Solving the integrated product mix-outsourcing problem using the Imperialist Competitive Algorithm

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1. Introduction

Maximizing the enterprise profit and fully utilizing the limited production resources is a key optimization problem in the strategic planning of enterprises. Furthermore, another major difficulty is how to optimize the enterprise's production throughput within the determined product types (Wang, Sun, Si, & Yang, 2008). Aggregating the above two optimization concerns results in the product mix optimization problem which involves identification of the type and quantity of product to produce in order to make the most profit (Coman & Ronen, 2000). In the competitive market enterprises try to meet the market demand as much as possible. Although, this increases the sale revenue and profit, but in most cases, the enterprise resources produce the total demand are insufficient. Therefore, the enterprises tend to outsource part of the demand to external suppliers. Hence, the product mix and the outsourcing decisions have to be made simultaneously in many of the real world cases and can be referred to as the integrated product mix-outsourcing optimization problem.

Küttner (2004) stated that to solve the product mix problem and decide on the production volume for each product, the demand’s forecast and the value of utilized resources should be considered. Lea and Fredendall (2002) demonstrated that the choice of product mix influences on the enterprise performance measures such as profit, work-in-progress (WIP), customer service, and manageability of the shop. To solve the product mix problem, there are two general approaches in the literature: heuristic and meta-heuristic algorithms.

One of the well-known heuristics is the Theory of Constraints (TOC) (Goldratt, 1987). The book by Eliyahu Goldratt “The Goal” explains that TOC is a management philosophy that focuses on constraints, which restrict the performance of an organization in achieving its goal (Goldratt & Cox, 2004). This Management philosophy originated from operation management (Watson, Blackstone, & Gardiner, 2007). Various applications of TOC are known to be product mix, logistics, scheduling, performance measurement, problem solving/thinking process, project management, market segmentation. Based on TOC, the throughput, due date, utilization, and other key performance measures of an enterprise can be controlled and optimized by controlling only the bottleneck resources in the enterprise (Goldratt, 1987). The TOC approach has been applied to solve the product mix problem by many researchers such as Goldratt (1990), Patterson (1992), Plenert (1993) and Lee and Plenert (1996). In most of previous studies related to the product mix problem, this problem has been extended either with increase in the number of constrained resources or products (Coman & Ronen, 2000). Balakrishnan (1999) claimed that when multiple constraints exist, the conventional TOC heuristic is not an appropriate solution method for product mix problem. They proposed that a linear-integer programming is a better tool than the TOC. Mabin (2001) showed that the TOC solution approach is improperly applied by Balakrishnan (1999) and argues that TOC and LP can be applied effectively in synergy. Plenert (1993) pro-
vided an example where the TOC heuristic does not produce an optimal or even feasible solution. In cases with more than one bottleneck, identification of the dominant bottleneck becomes very difficult (Fredendall & Lea, 1997). Based on TOC approach the resource that has the maximum value of planned utilization is considered as the dominant bottleneck resource (Fredendall & Lea, 1997), Patterson (1992) and Goldratt (1990) tested the TOC heuristic by a test problem that its solution involves 100% utilization of bottleneck resource. In some other studies such as Plenert (1993) and Lee and Plenert (1996), the TOC approach has been applied and examined in instances that the bottleneck resource has not 100% utilization in the optimal solution. In other words, idle time is considered for the bottleneck resource during optimization.

The main objective of TOC is to maximize the output that is obtained by determination and exploitation of the Critically Constrained Resource (CCR) (Onwubolu & Mutingi, 2001). However, the shortcoming of the above methods is that, in an attempt to make the TOC approach explicit, only small problem sizes can be solved to optimality within reasonable computation time (Onwubolu & Mutingi, 2001). Therefore, in addition to heuristic methods some meta-heuristics and intelligent search algorithms, such as Tabu search (TS) (Onwubolu, 2001), genetic algorithms (GA) (Onwubolu & Mutingi, 2001), hybrid Tabu-simulated annealing approach (Mishra, Tiwari, Shankar, & Chan, 2005), and other meta-heuristics algorithms are used to solve large-scale product mix optimization problem. Wang et al. (2008) developed an immune algorithm to solve the TOC product mix problem.

