An Affine Arithmetic Approach to Model and Estimate the Safety Parameters of AC Transmission Lines

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Abstract—With an increase in population of the country day by day and with high growing speed of geographical residential plots, the demand by the public for new set up of electrical power transmission system has become a common mandate. Therefore it is the responsibility of the concerned authority to protect the interests of common man in a smart manner and develop solutions for the growth of country in an intelligent way keeping safety of public as prime importance. This paper proposes an Affine Arithmetic approach of mathematical modelling to estimate the safety parameters of AC transmission line leading to sag, like, temperature, wind loading, ice loading, weight of the conductor, stress, tension, pressure etc., taking into account the uncertainty conditions so that the solutions developed address in real time. The proposed model is executed in MATLAB integrated development environment and gives the complete behavior of sag in transmission lines with respect to each of the safety parameters individually and closely ascertains the safety threshold limits considering 50% factor of safety and 5m ground clearance. The critical safety threshold limit for wind loading is found to be about 3.02 kg/m and the typical value is about 1.51 kg/m. Similarly, the critical safety threshold limit for weight of ice is found to be about 1.45 kg/m, whereas its typical value is about 0.7 kg/m. Extending further, the critical safety threshold limit for conductor weight is found to be about 3.07 kg/m, whereas its typical value is about 1.6 kg/m, which is a close approximation to the typical weight per unit length of industry-grade conductors like Aluminum Conductor Steel Reinforced (ACSR). The critical safety threshold limit for tension in the conductor and the span lengths are found to be 1850 kg and 230m respectively.

Index Terms—Transmission lines, Affine Arithmetic, Safety parameters, Sag, Ground clearance

I. INTRODUCTION

A network of cables that support in transmitting electric power from the site of production to the site of utilization forms the transmission lines. With the ever increasing demand for electric power, the amount of energy carried by these transmission lines is being pushed to the maximum limits. The overhead electricity transmission network must comply with the safety rules, quality and continuity as regulated by the government to make sure the safety of electric system for general public, distribution network staff operators and to limit the faults [1]. As transmission towers and lines are indispensable to normal day life, erection of these lines must be through numerous stringent guidelines set by government as well as safety agencies, so that it meets the standards of safety and to avoid potential hazards. Also as these lines are designed to carry voltages in excess of kV, there is a high risk of electrocution. Adding to the concern is that these overhead transmission lines are not insulated and any contact with them leads to fatalities in living beings and damage to property. There are certain parameters that limit the amount of electrical power that can be transmitted without compromising on critical safety issues and security regulations. Sagging is one of the primary safety aspects in transmission lines which is of major concern and is defined as decline of transmission line to lower level than usual due to expansion of its conductor [2] and it has to be within the permissible value as directed by the Regional Electricity Board regulations. The affecting parameters which lead to sag in AC transmission lines can be accounted for conductor weight, wind pressure, ice loading, and variation in span length, tension and temperature effects. If the conductors are too much expanded between the supports, the stress in the conductors reaches unsafe value resulting in breakage due to excessive sagging [2]. There are situations where flashover may also take place when phase to phase distance between conductors becomes small or phase to

tower distance becomes small and it is also certain that the general public is disturbed when the minimum ground clearance is not properly maintained. Many short circuits have also taken place because of phase to phase touch due to increased sagging and wind, ice getting warmer with temperature change etc., Ref. [3]. The conventional method followed to obtain sag value is from lookup curves given by design departments and are prone to make personal errors [4]. The authors of Ref. [5] have calculated the sag error as a weighted sum of partial errors in the state equation and have concluded that the sag error difference between designed and actual transmission line conductor is negligible due to creep and modulus of elasticity, whereas the loading effects on the conductor are large and contribute significantly to final sag error. The modern electric power transmission system for smart power grid has to be planned for improved efficiency, reliability, safety and automated communication technologies [6-7]. The unpredictable appearance of wind and ice accumulation increases the horizontal and vertical loading which makes the calculated data uncertain and hence the requirement for uncertainty model [8]. Thus from the above discussion we see the necessity for an accurate, efficient and error free modelling which bridges theoretical and practical values considering uncertainty, before erecting new transmission systems and is of utmost importance.

Motivated by the above facts the authors propose the affine arithmetic uncertainty model which intends to estimate the safety threshold values for the parameters involved in causing sag in transmission lines. From the previous and ongoing works, many mathematical models are developed with regard to estimating these safety parameters, out of which Affine Arithmetic method closely estimates to give accurate results, as it has a better approach to error and correction analyses for dynamic and probabilistic scenarios. The proposed model isolates each of the parameters when assessing the behavior of sag and plotting of graphs on that particular parameter, without the influence of other parameters. Thus, in the proposed model, the complete behavior of sag in transmission lines is obtained with respect to each of the parameters separately and closely ascertains the safety threshold limits.

II. Related Work

In the years to count, many mathematical models have been proposed by various authors which are comprehended in this section for their advantages and disadvantages, as well we see that considerable amount of ongoing research is noticed in the field of interest as elucidated below.

