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Active flow control of convectively unstable flows

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Abstract

In this article, we apply an optimal control over a convectively unstable flow to attenuate the disturbance amplitudes downstream of the flow. In this regard, we adopt Ginzburg-Landau (GL) equation to model the convectively dominated unstable flow over a flat plate. The motivation to design the appropriate controller is to delay the transition process over a flat plate. We integrate the GL equation with standard optimal control (LQR) to design the controller. We supply proper control actuation and observe a substantial decrease in the amplitude of the perturbations downstream of the flow.

Keywords - *Transition delay; Active flow control; Ginzburg-Landau (GL) equation; LQR Linear quadratic regulator*

I. INTRODUCTION

Linear amplification caused by initial fluctuations in any flow is among one of the several paths to induce the transition to turbulence. Other mechanism includes transient growth possessing higher amplitude to the eigenmode growth (TS path) [1], or transient growth directly result in secondary instability, or receptivity bypasses and undergoing breakdown to turbulence (bypass transition) [2]. Once the flow undergoes turbulence, it will be difficult to control as the flow is chaotic. We intend to control the convectively growing instabilities before the flow starts undergoing transition. The stability of a flow can be determined by calculating the response to an impulse at, say $x = 0$ and $t = 0$. If the response of the flow amplifies and convected away from $x = 0$, the flow is locally convectively unstable. In this regard, Huerre [3] and Monkewitz [4] extensively studied the global and convective instability behavior of the basic flow. Although in the convectively unstable flows, the disturbances eventually flush out of the domain, it causes unnecessary drag and noise in the system. Our aim in this paper is to limit the further increment of the disturbances in the flow downstream. Researchers have successfully achieved delay in transition process by limiting the amplitude of unstable waves downstream [5], [6], [7]. Nitsche [8] also studied transition delay over airfoil using adaptive system identification approach. We start with a simple mathematical model (one dimensional GL-equation) which mimics and preserves essential stability characteristic of fluid flow over a flat plate. Fabbiane [6] achieved the control of these unstable waves through the model based (KS equation model) and model free methods. In contrast we have adopted the similar approach with GL equation model the flow and integrated the optimal control to attenuate the growth of the convectively unstable disturbances in the system. Although the GL equation is not strictly used to model the flow over flat plate, but it can also be extended to model the parabolic flow, diverging flows as well. But in order to mimic the dynamical behavior of the real flows, we need to tune the parameters/co-efficients of the GL in such a way that it inherits the essential stability characteristics of the different flows. Nevertheless, the role of Reynolds number associated with the real flow is taken care by the parameters/coefficients in GL equation.

II. SYSTEM FORMULATION

We consider a steady uniform flow V over flat plate of finite length $L = 800$ as shown in FIG. 1. One dimensional domain is chosen at particular Y location = $\delta_0^\star$, where $\delta_0^\star$ is displacement thickness at $x = 0$. (see FIG. 1). It also shows a schematic of flow over flat plate with discrete sensors and actuators. Flow is subjected to initial disturbance signal $d(t)$ via disturbance actuator near the leading edge. Because of the convectively unstable nature, disturbance grows and convects downstream. The flow dynamics downstream is then measured by reference sensor as $y(t)$ signal. A control actuator is designed and kept downstream which modulates the signal as $u(t)$ based on the desired attenuation of error signal $z(t)$ measured by error sensor even further downstream. For a given reference signal $y(t)$, our aim of this work is to design the control signal $u(t)$ in order to achieve a desired attenuation of error signal $z(t)$.

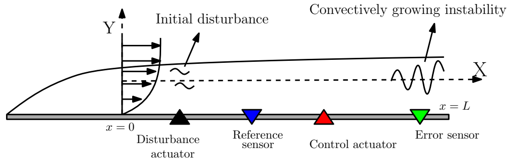

Figure 1: **Flow setup along with actuators and sensors.**

A. **Ginzburg-Landau equation model**

Landau-Ginzburg equation [9] for convection dominated flow, is described as,

$$\frac{\partial v'}{\partial t} = -\nu \frac{\partial v'}{\partial x} + \gamma \frac{\partial^2 v'}{\partial x^2} + \mu v' \tag{1}$$

