IJAREEIE

e-ISSN: 2278-8875
p-ISSN: 2320-3765

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

Volume 11, Issue 6, June 2022

INTERNATIONAL STANDARD SERIAL NUMBER INDIA

Impact Factor: 8.18

| e-ISSN: 2278 – 8875, p-ISSN: 2320 – 3765| www.ijareeie.com | Impact Factor: 8.18|

||Volume 11, Issue 6, June 2022||

|DOI:10.15662/IJAREEIE.2022.1106029 |

Solving Electric Transmission Line Wave Equations via Double Gupta Transform

Rahul Gupta[1], Rohit Gupta[2]*, Dr Rakesh Kumar Verma[3]

Lecturer, Dept. of Physics, SDMP Public Hr. Sec. School, Karwanda Balwal, Jammu, India [1]

Lecturer, Dept. of Applied Sciences (Physics), Yogananda College of Engg. and Tech., Jammu, India [2*]

Assistant Professor, Dept. of Applied Sciences (Mathematics), Yogananda College of Engg. and Tech., Jammu, India [3]

*Corresponding author

ABSTRACT: The term electric transmission line adds up to a set of wires belonging to upright electrical conductors furnished with outstanding electric insulation, and operated for transmission of electric energy. Normally, the popular wave equations of electric transmission lines are looked over by the Fourier integral transform, Laplace transform, and techniques of variation of parameter. These techniques are very convenient for looking up differential equations. The Gupta transform is a fresh integral transform that has been put in to look up initial value problems that appear in distinct areas of science and engineering. This paper hands out with solving the wave equations on an electric transmission line with inconsequential leakages through electric insulation to ground via Double Gupta transform. The Double Gupta transform will pan out to be a very fruitful mathematical device put in to look up the popular electric transmission line wave equations.

KEYWORDS*:* Double Gupta transform, Electric Transmission Line.

I. INTRODUCTION

The term electric transmission line adds up to a set of wires belonging to upright electrical conductors furnished with outstanding electric insulation and operated for transmission of electric energy. Normally, an electric transmission line has a resistance R, an inductance L, a capacitance C, and shunt conductance G. These four parameters structure the parameters of the electric transmission line and their values turn on the kind and erection of the electric transmission line. Normally, the popular wave equations of electric transmission lines are looked over by the Fourier integral transform, Laplace transform, and techniques of variation of parameter. These techniques are very convenient for looking up differential equations. This paper hands out with solving the wave equations on an electric transmission line with inconsequential leakages through electric insulation to ground via Double Gupta transform.

II. MATERIAL AND METHOD

Taking into account a semi-infinite electrictransmission linewith a perpetual voltage V_0 put in at its transmitting end at t = 0. If $v(z\,,t)$ *and* $i(z\,,t)$ are the voltage and the current at (z, t), then the equations reporting the advancement of current and voltage on a droppingelectrictransmission line [1-5]are written as

$$-\frac{\partial V(z\,,t)}{\partial z} = RI(z\,,t) + L\frac{\partial I(z\,,t)}{\partial t} \;\ldots\ldots\ldots\ldots\ldots\; (1)$$

and

$$-\frac{\partial I(z\,,t)}{\partial z} = GV(z\,,t) + C\frac{\partial V(z\,,t)}{\partial t} \;\ldots\ldots\ldots\ldots\ldots\; (2)$$

Differentiate (1) w.r.t. z and (2) w.r.t. t and simplify, we have

$$\frac{\partial^2 V(z\,,t)}{\partial z^2} = \{LC\frac{\partial^2}{\partial t^2} + (\mathrm{LG} + \mathrm{RC})\frac{\partial}{\partial t} + RG\}V(z\,,t) \;\ldots\ldots\ldots\; (3)$$

Differentiate (1) w.r.t. t and (2) w.r.t. z and simplify, we have

$$\frac{\partial^2 I(z\,,t)}{\partial z^2} = \{LC\frac{\partial^2}{\partial t^2} + (\mathrm{LG} + \mathrm{RC})\frac{\partial}{\partial t} + RG\}I(z\,,t) \;\ldots\ldots\ldots\ldots\; (4)$$