Outsourcing of production happens when an enterprise, instead of performing the entire production in-house, contracts with one or more external contractor to make part of the production outside (Sen & Zhu, 1996). In other words, the production outsourcing problem is the decision making problem of which products/parts should be produced in-house and which should be contracted out to be manufactured by one or more external suppliers (Küttner, 2004). Outsourcing is defined by Heshmati (2003) as the subcontracting relationship between firms, and the hiring of workers in non-traditional jobs. In practice, outsourcing is not only a “pure” make-or-buy decision, but also includes a shifting from internal production to external procurement (Lai, 2006). Outsourcing decision is always a de-integration decision, in which prior commitments to internal production should be ignored (Roodhooft & Warlop, 1999).

Generally, the top five reasons for outsourcing are: improvement of the company’s focus, access to world-class capabilities, acceleration of benefits from re-engineering, risk sharing, and freeing the resources for other aims (Deavers, 1997).

The strategic goal for outsourcing decision-makers should be minimizing the total costs required to receive a given quantity and quality of outsourced products (Lai, 2006). An enterprise could benefit from competitive advantages (reliability, quality and cost) by contracting out the production of products (Perry, 1997). Sharpe (1997) mentioned that outsourcing lowers the adjustment costs of responding to economic changes. Adjustments are normally needed due to technological innovation, customer preference changes, and other changes in supply or demand. Glass and Saggi (2001) mentioned that outsourcing decreases the marginal production cost, increases profit, and creates greater incentives for innovation.

Efficient enterprises usually assign their own resources to those activities that are in the value chain and create more profit than competitors (Shank & Govindarajan, 1992). Other activities not enjoying such advantages are increasingly outsourced to external suppliers. It is expected that with outsourcing the production cost savings relative to internal production happens because outside suppliers benefit from smoother production schedules and centralization of expertise (Chalos, 1995; Roodhooft & Warlop, 1999).

De Kok (2000) considered outsourcing as a measure for assigning the capacity. The excessive capacity needs are not delayed but are instead outsourced. Thus, in the cases that market demands exceed existing capacity, outsourcing may be a good way to achieve the benefits of cost saving and risk sharing.

In recent years, outsourcing has become an important strategy for many business enterprises. For a successful outsourcing decision, the benefit of cost savings is an important factor. Thus, the decision about outsourcing requires an accurate analysis of relevant costs (Lai, 2006). In his thesis, Lai (2006) develops a decision model which incorporates capacity expansions and outsourcing using a mathematical programming approach. In this work, the benefits of expanding resource capacities of various kinds or outsourcing simultaneously are evaluated. Gardiner and Blackstone (1991) solved a make-or-buy one-bottleneck-one-product problem using the bottleneck capacity for better decision making and integrated some capacity issues with financial issues. Their work was followed by Balakrishnan and Cheng (2005) who presented a method based on spreadsheet LP solver that generated better solutions for the outsourcing problem. Coman and Ronen (2000) also formulated a production mix-outourcing problem as a LP problem. They examined three solution approaches: standard cost accounting, standard TOC, and their solution. In their results, they showed that the TOC solution was worse than their LP enhanced solution.

One of the shortcomings of TOC and Standard Accounting solutions approaches is that these methods only consider the situations (constraints) present inside the enterprise. While in the product mix-outsourcing problem both internal and external constraints should be dealt with. Availability and cost of enterprise resources are examples for internal constraints and availability and cost of outsourcing resources are cases for external constraints. Thus, models for integrated product mix-outsourcing problem must consider both internal and external constraints simultaneously.

In the work at hand to solve large-scale integrated product mix-outsourcing problems a novel meta-heuristic is employed. This meta-heuristic is called Imperialist Colony Algorithm (ICA) that has recently been introduced by Atashpaz-Gargari and Lucas (2007) for dealing with different optimization tasks. The rest of this paper is organized as following: in Section 2 the integrated product mix-outsourcing problem is described and formulated as a LP model. In Section 3 the ICA algorithm is described in detail. Section 4 includes the simulation results. In this section results of applying the ICA algorithm and two other methods (TOC and Standard Accounting) are shown. Section 5 concludes the paper.

2. Problem statement

Here, we describe the linear programming (LP) formulation for the integrated product mix-outsourcing problem. In addition, an instance of this problem is given. Both formulation and instance are based on the work of Coman and Ronen (2000). The LP definition of product mix-outsourcing problem can be represented by:

$$
\text{Max} \sum_{i=1}^{n} X_i (OP_i - RM_i) \\
\text{s.t.,} \\
\sum_{i=1}^{n} a_{ij} X_i \leq b_j \in \{1, 2, \ldots, M\}, \\
X_i \leq D_i \in \{1, 2, \ldots, n\}.
$$

Where $X_i$ is the production decision variable that indicates the amount of production of product $i$ in the enterprise, $OP_i$ and $RM_i$ are the outsourcing cost and the raw material cost, respectively. $D_i$ indicates the demand of product $i$. The total capacity of resource $j$ in the
planning period is shown by $b_i$ and $a_{ij}$ is the capacity requirement for product $i$ in resource $j$. We assume that there are $M$ machines in the enterprise.

