Backlash Detection in Geared Mechanisms: Modeling, Simulation, and Experimentation

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ABSTRACT

Backlash is a common fault that occurs in geared mechanisms, and can produce inaccuracy or uncontrollability of the mechanism. It is shown that, by modeling backlash as microscopic impact, its presence can be detected and possibly measured using only simple sensors. The first stage of modeling establishes the need for the use of nonlinear elastic and damping forces in establishing the gear-teeth reaction force. The second stage of modeling used a detailed digital multibody simulation to develop and test the effectiveness of the proposed detection approach. Experiments verify the efficacy of the methodology in practice. Finally, the results can be quantified for a testbed mechanism, indicating that this methodology may be useful for condition monitoring of industrial mechanisms.

1. Introduction

The use of gears in power transmission, instrumentation, and as an integral part of position devices has become almost ubiquitous. Consequently, any faults in gear transmission severely affect the performance of such systems. These faults can be diverse and, depending on the situation, can affect the performance of the system considerably. Common examples of such faults are broken gear teeth, backlash between the teeth, interference between the teeth, and eccentric mounting of mating gears. But regardless of the source of degraded performance and the chances of preventing the occurrence of such faults altogether, early detection of the cause (faults) can prevent severe damaging consequences in many cases.

This work concentrates on the detection of backlash between mating gear teeth, for two reasons. First, it is the most common fault found in any geared systems; although some backlash is essential for any gear transmission, less than the appropriate amount results in interference between the teeth whereas excess backlash introduces looseness into the system. In either case, the result is poor performance and possible damage to the system. Second, the effect of backlash is most conspicuous when the system is subjected to non-continuous motion with frequent reversal of the direction of rotation. Mechanisms that fall into this category include robotic manipulators which have become an integral part of manufacturing automation. As the use of robotic manipulators increases, the detection of faults which can severely reduce their performance becomes increasingly important.

The primary objective of this paper is to propose a measure that can be used to detect backlash in a geared mechanism, a goal that is approached by successively elaborating a contact model. The first step is the development of a mathematical model for gear mechanisms with backlash. Second, a measure is developed that has
the potential both to detect small backlash in the mechanism and to describe the progression of backlash as the
gear-teeth wear. Third, a detailed digital multibody simulator is developed, to simulate backlash and to detect
its presence appropriately using the proposed measure. Finally, physical experiments are conducted to verify the
practicality of the proposed measure.

This paper is organized as follows. Relevant literature is surveyed in Section 2, and the elementary dynamics
of spur gears is reviewed in Section 3. Section 4 is devoted to the modeling of contact between gear teeth. A
diagnostic measure for backlash is presented in Section 5, and a model of one such mechanism is developed into a
digital dynamics simulator in Section 6. The results obtained from the simulations are presented in Section 7, with
an experimental testbed and experimental results described in Section 8. Quantification of the results is presented
in Section 9. Finally there is a summary of the contributions as well as drawbacks of this approach.

2. Literature Survey

When there is backlash between the mating gears the initial contact can modeled as an impact phenomenon.
Gear impact is generally approximated by a linear model [14, 16, 15], although the limitations of this approxima-
tion in impact modeling have been firmly established [13]. The study of the transient behavior of the gear pair,
however, requires an understanding of the exact form of the impacts between the gear teeth and consequently
the research into this area often bears more resemblance to impact studies [6, 7, 12, 13]. The dynamic model
of mating gears is actually a rotary version of the classic “impact pair”, first conceived by Kobrinskii [17] for
the study of vibrations in systems with clearances. Yang and Sun [27] proposed a rotary version of this impact
pair model. The effect of gear backlash has a destabilizing effect on the controller once the gears become part
of a closed-loop control system; transient effects of gear motion due to backlash and other nonlinearities act as
destabilizing forces, which Tustin [25] showed can result in limit-cycle instabilities.