The temperature distribution in different layers of transmission line conductors considering ACSR using ANSYS CFX are dealt in by the authors of Ref. [9]. Dynamic line rating is another area of research which gives the updates about the current carrying capacity of the transmission line, and the thermal stresses that occur on the conductors as well as on the insulators and on

other pieces of the equipment [10]. The authors of [11] have carried out an analysis that affect the sag tension model and have analyzed the effect of weather on the conductor tension values. The wind and the rain overload the conductor, besides, the wind cause vibration or movements of the conductor that result in fast variations of the measured tension values according to the authors. Wavelet refined parameters to model temperature using ANFIS has been proposed by the authors of Ref. [12]. Modelling differential equations using MATLAB for various approaches has been dealt by the authors of Ref. [13]. A design which considers thermal transient state of conductors by taking into account the thermal inertia of the line as well as prediction values of influencing parameters has been presented in [14]. WSN based efficient monitoring systems are being developed for transmission lines which are capable of transmitting the necessary data to the concerned [15].

The authors of [4] have proposed an iterative technique to estimate sag considering stress equations which are solved in an iterative manner, as computer programs handle iterative methods much better and as imaginary numbers and phase considerations can be avoided. The main drawback of this method is that it cannot address uncertainty and have not considered different safety parameters for calculation. An outdoor test setup for finding the variations in temperature due to ampacity and computer programs to determine sag was proposed by the authors of [16]. The authors have considered only temperature as the safety parameter. The authors of Ref. [17] have proposed a mathematical model using finite difference scheme and heat transfer equations and have suggested for increased electricity for melting ice and reduce sag which is difficult to accept in practical situations. Also ice loading is the only parameter considered. The authors of Ref. [18] have used an Analytical method to calculate the mechanical tension on transmission line conductors which results in sag. The authors have used a simple polynomial equation which cannot be fitted for real time uncertainty situations. Interval arithmetic is another method which is based on an approach called "Unknown but bounded" in which upper and lower limits on the uncertainties are assumed without probability distributions [19].

In comparison with the above methods, the proposed affine arithmetic method can be applied for unbounded conditions, models uncertainty, effectively keeps track of truncation errors and round off errors, giving accurate and error free results.

III. Mathematical Modelling

Affine Arithmetic (AA) is one among the several Self Validated (SV) algorithms developed, which can deal with real world problems with prevalence of uncertainty. Several constraints such as vagueness in data set, error assessment of results obtained, ease of assimilation into a probabilistic model etc., determine which method best suits the intended application. Affine Arithmetic is a modified form of Interval Arithmetic (IA). Several

problems have been addressed successfully using affine form with uncertain measurement which was previously solved by interval mathematics. AA like IA can be used to manipulate imprecise values due to input and measurement errors without upper and lower bounds as in the case of IA [8].

Affine arithmetic is represented by,

$$\hat{x} = x_0 + \sum_{i=1}^{n} x_i \epsilon_i \qquad (1)$$

Where,

$\hat{x}$ = affine form
x_0 = central value
x_i = partial deviation
ϵ_i = symbolic variable which lies in the interval [-1, 1]

In order to redefine basic operations and non-affine operations let us consider the following two affine forms,

$$\hat{x} = x_0 + \sum_{j=1}^{n} x_j \epsilon_j \qquad (2)$$

$$\hat{y} = y_0 + \sum_{j=1}^{n} y_j \epsilon_j \qquad (3)$$

Addition of two affine forms is represented as,

$$\hat{x} \pm \hat{y} = (x_0 \pm y_0) + \sum_{j=1}^{n}(x_j \pm y_j)\epsilon_j \qquad (4)$$

Addition of a constant to an affine form is given by,

$$b \pm \hat{x} = (b \pm x_0) + \sum_{j=1}^{n} x_j \epsilon_j \qquad (5)$$

Multiplication of an affine form by a constant is represented as,

$$b\hat{x} = bx_0 + \sum_{j=1}^{n} bx_j \epsilon_j \qquad (6)$$

The multiplication of two affine forms is formalized as,

$$\hat{x}\hat{y} = (x_0 y_0) + \sum_{j=1}^{n}(x_0 y_j + x_j y_0)\epsilon_j$$
$$+ \left\{\sum_{j=1}^{n}|x_j| \sum_{j=1}^{n}|y_j|\right\}\epsilon_{n+1} \qquad (7)$$

Many of the mathematical operations such as square, square root, trigonometric, logarithmic etc., are not affine operations. In order to approximate these functions, interval approximation, min range approximation and Chebyshev approximation are commonly used and also from literature Chebyshev method yields high accuracy with reduced error.

Consider Fig.1 which shows a typical transmission line conductor suspended between the two towers A and B, subjected to sag due to various loading such as wind, ice, own weight of the conductor and horizontal tension. The various notations used in the figure can be understood as we run through the modelling. It is very important to note that there should be a minimum clearance of 5m between the ground level and the lowest point of conductor sag as depicted in Fig.1, in order to ensure the safety of public.
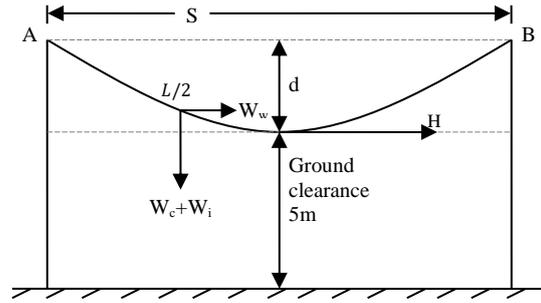

Fig.1. Transmission line conductor affected by sag

The wind loading on the conductor is given by,

$$W_w = P_w \, [\, D + 2t \,] \qquad (8)$$

Where,

W_w = Weight due to wind loading (kg/m)
P_w = Wind pressure (kg/m^2)
D = diameter of the conductor (m)
t = thickness of ice (m)

