Optimal Scheduling of Hydrothermal Power Generation in Wholesale Market Environment

I. Nechaev, R. Cherkaoui Senior Member, IEEE, S. Palamarchuk Senior Member, IEEE

Abstract—Generation scheduling is an important component of power systems operation in a market environment. Electricity demand in different areas depends on local marginal prices in the wholesale market. The paper presents a model for mid-term optimization of electricity generation in the hydrothermal power systems. The scheduling aims to minimize total production cost. Elasticity of consumer demand is taken into account in the model. A mathematical formulation of the problem supposes the application of dynamic programming method. An algorithm for the dynamic programming application is developed. Efficiency of the algorithm is demonstrated by the generation scheduling for a 14 bus system.

Index Terms—Consumer demand elasticity, dynamic programming, electricity generation price, Kuhn-Tucker conditions, Lagrange multipliers, mid-term generation scheduling.

I. INTRODUCTION

The market organization in any form in power industry of many countries is inevitable. To increase power generation efficiency, different generation companies compete with one another to get maximum profit. Power generation costs turn out one of the fundamental components of the electric network economic efficiency. Competitiveness is the most effective mechanism of cost reduction, although it is necessary to create conditions for rational rules and infrastructure formation, usage of effective forms of government regulation in the market [6].

The centralized control of power systems provides cost reduction, guarantees the best combination of different electric power sources while meeting the general constraints, and allows power distribution to be optimized.

The System Operator (SO) of Electric Power System (EPS) analyses electricity generation and electricity consumption for a mid-term perspective. Analysis is based on optimization of generation to get minimum of total production cost at thermal power plants (TPPs). Mid-term optimization supposes consideration of the EPS operation through a period from several weeks up to one year. The scheduling period is divided into the finite number of time intervals (days, weeks or months).

Mid-term scheduling is important for EPS with a large contribution of hydroelectric power plants (HPPs) with significant storage reservoirs electricity production.

Let us call the mid-term optimization problem with total production cost minimization a traditional optimization problem in EPS. For this problem, HPPs are of strategic importance, since they are a cheaper source of electricity.

In competitive terms the traditional problem is inefficient to reflect objectives of generating companies (GC) in the market environment where they are oriented to maximum profit. Also the traditional problem doesn’t take into account consumer opportunities and needs. As a consequence, traditional problem is incapable of guaranteeing optimal loading of generating capacities and the best resources utilization. Therefore, power plant generation can be not only non-optimal in general, but also unprofitable [1].

In this article the traditional problem takes into consideration consumer demand elasticity. This consideration allows one to determine to what extent electricity consumption depends on price fluctuation. The problem statements and solution algorithm of this task are shown in parts 2 and 3 of the paper.

The algorithm is tested on the example of the IEEE 14 bus power system. However, the demand characteristics of consumers, the cost functions of TPPs and water parameters of HPPs were chosen arbitrarily.

Nowadays there are numerous works on the problem of optimal scheduling of hydrothermal systems in the wholesale market environment. The researches done by J. Bushnell [7], M.V. Pereira [4], [10], [12], H. Rudnick [8] and others are aimed at developing models of hydrothermal systems for profit maximization of every generating company. The paper does not consider an oligopolistic model [5] of electricity markets. Hence, the problem of profit maximization from a position of every generating company isn’t considered either. The algorithms presented in the paper make it possible to consider similar problems in the future.

II. MATHEMATICAL STATEMENTS OF OPTIMIZATION PROBLEMS FOR MID-TERM GENERATION SCHEDULING

In the paper the model of pricing based on marginal costs is considered. The prices found on the basis of marginal costs determine the economic efficiency of electric power at certain time in a certain place. They are known as the nodal (local) marginal prices.
Steady state of the system in this case is modeled with certain simplifications:

- Only active power distribution in the system is taken into account (reactive power distribution is assumed as balanced);
- Power grid is transformed to one voltage level;
- Transmission losses are known as fractions of power flows. Values of losses are determined taking into consideration network configuration and actual power flows.

With these simplifications the calculation of the steady state of the system adds up to defining values of active generation of buses and power flows in the system ties.

