Finite Element Based Analytical Model for Controller Development of Switched Reluctance Machines

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Abstract—This paper presents a novel method of modeling SRM including mutual coupling effect based on Finite Element Analysis (FEA). Once the machine geometry and design is completed in FEA, it is required to analyze the performance of the machine at different operating conditions with different control methods. Designing control algorithm with FEA in the loop is not convenient because of the complexity and simulation time involved. An analytical model performing similar to the FEA model and including the mutual coupling effect is desired during tuning of the control algorithm at different modes and variable speeds of operation. The self and mutual $\lambda$-$i$-$\theta$ characteristics of the machine can be derived from the FEA model and stored in a look-up table. A state space model based on the $\lambda$-$i$-$\theta$ characteristics of the machine is developed which can be readily coupled to any controller model. The method incorporates the mutual coupling effect which cannot be neglected for accurate modeling and good controller performance.

I. INTRODUCTION

Switched Reluctance Machines (SRMs) have several advantages including lower cost and higher reliability for automotive applications [1]-[2]. Traditionally, electrical machine performance analysis is evaluated either through detailed mathematical models or finite element analysis. Both methods have certain disadvantages. In mathematical models, machine parameters are evaluated with formulas based on approximations of flux distributions in the machine. Consequently, physics based mathematical models lack accuracies in the analysis [3]-[18]. More importantly, the mutual coupling effects from other phases are ignored while developing the mathematical model.

Finite Element Methods are used to improve the accuracy of computations. The method has emerged with considerable success as a useful numerical method for solving a wide class of boundary value problems that arise in multidimensional multi-physics problems in the areas of electromechanical analysis, structural mechanics, fluid dynamics, thermal analysis, and acoustic analysis. FEA now serves as the basis for many of today's large general purpose multi-physics systems. It is now further convenient to verify and tune the control algorithm through coupling the finite element software with a dynamic simulation model [3]. FEA methods involve extensive computation and require significant simulation time. In this paper, an improved look-up table based SRM model utilizing the advantages of both formula based method and the FEA based method is proposed to avoid inaccuracies in performance prediction resulting from approximation. The method properly incorporates the mutual coupling effect from other phases.

II. MACHINE MODEL

The machine is designed and optimized in FEA for an application. Machine $\lambda$-$i$-$\theta$ characteristics depend on the geometry design and electromagnetic characteristics of the material used. In this research, the designed SRM from [1] is used as an example to demonstrate the development of the machine model from FEA results.

The $\lambda$-$i$-$\theta$ characteristics of the designed machine is presented in Fig. 1. The machine model can be based on curve fitted mathematical formulae of machine characteristics [4]. An exponential fitting was first proposed in [4] as described in equations (1) and (2).

\[
\lambda_j(i_j, \theta_e) = \lambda_e \left(1 - e^{-i_j f_j(\theta_e)}\right) \tag{1}
\]
\[
f_j(\theta_e) = a + b \cos \left(\theta_e - \frac{(j - 1)2\pi}{m}\right) \tag{2}
\]

where $\lambda_e$ is the saturated flux-linkage, $a$ and $b$ are tuned constants, and $m$ is the number of phases. The exponential fitting is an approximation and does not accurately portray the machine characteristics. The phase flux-linkage $\lambda_j$ is expressed as a nonlinear function of phase current $i_j$ and rotor electrical position $\theta_e$ to account for the magnetic saturation.

![Fig. 1. $\lambda$-$i$-$\theta$ characteristics from FEA for aligned to unaligned position.](image-url)
In this research, a high resolution $\lambda - i - \theta$ characteristics as in Fig. 1 is generated either from FEA design or actual built machine and stored in a look-up table. The corresponding incremental inductance $L_{inc}(i, \theta)$ and incremental flux-linkage $\lambda_{inc}(i, \theta)$, generated from the finite element results as described in equations (3) and (4) are also stored in separate look-up tables. This table is used while solving the machine state space model described later in this section.

The mutual flux-linkage generated from FEA is also required to calculate the mutual torque. The mutual flux-linkage between phase A and phase B generated from FEA is shown in Fig. 2. A fixed amount of current with a stepping command is passed through phase A, and the corresponding flux-linkage is obtained from phase B to capture the mutual flux-linkage component. The mutual flux-linkage for the other phases have the same table with correspondingly phase shifted values.

