacity of the $l$th bin for a typical user can be shown to equal

$$C_l = \rho \log(1 + \frac{E_\gamma |g_l|^2}{N_j/\rho + (K-1)E_\gamma/h}) + (1-\rho) \log(1 + \frac{|g_l|^2}{(K-1)h})$$

(1)

Eq. (1) can be rewritten as

$$C_l = \rho \log(1 + \frac{E_\gamma |g_l|^2}{N_j/\rho + (K-1)E_\gamma/h}) + (1-\rho) \log(1 + \frac{|g_l|^2}{(K-1)h})$$

(2)

where

$$\gamma_\rho = (\gamma_\rho + \gamma_m)/(\gamma_\rho \gamma_m), \quad \gamma_\rho = \rho \gamma, \quad \gamma_m = \gamma/L$$

$$\gamma = E/N_f$$

and

$$\gamma_m = h/(K-1).$$

(3)

$\gamma$ is regarded as the average signal-to-jamming-noise ratio (SJR) per block, while $\gamma = E/N_f$ is the average SJR per bin. The total capacity per bin for a typical user is then equal to

$$C_t = \frac{1}{N} \sum_{l=1}^{L} C_l.$$

(4)

For a static channel, (4) reduces to

$$C_t = 1/h[\rho \log(1 + \gamma) + (1-\rho) \log(1 + \gamma_m)].$$

(5)

For a fading channel, $C_t$ is a random variable; the average capacity per bin per user can then be shown to equal

$$E(C_t) = \frac{\log(e)}{h} \left[ \rho \exp(1/\gamma) \text{Ei}(1/\gamma) + (1-\rho) \exp(1/\gamma_m) \text{Ei}(1/\gamma_m) \right]$$

(6)

where $\text{Ei}(x)$ is the exponential integral given in [15]

$$\text{Ei}(x) = \int_{-\infty}^{x} \frac{\exp(-t)}{t} dt.$$  

(7)

It can be proved that $C_t$ in (5) and $E(C_t)$ in (6) are decreasing functions of $\rho$, and therefore, the best strategy for the jammer to minimize the (average) capacity is to spread its average energy per bin uniformly over all degrees of freedom in the channel. Also, examination of (5) and (6) reveals that for a fixed number of segments, or equivalently to decrease the number of bins per segment in terms of maximizing the (average) capacity. Hence, if we merely desire to increase the (average) capacity, we should set $h$ to 1 and avoid hopping type signatures. We will expound on this in more detail in subsequent sections.

Fig. 1 shows the (average) capacity per user per bin for static and fading channels when there are blocks of 32 segments with 8 bins each. When $\rho$ increases, the capacity deteriorates rapidly and the gap between static and fading capacities diminishes. For small values of $\rho$, the jammer effect is weak and the multiple access interference, i.e. the number of
users, determines the performance. On the other hand, for large values of $\rho$, the jammer is dominant and the capacity is primarily determined by $\gamma$.

For fast fading channels and systems with delay constraints, average capacity is no longer an appropriate measure of merit. We can consider outage probability instead, which is defined here as the probability of the capacity falling below a certain rate $R$. This rate is required for sustaining some quality of service in the system. From (2) and (4) and assuming that the $g_i$'s are independent for $i = 1, \ldots, L$, $C_f$ is almost normal for large $L$ with mean given in (6); the variance can be computed numerically. The outage probability for a given rate $R$ is then calculated straightforwardly. The results for $R = 0.05$ and blocks of 64 segments with 4 bins each are shown in Fig. 2. We observe that increasing $\rho$ not only decreases the average capacity but also increases the outage probability rapidly. Again, the dominant effects of $K$ and $\gamma$ are obvious at small and large $\rho$, respectively.

III. MC-FH SYSTEM MODEL

All the results obtained in Sec. II hold for MC-FH schemes. Therefore, in this section we describe the MC-FH system model as a practical and frequently cited example for the general framework developed previously. In the following sections, we examine the performance of coded MC-FH systems in static and fading jammed channels.