These (3) and (4) represent the normal wave equations for a lossy electrical transmission line [1-5].

| e-ISSN: 2278 – 8875, p-ISSN: 2320 – 3765| www.ijareeie.com | Impact Factor: 8.18|

||Volume 11, Issue 6, June 2022||

|DOI:10.15662/IJAREEIE.2022.1106029 |

In view of inconsequential leakages, we putG =0 andL = 0, because these parameters are responsible for leakages on the electrictransmission line [5]. Therefore, we can rewrite (3) and (4) as follows:

$$\frac{\partial V(z,t)}{\partial z} = -RI(z,t) \ldots\ldots\ldots (5)$$

and

$$\frac{\partial I(z,t)}{\partial z} = -C\frac{\partial V(z,t)}{\partial t} \ldots\ldots\ldots (6)$$

The initial conditions are

$V(z,0) = 0, I(z,0) = 0, V(0,t) = \mathrm{V}_0$ *and* $V(z,t)$ *is finite for all z ad t.*

We will solve (5) and (6) via Double Gupta transform.

Let f(z) be a function which is continuous for z ≥ 0. The Gupta Transform of f(z) is [6-13]

$\dot{\mathrm{R}}\{\mathrm{f(z)}\} = \frac{1}{q^3}\int_0^\infty e^{-qz} f(z)dz = G(q)$, provided that the integral is convergent.

The double Gupta transform off(z, t), z> 0 and t > 0 is [7-12] as follows

$$\dot{\mathrm{R}}_z\dot{\mathrm{R}}_t[f(z,t)] = G(q,r) = \frac{1}{q^3}\frac{1}{r^3}\int_0^\infty e^{-qz}\int_0^\infty e^{-rt}\, f(z,t)dzdt,$$

where q and r are complex numbers.

The double Gupta transform of the first orderderivative is as follows [8-11]:

$$\dot{\mathrm{R}}_z\dot{\mathrm{R}}_t\left[\frac{\partial f(z,t)}{\partial z}\right] = q\, G(q,r) - \frac{1}{q^3}\, G(0,r),$$

and

$$\dot{\mathrm{R}}_z\dot{\mathrm{R}}_t\left[\frac{\partial f(z,t)}{\partial t}\right] = r\, G(q,r) - \frac{1}{r^3}\, G(q,0).$$

The double Gupta transform of the second order derivative is as follows:

$$\dot{\mathrm{R}}_z\dot{\mathrm{R}}_t\left[\frac{\partial^2 f(z,t)}{\partial z^2}\right] = q^2\, G(q,r) - \frac{1}{q^2}G(0,r) - \frac{1}{q^3}\frac{\partial G(0,r)}{\partial z},$$

and

$$\dot{\mathrm{R}}_z\dot{\mathrm{R}}_t\left[\frac{\partial^2 f(z,t)}{\partial t^2}\right] = r^2\, G(q,r) - \frac{1}{r^2}G(q,0) - \frac{1}{r^3}\frac{\partial G(q,0)}{\partial t}.$$

III. SOLUTION OF ELECTRIC TRANSMISSION LINE EQUATIONS

Differentiating (5) w.r.t. z and using (6). We get

$$\frac{\partial^2 V(z,t)}{\partial z^2} = \mathrm{RC}\frac{\partial V(z,t)}{\partial t} \ldots\ldots\ldots (7)$$

Applying double Gupta transform of (7), we get

$$\dot{\mathrm{R}}_z\dot{\mathrm{R}}_t\{\frac{\partial^2 V(z,t)}{\partial z^2}\} = \mathrm{RC}\dot{\mathrm{R}}_z\dot{\mathrm{R}}_t\{\frac{\partial V(z,t)}{\partial t}\}$$

$$r^2\bar{\mathrm{v}}(\mathrm{r,s}) - \frac{1}{r^2}\bar{\mathrm{v}}(0,\mathrm{s}) - \frac{1}{r^3}\frac{\partial}{\partial z}[\bar{\mathrm{v}}(0,\mathrm{s})] = \mathrm{RC}[s\,\bar{\mathrm{v}}\,(\mathrm{r,s}) - \frac{1}{s^3}\bar{\mathrm{v}}\,(\mathrm{r},0)] \ldots\ldots (8)$$