The co-energy is calculated from the following equation

$$W'(i, \theta) = \int_{0}^{i} \lambda(i, \theta) di$$  \hspace{1cm} (5)

The co-energy for self torque uses a self flux-linkage table generated from the data of Fig. 1 and the co-energy for mutual torque uses a mutual flux-linkage table generated from the data of Fig. 2. The individual phase torque is evaluated from their corresponding co-energy calculated using Eq. (5). The phase torque is evaluated using the differentiation of the co-energy from self flux-linkage with respect to the position as follows

$$T_j(i, \theta) = \frac{\partial W'_j(i, \theta)}{\partial \theta}$$  \hspace{1cm} (6)

The total torque is the summation of individual phase torques and the mutual torques as follows

$$T_e = \sum_{j=1}^{m} T_j + T_m$$  \hspace{1cm} (7)

where

$$T_m = \sum_{p=1,p\neq j}^{m} \frac{\partial W'_m(i_p, \theta_{p,e})}{\partial \theta_{p,e}}$$  \hspace{1cm} (8)

$T_m$ is the lumped mutual torque due to currents in the other phases, $W'_m$ is the co-energy from self flux-linkage, $W'_m$ is the co-energy from mutual flux-linkage, and $m$ is the number of phases. The torque contributed from each phase depends on the rotor electrical position with respect to the phase ranging from a positive maximum to a negative maximum. The $m$-phase system is solved using the state space model described by the following equations:

$$\frac{d\omega}{dt} = \frac{V_j - iR - \frac{\partial W'_m(i, \theta)}{L_{inc}(i, \theta)}}{J}$$  \hspace{1cm} (9)

$$\frac{d\omega}{dt} = \frac{T_e - T_l}{J}$$  \hspace{1cm} (10)

$$\frac{d\theta}{dt} = \omega$$  \hspace{1cm} (11)

Here $V_j$ is the applied phase voltage, $R$ is the phase resistance, $\omega_e$ is the machine speed, $T_l$ is the load torque. The total shaft torque $T_e$ of the machine is the summation of all the phase torques. The model can be used to simulate both motoring mode and generating modes covering four-quadrant operation of the SRM drive system.

III. MUTUAL TORQUE

Mutual torque arises from the current in two or more phases and the flux-linkage among them. During generation of static $T - i - \theta$ characteristics, the other phases have zero current; during machine operation, the other phases have currents controlled by the controller. Thus, both the self-torque and the mutual torque are included for torque calculation in Eq. (6).

In applications where torque ripple is of less importance, mutual torque could be neglected during machine model development. For torque ripple sensitive applications, if the percentage of mutual torque contribution is more than the percentage of torque ripple requirement, then the mutual torque must be modeled during control algorithm development.
A simple FEA simulation can be used to determine the amount of mutual torque on top of the self-torque contributions. First, a fixed amount of current is passed through two phases separately. The torque produced by each phase is added. Then the same amount of current is passed through the same phases simultaneously. The process is explained graphically in Fig. 3. The mutual torque is generated when two or more phases conduct current simultaneously as in Fig. 3(a). In Fig. 3(b) and 3(c), the two phases are conducting currents separately. These two cases only generate self-torque from phase A and B respectively.

![Diagram of torque generation](image)

**Fig. 3.** Self and mutual flux-linkage resulting from currents in two phases, (a) Both Phase A and Phase B are carrying currents (b) Phase A is carrying current (c) Phase B is carrying current.

Equations (5) and (8) is used to model the mutual torque. First, the co-energy from mutual flux-linkage is calculated from Eq. (5) for each phase separately. Then the rate of change of the co-energy with respect to position from each phase is calculated. The mutual torque in phase A depends on the current in phase A. If there is no current in phase A, then the mutual flux-linkage from phase B does not generate any mutual torque in phase A. The result from Eq. (5) is zero in this case. When both phase A and B is conducting current then there are two portions of the mutual torque. One portion is generated in phase A due to the current in phase A. The other portion is generated in phase B due to the current in phase B.

The mutual torque calculated from the model can also be found from FEA simulation. In Fig. 4, $T_a$ and $T_b$ are the generated torque from phase A and phase B, respectively with a 50A fixed current on each phase separately. $T_{ab}$ is the torque with 50A fixed current conducting simultaneously on both phase A and phase B. The difference between the generated torques of the two simulation results provides the mutual torque as a function of rotor position. The results are presented in Fig. 4.

![Graph of mutual torque](image)

**Fig. 4.** Mutual torque for 50A of current in phase A and phase B.

The difference between the summation of $T_a$, $T_b$ and $T_{ab}$ is nearly 5% of the maximum value of $T_a$ and $T_b$. From this result it is obvious that for applications requiring less than 5% torque ripple mutual coupling effects should be considered during control algorithm development.

![Graph of mutual torque comparison](image)

**Fig. 5.** Mutual torque from FEA and analytical model using Eq. (5) and (8).

In Fig. 5 the mutual torque output from the model and the mutual torque from FEA for the simulation setup of Fig. 3 are presented to analyze the accuracy of the model. The results from the model correlates well with the FEA result.

