# Geometry of the Early Neolithic

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*Abstract*. Following a visit in the Winter of 2007 to the Alhambra — a Moorish palace built around 1350 CE in southern Spain — I have been digging up the roots of the *Alhambra Theorem*, which says that Moorish craftsmen in the Middle Ages knew that there were exactly 17 crystallographic groups in two dimensions. This fundamental result of modern group theory emerged into the mathematical literature only in 1891. Digging back from the Alhambra, we found an Ur source for the motifs of the Alhambra in the earliest wall-paintings of Çatal Hüyük, around 10 KYA (thousand years ago). Digging deeper led to the cave paintings of Upper Paleolithic Europe, beginning around 35 KYA. There appears to be a straight line of development from the Upper Paleolithic caves to the Neolithic villages of Anatolia, and on to the Alhambra. In this article we tell the story of the geometric patterns found in the Neolithic village of Çatal Hüyük.

NOTE: This first draft is written as a self-standing article, MS#132, and will be adjusted later as a chapter in the book, *Motif*.

# CONTENTS

# 1. Introduction

We were led, in a process of intellectual archeology, from the Alhambra (from 1350 CE) in southern Spain back in time to Çatal Hüyük (founded 7500 BCE).[1] This archeological site provides the first evidence of the patterns of Turkish carpets which are the models for the repeating patterns found in the Alhambra today. The trajectory from Çatal Hüyük to the Alhambra will be treated in a subsequent article. Çatal Hüyük (abbreviated CH in the following) is a major Neolithic site in southern Anatolia. and was discovered by James Mellaart, then Assistant Director of the British Institute of Archeology in Ankara, and his Turkish born wife, Arlette. We begin with their remarkable story.

# 2. Jimmie and Arlette

James Mellaart was born November 14, 1925 in London, grew up in Holland, and studied Egyptology at University College, London before joining the British Institute of Archeology at Ankara (BIAA). In the 1950s he made a survey of prehistoric sites in Anatolia. He led his first dig at Copper Age Hacilar in 1956, discovering Neolithic painted pottery and female figurines. He met Arlette Cenani, a Turkish archeologist, at a dig near Istanbul. They married and had a son, Alan.

In 1958 Jimmie met a Greek woman on a train who had a collection of antique gold ornaments called the Royal Treasures of Dorak. These were claimed to be from a clandestine dig near Dorak, in northwest Turkey. Accompanying her home, he made rubbings of the pieces, and published some drawings in *The Illustrated London News* of November 28, 1959. The pieces later disappeared, and some investigators claim that the Dorak find was a hoax perpetrated on the innocent Mellaart by thieves.[2]

Mellaart had noticed Çatal Hüyük (meaning forked mound) in 1952, while he was assistant director of the BIAA, and Arlette served as its secretary. He visited it and recognized it as a Neolithic site in 1958.[3] It comprises two mounds, CH East and CH West. The first main stage of excavation occupied three summer seasons, 1961-63. In 1964, excavation was interrupted, and Mellaart lectured on Anatolian archeology in Ankara. But after the 1965 season, the Mellaarts were expelled from Turkey, as an aftermath of the Dorak affair, and digging at the site was halted for many years. The Mellaarts moved back to London, where James served as a lecturer at the Institute of Archeology until his retirement in 1991. Reports on his Çatal Hüyük findings were published in various technical articles, and two popular books.[4]

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1 There are many variant spellings of this name. We follow the style of Mellaart.

2 See (Mazur, 2005) and (Balter, 2005; ch. 2).

3 (Mellart, 1967; p. 27)

4 In (Melaart, 1967), soon after the end of operations, and more recently, in (Mellaart, 1989; vol. 2).

Figure 1. Jimmie and Arlette Mellaart with kilims. (Hodder, 2005; frontispiece)

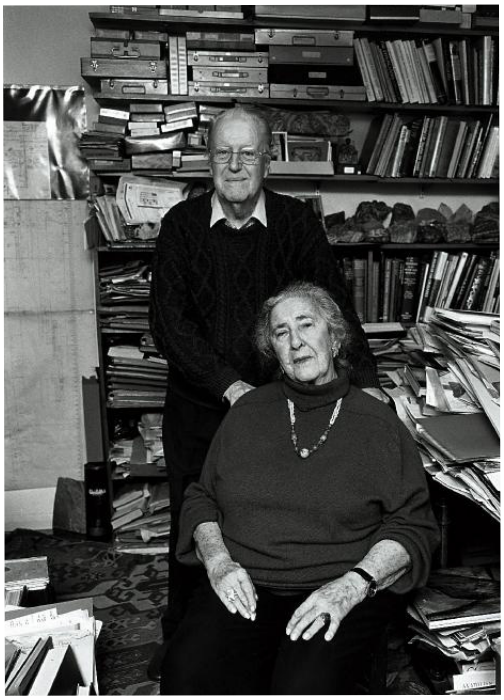

Figure 2. Jimmie and Arlette Mellaart, 2005.
Photo by Charles Hopkinson (www.cornucopia.net).