As $\overline{\mathrm{V}}(0,\mathrm{s}) = \frac{1}{s^4}\mathrm{V}_0$, $\overline{\mathrm{V}}(\mathrm{r},0) = 0$ *and put* $\frac{\partial}{\partial z}[\overline{\mathrm{V}}(0,\mathrm{s})] = D(s)$, *therefore*, (8) *becomes*

$$r^2\overline{\mathrm{V}}(\mathrm{r,s}) - \frac{1}{r^2}\frac{1}{s^4}\mathrm{V}_0 - \frac{1}{r^3}D(s) = RC[s\,\overline{\mathrm{V}}\,(\mathrm{r,s})]$$

| e-ISSN: 2278 – 8875, p-ISSN: 2320 – 3765| www.ijareeie.com | Impact Factor: 8.18|

||Volume 11, Issue 6, June 2022||

|DOI:10.15662/IJAREEIE.2022.1106029 |

Rearranging, we get

$$\overline{V}(r,s) = \frac{\frac{1}{r^2}\frac{1}{s^4}V_0}{r^2 - RCs} + \frac{\frac{1}{r^3}D(s)}{r^2 - RCs} \ldots\ldots\ldots\ldots (9)$$

Applyingdouble inverse Gupta transform [9-10] of (9) w.r.t t and then s, we get

$$\dot{R}_s^{-1}\dot{R}_r^{-1}\{\overline{V}(r,s)\} = \dot{R}_s^{-1}\dot{R}_r^{-1}\{\frac{\frac{1}{r^2}\frac{1}{s^4}V_0}{r^2 - RCs} + \frac{\frac{1}{r^3}D(s)}{r^2 - RCs}\}$$

Or

$$\dot{R}_s^{-1}\{\overline{V}(z,s)\} = \dot{R}_s^{-1}\{\frac{1}{s^4}V_0\cosh\sqrt{RCs}\,z + \frac{D(s)}{\sqrt{RCs}}\sinh\sqrt{RCs}\,z\}$$

Or

$$R_s^{-1}\{\overline{V}(z,s)\} = R_s^{-1}\{\frac{1}{s^4}V_0\frac{e^{\sqrt{RCs}\,z} + e^{-\sqrt{RCs}\,z}}{2} + \frac{D(s)}{\sqrt{RCs}}\frac{e^{\sqrt{RCs}\,z} - e^{-\sqrt{RCs}\,z}}{2}\}$$

Or

$$\dot{R}_s^{-1}\{\overline{V}(z,s)\} = \dot{R}_s^{-1}\{\left(\frac{1}{s^4}\frac{V_0}{2} + \frac{D(s)}{2\sqrt{RCs}}\right)e^{\sqrt{RCs}\,z} + \left(\frac{1}{s^4}\frac{V_0}{2} - \frac{D(s)}{2\sqrt{RCs}}\right)e^{-\sqrt{RCs}\,z}\}\ldots\ldots\ldots (10)$$

$$\text{Since } \overline{V}(z,s) \text{ is finite as } z \to \infty, \text{therefore, putting } \frac{1}{s^4}\frac{V_0}{2} + \frac{B(s)}{2\sqrt{RCs}} = 0 \text{ or } D(s) = -\frac{1}{s^4}V_0\sqrt{RCs} \text{ in (10) and simplifying, we get}$$

$$\dot{R}_s^{-1}\{\overline{V}(z,s)\} = \dot{R}_s^{-1}\{\frac{1}{s^4}V_0e^{-\sqrt{RCs}\,z}\}\ldots. (11)$$