To illustrate how the objective function in the above model indicates the profit of enterprise the following discussion is necessary. It is assumed that the enterprise meets the market demand by producing products inside enterprise and by outsourcing. The profit of enterprise can be calculated from subtracting costs (raw materials costs, outsourcing costs, and operating expenses) from income produced by selling the products:

$$\text{profit} = \sum_{i=1}^{n} \left( (D_i \times P_i) - (X_i \times RM_i) \right) - OE. \tag{2}$$

$P_i$ indicates the market price of product $i$ $(i = 1, \ldots, n)$ and $OE$ represents the operation costs that is a constant value. The enterprise policy is that the market demand of all products should be met because the maximum throughput and utilization are two main goals of the enterprise. Therefore, the enterprise produces $X_i$ number of product $i$ in the enterprise and orders the number $(D_i - X_i)$ of product $i$ to outsourcing resources. The term $(D_i \times P_i)$ indicates the income from selling the product $i$ in the market. The term $(D_i - X_i) \times OP_i$ indicates the outsourcing costs for product $i$. While $(X_i \times RM_i)$ represents the raw materials cost of producing product $i$ in the enterprise. We can rewrite the Eq. (2) as below:

$$\text{profit} = \sum_{i=1}^{n} \left( (P_i - OP_i) \times D_i + X_i \times (OP_i - RM_i) \right) - OE. \tag{3}$$

The value of $(P_i - OP_i) \times D_i$ is constant for each product $i$. Therefore, the profit of enterprise can be calculated as the value of objective function mentioned in Eq. (1).

3. Imperialist Competitive Algorithm

Imperialist Competitive Algorithm (ICA) is a new socio-politically motivated global search strategy that has recently been introduced for dealing with different optimization tasks (Atashpaz-Gargari & Lucas, 2007). This evolutionary optimization strategy has shown great performance in both convergence rate and better global optima achievement (Atashpaz-Gargari & Lucas, 2007; Rajabion et al., 2008b; Biabangard-Oskouyi, Atashpaz-Gargari, Soltani, & Lucas, 2009; Sepehri Rad & Lucas, 2008; Sepehri Rad, Rajaei Salmasi, & Lucas, 2008b). Nevertheless, its effectiveness, limitations and applicability in various domains are currently being extensively investigated. In Atashpaz-Gargari et al. (2008), ICA is used to design an optimal controller which not only decentralizes but also optimally controls an industrial Multi Input Multi Output (MIMO) distillation column process. Almost the same is done in Rajabion et al. (2008b) for a more complicated MIMO system which is a $3 \times 3$ model of Evaporator Plant. Biabangard-Oskouyi et al. (2009) use ICA for reverse analysis of an artificial neural network in order to characterize the properties of materials from sharp indentation test. In order to find the optimal priorities for each user in recommender systems, Sepehri Rad and Lucas (2008) use ICA in “Prioritized user-profile” approach to recommender systems, trying to implement more personalized recommendation by assigning different priority importance to each feature of the user-profile in different games.

Fig. 1 shows the flowchart of the ICA. Similar to other evolutionary algorithms, this algorithm starts with an initial population. Each individual of the population is called a country. Some of the best countries (in optimisation terminology, countries with the least cost) are selected to be the imperialist states and the rest form the colonies of these imperialists. All the colonies of initial countries are divided among the mentioned imperialists based on their power. The power of each country, the counterpart of fitness value in the GA, is inversely proportional to its cost. The imperialists states together with their colonies form some empires.

After forming initial empires, the colonies in each of them start moving toward their relevant imperialist country. This movement is a simple model of assimilation policy which was pursued by some of the imperialist states. The total power of an empire depends on both the power of the imperialist country and the power of its colonies. This fact is modelled by defining the total power of an empire as the power of imperialist country plus a percentage of mean power of its colonies.