The available bandwidth $W$ contains $N$ subcarriers in total divided into $L$ frequency subbands. Each frequency subband has exactly $h = N/L$ frequency subcarriers. For each user in any signaling interval, $L$ subcarriers, one from each frequency subband, are chosen based on the user’s hopping type signature code. Selected subcarriers are then transmitted simultaneously after phase modulation by the user’s encoded bits. Fig. 3 represents the transmitter block diagram [12]. The frequency hoppers operate at the same rate as the symbol rate, and are controlled by the user’s signature code. The received signal for user $k$ can be written as

$$r(t) = \sum_{k=1}^{K} s^{(k)}(t) + n(t)$$

(9)

where $K$ is the number of users and $n(t)$ is the complex Gaussian jamming noise. The jammer has total power $J$ and contaminates a portion $\rho$ of the total bandwidth $W$. Therefore, with $N_J = J/W$, $n(t)$ has power spectral density $N_J/\rho$ wherever in the spectrum that the jammer is present, and 0 elsewhere. It is assumed that $1/T_s \leq \rho W \leq W$ or $1/N \leq \rho \leq 1$.

IV. PERFORMANCE EVALUATION

Let User 1 denote the desired user, and assume the all 0’s sequence is transmitted. Then the outputs of the maximal-ratio combining decorrelators, after normalization and dropping the superscript (1) for simplicity, can be written as

$$y_i = -|g_i|^2 + \text{Re} \left[ \sum_{k=2}^{K} \eta_i^{(k)} d_i^{(k)} g_i g_i^* \right] + z_i \eta_i$$

(10)

where $z_i$ is 0 (in the absence of jamming) with probability $1-\rho$ and 1 with probability $\rho$, the $z_i$’s are considered to be iid random variables, $n_i$ conditioned on $g_i$ is a real zero-mean Gaussian random variable with variance $|g_i|^2/(2\rho \gamma_i)$, wherein $\gamma_i = ST_s' N_J$ is the SIR for one coded bit and $\eta_i^{(k)}$ is
A random variable which is 1 when there is a hit by user $k$ in the $l^{th}$ frequency subband and otherwise 0. From the statistics of the $\gamma_{l,i}^{(k)}$s, it can be shown that the $\eta_{l}^{(k)}$s are iid random variables. Each is 0 with probability $\alpha=1/1+1/h$ and 1 with probability $\beta=1/h$.

For this system, a class of low variable rate convolutional codes, called superorthogonal codes, is considered which has the characteristic function [12],[16]

$$T(D, M) = \frac{MW^{C+2}(1-W)}{1-W[1+M(1+W^{C-3}-2W^{C-2})]}$$

where $W = 2^{C-3}$, and $C \geq 3$ is the constraint length of the code. The free distance is determined as $d_{free} = 2^{C-3}(C+2)$. The code rate is $R = 2^{C-3}$ which is taken to equal $1/L$ such that no additional bandwidth is required for coding. Thus,

$$L = 2^{C-2}.$$  \hfill (12)

Note that the coding gain of the above code is equal to $Rd_{free} = (C+2)/2$. Now we have

$$P_d \leq P_t \leq \sum_{d=d_{free}}^{\infty} b_d P_t$$

where $b_d$ is the coefficient of $D^d$ in the expansion of $\partial T(D, M)/\partial M$ evaluated at $M = 1$, and $P_d$ is the pairwise error probability that a path of weight $d$ is taken in place of the all-zero sequence in the Viterbi decoder.

A. AWGN channels

In static channels, $g_{l,i}^{(k)} = 1$ and (10) reduces to

$$y_l = -1 + \sum_{k=2}^{K} \eta_{l}^{(k)} d_{l}^{(k)} + z_l n_l.$$  \hfill (14)

It is assumed that $y_l$’s with $z_l = 1$ are given a weight of $w_0$ and those with $z_l = 0$ are given a weight of $w_1$. Therefore, the quantity $P_d$ in (13) can be viewed as the probability of the sum of $d$ weighted $y_l$’s (i.e. $w_{y_l} y_l$) being positive. Now, from the result for a pairwise error probability in [17], we can write

$$P_d \leq D^d$$

where

$$D = \min_{\lambda \geq 0} D(\lambda')$$  \hfill (16a)

in which

$$D(\lambda') = \mathbb{E}[e^{\lambda w_{y_l} y_l}] = \rho \exp(-w_1 \lambda' + w_0 \lambda'^2/4\rho \gamma_s)
\times[\alpha + \beta \cosh(w_0 \lambda')]^{(K-1)} + (1-\rho)\exp(-w_0 \lambda')[\alpha + \beta \cosh(w_0 \lambda')]^{(K-1)}.$$ \hfill (16b)