Since$erf\left(\frac{a}{2\sqrt{y}}\right) = \frac{2}{\sqrt{\pi}}\int_0^{\frac{a}{2\sqrt{y}}} e^{-x^2}\,dx$, [14-15] therefore, taking Gupta transform [8-12] on both sides, we have

$$\dot{R}_y[erf\left(\frac{a}{2\sqrt{y}}\right)] = \frac{2}{\sqrt{\pi}}\frac{1}{r^3}\int_0^{\infty} e^{-ry}\int_0^{\frac{a}{2\sqrt{y}}} e^{-x^2}\,dx\,dy$$

$$\dot{R}_y[erf\left(\frac{a}{2\sqrt{y}}\right)] = \frac{2}{\sqrt{\pi}}\frac{1}{r^3}\int_0^{\infty}\int_0^{\frac{a}{2\sqrt{y}}} e^{-ry}e^{-x^2}\,dy\,dx$$

On interchanging the order of integration by keeping in mind that $0 < x < \frac{a}{2\sqrt{y}}$ and $0 < y < \infty$. Then $0 < y < \frac{a^2}{4x^2}$ and $0 < x < \infty$. This yields

$$\dot{R}_y[erf\left(\frac{a}{2\sqrt{y}}\right)] = \frac{2}{\sqrt{\pi}}\frac{1}{r^3}\int_0^{\infty}\int_0^{\frac{a^2}{4x^2}} e^{-ry}e^{-x^2}\,dy\,dx$$

$$\dot{R}_y[erf\left(\frac{a}{2\sqrt{y}}\right)] = \frac{2}{\sqrt{\pi}}\frac{1}{r^4}\int_0^{\infty} e^{-x^2}\left[1 - e^{-\frac{ra^2}{4x^2}}\right]dx$$

$$\dot{R}_y[erf\left(\frac{a}{2\sqrt{y}}\right)] = \frac{2}{\sqrt{\pi}}\frac{1}{r^4}\int_0^{\infty} e^{-x^2}dx - \frac{2}{\sqrt{\pi}}\frac{1}{r^4}\int_0^{\infty} e^{-x^2}e^{-\frac{ra^2}{4x^2}}dx$$

| e-ISSN: 2278 – 8875, p-ISSN: 2320 – 3765| www.ijareeie.com | Impact Factor: 8.18|

||Volume 11, Issue 6, June 2022||

|DOI:10.15662/IJAREEIE.2022.1106029 |

$$\dot{R}_y[erf\left(\frac{a}{2\sqrt{y}}\right)] = \frac{2}{\sqrt{\pi}}\frac{1}{r^4}\frac{\sqrt{\pi}}{2} - \frac{2}{\sqrt{\pi}}\frac{1}{r^4}\int_0^\infty e^{-x^2-\frac{r\,a^2}{4\,x^2}}dx$$

$$\dot{R}_y\left[erf\left(\frac{a}{2\sqrt{y}}\right)\right] = \frac{1}{r^4} - \frac{2}{\sqrt{\pi}}\frac{1}{r^4}\int_0^\infty e^{-x^2-\frac{r\,a^2}{4\,x^2}}dx \ldots\ldots(i)$$

$$Let\ I(r) = \int_0^\infty e^{-x^2-\frac{r\,a^2}{4\,x^2}}dx \ldots\ldots(ii)$$

Using Leibniz rule to differentiate under the integral sign w.r.t. r, we have

$$I'(r) = \int_0^\infty e^{-x^2-\frac{r\,a^2}{4\,x^2}}\left(-\frac{a^2}{4\,x^2}\right)dx \ldots..(iii)$$

$Put\ z^2 = \frac{r\,a^2}{4\,x^2}, dx = -\frac{a\,r^{\frac{1}{2}}}{2\,z^2}dz$, and simplifying, we have

$$I'(r) = -\frac{a}{2\sqrt{r}}\int_0^\infty e^{-z^2-\frac{r\,a^2}{4\,z^2}}dz$$

Using equation (ii), we have

$$I'(r) = -\frac{a}{2\sqrt{r}}\,I(r)$$

$$I'(r) + \frac{a}{2\sqrt{r}}\,I(r) = 0$$

The solution of this equation is written as

$I(r) = C\,e^{-a\sqrt{r}}$ … … (iv)

To find the constant C, from (ii), $I(0) = \int_0^\infty e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$. Therefore, C = $\frac{\sqrt{\pi}}{2}$.

