Abstract: In the last decade, the concern about significant changes in the business environment has led in designing the robust supply chain networks. This paper proposes a multi-stage stochastic programming model for supply chain network design (SCND) that uses the conditional Value-at-Risk (CVaR) criteria in its objective function to control the risk resulted from uncertain nature of model parameters. Firstly, a deterministic mixed integer linear programming (MILP) model that is able to consider capacity expansion for network facilities as well as different transportation modes beside the traditional features of SCND problem is developed. Secondly, the stochastic counterpart of the proposed deterministic model is developed by using scenario-based multi-stage stochastic programming approach. Finally, the proposed stochastic model is extended by incorporating the CVaR criteria into the objective function. Numerical results show the efficiency of the proposed stochastic programming model in controlling the risk level resulted from uncertain input data.

Keywords: Conditional Value-at-Risk (CVaR), Robust Facility location, Multi-stage stochastic programming, Supply chain network design.

I. INTRODUCTION

Nowadays, by increasing the uncertainties in business environments and more intense competitive pressures make the firms to seek for more reliable supply chains. Supply chain network design (SCND) is one of the crucial strategic decisions in supply chain management due to its significant impact on overall performance of supply chain. SCND in a dynamic and uncertain conditions leads that the organizations deal with turbulences as an integral component of today’s business environments. These turbulences may come from changing and diverse in demand of customers, changing in energy price, that will affect on the all configuration of SCN, and other risks such as wars and natural disasters. As the relevant literature shows (see [1] and [2]), most of the researches in this area ignore the uncertain and dynamic nature of SCND problem. Kilibi et al. [3] and Pishvaee et al. [4] mentioned that static and deterministic models are not able to handle the parameters tainted by dynamism and uncertainty and therefore the decisions resulted from these models may impose high costs to the firms. In addition most of previously proposed models consider fixed capacities for all facilities, whereas determining capacity for facilities is often difficult in practice [5]. Regarding the issue of uncertainty, it is crucial to measure and control the negative impact of uncertainty called risk [3]. Risk measurement has an important role in controlling the uncertainty in optimization problems, especially when the losses might be incurred in finance, insurance industry or other investments [6]. The literature of SCND also suffers from lack of models able to measure and control the risk resulted from dynamic and uncertain nature of this problem.

To overcome the above-mentioned drawbacks, this paper presents a multi-stage stochastic MILP model for supply chain network design by considering Conditional Value-at-Risk (CVaR) criteria as a risk measure to assure the robustness of the concerned supply chain network.

The remainder of this paper is classified as follows. In Section 2, some relevant papers are systematically reviewed and classified. Section 3 is devoted to problem description and formulation. In Section 4, the stochastic counterpart of the proposed model is developed. After introducing CVaR criteria in Section 5, the stochastic MILP by considering CVaR criteria is developed in this Section. The experimental results are reported in Section 6. Finally, Section 7 concludes this paper and offers some directions for further research.

II. LITERATURE REVIEW

Facility location decision is one of the most important strategic decisions in supply chain management. The literature covers many efficient models (e.g. [7] and [8]) are developed for locating facilities in supply chain network design. At the same line, Kilibi et al. [3] presented a comprehensive critical review on design of robust value-creating supply chain networks under uncertainty and also their review covers optimization models, uncertainty sources, and risk exposures, evaluation criteria of SCD and assessment of SCN robustness as a necessary condition to ensure robust value creation. Melo et al. [9] presented a general review on supply chain network design to identify basic features that such models must capture to support decision-making involved in strategic supply chain planning and support a variety of future research directions. Another interesting reviews in this field can be found in Dullaert et al. [10] and Snyder and Lawrence [11] works. In this paper, we survey relevant various papers about supply chain network design problems. A large part of the literature in supply chain network design is related to forward supply chain network design aimed to determine the configuration of a directed network covering suppliers, plants, distribution centers and customer zones. Azaron et al. [12] proposed a stochastic multi-objective model for SCND under risk and uncertainty conditions. In their model Demands, supplies, processing, transportation, shortage and capacity expansion costs are all considered as the uncertain parameters.
Melachrinoudis et al. [1] developed a multiple objective model based on linear physical programming to sustain the Warehouse Network. The capacity expansion issue is ordinary in real world, because the location of facilities cannot be changed in short-term. At the same line, Thanh et al. [13] proposed a dynamic MILP model for facility location in SCND. Competitive market pressures in the recent decades have caused business organizations are looking to increase responsiveness and agility. Bachlaus et al. [2] presented an integrated multi-objective MILP model to integrate of production, distribution and supply chain activities at the strategic decision considering agility as a key design criterion. Pan and Nagi [14] developed a robust integrated model of supply chain and production costs associated with the supply chain members in agile manufacturing systems. Wang and Watada [15] presented a Value-at-Risk (VaR) based fuzzy random facility location model which both the costs and demands are assumed to be fuzzy random variables, and the capacity of each facility is unfixed but a decision variable with continuous values.

Kenne et al. [16] proposed a control model of inventories for production planning of a hybrid manufacturing-remanufacturing system under uncertainty and disruption risk. Dal-Mas et al. [17] developed a dynamic MILP model of the entire biomass-based ethanol supply chain under uncertainty on biomass production cost and product selling price.

A more detail classification of the literature is illustrated in Table 1 (See Appendix 1). The characteristics of the problem discussed in this paper are presented in the last row of Table 1.

III. PROPOSED MODEL

The considered network in this study is a multi-period, multi-echelon and single product network which is consists of productions, distributors, and customer zones. As it is illustrated in Figure 1, new products are shipped from production centers to customer zones through distribution centers in a pull system to satisfy the demand of each customer.

The main issues to be addressed by this study are to choose the location and number of distribution, production centers and determine the capacity options are added to production or distribution centers and quantity of flow between network facilities. Each production and distribution sites have an initial capacity and a limited maximal extensible capacity. For example, a production center is built on a plot of 25,000 m² at the beginning, 21,000m² are used for the building and 4,000m² remain available for possible extension. In this example, the initial capacity is 21,000m² and the maximal extensible capacity is 25,000m².