# 3. Çatal Hüyük

This Neolithic site is among the earliest settlements to rank as a town. Chronologically, it follows nearby Göbekli Tepe[5] (11,000 BP) and Jericho (12,000 BP), 200 miles east in Jordan, and precedes Hacilar, 200 miles to the west in Anatolia. The earliest CH date found so far is 9,300 BP (at CH East), but older layers may yet be uncovered.[6] It is 35 miles southeast of present-day Konya, and consists of two riverside mounds (CH East and CH West) in a fertile landscape at an elevation of 3,000 feet. The older and larger mound, CH East, occupies 32 acres, with Neolithic remains at least 60 feet deep. At the time, it was the largest Neolithic site known in the Middle East. Twelve building-levels were uncovered, with heights up to seven feet or more, filling fifty vertical feet in all. New building-levels were constructed every century or so. Radiocarbon dating during the first stage of excavation, in the 1960s, spanned nine centuries, from 6500 to 5700 BCE. After this date, CH East was abandoned, and CH West created. After another seven centuries, this also was abandoned. The discoveries of Mellaart in both Hacilar and Çatal Hüyük East provide links from the cave art of the Upper Paleolithic in Europe to the Early Chalcolithic art of the Middle East.[7] These links involve primarily the goddess figurines and shaped cave paintings of the Upper Paleolithic.[8]

At CH East, each building had its own walls, of mud brick covered with layers of plaster. Buildings comprised houses, sanctuaries, cult rooms, and shrines. The buildings were crowded together in a matrix, with access from the flat roof, through a skylight joined by a wooden ladder. Each room had built-in furniture-- including benches, sleeping platforms, hearths, and ovens -- covered with fine, white clay. The walls of houses were sparsely ornamented, painted a flat red, or decorated with geometric patterns which may be related to to the iconography of the Paleolithic caves of Europe. Shrines exhibit elaborate wall-paintings of naturalistic subjects. Both houses and shrines are decorated with stylized bucrania. These combine a pillar of brick with the horn cores and skulls of wild bulls. These sculptures are similar to the Paleolithic animal images in the painted caves, where a shamanistic cosmology is represented, with the cave (or house) wall representing a membrane between worlds.[9]

Of the manifest art forms of CH East, we are especially interested in the wall-paintings. Many of these imitate woven woolen rugs, with intricate geometric ornaments.[10] These patterns represent an evolution of geometric imagery that is intermediate between the Paleolithic cave paintings, and the medieval tile decorations of the Alhambra. The abstract geometric imagery is of special interest, as common shamanic symbols from the walls of the caves reappear on the walls of CH East.[11]

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5 (Schmidt, 2008)
6 (Hodder, 2005; p. 6)
7 (Mellaart, 1967; p. 23)
8 See (Mellaart, 1967; p. 180), (Gimbutas, 1991; p. 7), and (Abraham, 2011b; Ch, 1).
9 (Lewis-Williams, 2002)
10 (Mellaart, 1967; p. 108, pl. 29)
11 (Abraham, 2011a)

# 4. Fiber arts from Paleolithic to Neolithic

.Mellaart found wall-paintings in the oldest levels of CH East. These wall-paintings are evolved from Paleolithic cave art, and are not surprising.[12] But the connection between the wall-paintings and woven woolen rugs as early as 7500 BCE may seem surprising, and we must consider now the evolution of fiber arts from the earliest known beginnings.

Natural fibers are abundant in biology for holding tissues together. natural fibers used by humans include vegetable fibers (such as reeds, cotton, hemp, jute, flax, and sisal), animal fibers (such as spider silk and wool).

As the fiber arts field has its own jargon, we might begin with a glossary of terms.

- **fiber**. A class of materials that are continuos filaments, or discrete pieces of filament.
- **thread, yarn**. Continuous filament made of twisted or spun fibers.
- **weaving**. Threads are organized in an interlaced matrix of rows and columns.
- **basket**. Utility vessel woven of reeds or other large plant fibers.
- **cloth, fabric, textile**. Flat piece of woven threads.
- **felt**. Woolen material made of fibers but without threads.
- **rug**. Textile floor covering,
- **carpet**. Rug with a woven backing and pile of twisted tufts knotted into the backing.
- **kilim**. Rug or wall hanging with no pile, the pattern is flatwoven into the cloth.

It is not easy to determine the earliest date of anything, as archeologists never quit digging. But at the present time, we can scan the literature and make this chronology for the fiber arts. (BP means before present.)

- 40,000 BP: probable needles
- 32,000 BP: earliest known threads (flax, wool)
- 27,000 BP: evidence of weaving
- 25,000 BP: representation of clothing on Venus figurines
- 19,000 BP: certain needles
- 10,000 BP: flax cultivation in the Middle East
- 9,500 BP: rugs at Çatal Hüyük (claimed by Mellaart)
- 9,000 BP: beginning of pottery at Çatal Hüyük
- 8,000 BP: surviving fragments of cloth at Çatal Hüyük
- 7,500 BP: linen cloth used in Ancient Egypt
- 7,000 BP: cotton clothing used in the Indus Valley
- 5,000 BP: breeding of sheep for wool, felt

The recent discovery of threads in a series of Upper Paleolithic layers of Dzudzuana Cave in

12 Mellaart, 1967; p. 131)

Figure 3. A wall-painting from Çatal Hüyük East, Level VII (8720-8610 BP), Shrine E.VII/21 showing a stitched border and kilim-type repeating pattern. (Mellaart, 1967; Pl. VIII, p. 152) (Mellaart, 1989; Plate V, #2)

The repeating pattern consists of three nearly identical rows (horizontal strips, or friezes). Each row alternates with red (up-pointing) and black (down-pointing) isosceles triangles with central dots. Ignoring the colors and the dots, the outline pattern amounts to a tesselation (covering) of a plane surface with triangular tiles. This outline pattern exhibits all of the usual symmetries of plane geometry.