The ice loading on conductor is given by the equation,

$$W_i = 3.14 \, W_{ice} \, [\, D + t \,] \qquad (9)$$

Where,

W_i = Weight due to ice loading (kg)
W_{ice} = weight of ice (kg/m)

The total weight on the conductor is given by the sum of the following,

(a) Bare conductor weight which acts vertically
(b) Ice loading on conductor which acts vertically
(c) Wind loading which acts horizontally

The total weight is now given by the resultant of all these i.e.,

$$W_t = \sqrt{(W_c + W_i)^2 + (W_w)^2} + q \qquad (10)$$

Where,

W_t = Total weight on conductor (kg/m)
q = factor of weight (constant)
W_c = bare conductor weight

The horizontal tension H is given by,

$$H = \frac{W_t S^2}{8d} \qquad (11)$$

Where,

S = span length of transmission line.
d = sag.

The initial conductor length L_0, without the effect of various loading is given by,

$$L_0 = S + \frac{2.667d^2}{S} \tag{12}$$

Equations (8) to (12) are applicable for fixed point calculations without considering various uncertainties occurring in overhead transmission line. Each of these equations can be modelled for uncertainty system using AA as below.

A. Modelling of wind pressure

The wind pressure varies with the wind speed. The wind pressure variation is uncertain in a given day due to frequent change in temperature. Let $P_{w \, (min)}$ and $P_{w \, (max)}$ are the minimum and maximum wind pressures. Then affine form of wind pressure is given by,

$$\widehat{P_w} = \left[\frac{[P_{w(min)} + P_{w(max)}]}{2} + \frac{[P_{w(max)} - P_{w(min)}] * \epsilon_k}{2} \right] \tag{13}$$

Where,

k is an element of natural number.

The affine form for the ith iteration can be written using equation (2) as,

$$\widehat{P_w} = P_{w,0} + \sum_{l=1}^{m} P_{w,l} \, \varepsilon_{p,l} \tag{14}$$

Where,

$\widehat{P_w}$ = wind pressure in affine form
$P_{w,0}$ = central value of wind pressure
$P_{w,l}$ = partial deviation of wind pressure
$\varepsilon_{p,l}$ = noise symbol

The size of index term is usually m≥1, in the proposed model m = 1 is used.

B. Ice thickness modelling

The ice thickness in affine form is written as,

$$\widehat{t_\iota} = t_{i,0} + \sum_{l=1}^{k} t_{i,l} \, \epsilon_{t,l} \tag{15}$$

Where,

$\widehat{t_\iota}$ = ice thickness in affine form
$t_{i,0}$ = central value of ice thickness
$t_{i,l}$ = partial deviation of ice thickness
$\epsilon_{t,l}$ = noise symbol

Once again the index size k is taken equal to 1

C. Wind loading

Observe from equation (8) that the wind loading on conductor depends on wind pressure P_w and ice thickness t. Now the wind load on conductor in affine form can be obtained by substituting equations (14) and (15) into

equation (8) and applying the affine rules given in equations, (2) and (4) to (7), the resulting equation is,

$$\widehat{W_w} = W_{w,0} + \sum_{l=1}^{m} W_{w,l} \, \epsilon_{p,l} + \sum_{l=1}^{k} W_{w,l} \, \epsilon_{t,l} + \sum_{l=1}^{r} W_{w,l} \, \epsilon_{w,l} \tag{16}$$

Where,

$\widehat{W_w}$ = Affine form of wind load
$W_{w,0}$ = central value of wind load
$W_{w,l}$ = partial deviation of wind load

$\epsilon_{p,l}$, $\epsilon_{t,l}$ and $\epsilon_{w,l}$ represents noise symbol due to wind pressure, ice thickness and affine approximation respectively.

D. Ice loading

Ice loading on conductor depends on weight of the ice W_{ice} and thickness of ice t as given in equation (9). Substitution of equation (15) into equation (9) results in affine representation of ice load as,

$$\widehat{W_\iota} = W_{i,0} + \sum_{l=1}^{k} W_{i,l} \, \epsilon_{t,l} + \sum_{l=1}^{s} W_{i,l} \, \epsilon_{i,l} \tag{17}$$

In equation (17), the symbols have their usual meanings.

E. Total load on conductor

From equation (10), it is evident that the total load on conductor is the resultant of the following loads,

(a) Bare conductor weight W_c
(b) Weight due to ice loading W_i
(c) Weight due to wind loading, W_w

Substituting equations (16) and (17) in equation (10), the affine form of total load on the conductor can be obtained as,

$$\widehat{W_T} = W_{T,0} + \sum_{l=1}^{m} W_{T,l} \, \epsilon_{p,l} + \sum_{l=1}^{k} W_{T,l} \, \epsilon_{t,l} + \sum_{l=1}^{r} W_{T,l} \, \epsilon_{T,l} \tag{18}$$

$\epsilon_{p,l}$, $\epsilon_{t,l}$ and $\epsilon_{T,l}$ represents noise symbol due to wind pressure, ice thickness, and affine approximation respectively.