A. Notation

The EPS consists of I TPPs, J HPPs and N buses. The number of time intervals t in the scheduling period is T. Let us designate the following parameters:

- \( Q_j^t \) - the water discharge through the turbines of the j-th HPP at interval t, m\(^3\);
- \( Q_{j,max}^t \) - the maximum admissible discharge value for \( Q_j^t \), m\(^3\);
- \( S_j^t \) - the spill discharge of water at the j-th HPP at interval t, m\(^3\);
- \( S_{j,max}^t \) - the maximum dam capacity at the j-th HPP with regard to the spill discharge of water at interval t, m\(^3\);
- \( V_j^t \) - the random water storage in the reservoir of the j-th HPP at the beginning of interval t, m\(^3\);
- \( V_{j,min}^t \), \( V_{j,max}^t \) - the minimum and the maximum admissible values of water storage \( V_j^t \), m\(^3\);
- \( V_{j+1}^t \) - the random water storage in the reservoir of the j-th HPP at the end of interval t (at the beginning of interval \( t+1 \)), m\(^3\);
- \( A_j^t \) - the lateral water inflow to the reservoir of the j-th HPP at interval t, m\(^3\);
- \( W_{HPP,m}^t \) and \( W_{HPP,max}^t \) - the minimum and maximum electricity generation at the j-th HPP at interval t, MWh;
- \( W_{HPP}^t \) - the random amount of the electricity generated at j-th HPP at interval t, MWh;
- \( W_{TPP,m}^t \) and \( W_{TPP,max}^t \) - the minimum and maximum electricity generation at the j-th TPP at interval t, MWh;
- \( W_{TPP}^t \) - the random amount of generation at the j-th TPP at interval t, MWh;
- \( W_{ab}^t \) - the random electricity flow from bus a to bus b at interval t, MWh;
- \( W_{ba}^t \) - the random electricity flow from bus b to bus a at interval t, MWh;
- \( W_{ab}^t \) - the random electricity consumption from the electric network at bus a at interval t, MWh.
- \( \Delta_{ab}^t \) - the power losses in the ties between buses a and b.

The variables of the problem are

\[ W_{iTPP}^t, W_{jHPP}^t, Q_j^t, S_j^t, V_j^t, t = 1, \ldots, T. \]

B. Nodal pricing

While searching minimum of total cost with constraints in the form of power balance at every bus, the duality theory allows one to obtain electricity price of every bus (nodal price) as a combination of Lagrange multipliers associated with relevant constraints [6].

In the market environment it is important to take into account the consumer influence on the price in the form of demand functions. Thus, for the traditional problem with consideration of consumer demand elasticity the nodal prices are included in constraints, being at the same time the combination of dual variables of the problem.

The unknown nodal prices can be obtained iteratively. First, the traditional problem can be solved without demand elasticity at the fixed demand amounts. Dual variables provide the values of nodal prices. Then the obtained nodal prices are substituted in demand functions to specify the demand. When the amount of the demand, used in solving, coincide with the value corresponding to the demand function - the problem is solved.

The iterative way is time-consuming enough and also requires considerable computing resources. The Kuhn-Tucker optimality conditions can be introduced into the problem to link dual variables of the problem with the demand functions.

In the theory of mathematical programming the Kuhn-Tucker conditions should be met at the solution point. They are necessary and sufficient conditions for the existence of optimal solution.

Let us calculate the maximum of objective function with nonnegative constraints at interval t, where all functions are differentiable. In accordance with the Kuhn-Tucker theorem [6] the following statements are right in optimal point (solution):

1) \( W_{iTPP}^*, W_{jHPP}^*, Q_j^*, S_j^*, V_j^*, t = 1, \ldots, T \) is an optimal solution point.

There are coefficients \( \lambda_{i,j}^*, a = 1, N, t = 1, T \), such that

2) \( \sum g_i (W_{iTPP}^*, W_{jHPP}^*, W_{ab}^*, W_{ba}^*), a = 1, N, t = 1, T; \)

3) \[ f(W_{iTPP}^*, W_{jHPP}^*, W_{ab}^*, W_{ba}^*) + \sum \lambda_{i,j}^* \cdot V g_i (W_{iTPP}^*, W_{jHPP}^*, W_{ab}^*, W_{ba}^*), a = 1, N, t = 1, T; \]

where \( \lambda_{i,j}^* \), \( a = 1, N, t = 1, T \) are Lagrange multipliers corresponding to constraints \( g_i \) with the objective function \( f \). \( V \) is a symbol of gradient.