where v', x, t are stream wise perturbation velocity, stream wise coordinate and time coordinate. The convective, dissipative and exponential nature of perturbations are associated with coefficients ν, γ and μ respectively. In general the coefficient $\mu = \mu(x)$ is a function of x for spatially developing flows, but we will consider it as constant coefficient for the sake of simplicity of the model. These coefficients are chosen in such a way that the dynamics closely matches with the two dimensional boundary layer behavior at Reynolds number $Re_{(\delta_0^\star)} = 1000$. Where $Re_{(\delta_0^\star)}$ is defined based on characteristic length scale as boundary layer thickness $\delta_0^\star$ at $(x = 0)$. Disturbance actuator placed at $x = 35$, impinges initial Gaussian disturbance to the system as $d(t)$. A reference sensor placed at $x = 300$ senses the initial dynamics of flow downstream as $y(t)$. An actuator placed at $x = 400$, supplies a control action as $u(t)$ followed by sensing of flow as $z(t)$ by an error sensor placed at $x = 700$. Coefficient values of $\nu = 0.4$, $\gamma = -0.0285$ and $\mu = -0.04$ are chosen so that the dynamics mimics the disturbance response of flow over flat plate. Inlet boundary conditions are considered as, $v'|_{x=0} = 0$ and $\frac{\partial v'}{\partial x}|_{x=0} = 0$, whereas outflow boundary conditions are set to be, $\frac{\partial v'}{\partial x}|_{x=L} = 0$ and $\frac{\partial^3 v'}{\partial x^3}|_{x=L} = 0$, to ensure the smooth exit of the disturbance wave. FIG. 2 shows variation of v'_{rms} with streamwise x location for KS equation model along with. The perturbation response of the system is obtained with initial random perturbation at $x = 35$. Root mean squared velocity $(v'_{rms}(x))$ is defined as, $\sqrt{(\overline{v'(x)^2} - \overline{v'(x)}^2}$, where $\overline{v'(x)}$ is time averaged perturbation velocity at particular streamwise x location. The perturbation response of Ginzburg-Landau equation model (with constant coefficients as $\nu = 0.4$, $\gamma = -0.0285$ and $\mu = -0.04$) is validated with KS (kuramoto-sivashinsky) equation model and Navier-stokes DNS model presented in [6] as shown in the FIG. 2.

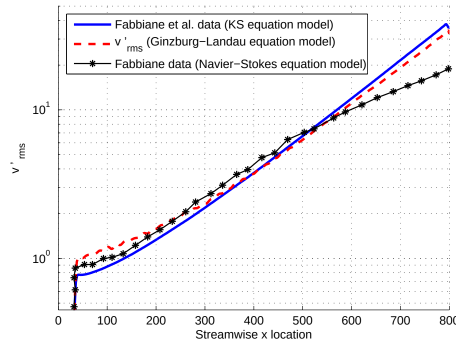

Figure 2: **Plot of perturbation responses** (v'_{rms}) **with streamwise x locations (at coefficient values of** $\nu = 0.4$, $\gamma = -0.0285$ **and** $\mu = -0.04$)**. Plot is also validated with the Fabianne's [6] KS-equation model data.**

B. **Linear stability approach**

Stability properties are analyzed through normal mode analysis by assuming,

$$v' = \hat{v} e^{i(kx - \omega t)} \tag{2}$$

where k, ω and $\hat{v}$ are spatial frequency, temporal frequency and amplitude of the traveling wave respectively. Substituting above assumed traveling wave solution in the governing GL equation, we get the dispersion relation as,

$$\omega = \nu k + i(\mu - \gamma k^2) \tag{3}$$

Positive values of (ω_i) are associated with exponential growth of disturbances. It is observed that for a particular interval of wavenumber, i.e. for $k > \sqrt{\frac{\mu}{\gamma}}$ we have $\omega_i > 0$. Also the $\omega_r/k = \mu$ determines the wave propagation speed.

C. **Control integration**

We modify GL equation by introducing a forcing function $f'(x,t)$ into it. This forcing function includes the disturbance $d(t)$ and control actuation $u(t)$ applied to the system.

$$\frac{\partial v'}{\partial t} = \left[-\nu \frac{\partial v'}{\partial x} + \gamma \frac{\partial^2 v'}{\partial x^2} + \mu v' \right] + f'(x,t) \tag{4}$$

we mathematical construct the model of actuators as,

$$f'(x,t) = b_d(x) d(t) + b_u u(t) \tag{5}$$

where b_d and b_u are spatial dependence parameters of actuators are given by Gaussian functions as,

$$b_d = \frac{1}{\sigma} e^{\left[-\left(\frac{x - x_d}{\sigma} \right)^2 \right]} \qquad \text{and} \qquad b_u = \frac{1}{\sigma} e^{\left[-\left(\frac{x - x_u}{\sigma} \right)^2 \right]}$$