Although much emphasis has been placed on the dynamic modeling of gear backlash, the research towards
the detection of such a fault has been minimal. Signal processing and noise analysis are well developed subjects
[21, 1], but they mostly deal with signals from continuously rotating machines. Dagalakis and Myers [5, 4, 3] were
the first to propose a technique to detect backlash in robotic systems. Their technique was based on the analysis
of a coherence function between the motor voltage and the acceleration of the robot arm, given joint excitation
by bandlimited random signals. Their method, although it may work in some settings, has three problems: 1) a
coherence function is sensitive to noise and may not be useful for noisy signals, 2) mounting accelerometers
on the driven shafts may pose instrumentation problems in some cases, and 3) since their method is empirical,
generalization of the technique is not readily apparent. To overcome these problems, Stein and Wang [22, 23]
proposed a technique based on the analysis of momentum transfer to detect backlash in mechanical systems.
They have derived an analytical expression relating the magnitude of the backlash to the change in the speed of
the primary gear due to impact with the secondary gear. Both simulations and experiments have been conducted
to verify their algorithm. Their algorithm is robust and easy to implement, since it requires only a velocity sensor
attached to the servomotor. There also have been some efforts towards mechanical fault detection using techniques
derived from artificial intelligence [2] as well as techniques such as building a Luenber ger state observer and used
it to detect incipient faults [26].

Our work is, in a sense, complementary to the work of Stein and Wang [22, 23]. Stein and Wang have
developed an algorithm to sense the change in velocity between the gear teeth, which can arise from tooth-tooth
impact due to backlash. Their method actually is an integration of the force of impact to obtain momentum. We,
on the other hand, concentrate on the force domain directly to obtain an impulse signal generated at the gear teeth
interface and analyse it to detect the presence of backlash. It is hypothesized here that backlash is a dynamic phenomenon that can be detected by physical manifestations of the small impacts of the gear teeth. This paper develops a technique for detection of incipient backlash that relies on impact signals.

3. Gear Dynamics

Geared mechanisms can vary considerably depending on the type of gears used in the systems. A pair of spur gears was chosen for study, because it is the most commonly found gear pair and is amenable to considerable modeling. Before presentation of the backlash detection model and related techniques, it is necessary to discuss some background concepts related to spur gear meshing.

3.1 Spur-Gear Dynamic Equations

When two involute gear teeth are meshed, the contact point will always travel along the common normal line, which is tangential to the two base circles. The details of spur gear dynamic equations have been developed by Yang [27] and Gerdes [10], but the final equations are presented here for the sake of completeness.

When there is no contact

\[ \tau_1 = J_1 \ddot{\theta}_1 \]  \hspace{1cm} (1)
\[ \tau_2 = J_2 \ddot{\theta}_2 \]  \hspace{1cm} (2)

and when there is contact

\[ \tau_1 - R_{b1}[F + G] = \ddot{\theta}_1 \]  \hspace{1cm} (3)
\[ \tau_2 + R_{b2}[F + G] = \ddot{\theta}_2 \]  \hspace{1cm} (4)
where $\tau_i$ is the applied torque, $R_{bi}$ is the base circle radius, $J_i$ is the moment of inertia and $\theta_i$ is the angular displacement of gear $i$. The contact force is composed of two components: $F$ — elastic restoring force and $G$ — damping force. The sign of Equations (3) and (4) will change if the direction of rotation changes.

To test whether there is contact or not, there is a contact constraint equation related to backlash:

$$|R_{b1}\theta_1 - R_{b2}\theta_2| \geq B$$

where $B$ is the backlash which must be measured along the common normal.

### 3.2 Number of Contacting Teeth

In order for gears to operate smoothly, a pair of teeth must come into contact before the previous pair ceases to touch. This requirement creates a period of overlap where two pairs of teeth are in contact and the effective stiffness of the gear pair increases. The interactions between a pair of meshing gears is a periodic process and it repeats after every base pitch, $P_b$, which is defined as:

$$P_b = \frac{2\pi R_b}{N}$$

where $N$ is the number of teeth. The base pitch is the same for each gear. Another important parameter is the contact ratio $m_c$, defined as

$$m_c = \frac{\Delta s_c}{P_b}$$

where $\Delta s_c$ is the length of contact measured along the line of action.

The detailed derivation can be found in Gerdes [10]. Suffice it to say that analytical expressions can be found to determine when there is one pair of teeth contact and when there are two pairs of teeth contact. It is important to note that these equations involve $\phi_0$, the operating pressure angle. Since $\phi_0$ is a function of the actual center distance between the gears given by

$$\phi_0 = \cos^{-1}\left(\frac{R_{b1} + R_{b2}}{C}\right)$$

the contact regions are dependent on the actual center-to-center distance ($C$) and therefore on backlash. When $C$ increases, the contact ratio decreases and eventually falls below unity, signifying that there will be periods when there will be no contact at all between the gears resulting in increased vibration.

Tooth thinning is another source of backlash, which must be accounted for in an accurate determination of the number of gear teeth in contact. An analytical expression for backlash at any center distance can be found in Gerdes [10], and from it the expression for linear backlash can also be determined.

