The purpose of this note is to correct an error in Li, Liu, Wang, and Lin (2013) in deriving inequality (33) from the state equation in (32). Inequality (33) establishes that the scalar state $y^T(n)Dy(n)$ in the state equation in (32) decays at an estimated rate of $\left(\frac{\rho_2}{\gamma^2}\right)^n < 1$, that is, the state equation in (32) is asymptotically stable. While the decay rate given in (33) is correct, (33) as written does not follow from (32). A correct derivation from (32) should have led to an estimated decay rate of $2 \left(\frac{\rho_2}{\gamma^2}\right)^n$ instead, due to an unnecessary factor of 2 that was introduced in the estimate of the right-hand side of the state equation in (32). This unwanted factor of 2 can be avoided by a tighter estimate of the right-hand side of the state equation in (32) as follows.

Instead of using the inequality $2a^Tb \leq a^T \tilde{a} + b^T \tilde{b}$, we will use the more general version $2a^Tb \leq \kappa a^T \tilde{a} + \kappa^{-1} b^T \tilde{b}$ with $\kappa = \frac{4N-1}{4N-2} > 0$ on the right-hand side of the state equation in (32). This way, the factor 2 will not be in the estimate and the rest of the proof remains valid. More specifically, Eq. (32) can be modified as,

$$y^T(n + 1)Dy(n + 1) = y^T(n)P^TDPy(n) + y^T(n)\kappa a^T \tilde{a} + y^T(n)\kappa^{-1} b^T \tilde{b} \kappa^{-1} b^T \tilde{b}$$

$$\leq y^T(n + 1) \kappa^2 + 2y^T(n)P^TDPy(n)$$

$$+ \kappa^{-2} y^T(n)\kappa a^T \tilde{a} + \kappa^{-1} y^T(n)\kappa^{-1} b^T \tilde{b} \kappa^{-1} b^T \tilde{b} \kappa^{-1} b^T \tilde{b}$$

$$\leq y^T(n + 1) \kappa^2 + 2y^T(n)\kappa a^T \tilde{a} + y^T(n)\kappa^{-1} b^T \tilde{b} \kappa^{-1} b^T \tilde{b}$$

Thus, using the inequality $\sqrt{\sum_{s=0}^{n} |a_s|^2} \leq \sum_{s=0}^{n} |a_s|$, we will arrive at the same estimate of the decay rate as given in inequality (33) of Li et al. (2013), but with the coefficients modified. That is, inequality (33) in Li et al. (2013) is slightly modified as

$$(\rho_1 \gamma)^{n+1} \|y(0)\|_D + \left(\frac{\rho_2}{\gamma^2}\right)^n y^{-1}(1 + \kappa^{-1})^2$$

$$\times \|\alpha(a(I - W))z(0)\|_D$$

$$+ \sum_{s=0}^{n} \left(\frac{\rho_2}{\gamma^2}\right)^s y^{-1}(1 + \kappa^{-1})^2 \|\alpha(a(I - W))z(n - s)\|_D.$$
Note that $\sqrt{\pi_{\min}} \|y\|_2 \leq \|y\|_D \leq \sqrt{\pi_{\max}} \|y\|_2$, where the $D$-norm of $y$ is defined as $\|y\|_D = \sqrt{y^T D y}$ and the diagonal matrix $D = \text{diag}(\pi_1, \pi_2, \ldots, \pi_N)$ is defined in (14) of Li et al. (2013). In view of the facts that $\|W\|_{\infty} = 1$ for the stochastic matrix $W$ and $\|W\|_2 \leq \sqrt{N} \|W\|_{\infty}$, $\|\alpha (W - I)\|_2 \leq \alpha (\|W\|_2 + \|I\|_2) \leq 2\sqrt{N}\alpha$. Correspondingly, the expressions for two parameters, Eqs. (22) and (24), in the statement of Theorem 1 in Li et al. (2013), can be slightly modified as follows:

$$M_1(\alpha, \gamma) = \frac{2\alpha^2 \sqrt{\pi_{\max}} N (1 + \kappa^{-1})^{\frac{1}{2}}}{\sqrt{\pi_{\min}} \gamma (\gamma - \rho_\eta)} + \frac{1 + 2\alpha}{2\gamma},$$

(22)

where $\rho_\eta = \left(1 - \frac{n}{4(N-1)}\right)^2$ and $\kappa = \frac{n}{4(N-1)-2\eta}$.

$$g_0 \geq \max \left\{ \frac{C_x}{K + \frac{1}{2}}, \frac{2\alpha C_x (1 + \kappa^{-1})^{\frac{1}{2}} + \gamma C_x (\gamma - \rho_\eta)}{\alpha (1 + \kappa^{-1})^{\frac{1}{2}}} \right\}.$$  

(24)

Acknowledgments

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References