A Modified Least Mean Square Algorithm for Adaptive Frequency Estimation in Power Systems

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Abstract—This research presents a method for frequency estimation in power systems using an adaptive filter based on the Least Mean Square Algorithm (LMS). In order to analyze a power system, three-phase voltages were converted into a complex signal applying \(\alpha\beta\)-Transform and its result was used in the adaptive filtering algorithm. The computing simulations were carried out using the ATP software. Four different situations were created for the performance analysis of the proposed methodology. The results were compared to a commercial relay for validation, showing the advantages of the new method.

I. INTRODUCTION

In order to protect Electrical Power Systems (EPS), a precise detection of faulty or abnormal situations is necessary to eliminate them and consequently to return to the normal operation condition as soon as possible. Taking this into account, protective relays constantly monitor the three-phase voltage and current signals, including their frequency.

The frequency is one of the main parameters to be monitored in an EPS, as during a faulty or undesired situation, it will experience significant alterations. In practice, a frequency variation range for the system operation is established as \(60 \pm 0.5\) Hz [1]. Variations on these limits are constantly observed as a consequence of the dynamic unbalance between the generation and load. However, larger variations may indicate faulty situations, as well as a system overload. Considering the latter, the frequency relay can help in the load shedding decision [2].

The importance of a correct frequency estimation for EPS is then observed, especially if the established limits for its normal operation are not reached. This can cause serious problems for the equipment connected to the utility, such as capacitor banks, generators and transmission lines, affecting the power balance. Therefore, frequency relays are widely used in the system to detect power oscillations outside the acceptable operation levels of the EPS.

It must also be remembered that taking into consideration the technological advances and a considerable increase in the use of electronic devices throughout the last decades, the concern of the frequency variation in EPS has been intensified. This is because these modern components are more sensitive to this kind of variation in frequency.

Taking this into account, the study of new techniques for a better and faster estimation of the system frequency has become extremely important for a power system operation. Having this in mind, several techniques were proposed, mainly based on the phasor estimation, using the Least Mean Square (LMS) Method, the Fast Fourier Transform - FFT, intelligent techniques and the Kalman Filter [3]–[7]. The adaptive filter based on LMS proposed by Pradkan [8] should be outlined. The LMS was first introduced by Widrow and Holf [9] for digital signal processing and has been widely used since because of its simplified structure, efficiency and computing robustness.

In this research, the LMS was used in its complex form [10] with an adaptive step-size [11], which provides an increased convergence speed. The complex signal analyzed is formed by the power system three-phase voltages applying the \(\alpha\beta\)-Transform.

It should also be highlighted that computing simulations were performed using the ATP software [12]. Some EPS equipment was modeled, including: a synchronous generator with speed control, transmission lines with parameters dependent on the frequency and power transformers. Extreme operational situations were tested in order to verify the behavior of the proposed technique and validate the obtained results.

II. THE ALGORITHM BASED ON LMS

The algorithm based on the LMS method, presented in Fig. 1, is a combination of the adaptive process with digital filtering. In this Figure, \(\hat{u}(n-1) = [\ u(n-1) \ u(n-2) \ ... \ u(n-M) \ ] \) is the vector with \(M\) past values; \(\hat{w}(n) = [\ w_1(n) \ w_2(n) \ ... \ w_N(n) \ ]^T \) is the vector with the filter coefficients; \(y(n)\) is the desired signal (the output filter) and \(e(n)\) is the error associated to the filter approximation.

Fig. 1. Adaptive filter based on LMS.
The input signal of the filter can be estimated by minimizing the squared error by the coefficient adaptations ($\bar{w}(n)$), which are recursively adjusted to obtain optimal values. At each iteration, the coefficients can be calculated by:

$$\bar{w}(n + 1) = \bar{w}(n) + \mu (-\nabla(n))$$

(1)

where $\mu$ is the convergence parameter and $\nabla$ is the gradient of error performance surface and is responsible for determining the necessary adjustments to the coefficients.

The LMS algorithm is very sensitive to $\mu$. This can be mainly observed by the speed of the estimation and processing time. The lesser the $\mu$ value, the longer the time to reach the aimed error and vice-versa.

However, it is important to respect the convergence interval given by [13]:

$$0 < \mu < \frac{1}{NS_{max}}$$

(2)

where $N$ is the filter size and $S_{max}$ is the maximum power spectral density value of the input signal.

### III. The Adaptive Algorithm and the Frequency Estimation

The study of digital filters is a consolidated research area. Regarding digital protection, digital filters provide the frequency component extraction used in digital relay algorithms. The information contained in the input data from a three-phase system can be processed simultaneously, making it possible to obtain more precise results, if compared to conventional methods.

