Match-Filtering Based TOA Estimation for IR-UWB Ranging Systems

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Abstract—Accurate geolocation techniques using IR-UWB (impulse radio ultra-wideband) signals have gained more and more interests recently. To exploit the fine time resolution of IR-UWB most, TOA (time of arrival) based ranging is the most appropriate technique. Considering the challenges for the design of TOA estimation algorithms in many aspects, a three-step match-filtering detection based algorithm is proposed. First, the peak detection on the match-filtering output, the search region for DP (direct path) detection is determined; then, a rough detection of DP is made by threshold comparison; last, a refined search process is carried out to obtain the precise location of DP. The threshold-factor in the second step is dynamically set using a joint-metric in terms of kurtosis and RMS (root mean square) delay spread of the match-filtering output. Performance comparisons with other algorithms prove that the proposed three-step algorithm can achieve a good trade-off between computation efficiency and estimation accuracy. The reliability of TOA estimation result is also discussed. Three levels of reliability are defined and the probability density functions for TOA estimation errors in each level are modeled. It is expected to improve the final location estimation accuracy further by properly incorporating the reliability information into the positioning algorithms.

Keywords – geolocation, IR-UWB, TOA estimation, match-filtering.

I. INTRODUCTION

Impulse radio ultra-wideband (IR-UWB) is very appropriate for high-precision ranging in dense multipath environments due to its extremely short pulse duration. It has now been regarded as the most promising physical layer technology for wireless sensor networks [1]. To exploit the fine time resolution most, ranging techniques based on estimating the time of arrival (TOA) of the received signal are most reasonable.

TOA estimation algorithms have been extensively studied these years, including those considering high sampling rate, match-filtering based coherent algorithms, and those considering lower sampling rate, energy-detection based non-coherent algorithms. All the algorithms try to detect the direct path (DP) component of the received signal to estimate the TOA. The fundamental advantage of non-coherent algorithms is the fast convergence [2]-[4], which is due to the low-rate sampling; but meanwhile, the estimation accuracy is not so high. In [5] and [6], the thresholds for DP block detection are optimized, where the kurtosis and the MMR (maximum to minimum energy sample ratio) of the energy samples are used as the optimization metrics respectively. The performances are improved consequently but still not so satisfactory due to the low time granularity of the energy samples. Given that the receiver sampling rate is high enough, match-filtering based coherent algorithms can make the most of the ranging ability of IR-UWB. Research on coherent algorithms was first conducted in [7], where a CLEAN [8] based iterative correlation algorithm was proposed; while high precision can indeed be achieved, the iterative amplitude estimating and adjusting process makes the algorithm extremely computational burdensome. Reference [9] optimized the process of amplitude adjusting, which greatly reduced the computation complexity without do harm to the performance; however, when severe multipath environments are encountered, the algorithm still has to carry out repetitious correlations and amplitude estimations. Therefore, some researchers considered detecting DP directly from the match-filtering output of the received signal. In [10], the peak of match-filtering output was considered as the location of DP, but this is only applicable for line-of-sight (LOS) situations with transceiver antennas being omnidirectional. Threshold detection was conducted in [11], but the threshold-factor for calculation of the threshold is fixed for all ranging signals. Reference [12] considered using dynamic thresholds, and the threshold setting model was derived by minimizing the probability of large errors; however, the setting model seemed related to so many metrics which are difficult to estimate from the received signals that the algorithm is not practically applicable.

Considering the multiple challenges (including the estimation accuracy, computation complexity and practicability) that practical applications have on TOA estimation algorithms, a three-step algorithm based on threshold detection on the match-filtering output is proposed in this paper. First, by peak detection on the match-filtering output, the search region for DP detection previous to the peak point is determined; second, threshold detection is carried out in the search region to estimate the rough location of DP; last, a refined search for the precise location of DP is conducted right after the detection point by the second step. The issue that affects the algorithm performance most is how the threshold in the second step is set. Here we proposed a dynamic threshold-factor setting method, which uses a joint-metric in terms of the kurtosis and the RMS delay spread of the match-filtering output to calculate the optimal threshold-factor. Since both kurtosis and RMS
delay spread are easy to obtain from the received samples, the proposed algorithm is highly practicable. Simulations under the IEEE 802.15.4a channel models [13] demonstrate that the threshold-factor setting model is well independent of the channel model, making the algorithm universally applicable for different situations.