### 4. Contact Models

The nature of contact force between the gear teeth is of utmost importance for the development of a backlash detection model. Contact models can be distinguished by different models of elastic force, damping force and coefficient of restitution. A brief discussion of some of the more important models follows.
4.1 Nature of Elastic Force

The elastic force developed in gear teeth contact can be approximated by applying Hertzian contact theory to the teeth, assuming that the involute profiles of the teeth are cylinders at the point of contact. The exact analytical expression for the half-width of the surface of contact can be obtained from Timoshenko [24] or Goldsmith [11]. Yang and Sun [27] have used this expression to derive an expression for penetration which gives a nonlinear relationship between the applied force and the elastic interpenetration. They have also linearized the relationship without appreciable loss of accuracy. In such a case, it is possible to obtain a linear force-displacement relationship in the form

\[ F = K\delta \]  

(9)

where the stiffness is \( K = \frac{\pi E L}{4(1-\nu^2)} \), \( L \) is the thickness of the gears, \( E \) is the Young’s modulus, and \( \nu \) is the Poisson’s ratio.

When two pairs of teeth are in contact, the stiffness is assumed to be double this value. It should be remembered that two main assumptions are made in deriving this expression: the radii of curvature are much greater than the half-width, and the axes of the cylinder are perfectly parallel such that ideal line contact occurs. Although the first of these assumptions seems safe with respect to the gear teeth, the validity of the second one may be questionable. To address this issue, we make use of Hertzian contact theory in its generalized form where force-displacement relationship obeys a power-law

\[ F = K\delta^n \]  

(10)

where \( n = 1 \) for line contact and \( n = \frac{3}{2} \) for point contact. Hunt and Crossley [13] have suggested that the law given in Equation (10) be used with a power \( 1 < n < \frac{3}{2} \).

4.2 Nature of Damping Force

Different methods have been proposed to simulate the damping force, including a linear model [6], a power-law relationship of interpenetration and approach velocity [13], and a direct solution of the surface equation [12, 18].

Although a linear model of the damping force is simplest, such a model exhibits two characteristics that are inconsistent with the physical systems. The Kelvin-Voigt model, which is a linear model of both stiffness and damping of the form \( F_c = K\delta + c\dot{\delta} \) predicts physically impossible force phenomena: a discontinuity in the force upon impact, and a tensile component in the hysteresis loop. (For a detailed study see Hunt [13]). A hysteresis loop that does not exhibit the inconsistencies of a linear model can be obtained [13] from a contact force of the form

\[ G = \frac{3}{2}\alpha K\delta^n \dot{\delta} \]  

(11)

where \( \alpha \) is found from a straight line approximation of the coefficient of restitution as a function of initial impact velocity \( \dot{\delta}_i \)

\[ e = 1 - \alpha \dot{\delta}_i \]  

(12)
A further refinement of this model, suggested by Herbert and McWhannell [12], incorporates a nonlinear coefficient of restitution of the form \( e = 1 - \alpha \dot{\delta}_i^\beta \).

A final approach for obtaining the contact force is to assume a coefficient of restitution and solve the surface equation directly under the boundary conditions such that the damping force goes to zero when \( \delta = 0 \) and when \( \dot{\delta} = 0 \) [12, 18]. Yang and Sun have taken this approach [27] and their expression for the contact force is:

\[
F_c = K\delta^n + \frac{6(1 - e)}{[(2e - 1)^2 + 3]} \frac{K}{\delta_i} \delta^n \dot{\delta} \tag{13}
\]

The hysteresis loop predicted by Equation (11) and Equation (13) differs by 1.5 percent [12]. Equation (11) is used to simulate the damping force for our backlash detection model because it exhibits greater numerical stability at low contact velocities.

### 4.3 Coefficient of Restitution

Approximation of the coefficient of restitution of an impact as a linear function of impact velocity causes a severe reduction in the accuracy of the contact model[13], which is especially significant when there are repetitive impacts. Herbert [12] showed that accuracy is improved considerably by modeling the coefficient of restitution as \( e = 1 - \alpha \dot{\delta}_i^\beta \). The values of \( \alpha \) and \( \beta \) can be obtained from experimental impact data, e.g. from Goldsmith [11]. One point that needs to be raised with respect to the use of this data is the dependence of the coefficient of restitution on the geometry of the impacting bodies. Since the vibration established in an object by an impact removes kinetic energy from the system, the coefficient of restitution decreases as the vibrational energy increases. Spheres, cylinders, and gear teeth are very different objects, so an attempt to use experimental data from one such object to find coefficient of restitution for another is not strictly valid. However, since there is no reliable experimental data for spur-gear tooth impact, this work uses the best published data [11].