It must be highlighted that the proposed algorithm, called Frequency Estimation Algorithm by the Least Mean Square (FEALMS), uses three-phase signals as inputs. Considering that the three-phase voltages from EPS can be represented by:

$$V_a(n) = A_{max} \cos(\omega n \Delta t + \phi) + \xi_{na}$$

$$V_b(n) = A_{max} \cos(\omega n \Delta t + \phi - \frac{2\pi}{3}) + \xi_{nb}$$

$$V_c(n) = A_{max} \cos(\omega n \Delta t + \phi + \frac{2\pi}{3}) + \xi_{nc}$$

(3)

where, $A_{max}$ is the peak value; $\omega$ is the signal angular frequency $1$; $n$ is the sample number of the discrete signal; $\Delta t$ is the time between two consecutive samples; $\phi$ is the signal phase and $\xi_n$ is the error between two consecutive samples. Fig. 2 illustrates the proposed relay algorithm.

#### A. Data Acquisition

All the stages in data acquisition are performed aiming a more realistic analysis of the obtained results. The input voltage signals simulated are characterized by a high sampling rate in order to represent more realistically the analog signals from the electric system analyzed.

Fig. 3 shows the data acquisition flowchart. A second order low pass Butterworth filter with a cut–off frequency of 200Hz was utilized. A sample rate of 1,920Hz and an analog–to–digital converter (ADC) of 16 bits were also used.

$1\omega = 2\pi f$, where $f$ is a power system frequency

The low pass filter was used to avoid the spectral spreading and to make sure that the digital representation after ADC conversion is a good representation of the original signal. It is worth commenting that a low cut–off frequency of $200Hz$ was used in order to stabilize the method and ensure the LMS algorithm convergence. Due to this situation, most of the harmonic components were eliminated, increasing the precision of the proposed method.

The data acquisition was performed in a moving window with one sample step. All the filter processing should be performed on one data window, respecting the available time for processing, which is the time between two consecutive samples. Fig. 4 illustrates this process.

#### B. Normalization

Normalization standardizes the data obtained from the electrical system, regardless of the voltage level analyzed. Consequently, if a either sag or a swell occurs in any phase, the algorithm will maintain its estimative without loss of precision or speed. Fig. 5 illustrates the normalization process implemented.
C. Pre-Processing

After normalization, a pre-processing stage was performed, obtaining the signal in its complex form for the digital filter. This was obtained by applying the \( \alpha \beta \)-Transform on the three-phase voltages, as represented in the following equation [14].

\[
\begin{bmatrix}
V_{\alpha}(n) \\
V_{\beta}(n)
\end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix}
1 & -\frac{1}{2} & -\frac{1}{2} \\
0 & \sqrt{\frac{3}{2}} & \sqrt{\frac{3}{2}}
\end{bmatrix} [V(n)]
\]  
(4)

where \( V(n) = [ V_{\alpha}(n) \ V_{\beta}(n) \ V_{\gamma}(n) ]^T \).

After pre-processing to obtain the \( \alpha \) and \( \beta \) components by (4), the complex voltage is defined as:

\[
u(n) = V_{\alpha}(n) + jV_{\beta}(n)
\]  
(5)

D. Coefficient Generator

The adaptation of the filter coefficients is simple and inherent to the algorithm. This adjustment is performed by sample in order to make sure that the squared average error is minimized. However, to improve the algorithm performance and minimize the processing time, the estimation filter coefficients are initialized, \( \hat{\beta}(n) \) with the estimation of the previous window. The first window is initialized with the fundamental frequency of the electrical system. The aim of this proceeding is to increase the speed of the estimation process.

E. Adaptive Filter

In the adaptive filter, the error or the difference between the estimate and desired signal is then used to regenerate the adaptive process, where the coefficients are updated recursively aiming at minimizing the squared error.

Reference [8] and textbook [13] show the steps and principles of the adaptive filter.

F. Frequency Estimation

The frequency estimation was performed according to [15]. To find the phase difference, we defined the complex variable \( \Gamma \) as:

\[\Gamma = y(n)y(n-1)^*\]  
(6)

The frequency of the estimated signal \( y(n) \) was calculated in function of the phase difference between two consecutive samples, and the latter was provided by the equation below:

\[f_{est} = \frac{f_s}{2\pi} \arctan\left(\frac{\Im(\Gamma)}{\Re(\Gamma)}\right),\]  
(7)

where \( f_{est} \) is the estimated frequency, \( f_s \) is the sample frequency and \( \Re() \) and \( \Im() \) are the real and imaginary parts, respectively.