The following are the assumptions considered in the proposed model:

1. Any facility which is opened in any period will be active up to the end of the time horizon.
2. Shortage is possible and based on predefined customer service level; some demand of customers may be not satisfied.
3. For capacity expansion reasons in different periods, modifying the initial capacity of a facility by adding capacity options permissible and also, it will be active up to the end of time horizon.
4. Facilities have limited maximal installable capacity.
5. Each facility will never be run at less than lower bound percentage of its capacities (initial capacity and added capacity options).
6. Initial Capacity of each location and capacity of options which added to facilities, are known and constant for all time periods.
7. The potential locations of production and distribution centers are known.
8. The location of Customer zones are known and fixed.
9. Products can be transported with different transportation modes.
10. For each facility, a set of available capacity options is determined.
11. Fixed opening and capacity expansion costs are constant for all periods, but costs of transportation and handling differ in any period.

The following notation is used in the formulation of the proposed model.
Sets

$I$ Set of potential production center locations $i \in I$

$J$ Set of potential distribution center locations $j \in J$

$K$ Fixed locations of customer zones $k \in K$

$N$ Set of transportation’s modes $n \in N$

$F$ Set of capacity option for production $f \in F$

$V$ Set of capacity option for distribution $v \in V$

$T$ Set of time periods $t \in T$

Parameters

$d_{kt}$ Demand of customer zone $k$ in period $t$

$f_{it}$ Fixed cost of opening production center $i$

$g_{jt}$ Fixed cost of opening distribution center $j$

$HR_{jt}$ Fixed cost of adding capacity option $f$ to production center $i$

$HQ_{vj}$ Fixed cost of adding capacity option $v$ to distribution center $j$

$HK_{kt}$ Percentage of customer service level

$c_{ijnt}$ Percentage of customer service level

$\alpha_{jkt}$ Shortage amount of customer $k$ in period $t$

$\rho_{it}$ Manufacturing cost per unit of product at production center $i$ in period $t$

$\phi_{jt}$ Processing cost per unit of product at distribution center $j$ in period $t$

$\tau_{it}$ penalty cost per unit of non utilized capacity at production center $i$

$\beta_{jt}$ penalty cost per unit of non utilized capacity at distribution center $j$

$\omega_{it}$ Initial Capacity of production for production center $i$

$\psi_{jt}$ Initial Capacity of handling at distribution center $j$

$L_{kt}$ Minimum percentage of utilization of production center $j$ in period $t$

$LBD_{jt}$ Minimum percentage of utilization of distribution center $j$ in period $t$

$KA_{it}$ Maximal installable production capacity at production center $i$

$KD_j$ Maximal installable distribution capacity at distribution center $j$

$R_f$ Capacity of option $f$ for production

$Q_v$ Capacity of option $v$ for distribution

Variables

$X_{ijnt}$ Quantity of product shipped from production center $i$ to distribution center $j$ with mode $n$ in period $t$

$U_{jkn}$ Quantity of product shipped from distribution center $j$ to customer zone $k$ with mode $n$ in period $t$

$\alpha_{jkt}$ Shortage amount of customer $k$ in period $t$

$W_{it} = \begin{cases} 1 & \text{if a production center } i \text{ is opened at period } t \\ 0 & \text{Otherwise} \end{cases}$

$Y_{jt} = \begin{cases} 1 & \text{if a distribution center } j \text{ is opened at period } t \\ 0 & \text{Otherwise} \end{cases}$

$RL_{jft}$ Shortage amount of customer $k$ in period $t$

$QL_{jft}$ Shortage amount of customer $k$ in period $t$

\begin{align*}
\text{Min} & \quad \sum_{i} f_{it} W_{it} + \sum_{j} g_{jt} Y_{jt} + \sum_{f} \sum_{t} R_{f} X_{if} + \sum_{v} \sum_{t} HQ_{vf} U_{jv} + \sum_{k} \sum_{t} HK_{kt} \alpha_{jkt} \\
& \quad + \sum_{i} \sum_{t} f_{it} (W_{it} - W_{it}) + \sum_{j} g_{jt} (Y_{jt} - Y_{jt}) + \sum_{f} \sum_{t} HR_{ft} (R_{ft} - RL_{jft}) + \sum_{v} \sum_{t} HQ_{vf} (QL_{jvf} - QL_{jvf}) \\
& \quad + \sum_{i} \sum_{j} \sum_{n} (\rho_{it} + c_{ijnt}) X_{i} + \sum_{j} \sum_{k} \sum_{n} (\phi_{jt} + \alpha_{jkt}) U_{j} \\
& \quad + \sum_{i} \sum_{j} \sum_{n} \sum_{t} HK_{kt} \omega_{it} \\
& \quad + \sum_{i} \sum_{j} \sum_{n} \sum_{t} (W_{it} \omega_{it} + \sum_{j} RL_{jft} - \sum_{j} X_{ijnt}) + \sum_{j} \sum_{k} \sum_{n} (\psi_{jt} \psi_{jt} + \sum_{n} QL_{jvf} - \sum_{n} U_{jvf}) \\
\text{S.t.} & \quad \sum_{j} \sum_{n} U_{j} = d_{kt} \quad \forall k \in K, t \in T \\
& \quad sh_{kt} \geq (1 - csl) d_{kt} \quad \forall k \in K, t \in T \\
& \quad \sum_{i} \sum_{n} X_{i} = \sum_{k} \sum_{n} U_{j} = 0 \quad \forall j \in J, t \in T \\
& \quad RL_{jft} \leq W_{it} \quad \forall f, i \in I, t \in T \\
\end{align*}
\( Q_{ij} \leq y_{ij} \quad \forall v \in V, j \in J, t \in T \) \hspace{1cm} (6)

\( W_i \leq W_{t(i,t)} \quad \forall i \in I, t \in T \) \hspace{1cm} (7)

\( Y_{ij} \leq y_{ij} \quad \forall j \in J, t \in T \) \hspace{1cm} (8)

\( R_{ij} \leq R_{t(j,t)} \quad \forall f \in F, i \in I, t \in T \) \hspace{1cm} (9)