Georgia[13] increases the credibility of woven rugs (kilims) being made in Çatal Hüyük in 9,500 BP, as reported (controversially) by James Mellaart. These threads twisted of wild flax and tur wool were spun, knotted, and died black, grey, turquoise, and pink, all by 32,000 BP.[14] The wall-paintings at Çatal Hüyük used pigments derived from minerals, such as iron oxides, copper ores, mercury oxide, haematite, manganese, and galena.[15] The colors found include all shades of red and brown, buff and yellow, pink and orange, mauve, grey, and black. Among the subjects reported are human figures, bulls, vultures, stags, goddesses, hands, architecture, symbols, kilim patterns, and simple patterns. This subjects are strongly suggestive of an evolution from Upper Paleolithic cave painting. The kilim patterns were reported from building-levels III, VI A, VI B, VII, IX, and X, indicating the earliest times of occupation.[16]

# 5. The kilim controversy

The kilim (Turkish for woven material) is a special kind of rug, traditionally associated with Anatolia. My immediate interest in them is for their repeating patterns, as they may be the earliest exemplars, preceding those of the wall-paintings and the frieze patterns of early pottery. This theory, proposed by James Mellaart, is based on the similarity between patterns of the wall-paintings discovered by him during his excavations of the 1960s and those of the Anatolian kilims found today in Anatolian homes and museums worldwide.[17] While it is agreed that the wall-paintings and the kilims exist, which is earlier? Mellaart's hypothesis on the precedence of the kilim was expressed very tentatively in 1967:

> There is one scrap of evidence that proves the use of red cloth; a number of broken beads in Level VI contained red stains of thread and as no red ochre was present in the grave, these can only have come from the thread. Nor is it at all impossible to assume that the red panels over the platforms imitate hangings of red cloth, just as the geometric patterns copy patterned textiles. Rows of holes for fixing such hangings occur in a number of shrines (VII.8; VI.A.10) in a position at the foot of the wall, where such panels with imitation hangings are most often found.[18]

And again in 1967:

> Another series, however, starting with a polychrome painting in shrine IX.8 and continuing with a whole group in levels VIII-VI A, evidently represent textiles and their resemblance to Anatolian 'kilims' (i.e. thin woven rugs) is striking. As weaving was widely practiced at Çatal Hüyük and dying would resent no difficulties, since all the necessary

13 (Kvavadze, 2009)

14 In the Renaissance textile industry, black dye was made from oak gall-nuts, blue from woad, red from madder, scarlet from murex shells, carmine red from tiny cochineal insects, and kermis red from an oriental louse. See (Greenblatt, 2011; p. 287).

15 (Mellaart, 1967; pp. 131-2)

16 (Mellaart, 1967; p. 81)

17 See for example those in (Valcarenghi, 1994).

18 (Mellaart, 1967; p. 150; pl. 8, 29; fig. 44)

> plants for the production of vegetable dyes grow wild all around the mound, the probability that kilims were already produced should seriously be considered. In this connection it is particularly interesting to note that two of these wall-paintings from shrine VII.21 imitate a stitched border.[19]

In fact, Plate VIII of 1967 (our Figure 3) shows one of these paintings in color, including not only the stitched border, but also a fine repeating wallpaper or frieze pattern. The date of Level VII is 8720-8610 BP.

An explicit statement of this "kilim connection" was presented by Mellaart at the 1983 International Conference on Oriental Carpets in London, and in print in a 1984 book on early Turkish tapestries. Controversy immediately sprang up, due to the lack of photographic or material evidence. In 1989 appeared Mellaart's self-defense.[20] As this work is relatively difficult to access, I will include here a brief inventory. In this volume of 100 pages, we find:

- Introduction, continuity from Upper Paleolithic art and religion through the Mesolithic to the Neolithic.
- Chapter 1, CH excavations, review and main findings: the people, economy, material culture, architecture, arts and crafts, and government.
- Chapter 2, CH wall-paintings: 301 rooms were uncovered in the 1960 excavations, 88 had been decorated with pictures, 150 surviving paintings created over a period of 600 years, frequently associated with skeletons buried in the buildings, colors, subjects, similarity to Upper Paleolithic cave paintings, goddesses and gods, compound compositions, mountains, caves, symbols, water nymphs, etc.
- Chapter 3, CH people wore woven clothes, probably both flax and wool, paintings of woven fabrics with geometric patterns.
- Chapter 4, CH wall-painting religious imagery versus later Anatolian kilims.
- Chapter 5, Present distribution of Anatolian kilims.
- Chapter 6, Survival of Anatolian kilims.
- Chapter 7. Place names in central Anatolia.

Mellart's defense of his thesis is spread over these seven chapters, but the main arguments are given in Chapters 3 and 4: it is certain, he claims, that CH people wore woven clothing, so looms must have existed, even though none have been found. Flax has been found. Floor imprints of slit-weave tapestries have been found, dating from 8,000 BP. All are agreed that the patterns found in the wall-paintings have survived in more recent kilims.