If we define $\lambda = w_1 \lambda'$, then with $w = w_0/w_1$ we have $D = D(\lambda_{opt})$. $\lambda_{opt} = \arg \min_{\lambda \geq 0} D(\lambda)$. and

$$D(\lambda) = A(\rho, \lambda) + B(\rho, w\lambda),$$

where

$$A(\rho, \lambda) = \rho \exp(-\lambda + \lambda^2/(4\rho \gamma_s))[\alpha + \beta \cosh(\lambda)]^{(K-1)}.$$  \hfill (19)

$$B(\rho, x) = (1-\rho)\exp(-x)[\alpha + \beta \cosh(x)]^{(K-1)}.$$  \hfill (20)

If the receiver has knowledge of the jammer state and complete side information, it can set $w$ to an optimal value in order to minimize $D(\lambda)$. Otherwise, $w$ can be determined empirically; however, choosing $w = 1$ seems intuitive when the receiver lacks information. Table 1 presents some detailed information about the values of $\lambda_{opt}$, $w_{opt}$ and $D$ for different choices of $K$ and $w$. In Table 1,

$$x_{opt} = \log\left[\frac{\alpha + \sqrt{[\beta(K-1)]^2 + \alpha - \beta}}{\beta(K-2)}\right]$$  \hfill (17)

is the point at which $B(\rho, x)$ reaches its minimum on $x \geq 0$.

A theoretical value of infinity for $w_{opt}$ in Table 1 means that the receiver chooses $w_0$ to be much greater than $w_1$. Note that $\lambda_{opt}$, the case for optimal weight and $K > 1$ can be found as the minimizer of $A(\rho, \lambda)$ on $\lambda \geq 0$.

It is interesting to explore the simpler results obtained by taking advantage of a central-limit theorem and assuming that the multiple-access term in (14) is Gaussian. In this case, $D(\lambda)$ becomes

$$D(\lambda) = \rho \exp\{-\lambda + \frac{\lambda^2}{2}[\beta(K-1) + \frac{1}{2\rho \gamma_s}]\} + (1-\rho)\exp\{-w_0 \lambda + \frac{\beta(K-1)w_0^2 \lambda^2}{2}\}.$$  \hfill (18)

For $K = 1$, one obtains the same previous results. For $K > 1$, first assume that the receiver knows the jammer state and utilizes the optimal weight $w_{opt}$. Then, it is straightforward to show that

$$w_{opt} = 1 + \frac{1}{2\beta(K-1)\rho \gamma_s}, \quad \lambda_{opt} = \frac{2\rho \gamma_s}{1 + 2\beta(K-1)\rho \gamma_s}.$$  \hfill (19)

On the other hand, for an arbitrarily selected $w$, $\lambda_{opt}$ is bounded by $2\rho \gamma_s/[1 + 2\beta(K-1)\rho \gamma_s]$ and 1/[$\beta(K-1)\rho \gamma_s$].

Note that an extreme case of MC-FH schemes is obtained when $h = 1$, i.e. when there is no hopping and the system is equivalent to a MC-CDMA system. Users should then be distinguished by a direct sequence type signature. In other words, $L = N$ encoded bits are modulated by the $N$ bits of the signature. The previous analyses also hold for the case of MC-CDMA systems if we set $h$ to 1.

B. Rayleigh Fading Channels

Similar to the analysis for AWGN channels, we can write

$$P_d \leq D^d$$

with $D = \min_{\lambda \geq 0} D(\lambda)$, where

$$D(\lambda) = \mathbb{E}[e^{\lambda w_{y_l} y_l}] = \rho \mathbb{E}[e^{\lambda w_{y_l} y_l} | z_l = 1] + (1-\rho)\mathbb{E}[e^{\lambda w_{y_l} y_l} | z_l = 0]$$

in which $w = w_0/w_1$ as before. Now we have
TABLE I

<table>
<thead>
<tr>
<th>Optimal weight and expression for $D$ for different cases</th>
<th>$w = 1$</th>
<th>$w &gt; 1$</th>
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<tr>
<td>$K = 1$</td>
<td>$w_{opt} = \infty$</td>
<td>$\lambda_{opt} = 2 \rho \gamma_s$ $D = \rho \exp(-\rho \gamma_s)$ $\lambda_{opt} \in (2 \rho \gamma_s, 2 \gamma_s)$ $D = A(\rho \lambda_{opt}) + B(\rho \lambda_{opt})$ $\lambda_{opt} \in (0, 2 \rho \gamma_s)$ $D = A(\rho \lambda_{opt}) + B(\rho \lambda_{opt})$ $\lambda_{opt} \in (0, 2 \rho \gamma_s)$ $D = A(\rho \lambda_{opt}) + B(\rho \lambda_{opt})$ $\lambda_{opt} \in (0, 2 \rho \gamma_s)$ $D = A(\rho \lambda_{opt}) + B(\rho \lambda_{opt})$</td>
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$E[e^{\lambda y_i} | z_l = 1] = E_{g_i, \eta_i^{(k)}, d_i^{(k)}}[E[e^{\lambda y_i} | z_l = 1, g_i, \eta_i^{(k)}, d_i^{(k)}]]$. From the statistics of $g_i^{(k)}$, $2 \leq k \leq K-1$, and $n_i$, it can be shown that