Hence $I(r) = \int_0^\infty e^{-x^2-\frac{r\,a^2}{4\,x^2}}dx = \frac{\sqrt{\pi}}{2}e^{-a\sqrt{r}} \ldots..(v)$

Using (v) in (i), we have

$$\dot{R}_y\left[erf\left(\frac{a}{2\sqrt{y}}\right)\right] = \frac{1}{r^4} - \frac{2}{\sqrt{\pi}}\frac{1}{r^4}\frac{\sqrt{\pi}}{2}e^{-a\sqrt{r}}$$

$$\dot{R}_y\left[erf\left(\frac{a}{2\sqrt{y}}\right)\right] = \frac{1}{r^4} - \frac{1}{r^4}e^{-a\sqrt{r}} \ldots\ldots(vi)$$

The Gupta transform of complementary error function is given by

$$\dot{R}_y\left[erf_c\left(\frac{a}{2\sqrt{y}}\right)\right] = \dot{R}_y\left[1 - erf\left(\frac{a}{2\sqrt{y}}\right)\right] = \dot{R}_y[1] - \dot{R}_y\left[erf\left(\frac{a}{2\sqrt{y}}\right)\right]$$

$$\dot{R}_y\left[erf_c\left(\frac{a}{2\sqrt{y}}\right)\right] = \dot{R}_y\left[1 - erf\left(\frac{a}{2\sqrt{y}}\right)\right] = \frac{1}{r^4} - \left(\frac{1}{r^4} - \frac{1}{r^4}e^{-a\sqrt{r}}\right) = \frac{1}{r^4}e^{-a\sqrt{r}}$$

$$\dot{R}_y\left[erf_c\left(\frac{a}{2\sqrt{y}}\right)\right] = \dot{R}_y\left[1 - erf\left(\frac{a}{2\sqrt{y}}\right)\right] = \frac{1}{r^4}e^{-a\sqrt{r}}$$

$$Hence\ inverse\ Gupta\ transform\ of\ [\frac{1}{r^4}e^{-a\sqrt{r}}]\ is\ \left[1 - erf\left(\frac{a}{2\sqrt{y}}\right)\right] = erf_c\left(\frac{a}{2\sqrt{y}}\right).$$

Hence equation (11) becomes

$$V(z,t) = V_0[1 - \mathrm{erf}\left(\frac{z\sqrt{RC}}{2\sqrt{t}}\right)]$$

Or

$$V(z,t) = V_0 erf_c\left[\frac{z\sqrt{RC}}{2\sqrt{t}}\right]$$

| e-ISSN: 2278 – 8875, p-ISSN: 2320 – 3765| www.ijareeie.com | Impact Factor: 8.18|

||Volume 11, Issue 6, June 2022||

|DOI:10.15662/IJAREEIE.2022.1106029 |

Or

$$V(z,t) = V_0 \frac{z\sqrt{RC}}{2\sqrt{\pi}} \int_0^t k^{-\frac{3}{2}} e^{-\frac{RCz^2}{4k}} dk \ \ldots\ldots\ldots (12)$$

From (6), we have

$$I(z,t) = -\frac{1}{R}\frac{\partial V(z,t)}{\partial z} \ldots (13)$$

Using (12) in (13), we can write

$$I(z,t) = -\frac{1}{R}\frac{\partial}{\partial z}\left(V_0 \frac{z\sqrt{RC}}{2\sqrt{\pi}} \int_0^t k^{-\frac{3}{2}} e^{-\frac{RCz^2}{4k}} dk \right)$$

Or

$$I(z,t) = -\frac{V_0}{R}\frac{\partial}{\partial z}\left(\frac{z\sqrt{RC}}{2\sqrt{\pi}} \int_0^t k^{-\frac{3}{2}} e^{-\frac{RCz^2}{4k}} dk \right)$$

By simplifying, we get

$$I(z,t) = \frac{V_0}{2\sqrt{\pi}} \sqrt{\frac{C}{R}}\, t^{-\frac{3}{2}} z e^{-\frac{RCz^2}{4t}} \ldots\ldots\ldots\ldots (14)$$

The (12) and (14) dispense the solutions of normal equations of electric transmission lines with inconsequential leakages through electrical insulation to the ground.