The Mellaart book of 1989 did not still the controversy, but both sides of the argument must concede reasonable doubt. Fortunately for us, it does not matter. Whether in wall-paintings or in weavings, repeated geometric patterns evolved to a degree manifesting considerable mathematical sophistication, and now at last we must turn to these patterns.

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19 (Mellaart, 1967; p. 152)

20 (Mellaart, Hirsch, and Balpinar, 1989, Vol. II,)

# 6. Geometric motifs

Photos of CH wall-paintings, and Mellaart's reconstruction of CH wall-paintings, are shown in many of the 23 plates (and 100 pages) of Volume I of the 1989 publication. More were published in 1990 and 1991. Keeping in mind that experts are not agreed on the authenticity of these reconstructions, we will use them as a tentative basis for the emergence of repeated patterns into human consciousness. We will now discuss these plates one-at-a-time.

- **Plate I. Çatal Hüyük.** Comprising 10 photos of the site from the 1960s, showing the site, excavations in progress, plaster reliefs of goddesses, bucrania, Mellaart working on a wall-painting, ochre burials, a piece of woven fabric, and, in slide 9, a wall-painting of great relevance to our enquiry into the prehistorical evolution of geometric thought.
- **Plate II. Hunting scenes and other naturalistic images.** Another 15 photos. The caption reads, in part, *Many of the Çatal Hüyük wall-paintings with hunting scenes are considered to be descendents of Paleolithic cave paintings*. And indeed the similarities are unmistakable. Figure 14 shows a scale copy, not a photo, of a wall-painting with kilim-type patterns. This is the earliest example of a frieze pattern known to us at present. Its location in the site is given as E.VIII/10, meaning site area E, building-level VIII, building number 10.[21] The date for level VIII is given as c. 8800-8720 BP.[22]
- **Plate III. Simple forms of fertility.** Seven photos, including five wall-paintings, all with repeating patterns. The earliest is from building E.VIII/14, 8800 BP.
- **Plate IV. Swastika, symbol of the act of creation.** Ten photos showing the swastika symbol from various cultures.
- **Plate V. The early 'yin-yang'; triangles and rhombs.** Five photos, including three of CH wall-paintings, all with repeating triangular patterns (one is shown above in Figure 3).
- **Plate VI. Rhomb with central point, the symbol of life.** Thirteen photos, including two wall-paintings from CH with repeating patterns.
- **Plate VII. The holy cave, rhomb with 'Parmakli' (Fingers).** Ten photos, including sketched reconstructions of CH wall-paintings paired with similar Anatolian kilims of more recent date. These comparisons are highly significant for our story of the emergence of geometric thinking from Neolithic times onward, but we overlook them here as the reconstructions have been questioned.
- **Plate VIII. The Goddess of fertility with hands on her breasts.** Fourteen photos including one sketched reconstruction of a CH wall-painting.
- **Plate IX. The Goddess of fertility in mirror image.** Twelve photos including four sketched reconstructions of CH wall-paintings. Each exhibits translation symmetry, and mirror symmetry through a central reflection line. The final image exhibits other symmetries as well.
- **Plate X. The Goddess giving birth.**

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21 (Mellaart, 1989; Vol. 1, p. 1)

22 (Mellaart, 1989; Vol. 1, p. xviii)

Four photos, including one sketched reconstruction of a CH wall-painting with two-dimensional translation symmetry.

- **Plate XI. The birth symbol.** Eleven photos, including four sketched reconstructions and one actual wall-painting from CH (incomplete).
- **Plate XII. The Goddess with leopards.** Fifteen photos, including six painted plaster reliefs illustration mirror symmetry. The oldest is from shrine E.VIII/27, dated c. 8800 BP.
- **Plate XIII. The Goddess with vultures.** Thirty-two photos, including two wall-painting photos and six sketched reproductions exhibiting many interesting symmetries which are repeated in later kilims.
- **Plate XIV. Vultures.** Eleven illustrations including two sketched reproductions of CH wall-paintings with interesting symmetries.
- **Plate XV. The bull.** Thirteen figures including six sketched reproductions.
- **Plate XVI. The Goddess and the mountain.** Eight figures, including one sketched reproduction.
- **Plate XVII. Caves and niches.** Sixteen figures, including ten sketched reproductions.
- **Plate XVIII. Different vertical multi-niches.** Ten illustrations, including one sketched reconstruction.
- **Plate XIX. Vertical multi-niches with bulls; heads.** Ten figures, including one sketched reconstruction.
- **Plate XX. Compartment design.** Two kilims.
- **Plate XXI. A late Mother Goddess form.** Three Bronze Age figurines.
- **Plate XXII. Tülü and filikli.** Earliest pile weavings from Anatolia, c. 5000 BP.
- **Plate XXIII. Cult kilim of the Goddess from Anatolia.** One exemplary kilim from later in Anatolia.

In summary, we know that both woven fabric and wall-paintings existed by 9000 BP, but the time sequence is uncertain. Mellaart's reconstructions of wall-paintings strongly suggest that the wall-paintings at CH were copies of kilims, but they are controversial because the originals no longer exist. For our story of the emergence of repeating geometric patterns in world cultural prehistory, the priority does not matter. But we could have a much fuller story if we used the reconstructions.[23]

In Volume II of the 1989 publication, we find photos of 14 actual CH wall-paintings with patterns of special interest (all are from shrines):

Plate I, #9. E.VIII/14.
Plate III, #2, 5. E.VIB/15. #3. E.VII/8. #4. E.VIII/14. #6. E.VIA/50.
Plate V, #1. E.VIII/14. #2, 3. E.VII/21.
Plate VI, #1. E.VII/1. #13. E.VIII/27.
Plate XI. #10. E.VIII/31.
Plate XIII, #1. E.VIII/10. #3. E.VII/23.