3.1. Creation of initial empires

The goal of optimisation is to find an optimal solution in terms of the variables of the problem. We form an array of variable values to be optimized. In the GA terminology, this array is called “chromosome”, but in ICA the term “country” is used for this array. In an $Nvar$-dimensional optimisation problem, a country is a $1 \times Nvar$ array. This array is defined as following:

$$\text{country} = [p_1, p_2, p_3, \ldots, p_{Nvar}].$$
where $p_i$'s are the variables to be optimized. The variable values in the country are represented as floating point numbers. Each variable in the country can be interpreted as a socio-political characteristic of a country. From this point of view, all the algorithm does is to search for the best country that is the country with the best combination of socio-political characteristics such as culture, language, economical policy, and even religion. From optimization point of view this leads to find the optimal solution of the problem, the solution with least cost value. Fig. 2 shows the interpretation of country using some of socio-political characteristics.

The cost of a country is found by evaluation of the cost function $f$ at variables($p_1, p_2, p_3, \ldots, p_{N_{var}}$). So we have

$$\text{cost} = f(\text{country}) = f(p_1, p_2, p_3, \ldots, p_{N_{var}}).$$

To start the optimization algorithm, initial countries of size $N_{Country}$ is produced. We select $N_{imp}$ of the most powerful countries to form the empires. The remaining $N_{col}$ of the initial countries will be the colonies each of which belongs to an empire.

To form the initial empires, the colonies are divided among imperialists based on their power. That is, the initial number of colonies of an empire should be directly proportional to its power. To proportionally divide the colonies among imperialists, the normalized cost of an imperialist is defined by

$$C_n = c_n - \max_i c_i,$$

where $c_n$ is the cost of the $n$th imperialist and $C_n$ is its normalized cost. Having the normalized cost of all imperialists, the normalized power of each imperialist is defined by

$$p_n = \frac{C_n}{\sum_{i=1}^{N_{im}} C_i}.$$

The initial colonies are divided among empires based on their power. Then the initial number of colonies of the $n$th empire will be

$$N_{C,n} = \text{round}(p_n N_{col}).$$

where $N_{C,n}$ is the initial number of colonies of the $n$th empire and $N_{col}$ is the total number of initial colonies. To divide the colonies, $N_{C,n}$ of the colonies are randomly chosen and given to the $n$th imperialist. Fig. 3 shows the initial empires. As shown in this figure, bigger empires have greater number of colonies while weaker ones have less. In this figure imperialist 1 has formed the most powerful empire and consequently has the greatest number of colonies.

3.2. Assimilation: movement of colonies toward the imperialist

Pursuing assimilation policy, the imperialist states tried to absorb their colonies and make them a part of themselves. More precisely, the imperialist states made their colonies to move toward themselves along different socio-political axis such as culture, language and religion. In the ICA, this process is modelled by moving all of the colonies toward the imperialist along different optimization axis. Fig. 4 shows this movement. Considering a 2-dimensional optimisation problem, in this figure the colony is absorbed by the imperialist in the culture and language axes. Then colony will get closer to the imperialist in these axes. Continuation of assimilation will cause all the colonies to be fully assimilated into the imperialist.

In the ICA, the assimilation policy is modelled by moving all the colonies toward the imperialist. This movement is shown in Fig. 4 in which a colony moves toward the imperialist by $x$ units. The new position of colony is shown in a darker colour. The direction of the movement is the vector from the colony to the imperialist.
In this figure \( x \) is a random variable with uniform (or any proper) distribution. Then

\[
x \sim U(0, \beta \times d),
\]

where \( \beta \) is a number greater than 1 and \( d \) is the distance between the colony and the imperialist state. \( \beta > 1 \) causes the colonies to get closer to the imperialist state from both sides.

Assimilating the colonies by the imperialist states did not result in direct movement of the colonies toward the imperialist. That is, the direction of movement is not necessarily the vector from colony to the imperialist. To model this fact and to increase the ability of searching more area around the imperialist, a random amount of deviation is added to the direction of movement. Fig. 5 shows the new direction. In this figure \( \theta \) is a parameter with uniform (or any proper) distribution. Then

\[
\theta \sim U(-\gamma, \gamma),
\]

where \( \gamma \) is a parameter that adjusts the deviation from the original direction. Nevertheless the values of \( \beta \) and \( \gamma \) are arbitrary, in most of implementations a value of about 2 for \( \beta \) and about \( \pi/4 \) (Rad) for \( \gamma \) results in good convergence of countries to the global minimum.

3.3. Revolution; a sudden change in socio-political characteristics of a country

Revolution is a fundamental change in power or organizational structures that takes place in a relatively short period of time. In the terminology of ICA, revolution causes a country to suddenly change its socio-political characteristics. That is, instead of being assimilated by an imperialist, the colony randomly changes its position in the socio-political axis. Fig. 6 shows the revolution in Culture-Language axis. The revolution increases the exploration of the algorithm and prevents the early convergence of countries to local minimums. The revolution rate in the algorithm indicates the percentage of colonies in each colony which will randomly change their position. A very high value of revolution decreases the exploitation power of algorithm and can reduce its convergence rate. In our simulations the revolution rate is 0.3. That is 30 percent of colonies in the empires change their positions randomly.