\( Q_{ij} \leq Q_{t(i,j)} \quad \forall v \in V, j \in J, t \in T \) \hspace{1cm} (10)

\[ \sum_{j \in J} X_{ij} \leq c_W W_i + \sum_{j \in J} R_{ij} \quad \forall i \in I, t \in T \] \hspace{1cm} (11)

\[ \sum_{j \in J} X_{ij} \leq c_Y Y_{ij} + \sum_{j \in J} Q_{ij} \quad \forall j \in J, t \in T \] \hspace{1cm} (12)

\[ \sum_{i \in I} X_{ij} \geq c_{LB} (c_W W_i + \sum_{j \in J} R_{ij}) \quad \forall i \in I, t \in T \] \hspace{1cm} (13)

\[ \sum_{i \in I} X_{ij} \geq c_{LB} (c_Y Y_{ij} + \sum_{j \in J} Q_{ij}) \quad \forall j \in J, t \in T \] \hspace{1cm} (14)

\[ c_W W_i + \sum_{j \in J} R_{ij} \leq K_A \quad \forall i \in I, t \in T \] \hspace{1cm} (15)

\[ c_Y Y_{ij} + \sum_{j \in J} Q_{ij} \leq K_D \quad \forall j \in J, t \in T \] \hspace{1cm} (16)

\[ W_i, Y_{ij}, Q_{ij}, R_{ij} \in [0,1] \quad \forall i \in I, j \in J, f \in F, v \in V, t \in T \] \hspace{1cm} (17)

\[ X_{ij}, U_{ij}, sh_{ij} \geq 0 \quad \forall i \in I, j \in J, k \in K, n \in N, t \in T \] \hspace{1cm} (18)

Objective function (1) minimizes the total costs includes fixed opening costs, capacity expansion costs, transportation and processing costs, shortage costs and penalty costs for non utilized capacities. Constraint (2) ensures that the demand for all customers is taken into account, either by being satisfied or by being allocated to the non-satisfied demand variable. Constraint (3) expresses the permissible shortage for each customer based on predetermined customer service level. Constraint (4) assures the flow balance at production and distribution centers. Constraints (5) and (6) cite the logical constraints. Constraints (7) and (8) ensure that when a facility is opened in any time period, it will be active for the end of time horizon. Constraints (9) and (10) ensure when a capacity option added to special facility, it will be active for the end of time horizon. Constraints (11) to (12) are initial capacity plus capacity expansions constraints on production and distribution centers. Constraints (13) to (14) express the utilization of lower bound percentage of all capacities of a facility. Equations (15) and (16) ensure the limited maximal installable capacity on production and distribution centers. Finally, Constraints (17) and (18) enforce the binary and non-negativity restrictions on corresponding decision variables. The resulting model is a MILP model with \((JNT+JKN+K)\) continuous variables and \((I+J+V+J+FI)\) binary variables. The number of constraints is \((6I + 7J + 2K + 2V + 2F + 15T)\), excluding constraints (17) and (18).

To deal with uncertainty in demand customers and also to configure a robust supply chain network a stochastic optimization model is developed in the next Section according to the proposed deterministic model.

IV. MULTI-STAGE STOCHASTIC LINEAR PROGRAMMING WITH RECURSIVE

Stochastic linear programming with recourse is a dynamic decision model with \(T \geq 2\) stages where a decision making in any period depend on previous periods and relevant scenarios. As it is shown in Figure 2, a first stage decision (i.e. \(x_1\)) is taken after observing the realization of a random variable \(\Omega_2\), a second stage decision \(x_2(\Omega_2)\) is taken after observing the realization of a random variable \(\Omega_3\) and thus the T-th stage decision is \(x_T(\Omega_2, \Omega_3, \ldots, \Omega_T)\) [18].

In other words, any decision in any period depends on the decisions that are taken based on the realization of the scenarios in previous periods.

In the following to work more convenient the compact form of the proposed deterministic model is presented:

\[
\text{Min} \sum_{i} f(y_i - y_{i-1}) + h(s_i - s_{i-1}) + c_ix_i
\]

s.t. \(Ax_i \geq d_i\)

\(Bx_i \leq C(y_i + s_i)\)

\(Bx_i \leq L(C(y_i + s_i))\)

\(M(y_i + s_i) \leq K\)

\(s_i \leq y_i\)

\(y_i, s_i \in [0,1], \quad x_i \in R^+\)

In the above compact form, \(f\) and \(h\) correspond to fixed opening and capacity expansion costs, respectively. \(C\) corresponds to transportation, processing, penalty and shortage costs. The matrices \(A, B, M\) and \(N\) are coefficient matrices of the constraints. \(K\) and \(L\) are the scalar in the related constraints. All \(y\) and \(s\) are the binary decision variables for the opening and adding capacity, respectively and all the continuous decision variables include into vector \(x\). Let \(\Omega\) be the set of all possible scenarios, \(\theta\) a particular scenario and \(\pi_{\theta}\) probability of occurrence scenario\(^\theta\) in period \(t\) because \(\theta\) is a finite number (number of scenarios) the expected value function become a summation on \(\theta\). The scenario-based multi-stage stochastic model for all scenarios has been introduced under demand uncertainty in the follow:

According to above explanations, the detailed stochastic model under demand uncertainty is defined in Appendix 2. In the detailed stochastic model, new sets, variables and parameters are defined as follow and so, others as the same those are defined in Section 3.

Fig. 2. Structure of Dynamic Decision
Min \( \sum_{i} (f(y_{i} - y_{i-1}) + h(s_{i} - s_{i-1}) + \sum_{\theta} \pi_{\theta} c_{\theta} x_{\theta}) \)

s.t. \( Ax_{\theta} \geq d_{\theta} \)
\( N x_{\theta} = 0 \)
\( B x_{\theta} \leq C (y_{i} + s_{i}) \)
\( B x_{\theta} \geq L (C (y_{i} + s_{i})) \)
\( M (y_{i} + s_{i}) \leq K \)
\( s_{i} \leq y_{i}, s_{i} \in \{0,1\}, \quad x_{\theta} \in R^{+} \)

Sets

\( \Omega \) Set of potential scenarios \( \theta \in \Omega \)
\( d_{2t} \) Demand of customer zone \( k \) in period \( t \) for scenario \( \theta \)
\( \pi_{\theta} \) Probability of scenario \( \theta \) in period \( t \)
\( X_{ij\theta} \) Quantity of scenario \( \theta \) from production center \( i \) to distribution center \( j \) with mode \( n \) in period \( t \) for scenario \( \theta \)
\( U_{ij\theta} \) Quantity of product shipped from distribution center \( j \) to customer zone \( k \) with mode \( n \) in period \( t \) for scenario \( \theta \)

It is worthy to note that the solution of the stochastic linear programming model is not optimal in general for the individual scenarios in different periods [19]. The probability of occurrence of the special scenario in any period is assigned based on its importance and the future periods and related scenarios are a schema of a future situation and the course of the events that enables one to progress from the original situation to the future situation.

V. CONDITIONAL VALUE-AT-RISK (CVaR)

In the following we refer the main general concept of CVaR from [6] and [20], so for more detail CVaR concept, interested readers are referred to them.