In addition, we have 52 scale-copies and reproductions (mostly from shrines):

23 (Valcarenghi, 1994)

Plate II, #14sc. E.VIII/10. (sc = scale copy)
Plate IV, #9. E.V/6.
Plate V. #4. A.III/8.
Plate VI, #3. E.VII/12. #12sc. E.VII/1.
Plate VII, #1. E.VIII/1. #3. E.VIB/22. #5. E.VIB/29. #8. E.VIB/15.
Plate VIII. #13. A.III/11.
Plate IX, #1. E.VIB/12. #2. E.VIB/3. #3. E.VIB/34. #12. E.VIB/2.
Plate X, #2. B.II/1.
Plate XI, #1L. E.VIA/15. #1R. E.VIB/31. #8, 9. A.II/1. #11. E.IX/31.
Plate XII, #8. A.III/2. #12. A.III/11. #16. A.III/4.
Plate XIII, #2sc. E.VIII/10. #4sc. E.VII/23. #5. E.VIB/3. #7. A.III/11. #13. A.III/14.
Plate XIII. #14. F.IV/15. #15. A.II/1. #19. E.VIB/4.
Plate XIV, #1. E.VIB/12. #10. E.V/9.
Plate XV, #2. E.V/3. #3. A.III/14. #4. E.VIB/28. #5. A.III/2. #6. A.III/1. #12. E.VIA/20.
Plate XVI. #7. E.IV/4.
Plate XVII. #1. A.III/11 lower. #2. A.III/11 upper. #3. E.VIB/1. #4. E.VIB/3. #5, 6. E.VIA/34.
Plate XVII. #7. E.VIB/4. #11. E.VII/24. #12. A.II/1. #13sc. E.VIB/1.
Plate XVIII. #1. A.III/11.
Plate XIX, #1. A.III/11.

We will begin by considering the photographed wall-paintings. In five cases we have a scale copy or a sketched reconstruction as well as a photo. In a seven other cases, the photo is clear enough that we may make a crude sketch for ourselves. In addition to the sketched reconstruction of the wall-painting from Shrine E.VIB/1 shown in Plate XVII #1 of the 1989 publication, there is a photo shown in Mellaart (1967).[24] So altogether, we have 13 reliable images of CH wall-paintings. These are:

Case 1. Shrine E.VIII/10: Photo in Plate XIII, #1. Scale copy in XIII #2.
Case 2. Shrine E.VIII/14: I #9.
Case 3. Shrine E.VIII/14: III #4.
Case 4. Shrine E.VIII/14: V #1.

Case 5. Shrine E.VII/1: VI #1.
Case 6. Shrine E.VII/21: V #2.
Case 7. Shrine E.VII/21: V #3
Case 8. Shrine E.VII/23: Photo in Plate XIII #3. Scale copy in XIII #4.

Case 9. Shrine E.VIB/1: Sketched reconstruction in XVII #3.
Case 10. Shrine E.VIB/15: Photo in Plate III #5. Sketched reconstruction in VII #8.
Case 11. Shrine E.VIB/15: III #2.
Case 12. Shrine E.VIA/50: Photo in Plate III #6. Scale copy in III #7.
Case 13. House A.VI//66. Photo in Plate IV #1. Scale copy in IV/2.

24 (Mellaart, 1967; pl. 8 following p. 32)

The interesting five cases of wallpaper patterns are: cases 2, 3, 4-6-7, 5, and 10. The oldest, cases 2, 3, and 4 from building level VIII (before c. 8600 BP), are especially relevant to our story. The three sketches are shown in the following figures. In case sketched reconstructions not supported by photos were considered, then the continuity with later kilims, as illustrated in (Mellaart, 1989, Volume I) would be even more convincing.

# 7. Symmetries of a frieze

It is time now to describe briefly the mathematics of symmetry in the plane. In geometry, the word *symmetry* is a synonym for isometry or rigid motion, meaning the entire infinite plane moves along itself without changing the distances between points. Let G be the set of all symmetries of the plane. Given any two symmetries, say S and T, then they may be combined by applying the motion S, and then applying the motion T. This resulting motion is also a symmetry, usually denoted T*S, and read "T after S" or the composition of S and T. The set G together with this way of composing any two symmetries into another symmetry is a *group*, that is, certain reasonable rules are satisfied. In geometry it has been proven that every symmetry may be expressed as a composite of the four basic symmetries: translation, rotation, reflection, and glide. A translation is a rigid motion by sliding, without any rotation. A rotation means one point is chosen as a center, and then the plane is rigidly rotated around that point. A reflection is determined by the choice a straight line in the plane, called the mirror line, and then the plane is reflected through that line like a mirror image. A glide is the composition of a reflection through a mirror line, followed by a translation along that same line.

Now consider a plane pattern, P. This means just that P is a subset of the plane. Then a symmetry S of G is called a *symmetry of P* if under its motion, the subset P is carried exactly onto itself. The subset of G consisting of all symmetries of a given pattern, P, is a group called the symmetry group of P.

