Benchmarking A Coupled Finite Element–Immersed Boundary–Lattice Boltzmann Method Solver for Simulations of Collapsible Tube Flows

Qiuxiang Huang*, Li Wang, Sridhar Ravi, Fang-Bao Tian, John Young and Joseph C. S. Lai

School of Engineering and Information Technology, University of New South Wales Canberra, ACT 2600, Australia.
qiuxiang.huang@adfa.edu.au, l.wang@unsw.edu.au, sridhar.ravi@adfa.edu.au, f.tian@adfa.edu.au, j.young@adfa.edu.au and j.lai@adfa.edu.au

Abstract. Fluid-structure interactions (FSIs) of fluid-conveying collapsible vessels produce rich physiologically significant phenomena in many biological systems. However, it remains challenging to compute the flow-induced vibration of collapsible vessels due to nonlinear FSIs involving large-deformations, three-dimensional (3-D) motion, unsteady flows and high Reynolds numbers. In this study a coupled finite element-immersed boundary-lattice Boltzmann method (FE-IB-LBM) solver is proposed to simulate collapsible tube flows. Validation of the solver was performed by simulating steady flow in a collapsible tube at $Re = 128$. Good agreements of the results between current computations and published data were observed. This study provides additional data for benchmarking of computational fluid dynamics (CFD) solvers in the simulation of collapsible tube flows. Detailed parameter investigations (e.g. effects of the Reynolds number and non-Newtonian fluid) of the collapsible tube flows will be reported in future works.

Key words: Finite element method, Immersed boundary method, Lattice Boltzmann method, Flow–vessel interactions, Collapsible tube flows, Blood flow

1 Introduction

Both the collapse of blood vessels and the blood flow in compliant vessels all involve FSIs between the blood and the vessel walls. In order to understand the physical mechanism for the onset of self-excited oscillations of collapsible blood vessels, various experimental and numerical studies have been conducted [1, 2]. CFD becomes an important tool for improving our understanding of the effect of normal physiological and pathological behaviors in the arterial system [3]. However, it remains challenging to compute the flow-induced vibration of collapsible vessels due to nonlinear FSIs involving large-deformations, 3-D motion, unsteady flows and high Reynolds numbers. Here, the computation of a steady flow in a collapsible tube at $Re = 128$ was conducted to validate the FE-IB-LBM solver.

2 Modeling of Fluid-structure Interaction in a Collapsible Tube

The coupled nonlinear system for the FSIs in a collapsible tube is solved by the FE-IB-LBM FSIs solver. The D3Q19 LBM with multi-relaxation-time (MRT) model is adopted for the fluid dynamics. An iterative feedback version of the immersed boundary method (IBM) is used to realize the no slip and no penetration boundary conditions of the tube wall [2]. The IB-LBM solver has been extensively validated and applied in confined flows (e.g. 2D collapsible channel flows and 3D stenosis) in our previous publications [2, 4, 5]. The structural equation is solved by an explicit finite element method (FEM). The strain of the wall is assumed to be small which enables the tube wall to be treated as a linear elastic structure. The dynamics for the solid is

$$\rho_m \frac{d^2 \boldsymbol{u}_s}{dt^2} + c\frac{d\boldsymbol{u}_s}{dt} - \nabla\cdot\boldsymbol{\sigma} - \rho_m \boldsymbol{b} = 0, \tag{1}$$

where ρ_m is the solid density, $\boldsymbol{u}_s(\boldsymbol{X},t)$ denotes the solid displacement field, c is the material damping, $\boldsymbol{\sigma}$ is the Cauchy stress tensor and $\boldsymbol{b}$ is the body force. To validate the 3D FE-IB-LBM FSI solver, a steady flow in a collapsible tube is considered. A 3D incompressible flow in a thin-walled collapsible tube of undeformed radius R, length L and wall thickness h is considered. As illustrated by Fig. 1, the elastic wall is subjected to an external pressure p_e, and the tube wall material has Young's modulus E and Poisson ratio ν_s. The rigid tube has a diameter of D. The averaged flow velocity at the inlet U_0, tube diameter D and fluid density ρ are used to non-dimensionalize this system, giving four non-dimensional parameters governing this FSI system: the Reynolds number, structure-to-fluid mass ratio, bending stiffness and external pressure are given respectively