$$E[e^{\lambda y_i} | z_l = 1, g_i, \eta_i^{(k)}, d_i^{(k)}] = \exp\left(\frac{\lambda^2}{4 \rho \gamma_s} - \lambda \right) |g_i|^2 \exp\left(\frac{\lambda^2 - \lambda}{4} \sum_{k=2}^{K-1} \eta_i^{(k)} |g_i|^2 \right).$$

Combining (21) and (22) results in

$$E(e^{\lambda y_i} | z_l = 1) = E_{g_i} \left\{ \exp\left(\frac{\lambda^2 - \lambda}{4} |g_i|^2 \right) \right\}.$$  

Using the binomial expansion gives

$$E(e^{\lambda y_i} | z_l = 1) = \sum_{k=0}^{K-1} \binom{K-1}{k} \frac{\lambda^{K-1-k}}{4(k+1/4)} \beta^k,$$

provided that all of the summands are positive. Similarly, we can show that

$$E(e^{\lambda w y_i} | z_l = 0) = \sum_{k=0}^{K-1} \binom{K-1}{k} \frac{\lambda^{K-1-k}}{4} \beta^k,$$

when all summands are positive. Combining (20)-(25) we conclude that

$$D(\lambda) = \sum_{k=0}^{K-1} \frac{\lambda^{K-1-k}}{4(k+1)} \beta^k S_k(\lambda, w, \rho),$$

where

$$S_k(\lambda, w, \rho) = \frac{\rho}{k+1/4} \lambda^2 + 1 - \rho - \lambda^2 + \lambda^2 + w \lambda + 1.$$ 

Now $D$ is obtained by minimizing $D(\lambda)$ over all values of $\lambda$ yielding positive summands in $S_k(\lambda, w, \rho)$. At the same time, the jammer, if smart, will try to maximize $D$ by selecting a suitable $\rho$, and the receiver, if aware of the jamming state, will countermeasure the jamming by choosing a suitable $w$ to minimize $D$. Note that for the special case of $K = 1$, we have

$$D(\lambda) = \frac{\rho}{\lambda^2/(4 \rho \gamma_s) + \lambda + 1} + \frac{1-\rho}{w \lambda + 1}.$$ 

Therefore, in this case the receiver, be it aware of the jamming state or not, opts for $w_{opt} = \infty$, which will result in

$$D(\lambda) = \frac{\rho}{\lambda^2/(4 \rho \gamma_s) + \lambda + 1} + \frac{1-\rho}{w \lambda + 1}.$$ 

and, thus, $\lambda_{opt} = 2 \rho \gamma_s$. Consequently, $D = \rho/(\rho \gamma_s + 1)$. A smart jammer in this case chooses to have $\rho = 1$ in order to increase $D$ to its maximum value, $1/(\gamma_s + 1)$. 

Another approach which will yield more direct yet useful results, is to use a central-limit theorem and assume that the multiple-access term in (10), conditioned on $g_i$, is Gaussian. The mean and variance of this term will then be equal to $0$ and $\beta(K-1) |g_i|^{2/2}$, respectively. Now by conditioning on $z_l$, we find $D(\lambda)$ defined in (20) as

$$D(\lambda) = \frac{\rho}{\beta(K-1) |g_i|^{2/2} + \lambda + 1} + \frac{1-\rho}{w \lambda + 1}.$$ 

provided that the denominators are positive. Clearly for $K = 1$, we have exactly the previous results. However, for $K > 1$, if the receiver has the knowledge of the jammer state, it can be shown that

$$w_{opt} = \frac{1}{\beta(K-1) \rho \gamma_s}, \quad \lambda_{opt} = \frac{2 \rho \gamma_s}{1+1/4}$$

and

$$D = \frac{\rho}{\beta(K-1) \rho \gamma_s + 1} + \frac{1-\rho}{w \lambda + 1}.$$ 

In this case, the optimal $\rho$ for the jammer is unity; i.e., the best strategy against a smart jammer is to force it to spread its spectrum over the entire frequency bandwidth. This rule is almost universal.