F. Horizontal tension

The conductor length changes with,

(a) Thermal expansion due to change in temperature
(b) Elastic elongation due to horizontal tension

The dependency of conductor length on temperature and horizontal tension is given by,

$$L = L_0[1 + \propto_T \, \Delta T]\left[1 + \frac{\Delta H}{EA} e_c\right] \tag{19}$$

Where,

$\Delta T = T - T_0$ = change in temperature
$\Delta H = H - H_0$ = change in horizontal tension
$\propto_T$ = co-efficient of thermal expansion
e_c = specific gravity of conductor material
E = young's modulus
A = cross sectional area of conductor

From equation (11),

$$d = \frac{W_T S^2}{8H} \qquad (20)$$

Substituting equation (20) in (12) and taking $H = H_0$,

$$L_0 = S + \frac{2.667}{S}\left[\frac{W_T S^2}{8H_0}\right]^2$$
$$= S + \frac{0.04167\, W_T{}^2\, S^3}{H_0{}^2} \qquad (21)$$

Also,

$$L = S + \frac{0.04167\, W_T{}^2\, S^3}{H^2} \qquad (22)$$

Substituting equations (21) and (12) in (19) results in,

$$S + \frac{0.04167\, W_T{}^2\, S^3}{H^2} = \left[S + \frac{0.04167\, W_T{}^2\, S^3}{H_0{}^2}\right]$$
$$\left[1 + \propto_T\ \Delta T\right]\left[1 + \frac{\Delta H}{EA}e_c\right] \qquad (23)$$

The interval value of H can be obtained at a given temperature using equation (23). The affine form of H is now given by,

$$\widehat{H_c} = h_{c,0} + \sum_{l=1}^{z} h_{c,l}\, \epsilon_{h,l} \qquad (24)$$

G. Sag

The sag depends on wind loading, ice loading and horizontal tension. Thus the sag in affine form can be represented by,

$$\widehat{d_c} = d_{c,0} + \sum_{l=1}^{m} d_{c,l}\, \epsilon_{p,l} + \sum_{l=1}^{k} d_{c,l}\, \epsilon_{t,l}$$
$$+ \sum_{l=1}^{z} d_{c,l}\, \epsilon_{h,l} + \sum_{l=1}^{v} d_{c,l}\, \epsilon_{d,l} \qquad (25)$$

H. Conductor Length

The conductor length also depends on wind loading, ice loading and horizontal tension. Therefore the conductor length in affine form is given by,

$$\widehat{L_c} = l_{c,0} + \sum_{l=1}^{m} l_{c,l}\, \epsilon_{p,l} + \sum_{l=1}^{k} l_{c,l}\, \epsilon_{t,l} +$$
$$\sum_{l=1}^{z} l_{c,l}\, \epsilon_{h,l} + \sum_{l=1}^{v} l_{c,l}\, \epsilon_{l,l} \qquad (26)$$

After finalizing and incorporating all the equations in a mathematical model, the same is programmed in the MATLAB Integrated Development Environment using GUI Processing and the graphs are plotted using MATLAB Graph Plotter as discussed in detail in the next section.

IV. IMPLEMENTATION METHODOLOGY

The equations which describe the uncertainty of various parameters like wind loading, ice loading, weight of the conductor, tension, span length etc., as seen from the previous section are mathematically modelled to obtain the critical safety threshold limit against each of the parameters considered individually and the sag dependence on each of these parameters through graphical output representations are described below in detail. Also the output comparison with respect to Ref. [9] is also shown. According to the authors of Ref. [20] explaining mathematical part using graphical method creates interest among readers.

A. Block Diagram

Fig.2 shows the proposed block diagram and we can observe the various functionality of each individual block sequentially. The various parameters acting on the transmission line conductor like wind loading, ice loading, conductor weight, tension and span length are modelled using affine arithmetic in MATLAB environment in order to estimate the critical safety threshold value of each parameter with uncertainty as depicted in the block diagram. The proposed mathematical model was executed in the MATLAB integrated development environment using GUI processing tools and the MATLAB graphing tools. The front-end of the GUI tool contains various fields used for inputting the data and the output is obtained in the field of Sag. The screenshot of the frontend of the MATLAB GUI tool – Sag Calculator is as shown in the Fig. 3. We can observe all the input data fields which are required to initialize the set up and also to calculate the sag values by varying the parameters of interest. After entering all the relevant data into their respective input fields, the final sag value is then obtained at the bottom of the GUI tool.
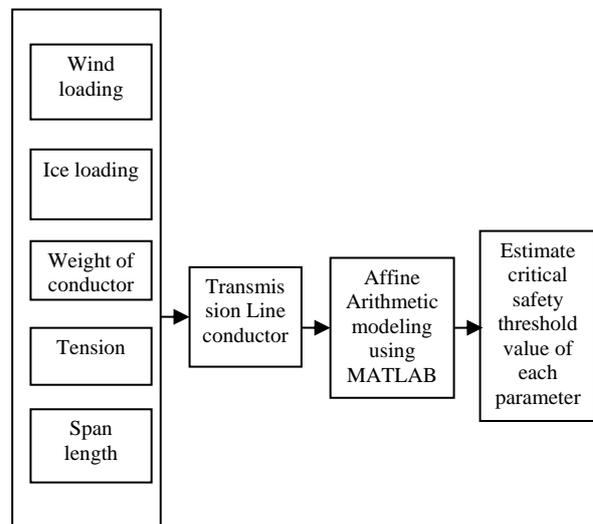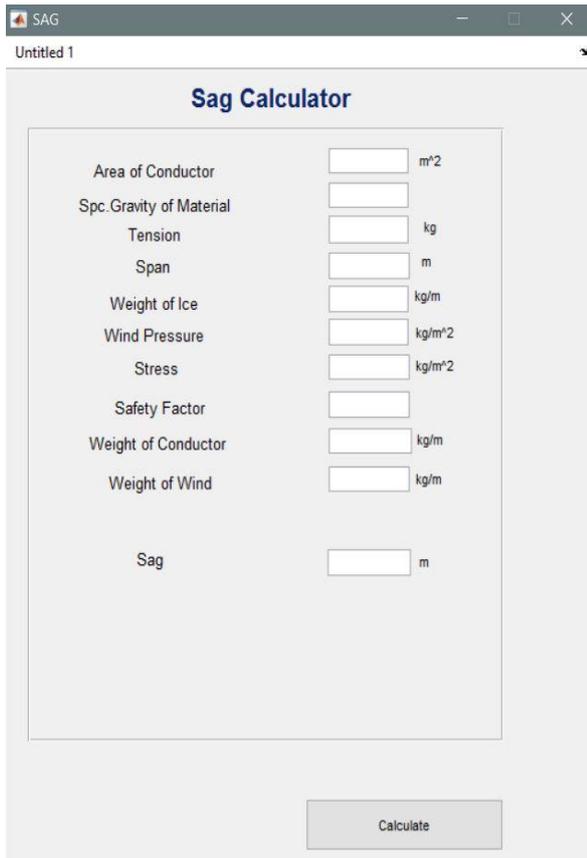