In recent years the attention towards coherent risk measurement has increased rapidly. Risk measures such as the VaR and CVaR can be countered by the stochastic optimization models. Considering the less favorable outcomes issue, the CVaR measures the risk of an investment in a conservative way. Value-at-risk (VaR), summarily, is a popular measure of risk which used in many of financial firms and Using of the VaR is ordinary in formal organizations. The VaR is the \( \alpha \) -percentile of the loss distribution or “\( \alpha \) -VaR” is a smallest value such that probability that loss exceeds or equals to this value is not larger than \( 1-\alpha \)”. For instance, 90%-VaR is an upper estimation of losses which is exceeded with 10% probability. For more detailed concepts of the VaR, the interested readers are referred to [21]. Furthermore in the scenario-based stochastic models, optimization of the VaR is difficult for the discrete distribution. \( \alpha \) -conditional Value-at-Risk (\( \alpha \)-CVaR) is the minimizing of “the expected value of the costs in the (1- \( \alpha \) )100% worst cases” [22].

where \( \alpha \in (0,1) \) is a confidence level and predetermined value.

The CVaR is a superior than the VaR. Because, it is able to quantify losses more efficiently than the discrete distributions, and moreover it is coherent against of the VaR.

In addition, optimization of the CVaR leads to optimize of the VaR and also Linear programming approaches can be used for minimization of the CVaR. And many large of scenarios can be handled with relatively small computational resources.

In the remaining of this Section, We introduce only the scenario-based calculation of the CVaR which is adapted to our work by following the [6] and [20].

We assume that positive values of \( f(x,\omega) \) represent losses. Assume \( \omega \) has a finite discrete distribution with \( N \) realizations and corresponding probabilities given as \( \pi_{\theta} \) for \( \omega_{\theta}, \theta = 1,...,N \), ( \( \theta \) is representative a individual scenario)

where \( \pi_{\theta} > 0 \forall \theta \) and \( \sum_{\theta=1}^{N} \pi_{\theta} = 1 \).

For \( f(x,\omega) \), the \( \alpha \) -CVaR can be stated by the following minimization formula:

\[
F_{\alpha} (x,\eta) = \eta + \frac{1}{1-\alpha} E \left[ (f(x,\omega) - \eta)^+ \right]
\]

Where,
\[
(f(x,\omega) - \eta)^+ = \max \{ f(x,\omega) - \eta, 0 \}
\]

Let \( \alpha \) -CVaR for loss random variable \( f(x,\omega) \) is denoted by \( \psi_{\alpha} (x) \), so, the \( \alpha \) -CVaR equation can be restated as follows:

\[
\psi_{\alpha} (x) = \min \left\{ \eta + \frac{1}{1-\alpha} E \left[ \max \{ f(x,\omega) - \eta, 0 \} \right] \right\}
\]

The above model is non-linear programming problem. And so, By considering additional variables \( z_{\theta} \) for representing \( \max \{ f(x,\omega) - \eta, 0 \} \) for all \( \theta = 1,...,N \) this non-linear equation can be transformed into a linear programming problem [18].

\[
\psi_{\alpha} (x) = \min \left\{ \eta + \frac{1}{1-\alpha} \sum_{\theta=1}^{N} \pi_{\theta} z_{\theta} \right\}
\]

s.t.
\[
f(x,\omega_{\theta}) - \eta - z_{\theta} \leq 0 \quad \forall \theta,
\]
\[
z_{\theta} \geq 0 \quad \forall \theta.
\]

It is should be noted that in the optimum solution \( \eta^{*} \) is corresponding to \( \alpha \)-VaR. Thus, by the above model optimal
values of both the CVaR and VaR can be achieved. Let the same assumptions and definitions in Section 4 and this Section, the compact model for the multi-stage stochastic linear programming (SLP) model by considering CVaR criteria in $\alpha$ confidence level can be stated as follow:

\[
\text{Min} \sum_{t} \left( f(y_t - y_{t-1}) + h(s_t - s_{t-1}) \right) + \sum_{t} \sum_{\nu} \pi_{\nu t} c_{\nu t} x_{\nu t} + \zeta (\eta + \frac{1}{1-\alpha} \sum_{t} \sum_{\nu} \pi_{\nu t} z_{\nu t})
\]

s.t. $Ax_{\nu t} \geq d_{\nu t}$

$Nx_{\nu t} = 0$

$Bx_{\nu t} \leq L(C(y_t + s_t))$

$M(y_t + s_t) \leq K$

$s_t \leq y_t$

$f(y_t - y_{t-1}) + h_1(s_t - s_{t-1}) + \pi_{\nu t} c_{\nu t} x_{\nu t} - \eta - z_{\nu t} \leq 0 \quad \forall \theta, t$

$y_t, s_t \in [0,1]^t$, $x_{\nu t}, z_{\nu t} \in \mathbb{R}^+$

$\zeta$ is the weighting factor for the risk measure and we assume that $\zeta = 1$ in all computations which is represented in the next Section.

According to above description, the detailed SLP model by considering CVaR criteria in $\alpha$ confidence level is defined in Appendix 3.

VI. COMPUTATIONAL EXPERIMENTS

In this section, to assess the performance of the proposed model two test problems are selected. The source of random generation nominal data for deterministic parameters is shown in Table 2. Each problem includes three periods and each period consists of four scenarios. The scenarios for the two test problems in different periods for uncertain demand are described in Table 3. It is should be noted that we assume the probability of occurrence a special scenario in particular period is independent from earlier periods and their scenarios. Test problems are solved with LINGO 8.0 on a Pentium dual-core 2.66 GHZ computer with 4 GB RAM. First scenario in each period of each problem that has a higher probability has been considered as nominal data for deterministic model.