### 5. Diagnostic Approach

This section describes the principle, the hypothesis, and a proposed qualitative measure for detection of backlash in geared mechanisms.

#### 5.1 Principle

A proposal of this paper is that backlash can be modeled as an impact generated between a pair of gear teeth. In the force domain, an impact will produce an impulse signal. Since such an impulse is due to a small impact caused by the clearance between the teeth, it is necessary that no other (large) signals overshadow the impulse signal. It is therefore further proposed that the output link of a gear-driven mechanism be fixed so that it cannot move in space, which would ensure that the effect of gravitational and inertial torques do not interfere with any measurement. To maximize the impulse response of the impact dynamics, the gear train should be driven by square wave signals and the impulse response of the gear teeth transduced by torque sensors attached to the gear shafts.
5.2 Hypothesis

It is hypothesized that, if the gear train is driven by square-wave excitation and torque signals are gathered by sensors mounted on the gear shafts, noticeable differences will appear between the signals between a gear pair with no backlash versus a pair with a small amount of backlash. The difference will be due to the detailed nature of the dynamics of the impacting bodies. The presence of backlash will result in an increased impact velocity and, consequently, an impact force that produces a larger torque signal. It is therefore proposed that backlash can be detected by exploiting this impact phenomenon.

5.3 Qualitative Measures: Amplitude Probability Functions

The amplitude probability density is a potentially useful means of characterizing continuous time-domain data from a statistical point of view [19, 20]. This function represents the probability a random variable will take on a specific amplitude value within a specified interval. Experimental data never comes from a continuous sample space because of sensor limitations, so one is always interested in probabilities connected with intervals and not with isolated points. If \( P(x) \) is the probability of \( x \) where \( x \) lies between \( x_1 \) and \( x_2 \) then

\[
P(x_1 \leq x \leq x_2) = \int_{x_1}^{x_2} p(x) \, dx
\]

where \( p(x) \) is the probability density function.

The values of a random variable must be acquired from a continuous space by sampling, so the set of discrete data can only approximate the actual physical variable. Consequently, the amplitude probability density must also be constructed from discrete values. Consider \( N \) data values of a variable \( x(t) \). The discrete probability density function can be estimated [1] by

\[
\hat{p}(x) = \frac{N_x}{NW}
\]

where \( W \) is a narrow interval centered at \( x \), and \( N_x \) is the number of data values that fall within the range \( x \pm \frac{W}{2} \). Hence an estimate \( \hat{p}(x) \) is obtained digitally by dividing the full range of \( x \) into an appropriate number of equal-width class intervals, tabulating the number of data values in each class interval, and dividing the number of values in a class interval by the class width \( W \) and the sample size \( N \). It should be noted that the estimate \( \hat{p}(x) \) is not unique, since it depends on the number of class intervals and their width selected for the analysis. We have used a fixed number of class intervals with equal width, to able to compare the results obtained for different cases. The window size, in our analysis, has been taken to be one complete wave length of the time waveform of the torque signal.

The discrete amplitude probability density can be considered as the amplitude probability distribution (APD), which is a measure that describes the amplitude distribution of the signal based on computation of the probability density function. The APD shows the percentage of time for which the signal lies within a certain amplitude range, and is particularly applicable to analysis of signals for machine monitoring.

In the context of backlash detection by gear-teeth impact, it is expected that for a constant square-wave excitation the dynamic response of the gears will differ with the amount of backlash (that is, with the clearance between the mating surfaces of the teeth). This is because the clearance permits “windup” of the driving gear, and an increase in its kinetic energy. As backlash increases, the driving gear will strike the loading gear with a greater initial velocity and there will be more energy to dissipate. Some researchers [22, 23] have analysed this
phenomenon in time domain. We, however, use an APD technique which we have earlier shown to be effective in signal analysis [20]. In the analysis of gear backlash, we suggest that the motion effects resulting from impact should produce marked changes in the APD of the torque signal from either gear shaft.

6. Modeling and Simulation

In order to test the veracity of the proposed diagnostic approach, a model of a single-link manipulator was developed. This model was then incorporated into a detailed multibody-dynamics simulator in order to digitally test the proposed approach.