G. Convergence

The stop rule adopted was the maximum number of iterations (1,000) or absolute error smaller than \( 10^{-5} \). This error can be estimated by:

\[e_{relat} = \text{abs}(y(n) - u(n)),\]  
(8)

where \( e_{relat} \) is the relative error between samples, \( \text{abs}() \) is the absolute value, \( y(n) \) is the estimate value and \( u(n) \) is the desired value or input sample.

H. Post-processing of the output signal

The output signal (estimated frequency) is additionally filtered by a second order low pass Butterworth digital filter with cut-off frequency of 5Hz. This procedure reduces the oscillation present in the proposed method output, avoiding errors due to abrupt variations of the frequency.

IV. THE SIMULATED ELECTRICAL SYSTEM

Fig. 6 shows the representation of the simulated electrical system, taking into account load switching and permanent faults in order to evaluate the frequency estimation technique proposed in this work.

The electrical system consists of a 13.8 kV and 76 MVA synchronous generator, 13.8:138 kV /138:13.8 kV and 25 MVA three phase power transformers, transmission lines between 80 and 150 km in length and loads between 5 and 25 MVA with a 0.92 inductive power factor. Power transformers have a delta connection in the high voltage winding and a star connection in the low voltage winding. Tables I and II show the parameters used in order to simulate the power system components using ATP (Alternative Transients Program) software.

In Table I, \( S \) is the total three-phase volt-ampere rating of the machine, \( N_p \) is the number of poles which characterize the machine rotor, \( V_L \) is the rated line-to-line voltage of the machine, \( f \) is the electrical frequency of generator, \( IFD \) is the field current, \( R_a \) is the armature resistance, \( X_f \) is the armature leakage reactance, \( X_o \) is the zero-sequence reactance, \( X_d \) is
the direct-axis synchronous reactance, \( X_d \) is the quadrature-axis synchronous reactance, \( X'_d \) is the direct-axis transient reactance, \( X''_d \) is the direct-axis subtransient reactance, \( \tau'_d \) is the direct-axis open-circuit transient time constant, \( \tau''_d \) is the direct-axis open-circuit subtransient time constant and \( \tau'_q \) is the quadrature-axis open-circuit transient time constant.

It is important to emphasize that the transmission line model used was JMARTI from ATP because it is possible to have a variation of the line parameters in function of the frequency consequently a better representation of the system behavior when facing disturbances resulting from unbalance between generation and load is reached.

It must also be emphasized that the synchronous generator was simulated with an automatic speed control for hydraulic systems [16], considering various electrical and mechanical parameters from the generator. Equation 9 shows the transfer function of the speed regulator used:

\[
\frac{\Delta F(s)}{\eta(s)} = \frac{1}{R \left[ 1 + sT_r \right]} \frac{1}{1 + sT_g} \left[ 1 + \frac{1}{\eta s T_r} \right]
\]

where \( \eta(s) \) is the servomotor position, \( \Delta F(s) \) is the frequency deviation, \( R \) is the steady-state speed droop, \( r \) is the transient speed droop, \( T_g \) is the main gate servomotor time constant and \( T_r \) is the reset time. Table III presents the parameters concerning the speed regulator.

### Table I

Synchronous generator data used in the simulation.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value (unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S )</td>
<td>76 (MVA)</td>
</tr>
<tr>
<td>( V_L )</td>
<td>13.8 (kV nom.)</td>
</tr>
<tr>
<td>( I FD )</td>
<td>250 (A)</td>
</tr>
<tr>
<td>( X_i )</td>
<td>0.175 (p.u.)</td>
</tr>
<tr>
<td>( X_d )</td>
<td>0.150 (p.u.)</td>
</tr>
<tr>
<td>( X''_d )</td>
<td>0.310 (p.u.)</td>
</tr>
<tr>
<td>( X'_q )</td>
<td>0.182 (p.u.)</td>
</tr>
<tr>
<td>( \tau'_d )</td>
<td>0.036 (sec)</td>
</tr>
</tbody>
</table>

### Table II

Power transformer data used in the simulation.