**Table 2: The source of random generation data**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_i$</td>
<td>$\sim \text{Unif}[4500000, 4900000]$</td>
<td>$\varphi_j$</td>
<td>$\sim \text{Unif}[25,35]$</td>
</tr>
<tr>
<td>$g_i$</td>
<td>$\sim \text{Unif}[160000, 200000]$</td>
<td>$\tau_j$</td>
<td>$\sim \text{Unif}[2000,2500]$</td>
</tr>
<tr>
<td>$HR_{fi}$</td>
<td>$\sim \text{Unif}[550000, 750000]$</td>
<td>$\beta_j$</td>
<td>$\sim \text{Unif}[1500,2000]$</td>
</tr>
<tr>
<td>$HQ_{ij}$</td>
<td>$\sim \text{Unif}[30000, 40000]$</td>
<td>$cw_i$</td>
<td>$\sim \text{Unif}[900,1100]$</td>
</tr>
<tr>
<td>$HK_{ij}$</td>
<td>$\sim \text{Unif}[1000,1200]$</td>
<td>$cy_j$</td>
<td>$\sim \text{Unif}[1500,2000]$</td>
</tr>
<tr>
<td>$c_{\text{var}i\text{stat}}$</td>
<td>$\sim \text{Unif}[40,55]$</td>
<td>$LB_i$</td>
<td>$\sim \text{Unif}[35%,45%]$</td>
</tr>
<tr>
<td>$\rho_j$</td>
<td>$\sim \text{Unif}[250,350]$</td>
<td>$LBD_j$</td>
<td>$\sim \text{Unif}[30%,40%]$</td>
</tr>
<tr>
<td>$KA_i$</td>
<td>$\sim \text{Unif}[1100,1300]$</td>
<td>$R_f$</td>
<td>$\sim \text{Unif}[100,200]$</td>
</tr>
<tr>
<td>$KD_j$</td>
<td>$\sim \text{Unif}[2200,2500]$</td>
<td>$Q_f$</td>
<td>$\sim \text{Unif}[200,400]$</td>
</tr>
</tbody>
</table>

**Table 3: Fluctuation of demand for two test problems**

<table>
<thead>
<tr>
<th>Problem size</th>
<th>Scenarios (\Omega)</th>
<th>T=1</th>
<th>T=2</th>
<th>T=3</th>
</tr>
</thead>
<tbody>
<tr>
<td>I<em>J</em>K<em>N</em>F<em>V</em>T</td>
<td>probability demand</td>
<td>probability demand</td>
<td>probability demand</td>
<td></td>
</tr>
<tr>
<td>10<em>15</em>20<em>2</em>3<em>3</em>3*3</td>
<td>1</td>
<td>0.4</td>
<td>$\sim \text{Unif}[350,450]$</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>$\sim \text{Unif}[360,460]$</td>
<td>0.2</td>
<td>$\sim \text{Unif}[450,550]$</td>
</tr>
<tr>
<td>3</td>
<td>0.3</td>
<td>$\sim \text{Unif}[400,500]$</td>
<td>0.15</td>
<td>$\sim \text{Unif}[460,530]$</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>$\sim \text{Unif}[420,520]$</td>
<td>0.15</td>
<td>$\sim \text{Unif}[440,540]$</td>
</tr>
<tr>
<td>10<em>10</em>15<em>2</em>3<em>3</em>3*3</td>
<td>1</td>
<td>0.5</td>
<td>$\sim \text{Unif}[200,280]$</td>
<td>0.45</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>$\sim \text{Unif}[320,440]$</td>
<td>0.25</td>
<td>$\sim \text{Unif}[350,480]$</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
<td>$\sim \text{Unif}[300,420]$</td>
<td>0.2</td>
<td>$\sim \text{Unif}[410,500]$</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>$\sim \text{Unif}[260,360]$</td>
<td>0.1</td>
<td>$\sim \text{Unif}[280,380]$</td>
</tr>
</tbody>
</table>
In the stochastic models by increasing the number of scenarios significantly increases the computational time with limited benefit in solution accuracy [23]. Our experiments on the proposed stochastic model by considering CVaR criteria also show the accuracy of this claim.

As it is shown in Table 4 stochastic model results in a higher objective function value compared with deterministic model. In addition the number of variables and constraints for the two models shows the higher degree of complexity of the stochastic model. Also, the stochastic model by considering CVaR criteria has a higher objective function value and higher degree of complexity respect to stochastic model without CVaR criteria. Share of different type of costs in objective function and optimum value of risk measures in 95% confidence level is depicted in Table 5.

It is should be noted that the stochastic model by considering CVaR criteria results the same solution compared with stochastic model without CVaR criteria in optimum solutions, exclude objective function value. As be mentioned in Section 5, the CVaR measure has a greater or equal value respect to VaR measure. The results acquired for risk measures in Table 5 confirm this judgment. The result for the VaR in test problem 1 shows that the smallest value such that probability that loss exceeds or equals to this value (39343606) is not larger than 1-α (5%). Because of discrete distribution of scenarios and little number of scenarios, in high confidence level the VaR and CVaR values lead to equal values; however, in less confidence level the difference of two these risk measures is appeared (See Table 9). In both two problems, Table 5 show the higher share of fixed costs in stochastic model compared with deterministic one. This difference is because of that the SLP model pays to avoid infeasibility or assure feasibility under all scenarios. To correct initial capacity of facilities with reasonable cost, deterministic model perform capacity expansion more than stochastic in test problem 1 against to test problem 2. In Table 6, the share of higher considerable costs in period one shows that the most of facilities are opened in this period or perform capacity expansion. Table 7 compares the binary variables of stochastic and deterministic model. In this table the asterisk (*) show that the opened facilities or added capacities are active in the subsequent periods. Also Wi=(3,5,6,7,8) express that the opened facilities or added capacities are active in the subsequent periods. Also Wi=(3,5,6,7,8) express that the opened facilities or added capacities are active in the subsequent periods.

<table>
<thead>
<tr>
<th>Problem size</th>
<th>Optimal value of objective function</th>
<th>Number of variables</th>
<th>Number of constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stochastic</td>
<td>Deterministic</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I<em>J</em>K<em>N</em>F<em>V</em> T</td>
<td>I<em>J</em>K<em>N</em>F<em>V</em> T</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10<em>10</em>15<em>2</em>3</td>
<td>41642580</td>
<td>3000</td>
<td>956</td>
</tr>
<tr>
<td>3*3</td>
<td>48342170</td>
<td>11,118</td>
<td>1,596</td>
</tr>
<tr>
<td>87685770</td>
<td>11,126</td>
<td>1,585</td>
<td>1,256</td>
</tr>
<tr>
<td>95%</td>
<td>796</td>
<td></td>
<td></td>
</tr>
<tr>
<td>95%</td>
<td>2537631</td>
<td></td>
<td></td>
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</table>

In Table 5 share of different type of costs in objective function and risk measure values in 95% confidence level is depicted.