In order to improve the final positioning accuracy to the most extent, the ranging module should provide not just the ranging results, but also the corresponding reliability information to the positioning module. In this paper, the threshold-factor setting metric, i.e. the joint-metric, is also considered as the metric for reliability evaluation. Through statistical analysis, the ranging results are ranked into three levels of reliability according to their joint-metrics, and the probability density functions (PDFs) for TOA estimation errors in each level are modeled. Therefore, when the ranging results are provided to the positioning module, their corresponding joint-metrics can be attached. By incorporating the reliability information (including the reliability levels and the error PDFs) implied by the joint-metric into the location estimation algorithms, the positioning accuracy will inevitably be improved.

The rest of the paper is organized as follows. Section II presents the three-step TOA estimation algorithm. In section III, the dynamic threshold-factor setting method is discussed. The performance of the proposed algorithm is simulated in section IV. In section V, reliabilities of the ranging results are discussed through error analysis and modeling, and finally in section VI conclusions are given.

II. THE TOA ESTIMATION ALGORITHM

A. The Received Signal Representation

When the transmitted pulse arrives at the receiver, the received signal is composed of DP, late arriving multipath components, noise, and interference. So the received signal can be represented as

\[ r(t) = \sum_{i=1}^{L} a_i p(t - \tau_i) + n(t) \]

\[ = a_{DP} p(t - \tau_{DP}) + \sum_{i=2}^{L} a_i p(t - \tau_i) + n(t) \]  

(1)

where \( \tau_{DP} = \tau_1 < \tau_2 < \ldots < \tau_L \) (assuming that the first arriving path is just DP), \( L \) represents the total number of multipaths; \( p(t) \) denotes the single path waveform, with a width of \( T_p \) nanoseconds; \( n(t) \) is the AWGN with zero mean and two-sided power spectral density \( N_0/2 \).

The receiver can take averages of \( N_S \) pulse transmissions over the symbol duration (i.e. the pulse repetition interval, denoted by \( T_f \)) to obtain a processing gain. Let \( r_{avg}(t) \) denote the averaged received signal, whose match-filtering output is

\[ c(t) = r_{avg}(t) \otimes p(t) \].  

(2)

The task of TOA estimation is just to detect DP from \( c(t) \).

B. The DP Detection Algorithm

The process of DP detection can be carried out in three steps. First, detect the peak point of the match-filtering output, and the region preceding the peak is determined as the search region for DP detection. Second, estimate the rough location of DP by threshold detection within the search region. Last, the precise location of DP, i.e. the center of DP, is obtained by a refined search process. Fig. 1 illustrates the general flows of the three-step algorithm, and next we are going to present the details of each step.

1) Step 1—determine the search region for DP detection:
The location of the strongest path (SP) can be detected by peak detection on the match-filtering output, i.e., \( \hat{t}_{SP} = \text{arg max}_{0 \leq t < T_f} |c(t)| \). In LOS situations with the transceiver antennas being omnidirectional, DP is just SP; in other situations, the received energy of DP may not be the strongest, so DP arrives earlier than SP. The search process for DP can therefore be limited in a certain region preceding SP. Two advantages can be introduced by this operation. First, the signal region after SP will not be processed, so computational complexity is reduced. Second, just a part instead of the entire region preceding SP is searched on, so the noise-only region far before SP will not be processed, which can restrain the probability of early false alarms to some extent. The length of the search region should be set based on the statistics of the arrival time difference of DP and SP, i.e., \( \delta = t_{SP} - t_{DP} \).