where, σ is the variance of the Gaussian profile of actuators and sensors and x_d and x_u are the locations where the disturbance and control actuations are applied respectively. Similarly measurement outputs $y(t)$ and $z(t)$are modeled as,

$$y(t) = \int_0^L c_y(x) v'(x,t) dx \tag{6}$$

$$z(t) = \int_0^L c_z(x) v'(x,t) dx \tag{7}$$

where spatially dependent kernal of sensors are also given by Gaussian functions as,

$$c_y = \frac{1}{\sigma} e^{\left[-\left(\frac{x-x_y}{\sigma}\right)^2\right]} \qquad \text{and} \qquad c_z = \frac{1}{\sigma} e^{\left[-\left(\frac{x-x_z}{\sigma}\right)^2\right]}$$

where x_y and x_z are the locations where reference and error signals are measured respectively.

D. **State space formulation and discretization**

Assuming spatial discretization as $v'(x,t) = v'(x_i,t)$, (where $i = 1, 2, ..m$). We may write it in vector form as, $\mathbf{v}^{'} = [v'(x_1), v'(x_2)...., v'(x_m)))]^T$. The governing GL equation (1) can be written in the form as,

$$\frac{d\mathbf{v}'}{dt} = [\mathbf{A}]\mathbf{v}' \tag{8}$$

where $[\mathbf{A}] = \left[-\nu\frac{\partial}{\partial x} + \gamma\frac{\partial^2}{\partial x^2} + \mu\right] = \left[-\nu\mathbf{D1} + \gamma\mathbf{D2} + \mu\right]$ is the GL operator. $\mathbf{D1}, \mathbf{D2}$ are first and second order finite difference operators and $\mathbf{v}'$ forms the state vector. Upon finite difference discretization, we can write,

$$\frac{\mathbf{v}_i^{'n+1} - \mathbf{v}_i^{'n}}{\Delta t} = [\mathbf{A}]\mathbf{v}_i^{'n} \tag{9}$$

Using Crank-Nicolson method, we rewrite,

$$\frac{\mathbf{v}_i^{'n+1} - \mathbf{v}_i^{'n}}{\Delta t} = [\mathbf{A}]\left(\frac{\mathbf{v}_i^{'n}}{2} + \frac{\mathbf{v}_i^{'n+1}}{2}\right) \tag{10}$$

$$\mathbf{v}_i^{'n+1} = \left(\mathbf{I} - \frac{\Delta t}{2}\mathbf{A}\right)^{-1}\left(\mathbf{I} - \frac{\Delta t}{2}\mathbf{A}\right)\mathbf{v}_i^{'n} \tag{11}$$

where index i represents the i^{th} spatial node with step length Δx and n is the index for temporal node discretization with step length Δt.

Substituting the discretized forms in the equation (4), we get the final state space form as,

$$\frac{d\mathbf{v}'}{dt} = [\mathbf{A}]\mathbf{v}' + \left[\mathbf{B}_d\right]d + \left[\mathbf{B}_u\right]u \tag{12}$$

and the measurement equations as,

$$\mathbf{Z} = \left[\mathbf{C}_z\right]\mathbf{v}' \tag{13}$$

where $\left[\mathbf{B}_d\right]$, $\left[\mathbf{B}_u\right]$ and $\left[\mathbf{C}_z\right]$ are kernals of actuators and sensors also discussed previously. Equation (11) is marched with time with the prescribed boundary conditions and initial condition as random perturbation (of gaussian profile at $x = 35$) all the times till (t = 5000).

E. **Optimal control**

Our goal is to design a controller which takes the measurement from error sensor $z(t)$ and supplies the control signal $u(t)$ in order to minimize the overall perturbation amplitude. Cost function J is chosen as the norm of output as,

$$J \;\; = \int_0^\infty norm\left(\begin{bmatrix} Z \\ u \end{bmatrix}\right) dt \tag{14}$$

where

$$\begin{bmatrix} Z \\ u \end{bmatrix} = \begin{bmatrix} C_z \\ 0 \end{bmatrix}\mathbf{v}' + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \tag{15}$$

$$J = \int_0^\infty (\mathbf{v}^{'T}\mathbf{Q}\mathbf{v}' + \mathbf{u}^T\mathbf{R}\mathbf{u})dt \tag{16}$$

where $\mathbf{Q} = \mathbf{C}_z^T\mathbf{w}\mathbf{C}_z$, $\mathbf{w}$ and $\mathbf{R}$ are the weight matrices (generally taken $\mathbf{I}$ identity matrix). In general, an optimization problem is formulated to minimize the cost function J over over an infinite time horizon subject to the constrain of equation (12). Solution of above optimization problem is seek through the standard solution of Riccati equation [10]. An optimal control gain $\mathbf{K}$ is then found and shown in Figure 4 and is supplied through feedback as $u = -\mathbf{K}\mathbf{v}'$.