3.4. Exchanging positions of the imperialist and a colony

While moving toward the imperialist, a colony might reach to a position with lower cost than the imperialist. In this case, the imperialist and the colony change their positions. Then the algorithm will continue by the imperialist in the new position and the colonies will be assimilated by the imperialist in its new position. Fig. 7a depicts the position exchange between a colony and the imperialist. In this figure the best colony of the empire is shown in a darker color. This colony has a lower cost than the imperialist. Fig. 7b shows the empire after exchanging the position of the imperialist and the colony.
3.5. Unitig similar empires

In the movement of colonies and imperialists toward the global minimum of the problem some imperialists might move to similar positions. If the distance between two imperialists becomes less than threshold distance, they both will form a new empire which is a combination of these empires. All the colonies of two empires become the colonies of the new empire and the new imperialist will be in the position of one of the two imperialists. Figs. 8a and b show the uniting process of two empires before uniting and resulting from uniting two empires, respectively.

3.6. Total power of an empire

Total power of an empire is mainly affected by the power of imperialist country. However the power of the colonies of an empire has an effect, albeit negligible, on the total power of that empire. This fact is modeled by defining the total cost of an empire by

\[ T.C. = \text{Cost(Imperialist)} + \xi \cdot \text{mean(Cost(Colony of empire))}. \]  

Where \( T.C. \) is the total cost of the \( n \)th empire and \( \xi \) is a positive small number. A little value for \( \xi \) causes the total power of the empire to be determined by just the imperialist and increasing it will increase to the role of the colonies in determining the total power of an empire. The value of 0.1 for \( \xi \) has shown good results in most of the implementations.

3.7. Imperialistic competition

All empires try to take the possession of colonies of other empires and control them. The imperialistic competition gradually brings about a decrease in the power of weaker empires and an increase in the power of more powerful ones. The imperialistic competition is modelled by just picking some (usually one) of the weakest colonies of the weakest empire and making a competition among all empires to possess these (this) colonies. Fig. 9 shows a big picture of the modelled imperialistic competition. Based on their total power, in this competition, each of empires will have a likelihood of taking possession of the mentioned colonies. In other words, these colonies will not definitely be possessed by the most powerful empires, but these empires will be more likely to possess them.

To start the competition, first a colony of the weakest empire is chosen and then the possession probability of each empire is found. The possession probability \( P_p \) is proportionate to the total power of the empire. The normalized total cost of an empire is simply obtained by

\[ N.T.C.n = T.C.n - \max(T.C.i). \]

Where, \( T.C.n \) and \( N.T.C.n \) are the total cost and the normalized total cost of the \( n \)th empire, respectively. Having the normalized total cost, the possession probability of each empire is given by

\[ P_p = \frac{N.T.C.n}{\sum_{i=1}^{N_{imp}} N.T.C.i}. \]

To divide the mentioned colonies among empires vector \( P \) is formed as following

\[ P = [p_{p1}, p_{p2}, p_{p3}, \ldots, p_{pn_{emp}}]. \]

Then the vector \( R \) with the same size as \( P \) whose elements are uniformly distributed random numbers is created,

\[ R = [r_1, r_2, r_3, \ldots, r_{N_{emp}}]; r_1, r_2, r_3, \ldots, r_{N_{emp}} \sim U(0, 1) \]

Then vector \( D \) is formed by subtracting \( R \) from \( P \)

\[ D = P - R = [d_1, d_2, d_3, \ldots, d_{N_{emp}}] = [p_{p1} - r_1, p_{p2} - r_2, p_{p3} - r_3, \ldots, p_{pn_{emp}} - r_{n_{emp}}]. \]

Referring to vector \( D \) the mentioned colony (colonies) is handed to an empire whose relevant index in \( D \) is maximized.
The process of selecting an empire is similar to the roulette wheel process which is used in selecting parents in GA. But this method of selection is much faster than the conventional roulette wheel. Because it is not required to calculate the cumulative distribution function and the selection is based on only the values of probabilities. Hence, the process of selecting the empires can solely substitute the roulette wheel in GA and increase its execution speed.