6.1 Modeling

To test the proposed method of backlash detection, we model a system consisting of:

- a DC electric servo-motor;
- a flexible coupling to the gear-box;
- a pinion gear;
- a load gear;
- a flexible coupling to the external load; and
- a rigid-body linkage.

In the above system, the servo-motor is used to excite the gear train that consists of the pinion and load gears. The flexible coupling, both at the driver and driven side, represent sensors whose torsional deformations are used to detect the torques generated due to impact. In reality, strain gauges are mounted on them to measure torque as will be seen in a later section describing experimental set-up. The rigid-body linkage represents load on the gear train. Thus the above system, even though simple, captures the essential elements of a real gear transmission mechanism.

Each component of the system is modeled as a linear second-order lumped-parameter system. The only source of nonlinearity comes from the modeling of contact interaction between the gear teeth. This contact interaction couples the driver and the driven side of the gear train. The result is a set of coupled nonlinear second-order differential equations which cannot be solved in closed form, but which can be numerically simulated. A schematic of such a system is given in Figure 2.

6.2 Dynamic Equations

The nomenclature for the dynamic model is given in Table 1. Using this nomenclature, the motor/gear/link system can be treated as a pair of multibody systems, which are coupled by the nonlinear tooth interaction force as described in Section 4. The equations of motion of the motor/input-coupling/pinion system are
Figure 2: A gear-train, with excitation and load link.

Table 1: Nomenclature For Dynamics Model

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_M(t)$</td>
<td>Time-varying motor excitation torque</td>
</tr>
<tr>
<td>$\theta_M$</td>
<td>Motor position</td>
</tr>
<tr>
<td>$J_M$</td>
<td>Motor inertia</td>
</tr>
<tr>
<td>$b_M$</td>
<td>Motor damping coefficient</td>
</tr>
<tr>
<td>$\theta_I$</td>
<td>Relative position of input coupling</td>
</tr>
<tr>
<td>$J_I$</td>
<td>Input-coupling inertia</td>
</tr>
<tr>
<td>$k_I$</td>
<td>Input-coupling stiffness</td>
</tr>
<tr>
<td>$b_I$</td>
<td>Input-coupling internal damping coefficient</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>Relative position of pinion gear</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>Relative position of load gear</td>
</tr>
<tr>
<td>$F$</td>
<td>Tooth-tooth elastic restoring force</td>
</tr>
<tr>
<td>$G$</td>
<td>Tooth-tooth internal damping force</td>
</tr>
<tr>
<td>$\theta_O$</td>
<td>Relative position of output coupling</td>
</tr>
<tr>
<td>$J_O$</td>
<td>Output-coupling inertia</td>
</tr>
<tr>
<td>$k_O$</td>
<td>Output-coupling stiffness</td>
</tr>
<tr>
<td>$b_O$</td>
<td>Output-coupling internal damping coefficient</td>
</tr>
<tr>
<td>$\theta_L$</td>
<td>Relative position of load link</td>
</tr>
<tr>
<td>$J_L$</td>
<td>Inertia of load link</td>
</tr>
<tr>
<td>$M_L$</td>
<td>Mass of link</td>
</tr>
<tr>
<td>$C_L$</td>
<td>Distance of center of mass of link from axis of rotation</td>
</tr>
<tr>
<td>$\ddot{g}$</td>
<td>Gravitational vector</td>
</tr>
</tbody>
</table>
where $M_1$ is the inertia matrix, computed in the usual manner. Similarly, the load/output-coupling/link system is described as

$$
M_2 \begin{bmatrix}
\ddot{\theta}_2 \\
\ddot{\theta}_O \\
\ddot{\theta}_L
\end{bmatrix} = \begin{bmatrix}
+R_{62}[F + G] \\
b_o \dot{\theta}_O + k_o \theta_O \\
M_L C_L \sin(\theta_L)\ddot{\theta}_L
\end{bmatrix}
$$

where, again, $M_2$ is a inertia matrix.

6.3 Digital Simulator

In order to verify the usefulness of this dynamics model – and of the idea that backlash can be detected by gear-tooth impact – an extensive C-language computer simulation was developed. The simulator is a multibody-dynamics code that is intended for applications where high accuracy is desired (as opposed to, for example, super-real-time performance). In order that future work could easily model backlash in complicated robotic linkages, the code employs Featherstone’s articulated-body inertias [9] as the fundamental dynamics representation.