Fig.2. Block Diagram of the Proposed Model

Fig.3. Frontend of MATLAB GUI tool -Sag Calculator

B. Outputs From Proposed Model

In order to deduce the relationship between the value of the parameter of interest and the sag in transmission lines, only that particular parameter field is varied, keeping rest of the parameters fixed to the typical values. In this method, a complete description of behavior of sag on a particular parameter is obtained independently without the influence from other parameters. After obtaining various graphs with respect to each parameter, the critical threshold limits of 5m clearance from lowest point in transmission line to ground level is taken for determining the maximum permissible limit a parameter can take, beyond which there maybe safety breaching concerns. Thus, the value of the parameter determined nearby to the sag value of approximately 5m from ground level determines its critical safety threshold limit. The various data related to transmission line conductors are taken from the standard ACSR data sheets.

1. Dependence of Sag on Wind Loading

In order to obtain the relationship between the sag and wind loading, the rest of the parameters are set to their respective typical values as in Table 1.

The graph of sag for different wind loading can be observed from Fig.4. The sag values for varying wind loading can be observed from Fig. 5. From the graph it follows that for the sag to be around 5m, the corresponding value of wind loading is found to be approximately 3.02 kg/m, which is the critical safety

threshold value. Taking about half of this critical safety threshold value gives the typical safety value, which is about 1.51 kg/m.

Table 1. Data related to transmission line conductor for wind loading dependence

Area of Conductor	0.0000469 m2
Specific Gravity	7.8140
Tension	2860.2 kg
Span	183 m
Weight of Ice	0 kg/m
Wind Pressure	100 kg/m2
Stress	15000000 kg/m2
Safety Factor	1
Weight of Conductor	1.628 kg/m
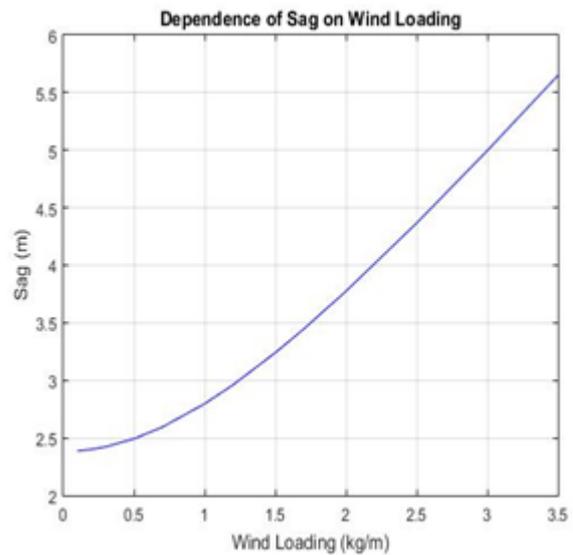
Fig.4. Dependence of Sag on Wind Loading

Wind Loading (kg/m)	Sag (m)
0.1	2.38720
0.2	2.40062
0.3	2.42282
0.5	2.49255
0.7	2.59362
1.0	2.79631
1.2	2.96004
1.5	3.23989
1.7	3.44497
2.0	3.77432
2.5	4.36637
3.0	4.99558
3.5	5.64956

Fig.5. Wind loading and its corresponding sag values

2. Dependence of Sag on Weight of Ice

In order to obtain the relationship between the sag and weight of ice, the rest of the parameters are set to their respective typical values as in Table 2.

Table 2. Data related to transmission line conductor for ice loading dependence

Area of Conductor	0.0000469 m2
Specific Gravity	7.8140
Tension	2860.2 kg
Span	183 m
Weight of Wind	1.51 kg/m
Wind Pressure	100 kg/m2
Stress	15000000 kg/m2
Safety Factor	1
Weight of Conductor	1.628 kg/m

The sag values for varying ice loading can be observed from Fig.6. The graph of sag for different ice loading can be seen from Fig.7. From the graph, it follows that for the sag to be around 5m, the corresponding value of weight of ice is found to be approximately 1.45 kg/m, which is the critical safety threshold value. Taking about half of this critical safety threshold value gives the typical safety value which is about 0.7 kg/m.