<table>
<thead>
<tr>
<th>Problem size</th>
<th>Optimal value of Risk Measure</th>
<th>value of fixed opening costs</th>
<th>value of transportation and processing costs</th>
<th>value of non utilized capacity penalty costs</th>
<th>value of capacity expansion costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>VaR</td>
<td>Deterministic</td>
<td>Stochastic</td>
<td>Deterministic</td>
<td>Stochastic</td>
<td>Deterministic</td>
</tr>
<tr>
<td>10<em>10</em>15<em>2</em>3</td>
<td>39343606</td>
<td>23838320</td>
<td>9539353</td>
<td>2326634</td>
<td>0</td>
</tr>
<tr>
<td>3<em>3</em>3*3</td>
<td>33491220</td>
<td>240.4</td>
<td>7274.12</td>
<td>1224811</td>
<td>2537631</td>
</tr>
<tr>
<td>24606271</td>
<td>19092780</td>
<td>2740.5</td>
<td>1747</td>
<td>29620760</td>
<td>48342170</td>
</tr>
<tr>
<td>19159060</td>
<td>54227030</td>
<td>5939353</td>
<td>4078.5</td>
<td>38620950</td>
<td>4932748</td>
</tr>
<tr>
<td>5265723</td>
<td>9553763</td>
<td>240.4</td>
<td>7274.12</td>
<td>1224811</td>
<td>2537631</td>
</tr>
<tr>
<td>54227030</td>
<td>9553763</td>
<td>240.4</td>
<td>7274.12</td>
<td>1224811</td>
<td>2537631</td>
</tr>
<tr>
<td>5265723</td>
<td>9553763</td>
<td>240.4</td>
<td>7274.12</td>
<td>1224811</td>
<td>2537631</td>
</tr>
<tr>
<td>139677</td>
<td>240.4</td>
<td>7274.12</td>
<td>1224811</td>
<td>2537631</td>
<td></td>
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<tr>
<td>1895389</td>
<td>240.4</td>
<td>7274.12</td>
<td>1224811</td>
<td>2537631</td>
<td></td>
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<td>1895389</td>
<td>240.4</td>
<td>7274.12</td>
<td>1224811</td>
<td>2537631</td>
<td></td>
</tr>
<tr>
<td>2426319</td>
<td>240.4</td>
<td>7274.12</td>
<td>1224811</td>
<td>2537631</td>
<td></td>
</tr>
</tbody>
</table>
To assess the performance of deterministic and stochastic models under each scenario, at the first, the models were solved by Lingo 8.0. Then, the solutions of the two models were obtained under realization of each scenario in any period by allowing the models to update their continuous decision variables with fixed binary variables which are acquired from the first step solution for all scenarios. Because of this, the solution is not optimal in realization of the individual scenarios. In realization of scenarios, to apply concept of multi-stage stochastic linear programming with recourse, we assume that occurrence of the individual scenario in the next periods is depend on the scenarios which were happened in prior periods. And also, it is assumed that the scenario with maximum likelihood was happened in previous period. The results described in Table 8 show the efficiency of multi-stage stochastic model in handling data uncertainty. Although, the objective function value of deterministic model is better than stochastic one; however, it is unable to handle the data uncertainty and lead to infeasibility in many scenarios.

In addition, in cases that two models are feasible, stochastic model has shortage cost much less than deterministic model (in 70% customer service level i.e. csl=0.7). So, the stochastic model has high degree of response which, nowadays it is very important in supply chain successful. measure is more conservative than the VaR measure, we performed sensitivity analysis on confidence levels. Since the CVaR is the expected losses exceeding the VaR, so, as be shown in Table 9 in certain confidence level, the CVaR is larger or equal to the VaR. Also, obviously, by increasing in confidence level the VaR and CVaR measures increase respect to measures with less confidence level. The results in Table 9 confirm this idea. From Table 9 results,

<table>
<thead>
<tr>
<th>Problem Size</th>
<th>Deterministic</th>
<th>Stochastic</th>
</tr>
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<tbody>
<tr>
<td>T=1</td>
<td>T=2</td>
<td>T=3</td>
</tr>
<tr>
<td>W=(1,3,5,6,7,8)</td>
<td>W=(1,3,5,6,7,8,9)</td>
<td>W=(3,5,6,7,8)</td>
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<tr>
<td>Y=(5,10,12)</td>
<td>Y=(3,12,14,15)</td>
<td>Y=(5,10,12)</td>
</tr>
<tr>
<td>QL(2,5)</td>
<td>QL(1,3)</td>
<td>QL(2,5)</td>
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<td>QL(3,5)</td>
<td>QL(3,14)</td>
<td>QL(3,5)</td>
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<td>QL(3,12)</td>
<td>QL(3,12)</td>
<td>QL(3,12)</td>
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<tr>
<td>RL(1,7)</td>
<td>RL(2,7)</td>
<td>RL(1,7)</td>
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<tr>
<td>RL(2,7)</td>
<td>RL(3,6)</td>
<td>RL(2,7)</td>
</tr>
<tr>
<td>RL(3,3)</td>
<td>RL(3,3)</td>
<td>RL(3,3)</td>
</tr>
<tr>
<td>RL(3,5)</td>
<td>RL(3,5)</td>
<td>RL(3,5)</td>
</tr>
<tr>
<td>RL(3,6)</td>
<td>RL(3,6)</td>
<td>RL(3,6)</td>
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</table>
### Table 8: Computational Results under Scenarios in Different Periods

<table>
<thead>
<tr>
<th>Problem size</th>
<th>Period</th>
<th>Scenario</th>
<th>Scenario probability (π(θ,t))</th>
<th>Value of objective function</th>
<th>Shortage costs</th>
</tr>
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<tr>
<td>10^15<em>20^2</em>3^3*3</td>
<td>1</td>
<td>1</td>
<td>0.4</td>
<td>31405110</td>
<td>2021985</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>0.2</td>
<td>31831210</td>
<td>2484080</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>0.3</td>
<td>Infeasible</td>
<td>1255989</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>0.1</td>
<td>Infeasible</td>
<td>1939027</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1</td>
<td>0.5</td>
<td>34636730</td>
<td>2738930</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>0.2</td>
<td>Infeasible</td>
<td>2766337</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>0.15</td>
<td>Infeasible</td>
<td>2713844</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>0.15</td>
<td>Infeasible</td>
<td>2363012</td>
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<tr>
<td></td>
<td>3</td>
<td>1</td>
<td>0.55</td>
<td>36857760</td>
<td>2427204</td>
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<tr>
<td></td>
<td></td>
<td>2</td>
<td>0.15</td>
<td>Infeasible</td>
<td>2320597</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>0.2</td>
<td>Infeasible</td>
<td>2788702</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>0.1</td>
<td>Infeasible</td>
<td>3474195</td>
</tr>
</tbody>
</table>