As shown in previous work [8], the choice of numerical integration routines can have dramatic effects on the simulated behavior of a system. A simple example of such differences arises when computing the hysteresis loop of gear-tooth interaction using Hunt and Crossley’s model [13]. Figure 3 shows a simulation conducted for 15ms of modeled time of the interaction when a back-differentiation formula is used – the BDF code, developed specifically for simulation of mechanisms. Choosing a high-order Runge-Kutta integration – the DVERK code – produces the correct behavior, as shown in Figure 4. Note that the BDF code overestimates the inertial effects of the gear, and the teeth rebound and begin to interpenetrate in the opposite direction. This phenomenon was also found in earlier work [8], where it led to dynamic instabilities. the DVERK integration routine was thus for the simulator.

7. Simulation Results

Extensive computer simulations were conducted to verify the backlash detection methodology. A number of parameter variations were studied; potentially relevant parameters were identified, and for various values the dynamics were simulated and the resulting time-domain signals were plotted for visual comparison. First, we varied the flexibility of both the input and output shafts by means of the couplers that represent torque sensors. It was observed that for material stiffness values typical of metals such as aluminum and steel, there was no appreciable qualitative change in the time waveform of the torque signal. Hence, we chose values of shaft flexibilities from standard engineering texts. Similarly, realistic values of viscous friction that were obtained from different motor data sheets did not cause any qualitative change in the torque signal. Both the amplitude and frequency of the

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1 All integrators used in these simulations are available on the Internet, from netlib@ornl.gov, and are refereed codes.
Figure 3: Impact hysteresis loop, back-differentiation simulation

Figure 4: Impact hysteresis loop, DVERK simulation
square wave excitation were varied, and it was found that the qualitative characteristics of the time waveforms remain largely unchanged provided that the amplitude and frequency are not extremely high or low. The amplitude of the excitation signal is chosen so that it provides enough impulse to be detected by the torque sensor, but not so high as to saturate the sensor. The frequency, on the other hand, is chosen so that it does not excite resonance or chaos in the system. Basically, our objective is to extract signatures using an excitation signal of small amplitude and moderate frequency.

We did not study the effect of Coulomb friction and assumed it to be negligible. Sensor noise, even though important, was also not studied. The rationale was to match the ideal simulation results with the experimental results: if the experimental results matches well, then neglected variables are not likely to be significant factors.

Our findings can be explained by using the results of a sample simulation. This simulation employed parameter values derived from specifications of the components of our experimental system, which will be presented in Section 8.² The values for the key parameters to this simulation are presented in Table 2.

<table>
<thead>
<tr>
<th>Table 2: Values of the key parameters used in the simulation.</th>
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<tbody>
<tr>
<td>No. of teeth in input gear</td>
</tr>
<tr>
<td>No. of teeth in output gear</td>
</tr>
<tr>
<td>Gear angle</td>
</tr>
<tr>
<td>Gear width</td>
</tr>
<tr>
<td>Gear pitch diameter</td>
</tr>
<tr>
<td>Gear inertia — input</td>
</tr>
<tr>
<td>Gear inertia — output</td>
</tr>
<tr>
<td>Motor inertia</td>
</tr>
<tr>
<td>Motor damping</td>
</tr>
<tr>
<td>Input-sensor stiffness</td>
</tr>
<tr>
<td>Input-sensor damping coefficient</td>
</tr>
<tr>
<td>Excitation frequency</td>
</tr>
<tr>
<td>Excitation Amplitude</td>
</tr>
</tbody>
</table>

Although the model and simulator have the provision for incorporating an output torque sensor (i.e. torque sensor at the driven side of the gear train), the output sensor was not used in the detection methodology.

The time waveforms of the torque signals with and without backlash are shown in Figure 5. When there is no backlash, there is no impact between the teeth and consequently the amplitude of the signal is low. The small ripples are due to the elasticity of the gear tooth. But, as soon as a very low (50 microns) backlash is introduced, a much higher amplitude of torque is predicted. If the APD of these two wave forms is plotted (Figures 6), two conspicuous features appear: the peaks are distinctly lower when there is backlash, and the amplitudes show a greater spread in the presence of backlash. Both of these results are the direct manifestation of the impact phenomenon.

²It can sometimes be instructive to tune the simulation parameters close to the real parameters and see how the simulation results fare against experimental results, but the effort is usually time-consuming and in high-order nonlinear systems can be very difficult. The goal of this paper is detection, rather than exact dynamics modeling, so parameter tuning was not performed.
Figure 5: Simulation results: Time waveform of the torque signals of the input torque sensor; solid line is without backlash and dotted line is with 50 microns backlash.