Weight of Ice (kg/m)	Sag (m)
0.1	3.0900
0.2	3.16085
0.3	3.35520
0.5	3.51047
0.6	3.63100
0.7	3.75323
0.8	3.82702
0.9	3.90222
1.0	4.12872
1.2	4.48518
1.5	4.97724
1.7	5.24268
2.0	5.74580
2.5	6.42815

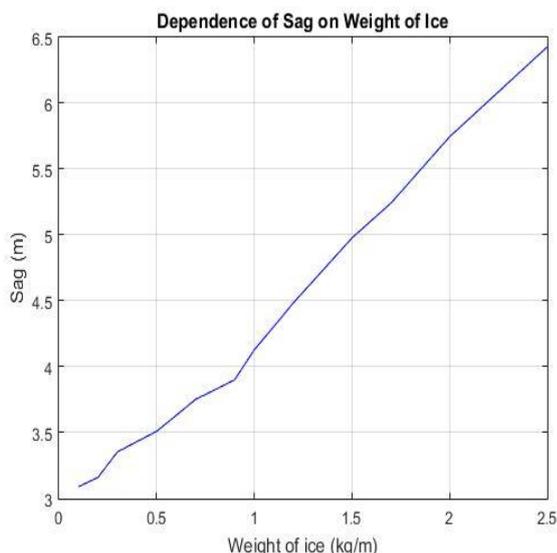Fig.6. Wight of Ice and its corresponding sag values

Fig.7. Dependence of Sag on Weight of Ice

3. Dependence of Sag on Conductor Weight

In order to obtain the relationship between the sag and conductor weight, the rest of the parameters are set to their respective typical values as in Table 3.

Table 3. Data related to transmission line for conductor weight dependence

Area of Conductor	0.0000469 m2
Specific Gravity	7.8140
Tension	2860.2 kg
Span	183 m
Weight of Ice	0 kg/m
Wind Pressure	100 kg/m2
Stress	15000000 kg/m2
Safety Factor	1
Weight of Wind	1.51 kg/m

The sag values for varying conductor weight can be observed from Fig. 8. The graph of sag for different weights on the conductor can be realized from Fig. 9. From the graph it follows that for the sag to be around 5m from ground level, the corresponding value of conductor weight per unit length between two poles is found to be approximately 3.07 kg/m, which is the critical safety threshold value. Taking about half of this critical safety threshold value gives the typical value, which is about 1.6 kg/m, which is a close approximation to the typical weight per unit length of industry-grade conductors like Aluminum Conductor Steel Reinforced (ACSR).

Conductor Weight (kg/m)	Sag (m)
1.0	2.6385
1.1	2.72241
1.2	2.81144
1.3	2.90512
1.4	3.00301
1.5	3.10472
1.6	3.20988
1.7	3.31816
1.8	3.42927
1.9	3.54295
2.0	3.65894
2.5	4.26703
3.0	4.90899
3.5	5.57314
4.0	6.25241

Fig.8. Sag values for different conductor weight
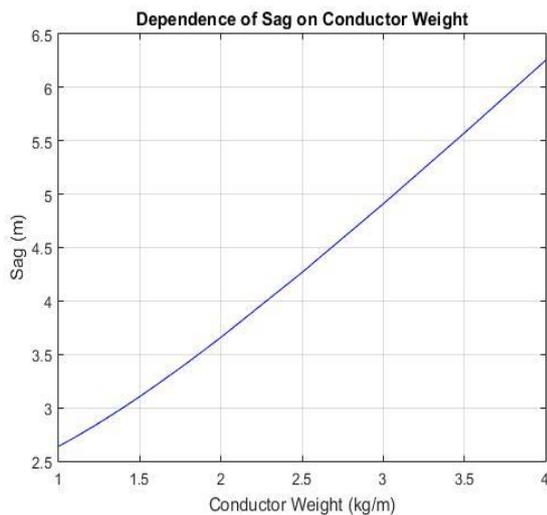
Fig.9. Dependence of Sag on Conductor Weight

Tension (kg)	Sag (m)
3000	3.08892
2950	3.14127
2900	3.19543
2860	3.24012
2800	3.30955
2700	3.43213
2600	3.56413
2500	3.70670
2300	4.02902
2000	4.63337
1850	5.00905
1800	5.14819
1600	5.79172
1500	6.17783

Fig.10. Sag values for different Tension

4. Dependence of Sag on Tension

In order to obtain the relationship between the sag and tension present in the conductor, the rest of the parameters are set to their respective typical values, as in Table 4.

Table 4. Data related to transmission line conductor for Tension dependence

Area of Conductor	0.0000469 m2
Specific Gravity	7.8140
Weight of Wind	1.51 kg/m
Span	183 m
Weight of Ice	0 kg/m
Wind Pressure	100 kg/m2
Stress	15000000 kg/m2
Safety Factor	1
Weight of Conductor	1.628 kg/m