In this paper, the IEEE 802.15.4a CM1 (residential LOS) and CM2 (residential NLOS) channel models are used for simulations. Statistical result of 1000 independent channel impulse responses shows that for both CM1 and CM2, the value of \( \delta \) satisfies \( P_{r}(\delta < 60) \text{ns} \approx 1 \), so by setting the length of the search region \( \delta_{max} \) to 80ns, it can be guaranteed that DP is within the search region. That is, the start search point can be denoted as \( \tau_{start\ point} = \hat{t}_{SP} - \delta_{max} \), and the search region is \([\tau_{start\ point}, \hat{t}_{SP}]\).

2) Step 2—estimate the rough location of DP by threshold detection: Comparing the samples within the search region
It is worth noting that in practice, due to the effects of noise, interference and nearby multipath components, the center of DP may still not be able to detect absolutely accurately, but the amplitude of the errors can be reduced to less than $T_p/2$.

### III. Dynamic Threshold-factor Setting

Throughout the process of TOA estimation, we can see that the step that affects the final estimation accuracy most is step 2, where the thresholds should be tuned in order to make the detection points fall within the region of DP as many as possible. In this section, the details of dynamic threshold-factor setting method are discussed.

#### A. The Selection of Setting Metric

For the sake of practical application, we should define a simple metric from the received ranging signal to dynamically set the threshold-factor. The metric should first be easy to obtain from the received samples, and in addition it must capture the characteristics of the ranging signal as much as possible. That is, the SNR information and the characters of the current channel (including both the amplitude and delay statistics) should all be reflected by the metric.

In [5] and [6], where energy-detection based non-coherent algorithms are discussed, the kurtosis and MMR of the signal energy samples are used as the metrics for threshold calculation respectively. However, kurtosis and MMR alone can just capture SNR information and amplitude statistics of the received multipath components (MPCs), while the information of the delay properties of the received MPCs is not captured. The RMS delay spread is a statistic which can characterize the delay information of the multipath channel [14], so we can consider using a metric that jointly exploits the kurtosis and the RMS delay spread of the received MPCs for threshold-factor setting. To formulate the joint-metric, the respective effects of kurtosis and RMS delay spread on DP detections should be taken into account. As far as kurtosis is concerned, larger kurtosis often implies higher SNRs [15], which makes the detection of DP easier; as far as RMS delay spread is concerned, the larger its value is, the severer the multipath effect is, which makes DP detection more difficult. To be brief, the effect of kurtosis on DP detection is positive, while that of RMS delay spread is negative, so we can formulate the joint-metric as

$$M = \frac{\tau_{\text{rms}}}{3k}$$

where $\tau_{\text{rms}}$ and $k$ just denote the RMS delay spread and the kurtosis, they are placed as nominator and denominator respectively to show that their effects on DP detection are contrary; the scaling coefficient $1/3$ is used to constrict the value of $M$ within a relatively small range. Assuming that the match-filtering output samples are $\{c_n\}, n = 1, 2, \ldots, N$ (where $N = \lceil T_f/t_s \rceil$ with $t_s$ denoting the sampling interval),

![Fig. 2. Possible detection areas of threshold detection on the match-filtering output of single pulse waveform.](image)
the kurtosis and RMS delay can be calculated as follows:

\[ k = \frac{N}{(\sum_{n=1}^{N} c_n^2)^2} \left( \sum_{n=1}^{N} c_n^4 \right) \]

\[ \tau_{rms} = \sqrt{\frac{\sum_{n=1}^{N} (nt_s)^2 c_n^2}{\sum_{n=1}^{N} c_n^2} - \left( \frac{\sum_{n=1}^{N} nt_s c_n^2}{\sum_{n=1}^{N} c_n^2} \right)^2} \]