III. RESULTS AND DISCUSSION

We have designed an optimal control to attenuate the convectively growing perturbation over a flat plate. The aforementioned technique is successful to attenuate the growing perturbation downstream of the flow. Figure 3 shows the perturbation evolution and its controlled

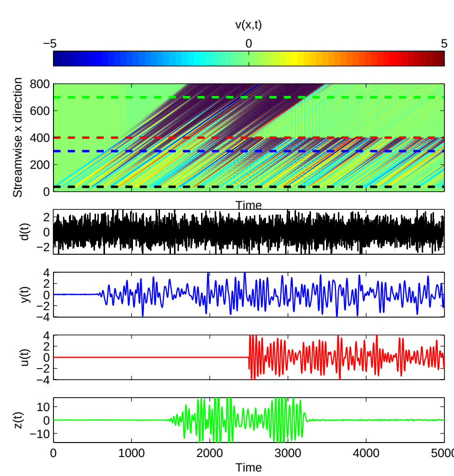

Figure 3: **Perturbation evolution after applying LQR control at time t = 2500.**

contour in space and time. The signal $d(t)$ is the applied disturbance signal (at $x = 35$) at all times. At a distance $x = 300$ the reference sensor reads the $y(t)$ signal. Further downstream (at $x = 400$) a control actuation signal is supplied to the system. This control signal is switched on after time $t = 2500$. Once the control action is supplied, the growing perturbations are getting canceled, but we see the error sensor $z(t)$ at $x = 700$, reads some fluctuations till the previous instabilities washes out from the domain. After supplying feedback after t = 2500, we get the attenuation of overall perturbation amplitude. To do so, we have found a static optimal control gain (shown in the Figure 4) and supplied the control actuation as $u = -\mathbf{K}\mathbf{v}'$. A comparative plot of v'_{rms} for uncontrolled

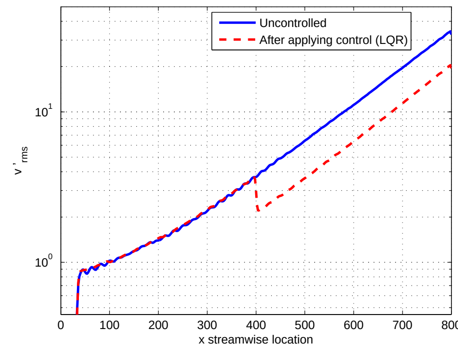

Figure 5: **Plot comparison of uncontrolled v'_{rms} with controlled v'_{rms} perturbation velocity.**

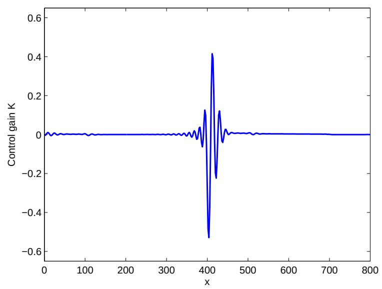

Figure 4: **Optimal control gain K**

and with control effort is also shown in the Figure 5. We see approximately 15 percent attenuation in the v'_{rms} of the perturbation data. However, it also evident that after control application, the perturbations did not entirely suppress forever, rather, the perturbations starts to grow again in the downstream of the flow (because of the growing dynamical nature of the flow). In other words we did not cancel the disturbances entirely, but we eventually delayed the transition process.

IV. CONCLUSIONS

We have designed an optimal controller in order to attenuate the disturbances growth in a convectively unstable flow. We have adopted Ginzburg-Landau (GL) equation to model the convectively dominated unstable flow over a flat plate. In order to delay the transition, we have integrated the GL equation with standard optimal control (LQR). After supplying the feedback control, we have achieved a substantial decrease in the amplitude of perturbation growth downstream of the flow. With the suppression of perturbation field, subsequently we achieve the delay in the transition process in the flow. This work can be extend to the parabolic flows, diverging flows too. It would be interesting to model different flow situations with The GL equation integrated with control, and at the same time preserving the essential stability characteristics of the flows.

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