The sag values for different tension can be observed from Fig.10. The graph of sag for different tension in the conductor is seen from Fig.11. It can be seen that the graph in Fig.11 represents a negative correlation between the magnitudes of tension and the sag, implying that more is the tension in the transmission line, lesser is the sag and vice versa. From the graph, it follows that for the sag to be around 5m from ground level, the corresponding value of tension present in the conductor is found to be approximately 1850 kg, which is the critical safety threshold value.
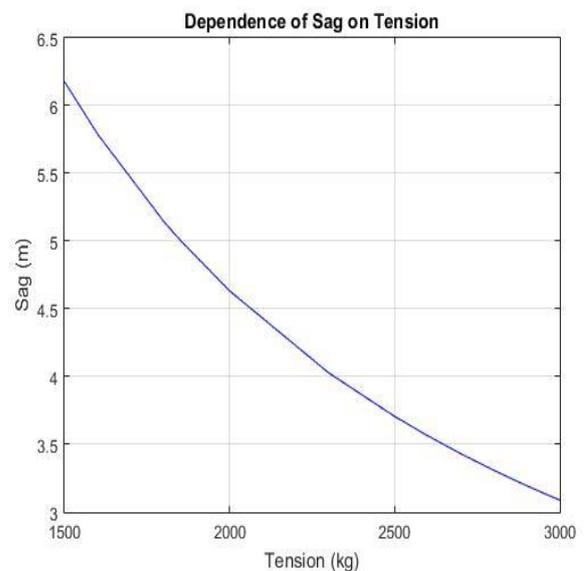

Fig.11. Dependence of Sag on Tension

5. Dependence of Sag on Span Length

In order to obtain the relationship between the sag and the span length, the rest of the parameters are set to their respective typical values, as in Table 5.

Table 5. Conductor data for span length dependence

Area of Conductor	0.0000469 m2
Specific Gravity	7.8140
Tension	2860.2 kg
Weight of Ice	0 kg/m
Wind Pressure	100 kg/m2
Stress	15000000 kg/m2
Safety Factor	1
Weight of Conductor	1.628 kg/m

The sag values for different span length can be observed from Fig.12. The graph of sag for different span lengths is seen from Fig.13. It can be seen that the curve denotes a non-linear relationship with positive correlation, implying that the increase in span length also increases the sag as the total weight acting throughout the length of the conductor between two support poles increases. From the graph of Fig.13 it follows that for the sag to be around 5m from ground level, the corresponding value of span length is found to be approximately 230m, which is the critical safety threshold value.

Span Length (m)	Sag (m)
100	0.96745
110	1.17061
120	1.39313
130	1.63499
140	1.89620
150	2.17676
160	2.47667
180	3.13454
200	3.86980
220	4.68246
230	5.11781
250	6.04656

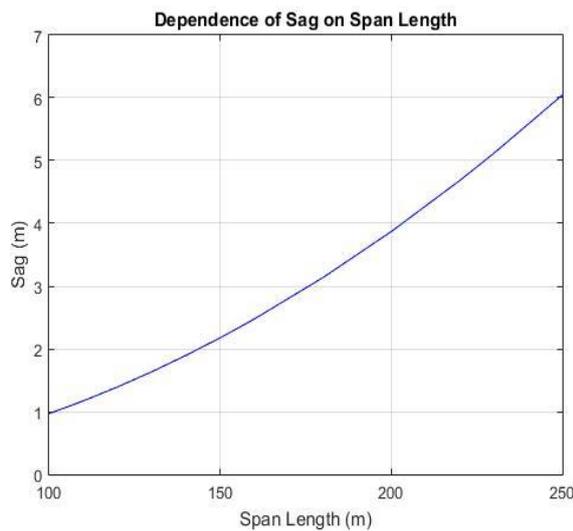Fig.12. Sag values for different span lengths

Fig.13. Dependence of Sag on Span Length

C. Comparison With Existing Models

Though there are various mathematical models proposed previously on the topic of interest, the Abebe's Sag Tension Model Ref. [8] gives the favorable theoretical base work and data sets best suited for comparison and validation of the proposed model, and also it is noteworthy to mention here that other models do not take uncertainty into consideration. The results obtained for the parameters-wind loading, weight of ice and conductor weight are compared with Abebe's Sag

Tension Model and we see that our proposed model has better accuracy and efficiency as it gives the critical safety threshold value at much lower levels from safety perspective which is validated through Fig. 14, 15 and 16. The other drawback of Abebe's Sag Tension Model is that it varies all parameters simultaneously, along with wind loading (Fig.14). This does not contribute to the resultant sag values because rest of the parameters act vertically downwards, whereas wind acts horizontally and thereby do not have any collateral effect on the sag in transmission lines subjected to varying wind loads. Also the authors of Ref. [8] follow a deterministic approach, whereas for situations like open transmission line conductors which are subjected to high level of uncertainty should be dealt in a probabilistic manner as followed in the proposed method.
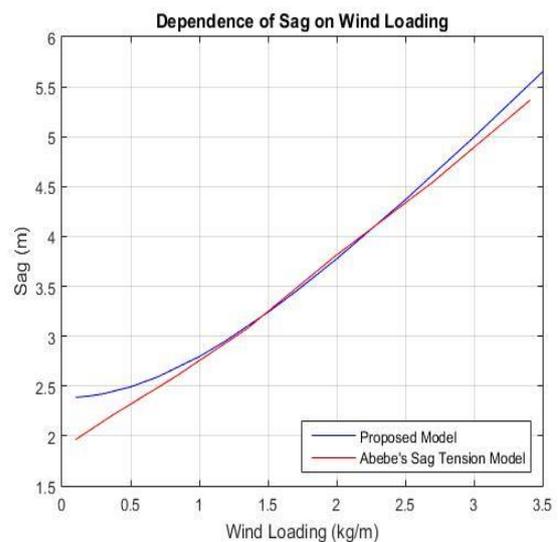