### Table 9: Optimal Value of Risk Measures for Different Periods

<table>
<thead>
<tr>
<th>Problem size</th>
<th>Period</th>
<th>Optimal value of Risk Measure under different confidence levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^15<em>20^2</em>3^3*3</td>
<td>1</td>
<td>α = 0.60 — 0.99, 0.55, 0.50, 0.45, 0.30, 0.20, 0.10, 0.05, 0.01</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>VaR(t), CVaR(t), VaR(t), CVaR(t), VaR(t), CVaR(t), VaR(t), CVaR(t)</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>Total</td>
</tr>
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</table>

**International Journal of Mechanic Systems Engineering (IJMSE)**
It can be concluded the CVaR measure is more conservative than VaR measure and so, it is more suitable for risk-averse organizations.

VII. CONCLUSIONS

In this paper we present a scenario-based multi-stage stochastic optimization model by considering CVaR criteria. Motivated from shortcomings in the literature, our model considers a dynamic MILP model for facility location with capacity expansion and different transportation modes in supply chain network design. In the proposed model demand of customers assumed to be uncertain. An efficient stochastic model is then developed based on the deterministic MILP model. Then, the multi-stage stochastic model is also extended by using the CVaR measure to control the risk level. Finally, the performance and behavior of the proposed models are investigated through numerical experiments. Computational results show the strength of stochastic model in handling data uncertainty and controlling risk level under uncertainty. Also, the results show that the deterministic model unable to handle uncertainty and has weaker response in dealing with shortage. So, the proposed stochastic model not only can be used as a powerful tool in practice cases but also can be used to control risk level of investment by using CVaR criteria.

Future research direction can be included the following statements.

Addressing multi-product or considering maximizing responsiveness, especially in agile supply chain (e.g. [25]), in uncertain condition that leads to multi-objective stochastic problems may be an attractive research direction. Also, considering other risk measure such as minimum variance (proposed by Markowitz, [26]) and comparing the performance with the CVaR and VaR measures can be considered for future research. Since the computational time increases significantly when the size of problem and the number of scenarios increased; developing an efficient solution algorithm in large cases can be covered in the future.

ACKNOWLEDGEMENT

The authors thank the anonymous referee for his/her valuable and constructive suggestions on this work, which improved the content substantially.

REFERENCES


APPENDIX 1

<table>
<thead>
<tr>
<th>Reference articles</th>
<th>Problem definition</th>
<th>Modeling</th>
<th>Objective function</th>
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<td>●</td>
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<td>●</td>
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<tr>
<td>Our work</td>
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</tr>
</tbody>
</table>

APPENDIX 2

\[
\text{Min } \sum_{i \in I} f_i W_i + \sum_{j \in J} c_j Y_j + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \sum_{n \in N} H_{ik} R_{ij} L_{jn} + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \sum_{n \in N} H_{ik} O_{ij} Q_{jn} \\
+ \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \sum_{n \in N} \beta_{ik} \zeta_{ij} + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \sum_{n \in N} \eta_{ij} U_{kn} \\
+ \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \sum_{n \in N} \alpha_{ik} \phi_{ij} + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \sum_{n \in N} \varphi_{ij} U_{kn} \\
+ \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \sum_{n \in N} \gamma_{ik} \theta_{ij} + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \sum_{n \in N} \theta_{ij} U_{kn} \\
+ \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \sum_{n \in N} \delta_{ik} \zeta_{ij} + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \sum_{n \in N} \zeta_{ij} U_{kn} \\
+ \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \sum_{n \in N} \epsilon_{ik} \zeta_{ij} + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \sum_{n \in N} \zeta_{ij} U_{kn} \\
+ \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \sum_{n \in N} \zeta_{ik} \zeta_{ij} + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \sum_{n \in N} \zeta_{ij} U_{kn} \\
+ \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \sum_{n \in N} \zeta_{ik} \zeta_{ij} + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \sum_{n \in N} \zeta_{ij} U_{kn} \\
\]

S.t.

\[
(19)
\]
\[
\begin{align*}
\sum \sum_{j=1}^{n} U_{j,k} + sh_{t,\theta} & \geq d_{t,\theta} \quad \forall k \in K, t \in T, \theta \in \Omega \\
sh_{t,\theta} & \leq (1-cs) d_{t,\theta} \quad \forall k \in K, t \in T, \theta \in \Omega \\
\sum \sum_{i=1}^{m} X_{i,t} - \sum \sum_{i=1}^{m} U_{i,t} & = 0 \quad \forall j \in J, t \in T, \theta \in \Omega \\
RL_{j,t} & \leq W_{j,t} \quad \forall f \in F, i,l \in I, t \in T \\
QL_{j,t} & \leq Y_{j,t} \quad \forall v \in V, j \in J, t \in T \\
W_{i,t} & \leq W_{i,t+1} \quad \forall i \in I, t \in T \\
Y_{j,t} & \leq Y_{j,t+1} \quad \forall j \in J, t \in T \\
RL_{j,t} & \leq RL_{j,t+1} \quad \forall f \in F, i \in I, t \in T \\
QL_{j,t} & \leq QL_{j,t+1} \quad \forall v \in V, j \in J, t \in T \\
\sum \sum_{i=1}^{m} X_{i,t} & \leq cw_i W_{i,t} + \sum \sum_{j=1}^{n} R_{j,t} RL_{j,t} \quad \forall i \in I, t \in T, \theta \in \Omega \\
\sum \sum_{i=1}^{m} X_{i,t} & \leq cy_j Y_{j,t} + \sum \sum_{v \in V} Q_{i,t} QL_{i,t} \quad \forall j \in J, t \in T, \theta \in \Omega \\
\sum \sum_{i=1}^{m} X_{i,t} & \geq LB_c (cw_i W_{i,t} + \sum \sum_{j=1}^{n} R_{j,t} RL_{j,t}) \quad \forall i \in I, t \in T, \theta \in \Omega \\
\sum \sum_{i=1}^{m} X_{i,t} & \geq LBD_c (cy_j Y_{j,t} + \sum \sum_{v \in V} Q_{i,t} QL_{i,t}) \quad \forall j \in J, t \in T, \theta \in \Omega \\
cw_i W_{i,t} + \sum \sum_{j=1}^{n} R_{j,t} RL_{j,t} & \leq KA \quad \forall i \in I, t \in T \\
cy_j Y_{j,t} + \sum \sum_{v \in V} Q_{i,t} QL_{i,t} & \leq KD \quad \forall j \in J, t \in T \\
W_{i,t} \cdot Y_{j,t} \cdot RL_{j,t} & \in \{0,1\} \quad \forall i \in I, j \in J, f \in F, v \in V, t \in T \\
\sum \sum_{i=1}^{m} X_{i,t} & \geq 0 \quad \forall i \in I, j \in J, k \in K, n \in N, t \in T, \theta \in \Omega
\end{align*}
\]