In the method torque ripple minimization using current profiles described in [1], torque feedback is required to fine tune the current profile. The torque feedback can be avoided if the mutual torque model based on mutual flux-linkage is available. The inverse of the static $T_s = i - \theta$ characteristics is
the \( i - T_s - \theta \) characteristics, where \( T_s \) represents the self-torque. The required current profile for a fixed torque command can be obtained from the \( T_s - i - \theta \) characteristics according to the method described in [1]. The final current profile is obtained through torque feedback and running the machine for several electrical cycles. A mutual torque model of \( T_m(i_l, \theta_e) \) will eliminate the iterative fine tuning process and provide the final current profile more readily. The percentage of difference related to mutual torque found from the results presented in Fig. 4 do not represent the torque ripple with current profiles. This result is obtained with constant currents in the phase, which is different from errors obtained with current profiles. The percentage depends on the value of the current and rotational position. The shaft torque ripple can be minimized with accurate modeling of the torque that includes mutual torque contribution.

IV. SYSTEM MODEL

For the development of a controller algorithm it is important to have a reliable model which performs similar to the FEA model, but requires significantly less computation time. For example, the current control algorithm developed in [3] can be easily analyzed with the developed high fidelity model. For solving the machine model the block diagram presented in Fig. 6 is used. The machine model can be based on the final designed FEA characteristics or the built machine characteristics.

To analyze the model accuracy, the \( T - i - \theta \) characteristics of the designed machine was generated by solving the system of equations given in Fig. 6(a). A fixed current command from the controller from 0A to 100A with steps of 4A was used and the system was solved in the sequence shown in the block diagram of Fig. 6. The result of the solved \( T - i - \theta \) characteristics is presented in Fig. 7. The \( T - i - \theta \) characteristics generated from the FEA design is found to accurately match the results found in Fig. 7. For generating the \( T - i - \theta \) characteristics, a fixed amount of current was passed through that phase, while the other phases were held at zero currents. As a result the \( T - i - \theta \) characteristics don’t incorporate any mutual torque.

![Fig. 7. \( T - i - \theta \) characteristics from 0 to 100A.](image)

![Fig. 8. Simulation result of machine torque and phase currents using the proposed model at 1000 rpm.](image)
The model is useful for high performance controller development, especially for systems where the mutual coupling effects cannot be neglected. Another set of simulation was performed applying the predictive current control algorithm described in [3] to the high performance torque ripple minimization algorithm in [1] with SRM models with and without mutual torque incorporated in the modeling. The real time torque feedback is no longer necessary for controller development with modeling of the mutual torque. Mutual torque, which varies with position, is the primary source of torque error. The other sources of torque errors do not contribute as a torque ripple in the shaft, since those are mainly speed and torque level dependent. These torque errors do not vary with position.

The results are presented in Fig. 8 with the simulation performed at 1000 rpm. A 50µs time step equivalent to 20 kHz has been used in the simulation similar to the real time DSP controller. The current profile shown is the one tuned with the mutual coupling effect considered; the controller with this profile produces output torque with minimal torque ripple. The results without the mutual torque model with the same current profile has a significant ripple as shown in the figure. For torque ripple minimization applications, the mutual torque is the key component that necessitates real time torque feedback from the machine to fine tune the current profiles. It has been found that the controller algorithm with the developed model performs similar to the coupled FEA and dynamic simulation approach.

V. MACHINE MODEL FROM EXPERIMENTAL CHARACTERISTICS

The \( \lambda - i - \Theta \) characteristics of the designed machine generated from FEA was used in the simulation model. However, to analyze the performance of the real built machine the \( \lambda - i - \Theta \) characteristics of the built machine is required. A 3-phase, 12/8 SRM was built following the design procedure in [1]. The \( T - i - \Theta \) characteristics of the built machine shown in Fig. 9 is collected using a highly calibrated torque sensor. The \( \lambda - i - \Theta \) characteristics is calculated from

\[
M'(i_j, \theta_e) = \int_0^{\theta_e} T(i_j, \theta_e) d\theta_e
\]

\[
\lambda_j(i_j, \theta_e) = \frac{\partial M'(i_j, \theta_e)}{\partial i_j} + \int_0^{i_j} L_{min} d\theta
\]

Here, \( M' \) is the intermediate co-energy and \( L_{min} \) is the machine inductance at the unaligned linear region. The corresponding \( \lambda - i - \Theta \) characteristics of the built machine is shown in Fig. 10. The model for the built machine can be established to analyze the performance of the controller using the \( \lambda - i - \Theta \) characteristics shown in Fig. 10 which has been developed from the measured \( T - i - \Theta \) characteristics.

VI. CONCLUSION

The SRM machine model can be built based on FEA based characteristics or measured machine characteristics. The results presented in section III show how the model can be used to analyze a predictive current control algorithm [3] and to improve the algorithm for four quadrant torque ripple minimization [1]. Indeed the model can be used for performance analysis of other high performance applications such as precision tooling, robotic automation, trajectory tracking, actuator control, etc. The results generated from the finite element analysis compares accurately to the combined model with mutual coupling effect incorporated.
REFERENCES