Now we may illustrate these concepts using some frieze patterns. A frieze is a plane pattern lying within a thin strip of the plane, and created by printing a row of images as if with a rubber stamp. That is, a rectangle containing an image is repeated, side-by-side, along the strip. Consider a frieze pattern in a horizontal strip. For such a frieze pattern, five special symmetries are particularly important. These are horizontal translations along the central axis of the frieze (HT), vertical reflections through any vertical line (VR), horizontal reflections through the central axis (HR), glides through the central axis (G), and half-turns (R2, rotation by 180 degrees around a point on the central axis).

Mathematicians proved in the 19th century that any symmetry of a frieze pattern must be a composition of these special symmetries, and that there are exactly seven types of friezes determined by their symmetry groups. Each of these is traditionally described by its crystallographic code, of four symbols. The first symbol is always p. The second symbol is m if there is a vertical reflection symmetry, otherwise I. The third symbol is m if there is a horizontal reflection, a if there is a glide, or I if neither. The fourth symbol is 2 if there is a half-turn symmetry, otherwise I. The

seven frieze types, with their codes, are illustrated in Figure 7.[25]

## 8. Symmetries of Çatal Hüyük

The symmetries represented (all have HT) are:

1. Vultures (VR)
2. Swastikas (all)
3. 2D array of circles (all)
4. Checkerboard (all)
5. Squares, rhombs (all)
6. Red and black triangles with dots (all)
7. Similar to 4 and 6 (all)
8. Vultures (VR)
9. Frieze (linear) structure (HT only)
10. Red and black rhombs (all)
11. Hand prints (VR)
12. Frieze type pattern (VR)
13. Swastikas (HT only).

25 (Coxeter, 1969; p. 48)

Figure 4. Wall painting from CH East, Shrine E.VIII/14 (8800-8720 BP) showing swastika-type forms (one with our outline in black). This is a frieze form, that is, a single row (horizontal strip) with a pattern repeated side-by-side. In this example, the patterns are displaced vertically in each repetition. (Mellaart, 1989, vol. 1; Plate I, #9) This pattern is thought to represent the act of creation. (Mellaart, 1989, vol. 1; Plate IV)

The frieze form occurs in the borders of weavings, and also around the necks of pottery. However, pottery occurs in Anatolia first, as far as we know, in the Chalcolithic period, 8300-7650 BP. Among the important sites for the pottery revolution are CH West and Hacilar. (Mellaart, 1978; p. 23) See frieze pattern 7 in Figure 7.

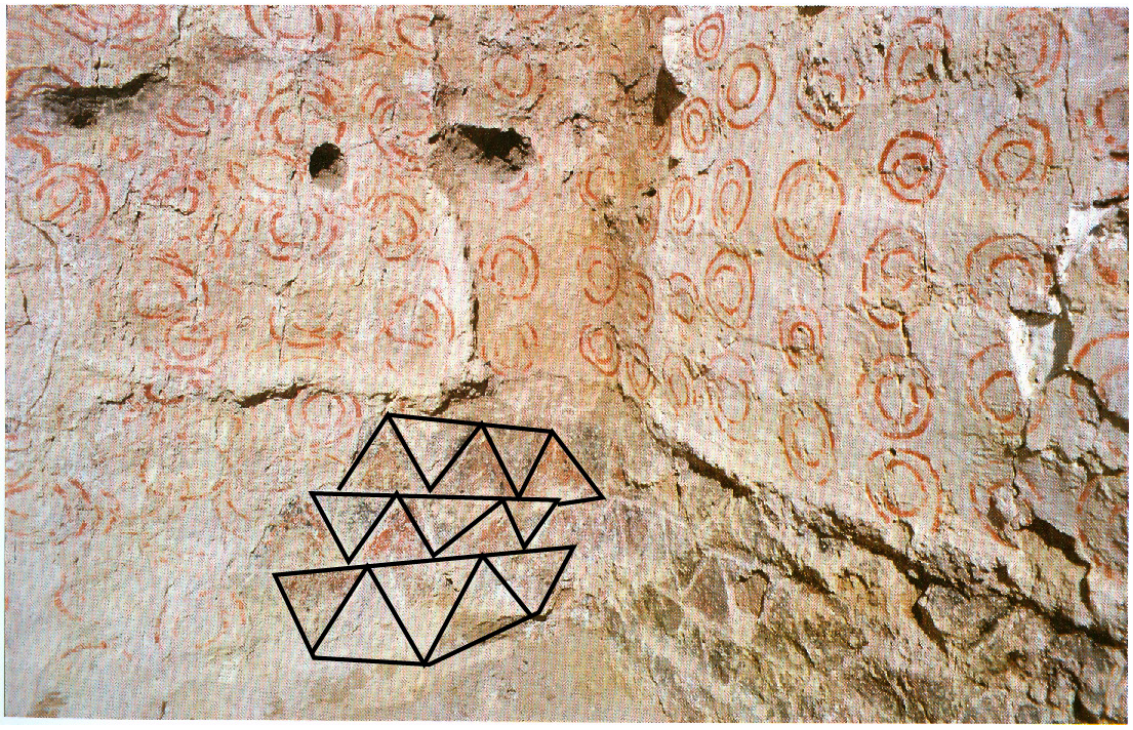

Figure 5. Another wall painting from CH East, Shrine E.VIII/14 with red roundels and red and black triangles, some outlined (our addition) in black. (Mellaart, 1989, vol. 1; Plate III, #4)

These forms are less regular than those of Figure 3, which is from Level VII, a century later. This suggests the possibility that symmetric frieze patterns were evolving in weavings during the time frame of Level VIII. See frieze pattern 5 in Figure 7.