**B. The Setting Model**

The simulation results obtained for 700 channel realizations at \( E_b/N_0 = \{10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30\} \) dB, i.e. for 7700 simulation cases, are used for modeling. Fig. 3 shows the TOA estimation MAE (mean absolute error) with respect to the threshold-factor under different joint-metrics (which have been rounded to integers) for CM1 (the plot for CM2 is similar). Decreasing MAE with respect to decreasing \( M \) can be observed; for each \( M \), the threshold-factor that yields the minimum MAE is just the optimal threshold-factor. For better visualization and the convenience of relationship modeling, the optimal threshold-factor for each \( M \), i.e. the abscissa of the lowest point of every curve in Fig. 3, is extracted and plotted in Fig. 4. It can be observed that the optimal threshold-factors for CM1 and CM2 almost overlap, which means that the relationship between the optimal threshold-factor and \( M \) is well independent of the channel model. When \( M \) increases, the optimal threshold-factor shows an increasing trend on the whole (slightly decreases when \( M > 10 \)). Considering these characteristics, the joint-metric based optimal threshold-factor calculation function can be modeled as

\[ \gamma_{opt} = p_1 M^4 + p_2 M^3 + p_3 M^2 + p_4 M + p_5 \quad (9) \]

where \( p_1 = -0.0003217, p_2 = 0.006987, p_3 = -0.04731, p_4 = 0.1596 \) and \( p_5 = 0.2068 \).

**IV. PERFORMANCE SIMULATIONS**

To verify the validity of the joint-metric based threshold-factor setting method as well the effectiveness of the proposed three-step TOA estimation algorithm, performance simulations are carried out in this section. The channel models CM1 and CM2 of IEEE 802.15.4a are employed. 700 independent realizations for each channel model are generated, with TOAs uniformly distributed within \((0, T_f)\), Gaussian doublet pulse with 1ns duration and 2.7GHz bandwidth is considered throughout the simulations. The other simulation parameters are: \( T_f = 200\)ns, \( t_s = 0.025\)ns, and \( N_s = 1 \).

**A. Compared with Using Fixed Threshold-factor**

In Fig. 5, the performance of the proposed three-step algorithm that generated by using joint-metric based dynamic threshold-factor is compared with that generated by using fixed threshold-factor under CM1 (the plot for CM2 is similar). It can be observed that the performance generated by fixed threshold-factor is not satisfying. In particular: when \( \gamma \) is small \((\gamma \leq 0.5)\), the algorithm performs badly at low-to-moderate SNRs due to the high probability of early false alarms; when \( \gamma \) becomes relatively large \((\gamma > 0.5)\), the performance at high SNRs turns worse due to the high probability of missed detections.In a word, there not exists any fixed threshold-factor that can generate good performance at all SNRs. As for the joint-metric based dynamic threshold-factor setting method, the resulting performance shows to be the best at nearly all SNRs, which validates the idea of using the joint-metric to set the threshold-factor. And the validity of the calculation model built in section III-B is also endorsed.
Fig. 5. The TOA estimation performance generated by using joint-metric based threshold-factor setting vs. that generated by using fixed threshold-factor.

Fig. 6. Performance comparison of three different algorithms.

B. Comparison to Two Other Algorithms

Of the existent non-coherent and coherent algorithms, the algorithms proposed in [6] and [9] have the best performance. The performance of our algorithm was compared with these two algorithms in Fig. 6. The plot denoted by MF-TC-JM (match-filtering based threshold crossing algorithm with joint-metric based dynamic threshold-factor) just corresponds to the three-step algorithm proposed in this paper, while the one denoted by ED-TC-MMR (energy detection based threshold crossing algorithm with MMR based dynamic normalized threshold) is for the algorithm in [6] and I-CLEAN-GML (improved CLEAN based GML algorithm) for the algorithm in [9]. It can be observed from Fig. 6 that MF-TC-JM outperforms ED-TC-MMR, which just agrees well with the theoretical analysis — MF-TC-JM adopts higher sampling rate thus the time resolution of the received samples is much higher. Compared to I-CLEAN-GML, MF-TC-JM shows a slightly weaker (on the order of 2dB) performance, which can also be predicated—the implementation of I-CLEAN-GML involves iterative correlation, amplitude estimating and adjusting, i.e., the high precision of I-CLEAN-GML is gained at the expense of great computation efficiency. While for MF-TC-JM, threshold detection is directly applied to the match-filtering output, the computation load is only about 1/10 of that needed by I-CLEAN-GML. Although the performance of MF-TC-JM is a little weaker than I-CLEAN-GML, its precision is still high enough for high accuracy positioning. In particular, the performance of less than 1m error (about 3.3ns TOA estimation error) can be obtained when $E_b/N_0 > 19$dB under CM1 and $E_b/N_0 > 22$dB under CM2. Therefore, MF-TC-JM achieves a good trade-off between TOA estimation accuracy and computation efficiency, which makes it more appropriate for current applications.