The main steps of ICA is summarized in the pseudo-code are given in Fig. 10. The continuation of the mentioned steps will hopefully cause the countries to converge to the global minimum of the cost function. Different criteria can be used to stop the algorithm. One idea is to use a number of maximum iteration of the algorithm, called maximum decades, to stop the algorithm. Or the end of imperialistic competition, when there is only one empire, can be considered as the stop criterion of the ICA. On the other hand, the algorithm can be stopped when its best solution in different decades cannot be improved for some consecutive decades.

4. Simulation results

In this section, ICA is applied to the integrated product mix-outsourcing problem. The sample problem from Coman and Ronen (2000) is used. In the following the problem is described and each part of the algorithm is restated to show how ICA can be applied to the outsourcing problems. Next the results of applying ICA are compared to the results obtained from TOC and Standard Accounting solution approaches.

4.1. A sample outsourcing problem

To illustrate the solution method, the example from Coman and Ronen (2000) is used here. In this example an enterprise has to decide on the proportion of the demand for each of its products to be produced inside and then the remainder will be outsourced. The enterprise produces three products A, B, and C. The enterprise has four resources E, F, G, and H that are used for producing these products; the market demand of each product is assumed to be 100 units per week. In addition, the capacity of each resource in each week is 2400 min. Each product uses two raw materials units and the raw materials costs for each product is $40. The outsourcing prices for products A, B, and C are $66, $68, and $98, respectively. Fig. 11 shows the process route, processing time of each product in each resource, market demand for each product, market price for each product, and raw material unit cost of each product. Therefore, we can formulate the product mix-outsourcing problem for this enterprise as below:

Max \( X_A(66 - 40) + X_B(68 - 40) + X_C(98 - 40) \)

s.t.

\( 2X_A + 4X_B + 13X_C \leq 2400, \)
\( 14X_A + 12X_B + 18X_C \leq 2400, \)
\( 4X_A + 10X_B + 10X_C \leq 2400, \)
\( 4X_A + 6X_B + 10X_C \leq 2400, \)
\( 0 \leq X_A, X_B, X_C \leq 100. \)
4.2. TOC approach

As mentioned earlier, TOC approach considers only the throughput and the utilization of bottleneck resources to control any performance measures. According to TOC approach for product mix problem, the resources of the enterprise are divided into two categories: bottleneck and non-bottleneck resources. To identify the bottleneck resource, the amount of time that each resource is loaded to meet the demand should be determined. If the amount of the total load on a resource is more than the amount of available time, this resource is a bottleneck otherwise a non-bottleneck resource. Based on the TOC, the cost of missing an hour of bottleneck equals to the cost of missing an hour in the whole system (Goldratt, 1987). Now, for the abovementioned instance in previous section the bottleneck is identified as following:

Table 1 shows how the bottleneck is identified. The second row in Table 1 indicates the total processing time needed for production of each unit of products A, B, and C in each resource. For example, for resource E the products A, B, and C consume 2, 4, and 13 min. Therefore, the total processing time for this resource is 19 min. The third row shows the total available time for each resource in the week is 2400 min for each resource. The forth row indicates the utilization percentage of each resource. The utilization percentage of each resource is determined by the following equation:

\[ Utilization \text{ percent} = \frac{Total \text{ Processing Time} \times Demand}{Total \text{ Available Time}} \times 100 \]  \tag{17}

For example, for resource E based on Eq. (17) the utilization percentage is
In Table 1 for resource $F$ the utilization percentage is 175 and more than 100%, therefore, this resource is a bottleneck resource.

Now, the amount of internal production of each product is determined. Table 2 shows this for each product in its sixth row. As discussed above, the subtraction of raw material costs from market price for each product indicates the value of profit resulted from producing one unit. Dividing the value of each product unit profit to its processing time in the bottleneck resource indicates the unit profit obtained from 1 min bottleneck resource for that product. Considering that 1 h of bottleneck resource equals to 1 h of the whole system, the higher the value of unit profit per unit time of bottleneck resource, the higher the profit of the whole system. Based on Table 2 the priority of internal production of each product is $B > C > A$. Thus, the product $B$ should be produced first. The demand of product $B$ is 100 units; therefore, 100 units of product $B$ are produced first. Production of 100 units of product $B$ takes $100 \times 12 = 1200$ min from the bottleneck resource $F$. The remaining time for resource $F$ is 1200 min. Hence, only $1200/18 = 66$ units of product $C$ can be produced internally. The rest of product $C$’s demand should be outsourced. As the remaining time for the bottleneck resource is zero, the whole demand of product $A$ has to be outsourced. Thus, 34 units of product $C$ and 100 units of product $A$ are outsourced. Therefore, the net profit based on Eq. (2) is $18428$.