In other cases where $K > 1$ and $w$ is not chosen to be $w_{opt}$, $\lambda_{opt}$ is less than the positive roots of the denominators in (26), and is bounded by the values of $\lambda$ that maximize the denominators, i.e. $2 \rho \gamma_s / (\beta(K-1) \rho \gamma_s + 1)$ and $2/(\beta(K-1) w)$. 

Note that all of the results above hold for MC-CDMA systems when $h = 1$.

V. NUMERICAL RESULTS

In this section, some numerical examples are presented to show the performance of the system under different condi-
The Gaussian approximation appears to work well for static and fading channels even for small numbers of users, although comparison graphs are not presented owing to space limitations. All the graphs shown for the static channels in this section have been obtained from the exact calculations. For the fading channels, however, due to its simplicity, the Gaussian approximation has been adopted. Note that in all of the following examples, a performance overbound has been utilized to provide a conservative design. In all of the following graphs, $SJR = L^\gamma$ is the signal-to-jamming-noise ratio for one signaling interval.

Fig. 4 shows the bit error rate (BER) of the system versus the number of active users for the static channel with two values of the SJR when the weight factor $w$ is either 1 (unknown jammer state) or optimum. The gain obtained from knowing the jammer state decreases as the number of users or the required BER increases. In fact, for a larger number of users, stronger multiple access interference cancels out some of the gain acquired by the receiver when it knows the jammer state.

Fig. 5 shows the BER versus $\rho$ for different values of the SJR in the two cases of known and unknown jammer states for static channels. The performance improvement when increasing the SJR at a fixed $\rho$ is almost linear. Also, the amount of the improvement when the jammer state is unknown appears to be independent of $\rho$. From this figure it is clear that the optimal $\rho$ for the jammer when $w = 1$ tends to be the smallest possible value. On the contrary, it tends to be the largest possible value, i.e., 1, when $w = w_{opt}$.

Fig. 6 presents the optimal weight factor in dB versus $\rho$ for different values of the SJR, $K$ and $N$ in fading channels. When the jammer chooses to cover a greater fraction of the entire bandwidth, a greater number of subcarriers are hit by the jammer which now has a weaker power spectral density. Therefore, it would be to the receiver’s advantage to lessen the weight factor appreciably in such cases.

From Fig. 7 we can observe that an increase in $L$ always improves the performance significantly in a linear fashion in fading channels. The same result is obtained for static channels. In fact, when $L$ is increased at a fixed processing gain, on the one hand $h$ will be decreased, which consequently results in an increase in multiple access interference, while on the other hand, the diversity order and the coding gain are increased. The latter effect appears to be strongly dominant. This result implies that MC-CDMA schemes, obtained as an extreme case of MC-FH schemes, always surpass MC-FH systems in terms of the BER performance in jamming environments. From the capacity results in the preceding sections, we know that choosing $h = 1$ or $N = L$ leads to the maximum available capacity. This means that MC-CDMA systems are superior to MC-FH systems also in terms of the achievable capacity. Nevertheless, one should note that frequency hopping concept is considered mandatory in some applications [10]. Furthermore, peak-to-average power ratio is expected to be better in MC-FH schemes than MC-CDMA systems due to the smaller number of transmitted subcarriers in a transmitted symbol in MC-FH systems. Another advantage of MC-FH systems over MC-CDMA systems is reduced sensitivity to phase jitter owing to the larger average frequency subcarrier spacing.

VI. CONCLUSION

The performance of a wide range of multicarrier multiple access systems in static and fading jamming channels such as ultrawide bandwidth time hopping CDMA and MC-FH systems in terms of achievable user capacity was analyzed. We conclude that the best strategy for the jammer to minimize the capacity is to spread all of its energy evenly over all degrees of freedom in the channel. This strategy also leads to maximum outage probability in fading channels. Also, it is best for the communicator to use all available degrees of freedom and avoid hopping type signatures.

The performance of coded MC-FH multiple access schemes in the presence of jamming in static and fading channels was analyzed. It was found that optimal weights for the receiver soft outputs in countering smart jamming will generally result in $\rho = 1$. On the other hand, equal weights cause a smart jammer to minimize its $\rho$. If the receiver is aware of the jammer state but cannot compute the optimal weight, it is better to choose a weight factor other than unity based on experimental results.

It was shown that although it causes an increase in multiple access interference, an increase in the number of subcarriers always improves the BER performance significantly. This result is interesting in that MC-CDMA systems, in spite of not utilizing frequency hopping, outperform MC-FH schemes even in jamming environments.

REFERENCES