C. Consumer demand functions

The consumer demand functions for nodal system of pricing become:

\[ W_{aD}^t = d_{a} - d_{a} \cdot (\lambda_{a}^1 + \lambda_{a}^2), a = 1, N, \]

where \( d_{a} \) and \( d_{a} \) are linear function coefficients for a-bus with consumer; \( \lambda_{a}^1 \) – Lagrange multiplier associated with constraint in the form of electricity balance in the whole system at inter-
val $t$; $\lambda^i_t$ – Lagrange multiplier associated with constraint in the form of electricity balance at bus $a$ at interval $t$.

D. System constraints

When solving optimization problems with HPPs two types of constraints are used: the first type includes constraints linked to technological features of HPPs (water constraints). The second type contains constraints based on physical laws (Kirchhoff's laws and energy conservation law) and constraints on a range of generation capacities [2].

The water constraints are represented by the following constraints at interval $t$:

1) on turbine capacities

$$Q'_j \leq Q'_{j_{\text{max}}}, \ j = \overline{1, J};$$

(3)

2) on water spill discharges:

$$S'_j \leq S'_{j_{\text{max}}}, \ j = \overline{1, J};$$

(4)

3) on water reserves in the HPP reservoirs:

$$V'_{j_{\text{min}}} \leq V'_j \leq V'_{j_{\text{max}}}, \ j = \overline{1, J};$$

(5)

4) on water balance:

$$V'_j = V'_{j-1} - Q'_j - S'_j + \Delta'_j, \ j = \overline{1, J}. $$

(6)

For EPS with cascades of HPPs it is necessary to take into account inflow of water from upstream HPPs in constraint (6).

Other constraints are represented as a range of generation capacities of HPPs and TPPs:

$$W'_{j_{\text{HPP min}}} \leq W'_j \leq W'_{j_{\text{HPP max}}}, \ j = \overline{1, J};$$

(7)

$$W'_{j_{\text{TPP min}}} \leq W'_{j_{\text{TPP}}}, \ j = \overline{1, J}. $$

(8)

The expressions of the EPS steady state according to Kirchhoff’s first law in the form of electricity balance for network buses are as follows:

$$W'_{j_{\text{HPP}}} + W'_{j_{\text{TPP}}} = \left[ d_{0_a} - d_a \cdot (\lambda^i_a + \lambda^i_b) \right] - \sum_{b=1}^{Z} W'_{ab} +$$

$$+ \sum_{b=1}^{Z} (1 - \Delta^{ab}) \cdot W'_{ab} = 0, \ b = \overline{1, Z}, \ a = \overline{2, N},$$

(9)

where $Z$ is the total number of buses connected to the bus $a$; $b$ is an index of the buses connected to the bus $a$.

The last bus is considered to be slack one in terms of active power. Expression (9) for this bus is not set up, and instead the expression in the form of active power balance in the whole EPS is given:

$$\sum_{i=1}^{I} W'_{i_{\text{TPP}}} + \sum_{j=1}^{J} W'_{j_{\text{HPP}}} - \sum_{a=1}^{N} \left[ d_{0_a} - d_a \cdot (\lambda^i_a + \lambda^i_b) \right] - $$

$$- \sum_{a=1}^{N} \sum_{b=1}^{Z} W'_{ab} \cdot \Delta^{ab} = 0, \ b = \overline{1, Z}, \ a = \overline{2, N}. $$

(10)

Electricity consumption from the electric network at bus $a$ at interval $t$ in expressions (9) and (10) is represented by expression (2).

In the simplified steady state modeling it is considered that electricity flows $W'_{ab}$ and $W'_{ba}$ cannot occur simultaneously in the line between buses $a$ and $b$. One of these power flows should be equal to zero. To fulfill this condition for every branch additional constraints are introduced:

$$W'_{ab} = W'_{ba} = 0, \ b = \overline{1, M}, \ a = \overline{1, N}. $$

(11)

Constraints (11) are bilinear, they make the mathematical model of EPS steady state nonlinear and they cause additional computational obstacles.