4.3. Standard Accounting approach

Unlike TOC approach which focuses on bottleneck resource, Standard Accounting approach considers all resources in the system. In this approach, the profit from producing one unit of each product is calculated and this value determines the priority of production. The equation:

\[
\text{unit product profit} = \frac{\text{market price} - \text{raw material cost}}{\text{total processing time}} - \text{unit operating cost.} \tag{19}
\]

The value of “market price – raw material cost” shows the profit of producing one unit of product disregarding the operating costs. To determine the unit operating cost for each resource, operating expense ($OE$) must be divided to the number of resources in the enterprise. Then, this value is divided to the available time of the resource to calculate the unit operating cost of each resource per minute. For instance $OE$ is $12,000$, therefore, the value of operating cost for each resource per minute is $12,000/(4 \times 2400) = 1.25$. Table 3 shows the results of applying the standard accounting approach.

For example, the value of “market price – raw material cost” for product $A$ is $130 - 40 = 90$. The total processing time for product $A$ is the sum of its processes times in the process route including resource $E$ (2 min), $F$ (12 min), $G$ (4 min), and $H$ (4 min) that is 22 min. The value of unit product profit for product $A$ according to Eq. (19) is $(90/22) = 1.25 \approx 2.84$. As the value of unit product profit

<table>
<thead>
<tr>
<th>Product</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit per unit ($)</td>
<td>90</td>
<td>110</td>
<td>150</td>
</tr>
<tr>
<td>Processing time in the bottleneck resource $F$ (min)</td>
<td>12</td>
<td>12</td>
<td>18</td>
</tr>
<tr>
<td>Profit per unit time in the bottleneck ($/min$)</td>
<td>7.5</td>
<td>9.17</td>
<td>8.33</td>
</tr>
<tr>
<td>Demand</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Internal production</td>
<td>0</td>
<td>100</td>
<td>66</td>
</tr>
<tr>
<td>Outsource</td>
<td>100</td>
<td>0</td>
<td>34</td>
</tr>
</tbody>
</table>

Table 3

The results of Standard Accounting approach.

determines the priority of production, the production priority is $A > B > C$. Thus, product $A$ should be produced internally based on the resource availability constraint. Because the full demand for product $A$ (which is 100) can be produced internally, outsourcing is not necessary for this product. Doing so the remaining available time for each resources $F, G, H$ and $C$ are 2200, 2400, 2000, and 2000 min, respectively. Because the remaining available time for resource $F$ is the least among the other resources, this resource determines the amount of internal production of product $B$. The processing time of product $B$ in resource $F$ is 12 min. Therefore, $(1200/12) = 100$ units of product $B$ can be produced internally. Hence, none of product $B$ is also outsourced. The remaining available time of resource $F$ is now zero and all of product $C$ has to be outsourced. Considering this, the net profit based on Eq. (2) will be $17,200$. 4.4. ICA steps for solving the outsourcing problem

The problem studied in previous section consists of the three $X_p$, $X_q$, $X_r$ parameters, each of which is an integer between 0 and 100. The objective is to maximize profit satisfying the constraints. The search space consists of $(101)^3$ set of solutions. To use the ICA to solve this problem, at first a proper definition of a country should be stated. As mentioned before the country is a set of unknown parameters of the problem. In this problem a country includes three parameters as following:

\[
\text{Country} = [X_p, X_q, X_r].
\]

For example $[X_p, X_q, X_r] = [56, 13, 42]$ indicate a country. Each country has a power which is inversely proportionate to its cost. The algorithm looks for a country which has the minimum value of cost function. To convert the maximization problem of outsourcing to a minimization one, the cost function is considered to be the negative of profit value. That is,

\[
\text{Cost(Country)} = -\text{Profit}(X_p, X_q, X_r) \tag{19}
\]

The best country, $\text{country}^*$, is defined as:

\[
\text{Country}^* = [X_p, X_q, X_r] \mid \text{Min : Cost(Country)} \text{s.t.}
\]

\[
2X_p + 4X_q + 13X_r \leq 2400, \\
14X_p + 12X_q + 18X_r \leq 2400, \\
4X_p + 10X_q + 10X_r \leq 2400, \\
4X_p + 6X_q + 10X_r \leq 2400, \\
0 \leq X_p, X_q, X_r \leq 100.
\]