Figure 6: Simulation results: APD of the input torque sensor signal when there is no backlash (solid line) and when there is 50 microns backlash (dotted line).
8. Experimental Results

In addition to the detailed simulation studies, physical experimentation was conducted to test the proposed diagnostic approach. A single degree-of-freedom manipulator was built, comprised of the following components:

- one Yasakawa DC servo motor (DM-373) with a peak torque of 0.7 N-m,
- two same size #303 stainless steel spur gears (NORDEX LAS-D1-40), each with 40 teeth, 20 degree pressure angle, and 0.03175 meter (1.25 inch) pitch diameter,
- input torque sensor which is a hollow aluminum cylinder instrumented with four strain gauges (CEA-13-062UV-350), mounted diametrically opposite (two gauges together on each side) to eliminate bending effects. This is attached to the motor output shaft and input gear shaft,
- several bearings, and
- an output link

A function generator (WAVETEK model 19) and a power amplifier (Brüel & Kjaer power amplifier type 2706) were used to excite the manipulator at 10Hz., and WAVEPAK data acquisition and analysis software were used to detect incipient backlash. The experimental data presented here is obtained with a sampling frequency of 1000Hz. A picture of the testbed is given in Figure 7.

![Experimental testbed](image-url)
Figure 8: Experimental results: Time waveform of torque signals from input torque sensor; when there is no backlash (solid line) and when there is 50 microns backlash (dotted line).

Figure 9: Experimental results: APD of torque signals from input torque sensor; when there is no backlash (solid line) and when there is 50 microns backlash (dotted line).
The time-domain waveform and APD data from the experiment are shown in Figures 8 and 9 respectively.

These results confirm the validity of the proposed backlash detection diagnostic, as do the simulation results. As a very small backlash (50 microns) is introduced by changing the center distance between the two gears by inserting precise shims, there is a marked increase in amplitude of the torque signal because of the impact. The APD results confirm the lowering of peaks and spreading of the data away from the peak with the introduction of backlash.

8.1 Discussion

It is evident from the experimental results that it is possible to detect backlash in a physical system by the proposed diagnostic approach. Additionally, the results obtained from the computer simulations closely match those from the physical experiments. This conformity indicates that the mathematical modeling embodied in the digital simulator captures the essential features of the physical world that is analysed.

The efficacy of the diagnostic approach is evident. It is a qualitative measure, but it can be made more meaningful by quantification of the results so that the shape of the APD can be translated into a measure that gives a better perspective to the users. The following section describes a means of quantifying the change in shape of the APD of the torque signal associated with increasing backlash.

9. Quantification of the Results

The variations in the shape of the APD curves are visually apparent, but it is necessary to develop a means of quantifying such differences if this information is to be employed in an automated diagnostic system. With this goal in mind, two measures were examined. The first was kurtosis which describes the “peakedness” of the function and is defined as the 4th moment [19]. This measure is sensitive to changes that affect the tails of the function, and is an excellent candidate for detecting the appearance of isolated peaks in the signal; however, since the occurrence of isolated peaks was not observed in backlash APD’s the kurtosis measure was not investigated in detail. The second measure was skewness, which characterizes a function’s degree of asymmetry around its mean value [19]. Although the time-domain data have almost the same mean regardless of backlash, the APD data is vastly different and provides a well defined trend if the skewness measure is applied to the APD data directly.

Skewness appears to capture the variations of APD in a quantitative manner. Because the simulation results closely match physical experimentation, and it is easier to exactly vary simulation parameters than to vary physical parameters, the quantitative analysis of backlash was conducted using digital simulations.

The expression used to compute skewness is:

\[
\text{skewness} = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{x_i - \bar{x}}{s} \right)^3
\]

where \( N \) is the number of data points, \( x_i \) is the individual data value, \( \bar{x} \) is the mean of the \( N \) data values and \( s \) is the standard deviation of the \( N \) data values.