Besides, electricity flows in the branches are considered as nonnegative:

$$0 \leq W'_{ab} = W'_{ba}, \ b = \overline{1, M}, \ a = \overline{1, N}. $$

(12a)

Also transfer capabilities should be include in considerations:

$$W'_{ab} \leq W'_{ab_{\text{max}}}, \ b = \overline{1, M}, \ a = \overline{1, N}. $$

(12b)

where $W'_{ab_{\text{max}}}$ - maximum transfer capabilities in transmission line between buses $a$ and $b$, MWh.

E. Objective function

The problem of total cost minimization deals with fuel consumption decrease at TPPs. The problem of the kind is associated with the problem of profit maximization, since the cost saving for producers assumes getting more benefit from sale in the competitive environment. In the absence of possibility for consumers to influence electricity prices and at limited generating capacities, the GCs have no motivation for saving costs. Therefore the consumer demand elasticity is able to induce generating companies to reduce the costs. For this purpose the traditional problem with consumer demand elasticity has an objective function:

$$\min E \left[ \sum_{t=1}^{T} B'_{i_{\text{TPP}}} (\bar{V}'_{i_{\text{TPP}}}) + f_{i_{\text{HPP}}} (\bar{V}'_{i_{\text{HPP}}}) \right], \ i = \overline{1, I}, \ t = \overline{1, T},$$

(13)

where $B'_{i_{\text{TPP}}} (\bar{V}'_{i_{\text{TPP}}})$ is a quadratic function of production cost of the $i$-th TPP at interval $t$, $\$/MWh; $f_{i_{\text{HPP}}} (\bar{V}'_{i_{\text{HPP}}})$ is the function of production cost at TPPs during the time, starting at interval $t+1$ to the end of the scheduling period, $\$/MWh; $\bar{V}'_{i_{\text{HPP}}}$ is a vector of random water storages in the HPPs reservoirs at the beginning of the $(t+1)$-th interval (at the end of interval $t$). $E$ is a symbol of mathematical expectation.

The objective function (13) is presented as an expectation value of summing two cost functions: current function for interval $t$ and future function starting at interval $t+1$ to the end of the scheduling period.

The model (13), (1)-(12) is one of the presentation of the dynamic DC-OPF problem.

III. METHODS FOR SOLVING OF MID-TERM OPTIMIZATION PROBLEMS

A. Description of algorithm

The algorithms for solving the given problem are based on the dynamic programming method. The essence of this method consists in decomposition of the problem into two subproblems: direct and inverse. Besides, every subproblem is solved for every time interval. The values of variables at the beginning and the end of the scheduling period are considered to be known [10].
The algorithm for solving the traditional problem, that takes into consideration consumer demand elasticity is shown in Fig 1.

At first the inverse problem is solved. The last interval of the scheduling period is considered. The water supply in the reservoirs at the end of the period is known. The presence of several HPPs in EPS significantly increases the complexity of calculations. Therefore the aggregation of reservoirs, i.e. an equivalent replacement of several reservoirs by one which can reflect regulation and power possibilities of several reservoirs (HPPs), is applied to simplify the calculations. The technique of HPPs system reservoir aggregation is given in [11].

It is supposed that at the beginning of the last interval several values of water supply in a reservoir are available \( \tilde{V}_{i,t} \) (\( t \) is the number of water-supply value, \( L \) –the total number of values). For each value of water supply \( l \) several scenarios of lateral water inflow \( \tilde{A}_{j,t} \) (step 3) are set with the determined probability. These scenarios are determined by the long-term data of hydrological measurements (\( m \) is the number of scenario of lateral water inflow, \( M \) is the total number of scenarios). Whereas the costs of electricity generation in HPPs are minimum, for the traditional problem solving HPPs should be loaded foremost to obtain the best economic effect. At step 4 the maximum generation of HPPs is determined taking into account constraints on water (3)-(6) and constraints on generation range (7) is defined. The maximum generation of HPPs in this problem is represented by the function:

\[
\max_{\tilde{V}_{i,t}} \left[ \sum_{j=1}^{J} \sum_{l=1}^{L} W_{j,t} (\tilde{V}_{i,t}, \tilde{Q}_{l,t}) + EW_{j,t} (\tilde{V}_{i,t}, \tilde{Q}_{l,t}, \tilde{Q}_{l,t+1}) \right]
\]

(15)

where \( EW_{j,t} (\tilde{V}_{i,t}, \tilde{Q}_{l,t}, \tilde{Q}_{l,t+1}) \) is the expectation value of the future generation function HPPs starting at the interval \( t+1 \) to the end of the scheduling period \( T \).