For example the sample solution $[56, 13, 42]$ that satisfies all of the constraints has the cost value of $-4256$. The constraints of the problem are considered in the definition of the cost function by defining a penalty value for the non-feasible solutions. Other approaches of constraint handling can also be used which might enhance the convergence of the algorithm. To start the algorithm a set of random solutions (random countries) is generated. The costs of all countries are calculated and some of those with least cost values are chosen.
to be the imperialists of the algorithm. The rest of the countries are divided among these imperialists as described in Section 3. After forming the initial empires, the assimilation is used to evolve the poor countries in each of the empires. As described earlier, in assimilation a colony moves toward imperialist in different axis. Fig. 12 shows the assimilation of a colony by the imperialist in $X_p$, $X_q$, $X_r$ axis. In this figure a sample poor solution $[10, 10, 10]$ is assimilated by a sample imperialist $[60, 60, 60]$ and moves to new position $[30, 30, 50]$.

After assimilating all of the colonies by imperialists in each empire, revolution takes place in some of the countries. Consequently some countries move to some positions that were not accessible by the assimilation process. After assimilation and revolution, the cost of each colony is calculated in its new position. Some of the colonies in each empire might have reached to better positions than the imperialist itself. In this case the imperialist is substituted by

### Table 4
Parameters of ICA approach.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of initial countries</td>
<td>30</td>
</tr>
<tr>
<td>Number of initial imperialists</td>
<td>5</td>
</tr>
<tr>
<td>Number Of decades</td>
<td>50</td>
</tr>
<tr>
<td>Revolution rate</td>
<td>0.3</td>
</tr>
<tr>
<td>$\beta$</td>
<td>2</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.5</td>
</tr>
</tbody>
</table>

### Table 5
Comparison of the net profit resulted from different approaches.

<table>
<thead>
<tr>
<th>The solution method</th>
<th>Standard Accounting</th>
<th>TOC</th>
<th>ICA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net profit</td>
<td>17,200</td>
<td>18,428</td>
<td>19,000</td>
</tr>
</tbody>
</table>

Fig. 12. Assimilation of a country by an imperialist in $X_p$, $X_q$, $X_r$ axis.

Fig. 13. convergence of the ICA to the optimal solution.
the best colony in the empire (The colony with the least value of cost function). The total cost of empires is calculated as in Eq. (4) and then imperialistic competition starts and a colony of poor empires are possessed by another one. The continuation of these processes converge the algorithm and it might reach to the global minimum of the cost function.

Table 4 shows the parameters of ICA used to find the optimal solution for the sample outsourcing problem defined before. Fig. 13 shows the convergence of the ICA to the optimal solution. As shown in this figure ICA has reached to the optimal values of [Xp, Xq, Xr] in about 10 decades. It is also notable that using 30 initial countries and 10 numbers of decades, the total number of cost function calls of ICA to reach to the global optimum of the function is 10 × 30 × 300 which is very small in comparison to (101)^6 number of entire solutions in the search space. The algorithm reached to optimal solution of [0, 50, 100] which leads the cost value $-7200 and satisfies all of the constraints. As the internal production of the product A, B, and C are 0, 50, and 100, respectively, and the demand for each product is 100 units, the outsourcing quantity for these products will be 100, 50, and 0. Based on Eq. (2) the net profit for the sample problem by the ICA is $19000.

### 4.5. Comparison of the methods

In previous sections three solution methods for the integrated product mix-outsourcing problem have been described and applied to the same example. Table 5 summarizes the results and illustrates that the ICA method leads to the maximum net profit among other methods for the given example. Many other examples were also solved to further test the ICA algorithm. All of the cases had similar results and demonstrated the superiority of ICA over TOC and Standard Accounting.

The resource utilization rate for the example discussed for the three solution approaches is shown in Fig. 14 below.

### 5. Conclusion

In this paper, we formulated the integrated product mix-outsourcing problem in LP format. Then, we applied a meta-heuristic algorithm called Imperialist Competitive Algorithm (ICA) for the first time to solve such problems. Then we examined ICA against another two well-known approaches in the literature (TOC and Standard Accounting) and obtained better solution. ICA is capable of solving large-scale integrated product mix-outsourcing problem. Furthermore, contrary to TOC and Standard Accounting methods, ICA considers all constraints inside and outside the enterprise. Availability and cost of outside resources as well as the resources in the enterprise are considered simultaneously. To name a few future research possibilities new constraints and multi-objective cases can be mentioned. In general, ICA has a promising potential to be used as a new solution approach in a variety of problems.

### References


![Fig. 14. Resource utilization rate of different resources obtained by three methods.](image-url)