<table>
<thead>
<tr>
<th>Element</th>
<th>( R_+ (\Omega) )</th>
<th>( L_+ (mH) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary impedance of transformer</td>
<td>1.7462</td>
<td>151.37</td>
</tr>
<tr>
<td>Secondary impedance of transformer</td>
<td>0.0175</td>
<td>1.514</td>
</tr>
</tbody>
</table>

### Table III

Parameters concerning the speed regulator.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value (unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main gate servomotor time constant</td>
<td>0.600 (sec)</td>
</tr>
<tr>
<td>Reset time (( T_r ))</td>
<td>0.838 (sec)</td>
</tr>
<tr>
<td>Transient speed droop (( r ))</td>
<td>0.279</td>
</tr>
<tr>
<td>Steady-state speed droop (( R ))</td>
<td>0.100</td>
</tr>
<tr>
<td>Moment of inertia (( M ))</td>
<td>1.344 (sec)</td>
</tr>
<tr>
<td>Water starting constant (( T_W ))</td>
<td>0.150 (sec)</td>
</tr>
</tbody>
</table>

V. Simulation Results

The purpose of this section is to present some results regarding the proposed algorithm, comparing it to a commercial relay, as well as the actual frequency variation curve given by the ATP software. Several different tests were performed on the system shown in Fig. 6. It should be emphasized that the sample rate of 1,920 Hz and 1,000 Hz were used in the FEALMS software and a commercial relay (having the 81 function), respectively.

Due to a great influence from the adjustment of the filter parameters in the results, these parameters were selected according to [17], and they are: \( \mu_{max} = 0.18 \), \( \mu_{min} = 0.001 \), \( \mu_{inicial} = 0 \), \( \lambda = 0.97 \), \( \gamma = 0.01 \) and \( \rho = 0.99 \). Based on Fig. 6, the simulated situations were:

1) sudden connection and disconnection of load blocks;
2) permanent fault at 50% of line length 1;
3) permanent fault involving phase A and ground (AG) in the BLT2 busbar at 0.5s;
4) energization of the TR3A transformer under an external fault.

A. Sudden connection and disconnection of load blocks

Fig. 7 shows the estimation of the synchronous generator frequency using the commercial relay (function 81), the ATP software (correct answer) and the FEALMS software for the
connection and disconnection of the load blocks in the BGCH3 busbar. A slight delay in the frequency estimation by the relay, can be observed in the figure when compared to the correct result given by the ATP software. In this situation, very good precision concerning the FEALMS algorithm can be observed, even at critical points of the system’s behavior.

**B. Permanent fault at 50% of line length 1**

In Fig. 8, a frequency variation is observed considering an AG fault at 50% of transmission line length 1. In this situation, the correspondent three-phase circuit breaker trip was considered after 80ms of the fault inception.

![Fig. 8. 3φ fault in 50% of the length line 1.](image)

It should also be observed that the recovery of the machine synchronization was achieved without the need of the relay action concerning the Brazilian electrical system it should happened when the frequency goes outside the range of 60 ± 0.5 Hz [1].

**C. Permanent fault involving phase A and ground (AG) in the BLT2 busbar at 0.5s**

Fig. 9 shows the proposed technique, as well as the commercial relay (function 81) performances, for an AG fault in the BLT2 busbar at 2.5s and circuit breaker closed at 2.6s, respectively.

![Fig. 9. AG fault in busbar BLT2 at 2.5s and circuit breaker closed at 2.6s.](image)

It should be observed that despite the fact that the fault was cleaned at 100ms, the frequency goes outside the range established before, probably causing the protection action by the presented techniques.

**D. Energization of the TR3A transformer under an external fault**

Fig. 10 illustrates the commercial relay (function 81) and the FEALMS algorithm responses for an energization of TR3A transformer under an external fault in its secondary winding. Analyzing of this condition is fundamental when testing the algorithm robustness facing actual field situations, taking into account the high level of distortion present in the input signals.

![Fig. 10. Energization of the TR3A transformer under a external fault.](image)

It can be clearly observed that the commercial relay (function 81), as well as the FEALMS software, presented similar
responses. Once more, a delay can be observed for the commercial relay (function 81), possibly due to the filtering process.

VI. CONCLUSIONS

This work presented an alternative method for the frequency estimation in electrical power systems using the LMS algorithm. The implementation of digital filtering and the \( \alpha/\beta \) algorithm. The implementation of digital filtering and the \( \alpha/\beta \) algorithm made the simultaneous use of the three-phase power system voltages for the estimation purpose possible.

In the complex LMS algorithm considered, the adjustment of the step size was used as being adaptive based on the estimations of the error.

The studied situations for testing the proposed algorithm were obtained using ATP software. The adaptive filter theory applied to the digital protection was fast and reliable. Some points should be observed:

1) The FEALMS algorithm can be applied to various situations and voltage levels, as it is not influenced by the magnitude of the input waveforms.
2) It should be emphasized that the three-phase voltages were analyzed contrary to a single phase (used in commercial relays), making the proposed algorithm more robust.
3) The technique is easy to implement and does not need adjustments and knowledge of further functions, as is the case of the commercial relay (function 81) utilized.

It is also important to highlight the feasibility and computing efficiency of this method, making it suitable for commercial applications.

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