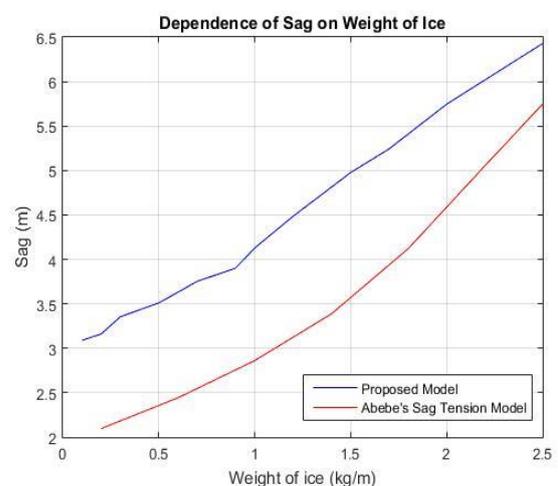Fig.14. Comparison Graph of Sag v/s Wind Loading

Fig.15. Comparison Graph of Sag v/s Weight of Ice

The comparison graph plots of both the proposed model and Abebe's Sag Tension Model are varied for the values of weight of ice over the same intervals(Fig. 15), keeping the rest of the parameters at their respective typical values. It can be noticed that there is a small

discrepancy between the two plots; the reason being that Abebe's Sag Tension Model took into account varying all the parameters simultaneously, including tension. It has been noticed that tension has a negative correlation with respect to the sag, implying that higher the values of sag present in the conductor, lower is the resultant sag values. Hence, the dip in the graph of Abebe's Sag Tension Model is because of the effects acting on behalf of tension variances in the transmission lines. However, the graph obtained from the proposed model accurately indicates all the values of the parameters which the transmission line is subjected to while computing the sag values by varying the weight of ice independently.

The proposed model keeps all other parameters constant (Fig. 16) while only the weight of the conductor is varied, which accounts for different industry grades of conductors being used; unlike in Abebe's Sag Tension Model where the sag values are calculated by varying other parameters simultaneously to obtain the results.
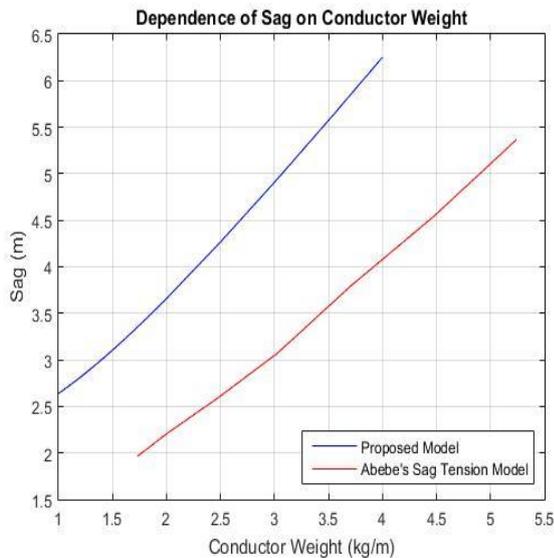

Fig.16. Comparison Graph of Sag v/s Conductor Weight

Table 6. Comparison of Proposed Model with other Existing models

Reference Paper	Methodology	Safety Parameter Considered	Uncertainty Addressed
[4]	Iterative technique	Stress	No
[16]	Computer Program	Temperature	No
[17]	Finite difference scheme	Ice loading	No
[18]	Analytical method	Tension	No
[8]	Interval Arithmetic/Basic Affine Arithmetic	Wind Loading, Ice Loading, Conductor weight	Yes
Proposed Model	Affine Arithmetic approach	Wind Loading, Ice Loading, Weight of conductor, Tension, Span Length	Yes

Therefore, the results obtained from the proposed model can be interpreted for the dependency relationship between sag and the conductor weight with more accuracy as it is not influenced by the effects of other parameter variations. The proposed model is also compared with other related works as shown in Table 6.

From the table we see that the proposed method addresses all safety parameters giving a complete behavioral model with respect to sag and its dependency giving importance to uncertainty to solve real time issues.

V. CONCLUSION

The authors have proposed an affine arithmetic model to estimate the safety parameters of AC Transmission lines and are successful in envisaging the complete behavior of sag in conductors incorporating the feature of isolating each of the parameters like wind loading, ice loading, conductor weight, tension and span length, whose effects are studied in detail, so that dependence of sag in transmission line can be calculated independently for that particular parameter and are executed in the MATLAB integrated development environment with GUI processing and graphing tool. The overall benefit of the proposed method can be attributed to the efficacious findings of critical safety values against each of the many parameters considered with high degree of uncertainty modelled in a probabilistic approach rather than in a deterministic way. It is noteworthy to mention here that a comparative study was also carried out taking existing work into consideration and the results after comparison showed a positive correlation. With 50% factor of safety and for a ground clearance of 5m, it can be verified from the graph results that the critical safety threshold limit for wind loading is found to be about 3.02 kg/m and the typical value is about 1.51 kg/m, the critical safety threshold limit for weight of ice is found to be about 1.45 kg/m, whereas its typical value is about 0.7 kg/m, the critical safety threshold limit for conductor weight is found to be about 3.07 kg/m, whereas its typical value is about 1.6 kg/m, which is a close approximation to the typical weight per unit length of industry-grade conductors like Aluminum Conductor Steel Reinforced (ACSR). The critical safety threshold limit for tension in the conductor and the span lengths is 1850 kg and 230m respectively.

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