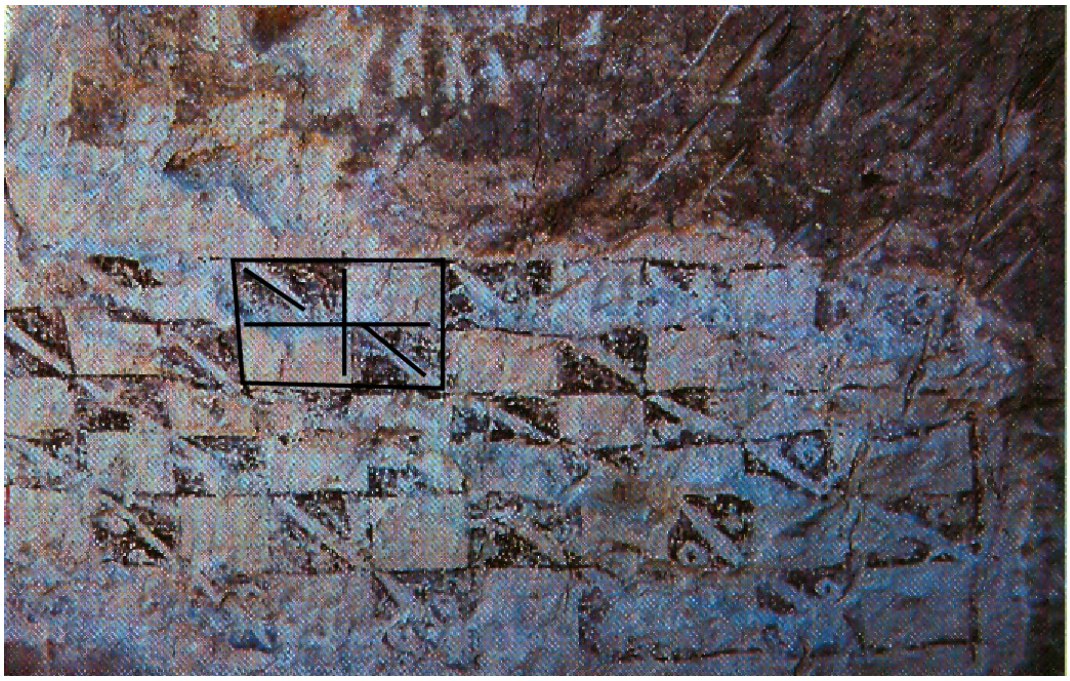

Figure 6. A third wall painting from CH East, Shrine E.VIII/14 showing a very regular repeated pattern. (Mellaart, 1989, vol. 1; Plate V, #1)

One cell (basic unit) is outlined in black (our addition). The cell is repeated with impressive regularity, suggesting as much as anything that a weaving has been copied to the wall. Weaving is the technical innovation which makes precise two-dimensional repeated patterns (which we call wallpaper patterns) possible. See frieze pattern 4 in Figure 7.