V. ERROR ANALYSIS AND MODELING

After TOA estimation, the ranging results will be sent to the positioning module for location estimation. To improve the positioning accuracy as much as possible, the reliability information of every ranging result should also be provided. As for the algorithm proposed in this paper, the threshold-factor directly determines the ranging precision, so its setting metric, i.e. the joint-metric, can also serve as a metric for reliability evaluation.

All the ranging results are grouped by joint-metric, and the TOA estimation MAE for each group is calculated and plotted in Fig. 7. It can be observed that under both CM1 and CM2 MAE increases with $M$. In particular: when $M < 5$, MAE is less than 1ns (i.e. 30cm ranging error), indicating that the ranging results are highly reliable; when $5 \leq M \leq 9$, MAE is about 1~4ns (i.e. 30~120cm ranging error), meaning that the ranging results become less reliable, but still tolerable for positioning; when $M > 9$, MAE increases up to more than
10 ns (i.e., 3 m ranging error), indicating that the ranging results are unreliable. Therefore, the ranging results can be ranked into three levels of reliability based on their joint-metrics: low-reliability ($M > 9$), medium-reliability ($5 \leq M \leq 9$) and high-reliability ($M < 5$).

The reliability level implied by the value of the joint-metric is just a rough reliability evaluation. Associated with the probability density of the TOA estimation errors, more detailed reliability information can be obtained. Based on whether DP is successfully detected, the TOA estimation errors can be classified into three types: the error for successful detections (SD_error), for early false alarms (EFA_error), and for missed detections (MD_error). When DP is successfully detected, due to the effects of noise and neighboring multipath components, the center of DP may not be accurately located, i.e., errors on the order of the pulse width still exist, and this kind of error is just the SD_error, whose distribution range is $[-T_p/2, T_p/2]$. EFA_error occurs when a false detection in the noise-only portion of the signal is regarded as the actual DP, and its distribution range is $(T_p/2, +\infty)$. For the convenience of PDF modeling, we shift the original EFA_error and MD_error to $-EFA_error - T_p/2$ and $EFA_error - T_p/2$, respectively, thus the distribution range of both types of error can be shifted to $(0, +\infty)$, which matches that of most probability distribution models. In the following subsections, the three types of error for each reliability level are modeled separately. Due to space constraints, we will just present the modeling details of CM1, while the modeling results for both CM1 and CM2 will be summed up in Table 1 last.

A. PDF Modeling for Low-reliability Errors (CM1)

The low-reliability errors are just those satisfying $M > 9$. Let the TOA estimation error be denoted by $\tau_{err} = \hat{\tau}_{DP} - \tau_{DP}$, the probability of the EFA_error, MD_error and SD_error occurrence can be calculated as

\begin{align}
P_{\text{EFA}} &= \Pr(\tau_{err} < -T_p/2) \\
P_{\text{MD}} &= \Pr(\tau_{err} > T_p/2) \\
P_{\text{SD}} &= \Pr(-T_p/2 \leq \tau_{err} \leq T_p/2)
\end{align}

Fig. 8-a, -b and -c are the normalized histograms of the EFA_error, MD_error and SD_error; the probability density of each type of error can be modeled by curve fitting on these normalized histograms.