At step 5 of the algorithm (Fig. 1) the minimization problem of total production costs at TTPs with already known values of generation at HPPs is solved. For the last time interval the future cost function is equal to zero in expression (13). Except constraints (8)-(12) Kuhn-Tucker optimality conditions (1a) and (1b) are added. To associate the nodal prices with demand functions in balance constraints (9)-(10). As a result of this problem solution, the total production costs for electricity generation are obtained for the certain lateral water inflow scenario and the value of water supply in reservoirs.

Similar calculations are carried out for other lateral water inflow scenarios (steps 6-12). After the probabilistic assessment of these results, the first point of the future cost function is obtained (step 8).

The calculations proceed with the following values of water supply in the reservoirs (steps 2-9). The future cost function for the considered time interval is plotted at the obtained points (step 11) and is approximated by the analytical expression through any known approach. The future cost functions for the remaining time intervals are plotted similarly (steps 1-12) with the use of expression (13). The inverse subproblem is considered to be solved when the future cost functions for all

---

Fig. 1. The control flow chart of the inverse subproblem calculation of traditional problem with consumer demand elasticity
time intervals of scheduling period are obtained [2].

Consideration of the direct subproblem begins with the first time interval (Fig. 2). In expression (13) the future cost function plotted at the inverse subproblem is used. Constraints (1)-(12) are taken into account as well. As a result of the solution the output data corresponding to an optimal steady state at each time interval are obtained, and the problem is considered to be solved.

**THE BEGINNING OF THE DIRECT PROBLEM CALCULATION**

1. Consideration of the first time interval of scheduling period $T : t$

2. Solution of problem (3)-(13) with Kuhn-Tucker conditions (1a) and (1b) by using the future function $f^{t+1}_w (\beta^{t+1})$ :

$$Q_t, S_t, W^{t+1}_{TPP}, \bar{W}^{t+1}_{TPP}, \bar{W}^{t}_{TPP}, \bar{W}_a, W_{aW}, \lambda_a, \lambda_{\bar{a}}; j=\bar{a}, j=\bar{a}, \bar{a}; a=\bar{a}, N, h=\bar{a}, Z$$

12. If $t=T$

No

Yes

**THE END OF THE DIRECT PROBLEM CALCULATION**

Fig. 2. The control flow chart of the direct subproblem of traditional problem calculation with consumer demand elasticity

**B. Nodal price calculation method**

The pricing mechanism in the mid-term optimization problem is based on marginal costs. The electricity price is a marginal price deal with the change of the expectation value of the total costs at a small change in electricity generation [9], [3].

In the traditional problem the nodal prices correspond to the combinations of Lagrange multipliers on the balance constraints of buses:

- **buses with TPPs:**
  $$N_{TPP}^P = \frac{\partial E_{TPP}}{\partial W_{TPP}} (W_{TPP}) = -\delta \bar{\lambda}_i - \delta \bar{\lambda}_a, a = i, i = \bar{1}, \bar{T},$$
  $$a = \bar{1}, N, t = \bar{1}, \bar{T};$$

- **load buses:**
  $$N_{\bar{P}} = -\delta \bar{\lambda}_i - \delta \bar{\lambda}_a, a = \bar{1}, N, t = \bar{1}, \bar{T}. $$
  (18)

For HPPs it is necessary to add a concept of the discharge water price. This price depends on the cost of fuel used for electricity generation at TPPs, if the whole amount of electricity HPPs do not generated. The discharge water price corresponds to Lagrange multiplier on the water balance constraint (6):

$$N_{\bar{P}}^j = \frac{\partial E^{t+1}_{TPP}}{\partial W_{TPP}} (\bar{W}^{t+1}_{TPP}) = -\delta \bar{\lambda}_j, j = \bar{1}, J, t = \bar{1}, \bar{T}. $$
  (19)

Expressions (17) - (19) are used to calculate nodal prices at various stages of the algorithm (Fig. 1−2).