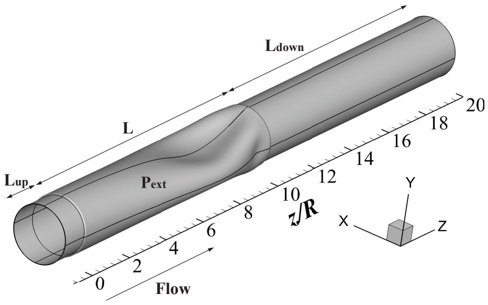
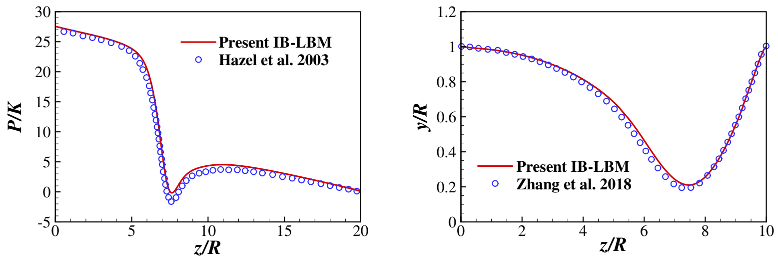


Fig. 1: Schematic diagram of fluid flow through a collapsible tube.



(a) Pressure distribution along the tube centreline.

(b) Wall shape of the tube projected to the symmetry planes (Y-Z plane).

Fig. 2: Comparison of steady solutions between present IB-LBM computations and FEM computations of Hazel et al. [6] and Zhang et al. [7]. The coordinates (x,y,z) are scaled to the radius R and the pressure is scaled to the bending modulus K of the tube, as in Hazel et al. [6].

by

$$Re = \frac{U_0 D}{\nu}, \quad M = \frac{\rho_m h}{\rho D}, \quad K_b = \frac{E h^3}{12\left(1-\nu_s^2\right)\rho U_0^2 D^3}, \quad P_e = \frac{p_e - p_d}{\rho U_0^2}. \tag{2}$$

A no-slip boundary condition is applied along the tube wall. The tube is clamped at both ends and is discretized with 26400 isoparametric 3D solid elements (20 nodes for each element and one element in the thickness direction), giving 185460 nodal points with three degrees of freedom (displacements) for each structural element. A steady Hagen-Poiseuille velocity profile with average velocity U_0 is imposed at the inlet, and a constant pressure $p_d = 0$ is specified at the downstream outlet. The computation parameters are $R = 0.5m$, $L_u = R = 0.5m$, $L = L_d = 10R = 5m$, $\nu_s = 0.49$, $E = 4559.4Pa$, $h = 0.05R = 0.025m$ and $\rho_m = 1000kg/m^3$, giving the same nondimensional parameters used by Hazel et al. [6]: $Re = 128$, $M = 25$, $K_b = 0.030517$, $P_e = 5.46875$. To approximate the steady flow, an empirical damping ratio $\zeta = c/c_c = 0.05$ ($c_c = 2\sqrt{Km}$ is the critical damping, $K = E(h/R)^3/12\left(1-\nu_s^2\right)$ is the bending modulus and m is the mass of the elastic tube) is used here to damp out the oscillations.

3 Results

Fig. 2 shows the comparison of present computations with those of Hazel et al. [6] and Zhang et al. [7]. Excellent agreement is achieved for both the pressure distribution and the wall shape. Fig.3 shows the details of the flow fields and tube configurations. For tube configurations in the X-Z plane, the elastic tube collapses into a two-lobed shape. For the flow fields in the Y-Z plane, the pressure contours show that a strong adverse pressure gradient develops at the reopening region ($7.5 \leq z/R \leq 9$) caused by the collapse of the tube, leading to a reversed flow between the upper and lower walls of the tube. The pressure contours and streamwise velocity contours at the X-Z plane reveal axially decaying twin jets emerging from the two-lobed throat with a region of reversed flow developed between them. The 3D flow structure is visualized by Q-criterion in Fig. 4 as the flow is highly 3D. It shows that the two-lobed shape of the collapsed tube drives the formation of two axial "jets".