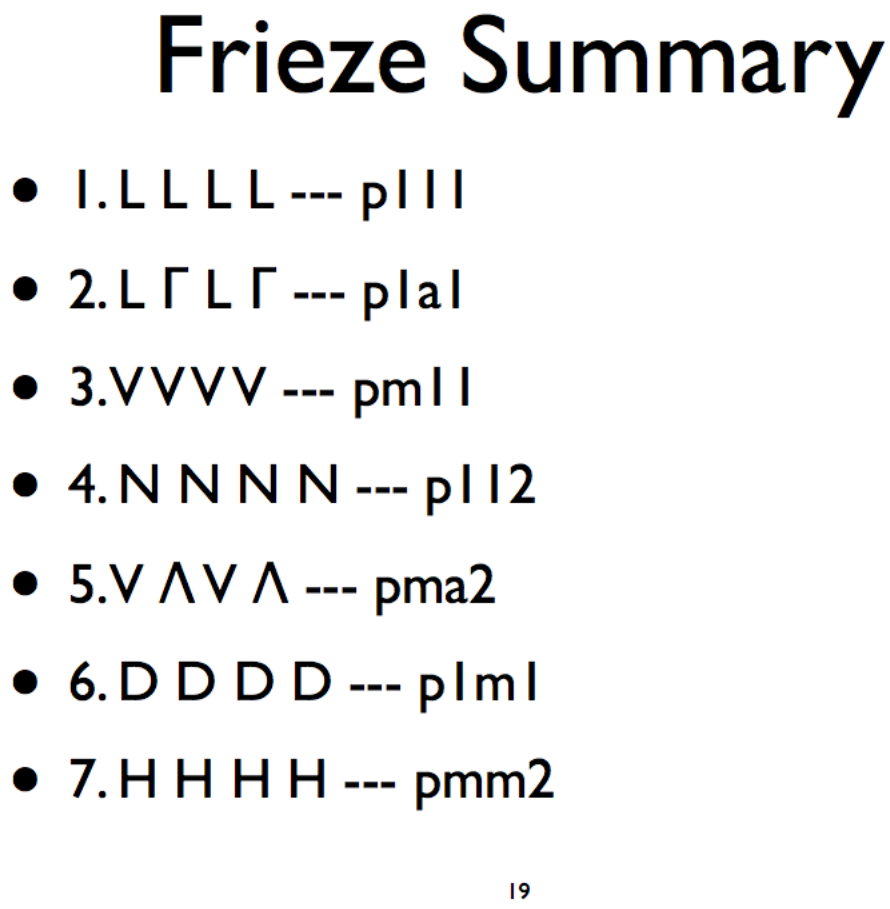


Figure 7. The seven types of frieze symmetry, with an example and crystallographic code for each. Here “LLLL” indicates an endless row of L’s.

# 9. Conclusion

We are interested in the trajectory from the earliest abstract geometric paintings of the Upper Paleolithic caves of Europe to the creation of modern algebra as a new branch of mathematics in the 19th century. The main way-stations are:

*** Chauvet cave, near Toulouse (France)
*** Çatal Hüyük, Anatolia (Turkey)
*** Early Islam, Baghdad (Iraq)
*** The Alhambra, Grenada (Spain)
*** Modern Europe

In this article we have established Çatal Hüyük as a midpoint between France and Spain. Although the Chauvet cave is only 500 miles from the Alhambra (as the crow flies), the path of geometry, clockwise around the Mediterranean, is nearly ten times further, as shown in Figure 8.

The transmission eastward from France to Turkey is indicated by the similarity of the wall paintings. First, the animal paintings are suggestive, at both times and places, of the shamanistic cosmology, as spirit animals emerge from the subterranean world into ordinary reality by bursting through the walls. Secondly, the most common abstract painted images of both include circles, triangles, and hands.[1] By 8,700 BP, the knowledge of frieze patterns had evolved to include at elast four of the seven possible tyes. The further journey westward from Turkey to Spain is supported by the symmetries of the frieze patterns.

1 See (Lewis-Williams, 2002) and (Abraham, 2011).

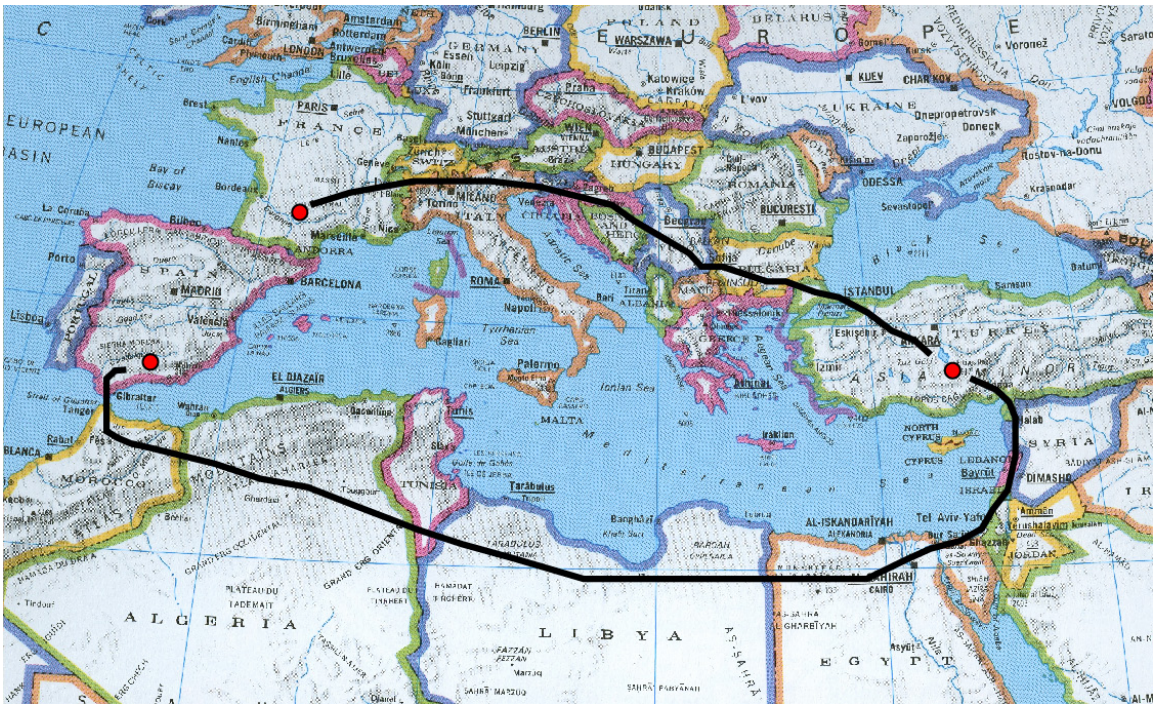


Figure 8. Trajectory of geometric patterns. From the cave paintings of the Upper Paleolithic in the Franco-Calabrian region of southern France, 1500 miles eastward skirting the northern edge of the Mediterranean, to the Early Neolithic in Anatolia. This diffusion took about 25,000 years. The geometric journey then went back westward skirting the southern edge of the Mediterranean to southern Spain, taking another 8,000 years. Only about 500 miles separates the two endpoints.

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