**IV. NUMERICAL EXAMPLE FOR MID-TERM OPTIMIZATION PROBLEM**

Efficiency of the offered algorithm is tested by the numerical example. The factors influencing the structure of costs and profit are analyzed for mid-term generation scheduling. The IEEE 14 bus EPS has been chosen (Fig. 3) as the most demonstrative and representative example. This system contains two power plants: TPP and HPP with mid-term operation. Other buses have electricity consumption. Three time intervals are considered. At each interval the scenario of lateral water inflow is determined based on hydrological measurements. Water storage in the reservoir at the beginning and the end of the scheduling period is also known. Consumer demand functions, production cost function for TPP and power-water consumption curve for HPP are also known.

Mathematical formulation of the problem contains 134 variables and 130 constraints.

The results of the mid-term scheduling are presented in Fig. 4 and Fig. 5. The obtained results testify that for correct forecasting of the nodal prices, structure of costs and also generation scheduling, it is necessary to use approaches for optimization of the steady state at power plant generation in the market environment. It will decrease possible risks of the lack of electricity supply to consumers and increase efficiency of electricity production as a whole. Also it will decrease probability of various system blackouts associated with the drop of consumption and generation in the system.

Despite the mentioned advantages, the algorithm is very labour-intensive. The mathematical model contains numerous variables and a great number of constraints. Further studies should be directed to simplification of the presented algorithm and to its adaptation to generation scheduling in large EPSs.

Optimization of the numerical example shows that at the
beginning of the scheduling period high generation at HPP results in low nodal prices (Fig. 5). At the next time interval electricity production by TPP increases (Fig. 4), leading to the rise in nodal prices. Thus, substitution of hydro generation by thermal generation results in reduction of consumption as buying capacity of consumers decreases at a rise in prices. At the third interval electricity consumption remain to be covered mainly by TPP, which causes a high price level at load buses with consumers.

Additionally, the values of water discharge through turbines, water storage in reservoirs at the beginning of each time interval and spill discharge of water can be obtained for HPPs. The optimization procedure for plotting the future cost functions is based on sustainable use of water resources. It assumes that spill discharge of water in HPPs will not take place during scheduling period.

The Kuhn-Tucker optimality conditions are introduced into the optimization problem to obtain the nodal prices. Bilinear constraints add nonlinearity to the considered problem and complicate the search for the objective function extremum.

V. CONCLUSION

The statement of optimization problem for total production costs minimization is proposed taking into consideration consumer demand elasticity. The developed algorithm allows obtaining optimal amounts of generation at hydro and thermal power plants during multi interval scheduling period. The calculations are carried out for the IEEE 14 bus test system.

Steady state of the power system is modeled by the equations reflecting nodal balances of electricity to be generated and consumed. Electricity flows in the branches are considered taking into account electricity losses.

The developed algorithm is based on the dynamic programming approach. For each time interval future cost functions are plotted. Electricity generation at power plants and electricity flows in all branches of EPS with losses being taken into consideration determine the nodal prices based on marginal costs.

VI. REFERENCES


VI. BIOGRAPHIES

Igor Nechaev graduated from the Energy Department of Irkusk State Technical University in 2008. In 2009 he defended the master’s diploma at the Irkusk State Technical University, Russia. His professional fields of research include optimal scheduling of the power plants, electricity pricing and hydro-generation scheduling.
Rachid Cherkaoui received the M.S. degree in electrical engineering and the Ph.D. degree from Swiss Federal Institute of Technology (EPFL), Lausanne, in 1983 and 1992 respectively. Since 1993, he is leading the research activities in the field of optimization and simulation techniques applied to electrical power and distribution systems. Presently his main research topics are electricity market deregulation, distributed generation and storage, and, power system vulnerability mitigation.

Sergey Palamarchuk graduated from Irkutsk Polytechnic Institute in 1968. In 1997 he defended the postdoctoral thesis in electric power engineering at the Energy Systems Institute (ESI). At present, he is a chief researcher at ESI and part-time Full Professor at Irkutsk State Technical University. His principal areas of research are: power industry restructuring, market design and organization.