IV. CONCLUSION

In this paper, the double Gupta Transform has been put in strongly for scrutinizing the wave equations of electrical transmission lines with inconsequential leakages through electrical insulation to the ground. The Double Gupta transform has been panned out to be a very fruitful mathematical device put in to look up the popular electric transmission line wave equations. The results come by the Double Gupta transform are the same as came with other integral transforms or techniques [1-5].

REFERENCES

[1] Wadhwa, C.L., Electrical Power Systems, New Age International Limited, New Delhi (2009)

[2] Michael Olufemi OKE, Solution of Wave Equations on Transmission Lines where Leakage to Ground on the Line is Negligible. American Journal of Applied Mathematics. Vol. 3, No. 3, 124-128 (2015)

[3] Mehta, V.K. and Mehta, R. (2008). Principles of Power Systems, S. Chand and Company Limited, New Delhi.

[4] Hayt, W.H. and Buck, J.A, Engineering Electromagnetics, McGraw-Hill Company Inc. 2006

[5] R. Gupta, D. Verma, A. P. Singh, Double Laplace Transform Approach to the Electric Transmission Line with trivial Leakages through electrical insulation to the Ground, Compliance Engineering Journal, Volume 10, Issue 12 , 301-304 (2019)

[6] R. Gupta, R. Gupta, D. Verma, Propounding a New Integral Transform: Gupta Transform with Applications in Science and Engineering, International Journal of Scientific Research in Multidisciplinary Studies, volume 6, issue 3, March (2020), 14-19.

[7]R. Gupta, R. Gupta, D. Verma, Application of Novel Integral Transform: Gupta Transform to Mechanical and Electrical Oscillators, ASIO Journal of Chemistry, Physics, Mathematics & Applied Sciences, 4(1), 2020, 4-7.

[8]R. Gupta, R. Gupta, L. Talwar, Response of Network Circuits Connected To Impulsive Potential Source via New Integral Transform: Gupta Transform, ASIO Journal of Engineering & Technological Perspective Research, 5(1), 2020, 18-20.

[9]R. Gupta,R. Gupta, D. Verma, Analysis of series RL and RC networks with sinusoidal potential sources by Gupta transform, ASIO Journal of Engineering & Technological Perspective Research, 5(1), 2020, 28-30.

[10] R. Gupta, R. Gupta,L. Talwar, Gupta Transform Approach to the Series RL and RC Networks with Steady Excitation Sources, Engineering and Scientific International Journal (ESIJ), Volume 8, Issue 2, 2021, 45-47.

| e-ISSN: 2278 – 8875, p-ISSN: 2320 – 3765| www.ijareeie.com | Impact Factor: 8.18|