Changes in the skewness of the APD are correlated with changes in the amount of backlash in the gears. This is shown by starting from zero backlash in the gears, computing the dynamic response of the geared mechanism to a square-wave excitation, finding the APD of the input torque sensor for a full period of excitation, and calculating the skewness of the APD. The backlash is then introduced in steps of 25 microns up to a total of 100 microns. The corresponding APD plots are shown in Figure 10; note the corresponding skewness values. Since the amplitude
Figure 10: APD of torque signals from input torque sensor for different backlash values — (a) zero, (b) 25 microns, (c) 50 microns, (d) 75 microns, and (e) 100 microns. (f) shows the corresponding condition monitoring curve
of the time waveform of the torque signal increases with backlash, the peaks of the APD are smaller, and the spans are broader. Also, because there is more time of free flight of the pinion gear for greater backlash-induced clearance, there is a steady development of a peak in the zero-amplitude region as backlash increases (Figure 10 (e)). All of these phenomena are captured by the skewness measure and, as a consequence, the skewness value decreases monotonically with the increase of backlash. Based on this observation one can construct a condition monitoring curve (Figure 10 (f)) which can play an important role in diagnosing the incipience of unwanted backlash.

9.1 General Discussion

The proposed approach may be appropriate for use in monitoring machine condition, but as with most approaches some care must be taken. First, since it is a relative measure, the skewness value of the APD should be recorded when the mechanism is in good operating condition (e.g. when new). Then this value can be used as a basis for comparison. Second, users can construct their own condition monitoring curve by using an authentic simulator and can use it as a guide to analyze the physical situation. Alternatively, it is possible for the manufacturer of the mechanism to provide such monitoring curve so that at any time the user can estimate the magnitude of the backlash in the system. Third, based on the applications and the experience of the user, such monitoring curve can be divided into regions corresponding to different grades of “good” regions to “shut down” regions. Finally, again based on experience, this approach can be automated using a computer hooked up with a data acquisition system. Because all the computer must do is to check the skewness value of the APD, this approach is ideal for automated monitoring.

The proposed approach indicates that it is possible to detect backlash using force-sensor signals. Unlike the approach of Stein and Wang [22, 23], in which backlash is detected using velocity signal sensed by a tachometer mounted on the servomotor, our approach uses a torque signal obtained from a torque sensor mounted on the primary shaft. Our backlash detection method may be harder to implement on existing systems, because it is usually easier to mount a tachometer on a motor than to sense torque from a motor-output shaft. It can be argued however, that since impact is the fundamental manifestation of backlash, one is better equipped to detect minute backlash by sensing a force signal than by sensing or estimating a velocity signal. Alternatively, the invasive instrumentation our method requires could be overcome by using accelerometers instead of torque sensors.

10. Conclusions

We have developed a novel backlash-detection approach that is capable of detecting extremely small backlash without any ambiguity. This approach is especially applicable to robotic manipulators which play an increasing role in manufacturing automation. A detailed model and computer simulation have been developed that incorporate a complicated nonlinear contact model to simulate realistic contact conditions. A qualitative measure has been presented that is capable of detecting the incipience of backlash in the gears, and simulations showed that the measure corroborates intuition. A single degree-of-freedom manipulator was constructed, and extensive experimentation clearly demonstrated the efficacy of the proposed diagnostic approach. Finally a quantitative analysis was explored so that this approach can potentially be automated.

The disadvantage of the approach examined here is that it is invasive. However, preliminary experimentation with an alternative instrumentation – accelerometers mounted on the gear shafts via lever arms – indicate that the invasive nature of the original instrumentation can be surmounted. There are many issues to be explored
when changing sensor technologies, not the least being accelerometer sensitivity to vibrations from physically adjacent mechanisms other than the one under diagnosis; thus, a user interested in detecting incipient backlash is not restricted to one sensor but faces trade-offs.

It is instructive to note that although it is may be difficult to insert a torque sensor into the existing robots it is quite easy to make provisions for built-in torque sensors in the input shafts of various joints for future robots. In such cases, with so simple a methodology as presented here, backlash can readily be detected and its effects nulled by appropriate control schemes.

Finally, the procedure described in this paper requires fixing the output link for the diagnostic test. Although such a requirement may seem artificial and contrived, most calibration methodologies also require that each robot undergoes a calibration test at various times during its operation. During such tests, robots are made to move in an artificial and constrained way, in order to set the reference frames for each joint; if backlash induces similarly undesirable behavior in a mechanism that must be precise or safe, fixation (or inertial loading) of the output link may be a small price to pay for accurate assessment of the condition of the mechanism.

11. Acknowledgments

This research was supported in part by the Manufacturing Research Corporation of Ontario, the Natural Sciences and Engineering Research Council of Canada, and the federal government under the Institute for Robotics and Intelligent Systems (which is a National Centres of Excellence programme).

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