**APPENDIX 3**

\[
\text{Min} \quad \sum \sum_{i=1}^{m} f_i W_{i,t} + \sum \sum_{j=1}^{n} g_j Y_{j,t} + \sum \sum_{j=1}^{n} HR_j RL_{j,t} + \sum \sum_{j=1}^{n} HQ_{t,j} QL_{i,t,j} \\
+ \sum \sum_{i=1}^{m} f_i (W_{i,t+1} - W_{i,t}) + \sum \sum_{j=1}^{n} g_j (Y_{j,t+1} - Y_{j,t}) \\
+ \sum \sum_{j=1}^{n} HR_j (RL_{j,t+1} - RL_{j,t}) + \sum \sum_{j=1}^{n} HQ_{t,j} (QL_{j,t+1} - QL_{j,t}) \\
+ \sum \sum_{t=1}^{T} \sum \sum_{k=1}^{K} \sum \sum_{\theta=1}^{\Omega} \pi_k \ (\rho_i + c_{i,t}) X_{i,\theta} + \sum \sum_{j=1}^{n} \sum \sum_{k=1}^{K} \sum \sum_{\theta=1}^{\Omega} \pi_k (\phi_j + a_{j,\theta}) U_{j,\theta} \\
+ \sum \sum_{t=1}^{T} \sum \sum_{k=1}^{K} \sum \sum_{\theta=1}^{\Omega} \pi_k \ HK_{i,k,\theta} sh_{t,\theta} \\
+ \sum \sum_{i=1}^{m} \sum \sum_{j=1}^{n} \sum \sum_{k=1}^{K} \sum \sum_{\theta=1}^{\Omega} \pi_k \ (W_{i,t} \cdot cw_i + \sum \sum_{j=1}^{n} RL_{j,t} R_j - \sum \sum_{j=1}^{n} X_{i,\theta} ) \\
+ \sum \sum_{i=1}^{m} \sum \sum_{j=1}^{n} \sum \sum_{k=1}^{K} \sum \sum_{\theta=1}^{\Omega} \pi_k \ (Y_{j,t} \cdot cy_j + \sum \sum_{v \in V} QL_{j,t} Q_i - \sum \sum_{j=1}^{n} U_{j,\theta}) \\
+ \zeta(\eta) + \frac{1}{1-\alpha} \sum \sum_{i=1}^{m} \sum \sum_{k=1}^{K} \sum \sum_{\theta=1}^{\Omega} \pi_k \ z_{i,\theta}
\]

\[ \sum_{i=1}^{n} \sum_{k=1}^{m} X_{ijk} \leq \sum_{i=1}^{n} \sum_{k=1}^{m} X_{ijk} - \sum_{i=1}^{n} \sum_{k=1}^{m} U_{ijk} = 0 \quad \forall j \in J, t \in T, \theta \in \Omega \] (40)

\[ RL_{jt} \leq W_a \quad \forall f \in F, i \in I, t \in T \] (41)

\[ QL_{jt} \leq Y_a \quad \forall v \in V, j \in J, t \in T \] (42)

\[ W_a \leq W_{a(i+1)} \quad \forall i \in I, t \in T \] (43)

\[ Y_a \leq Y_{a(j+1)} \quad \forall j \in J, t \in T \] (44)

\[ RL_{jt} \leq RL_{jt(j+1)} \quad \forall f \in F, i \in I, t \in T \] (45)

\[ QL_{jt} \leq QL_{jt(j+1)} \quad \forall v \in V, j \in J, t \in T \] (46)

\[ \sum_{i=1}^{n} \sum_{k=1}^{m} X_{ijk} \leq cW_a + \sum_{j=1}^{m} R_j RL_{jt} \quad \forall i \in I, t \in T, \theta \in \Omega \] (47)

\[ \sum_{i=1}^{n} \sum_{k=1}^{m} X_{ijk} \leq cy Y_a + \sum_{v=1}^{N} QL_{vtj} \quad \forall j \in J, t \in T, \theta \in \Omega \] (48)

\[ \sum_{i=1}^{n} \sum_{k=1}^{m} X_{ijk} \geq LB_a (cw W_a + \sum_{j=1}^{m} R_j RL_{jt}) \quad \forall i \in I, t \in T, \theta \in \Omega \] (49)

\[ \sum_{i=1}^{n} \sum_{k=1}^{m} X_{ijk} \geq LBD_0 (cw Y_a + \sum_{v=1}^{N} QL_{vtj}) \quad \forall j \in J, t \in T, \theta \in \Omega \] (50)

\[ cw W_a + \sum_{j=1}^{m} R_j RL_{jt} \leq K_0 \quad \forall i \in I, t \in T \] (51)

\[ cy Y_a + \sum_{v=1}^{N} QL_{vtj} \leq K_0 \quad \forall j \in J, t \in T \] (52)

\[ \sum_{i=1}^{n} \sum_{j=1}^{m} f_i W_{a(i+1)} - W_a + \sum_{j=1}^{m} g_j Y_{a(j+1)} - Y_a \] (53)

\[ \sum_{j=1}^{m} \sum_{j=1}^{m} HR_{jt} RL_{jt(j+1)} + \sum_{v=1}^{N} HQ_{jtj} QL_{vtj} + \sum_{i=1}^{n} \sum_{k=1}^{m} H_{ikt} \pi_{ikt} \] (54)

\[ \sum_{j=1}^{m} \sum_{j=1}^{m} \beta_i \pi_{ikt} (Y_a + c_{ijkl} X_{i,j,k} + \sum_{j=1}^{m} \sum_{k=1}^{m} \sum_{l=1}^{n} \pi_{ikt} (\phi_j + \alpha_{ijkl}) U_{ijkl}) \] (55)