||Volume 11, Issue 6, June 2022||

|DOI:10.15662/IJAREEIE.2022.1106029 |

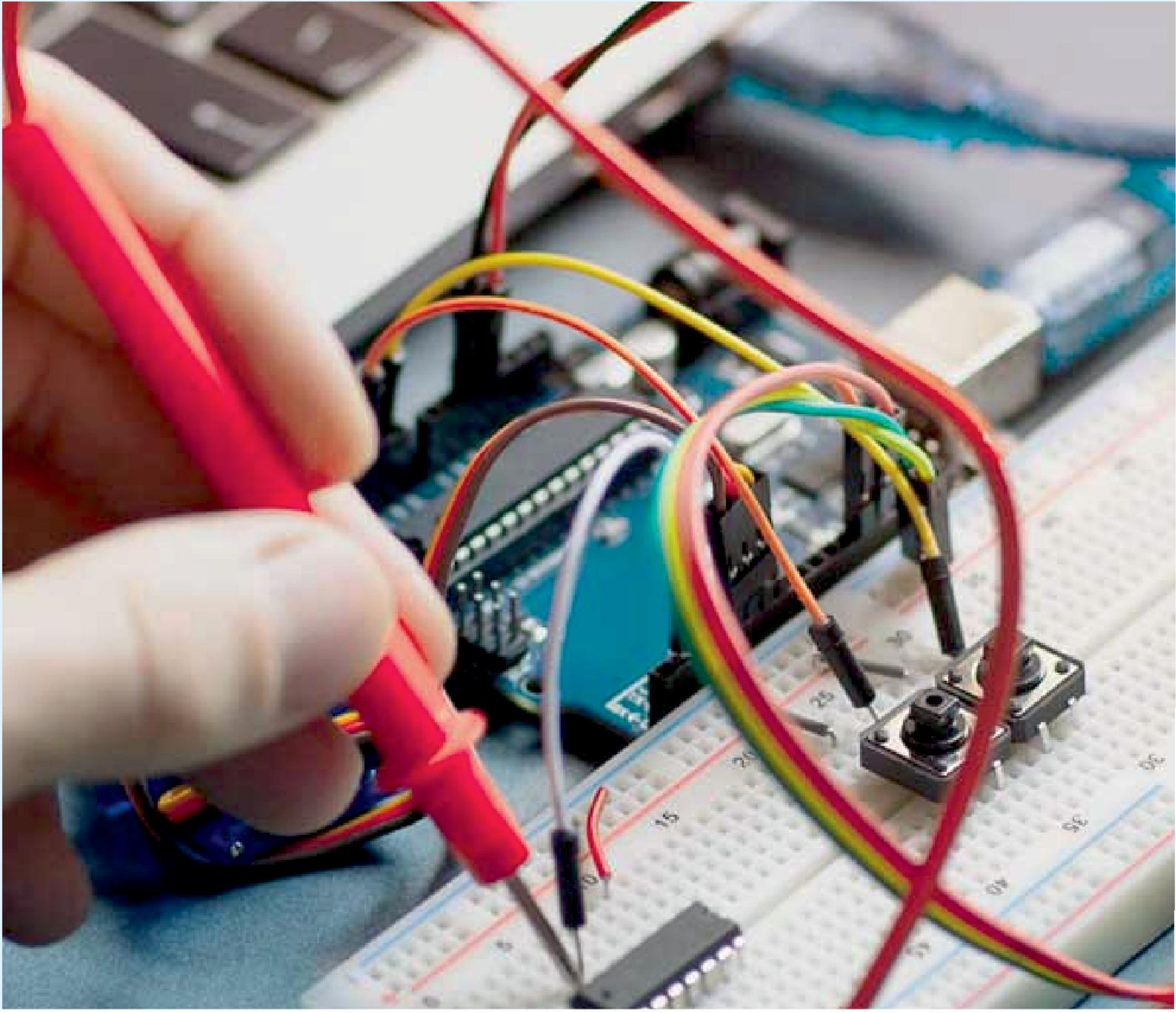
[11] R. Gupta, R. Gupta, D. Verma, Determining Rate Of Heat Convected From A Uniform Infinite Fin Using Gupta Transform, Roots International Journal Of Multidisciplinary Researches, Vol.7, No. 3, 2021, 66-70.

[12] R. Gupta, S. Sharma, R. Gupta, Study of Laminar Flow between Parallel Plates via Gupta Integral Transform, International Journal for Research in Applied Science & Engineering Technology (IJRASET)*, Volume 10, Issue VI, June 2022, pp. 385-389, doi:* https://doi.org/10.22214/ijraset.2022.43711

[13] R. Gupta, N. Pandita and R. Gupta, "Solving One-Dimensional Heat and Wave Equations Via Gupta Integral Transform," *2022 International Conference on Sustainable Computing and Data Communication Systems (ICSCDS)*, 2022, pp. 921-925, doi: 10.1109/ICSCDS53736.2022.9760823.

[14] E. Butkov. Mathematical physics, Addison-Wesley Pub Co, 1968.

[15] N.P. Bali and Dr. Manish Goyal. A Text Book of Engineering Mathematics, S Chand and Company Limited, 2014.



INNO SPACE
SJIF Scientific Journal Impact Factor
Impact Factor: 8.18

INTERNATIONAL STANDARD SERIAL NUMBER INDIA

International Journal of Advanced Research

in Electrical, Electronics and Instrumentation Engineering

9940 572 462 6381 907 438 ijareeie@gmail.com

www.ijareeie.com

Scan to save the contact details