1) PDF of the EFA_error: The modeling result is Nakagami distribution, i.e.

\begin{align}
f(\tau_{err_{EFA}}) &= 2 \left( \frac{\mu_{EFA}}{\omega_{EFA}} \right) \frac{1}{\Gamma(\mu_{EFA})} e^{(\mu_{EFA} - 1)} e^{\frac{\mu_{EFA}^2}{\omega_{EFA}^2} \tau_{err_{EFA}}^2}, \tau_{err_{EFA}} > 0
\end{align}

2) PDF of the MD_error: The modeling result is Lognormal distribution, i.e.

\begin{align}
f(\tau_{err_{MD}}) &= \frac{1}{\tau_{err_{MD}} \sigma_{MD} \sqrt{2\pi}} e^{-\frac{(\ln \tau_{err_{MD}} - \mu_{MD})^2}{\sigma_{MD}^2}}, \tau_{err_{MD}} > 0
\end{align}

3) PDF of the SD_error: It has been analyzed in section II-B that if DP can be successfully detected, there are two possible detection areas. We can observe from Fig. 8-c that the TOA estimation errors corresponding to the DP center (i.e. within area 2) are much more than that detecting close to the first minor peak (i.e. within area 1), this is just due to the function of step 3 of the DP detection algorithm — the refined search process makes the final detection point close to the DP center as much as possible. Let $P_1$ and $P_2$ denote the error distribution probability within area 1 and area 2 respectively, then the PDF of SD_error can be modeled as weighted sum of two Gaussian distributions, i.e.

\begin{align}
f(\tau_{err}) &= P_1 \cdot \frac{1}{\sigma_1 \sqrt{2\pi}} e^{-\frac{(\tau_{err} - \mu_{1})^2}{2\sigma_1^2}} + P_2 \cdot \frac{1}{\sigma_2 \sqrt{2\pi}} e^{-\frac{(\tau_{err} - \mu_{2})^2}{2\sigma_2^2}}, \tau_{err} \in [-0.5, 0.5]
\end{align}

B. PDF Modeling for Medium-reliability Errors (CM1)

The medium-reliability errors correspond to those satisfying $5 \leq M \leq 9$. The probability of EFA_error occurrence approximately equals zero so that its density modeling is skipped, while that of MD_error decreases greatly compared to the low-reliability level. The density modeling results for MD_error and SD_error are just the same as that for the low-reliability level. That is, the density of MD_error can be...
modeled as Lognormal distribution, while that of SD_error can be modeled as the sum of two weighted Gaussian distributions.

C. PDF Modeling for High-reliability Errors (CM1)

The high-reliability errors are just those satisfying $M < 5$. Both the probabilities of EFA_error occurrence and MD_error occurrence are close to zero, so we believe that when $M < 5$ DP will surely be successfully detected. The PDF of SD_error is naturally modeled as the sum of two weighted Gaussian distributions.

D. Summarization of the Modeling Results

The modeling results for CM2 are just the same as their counterparts for CM1, except that the parameter values are different. All of the modeling results are summarized in Table 1, where the parameter values of the error PDFs for each reliability level under both CM1 and CM2 are displayed. From Table 1 we can observe that most model parameters have approximate values under CM1 and CM2, which suggests a simpler way of using the PDFs — using the mean parameter values of CM1 and CM2 for all ranging situations. While it means that the error PDFs are considered to be independent of the channel model, the resulting positioning performance may decrease to a certain extent, and this is what we plan to discuss in the future work.

VI. CONCLUSIONS AND FUTURE WORK

Estimation accuracy, computation complexity and practicability are the three aspects that we care most when designing TOA estimation algorithms. In this paper, a three-step algorithm based on threshold-detection on the match-filtering output of the received ranging signal is proposed. The threshold-factor used to calculate the threshold is dynamically set using a joint-metric in terms of the kurtosis and the RMS delay spread of the match-filtering output. The joint-metric is easy to get from the received samples and the setting model is well independent of the channel model, indicating that the algorithm is highly practicable. Performance comparisons with two other existing algorithms prove that the proposed three-step algorithm can achieve a good trade-off between computation efficiency and estimation accuracy. The idea of attach reliability information to the ranging results for positioning module to use is also proposed. The joint-metric for threshold-factor setting is also used as the metric for reliability evaluation. Based on statistical analysis, three levels of reliability are adopted, and the error PDFs for each reliability level are modeled. By incorporating the reliability information into the location estimation algorithms, it is expected to improve the positioning accuracy. As the future work, we plan to set up a joint-ranging-positioning simulation platform, by which the detailed strategy of how to use the reliability information will be discussed.

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