Distributed Non-Parametric Estimation in a Bandwidth-Constrained Sensor Network

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***Abstract*—Non-parametric estimation of an unknown position parameter in a bandwidth-constrained wireless sensor network (WSN) is considered in this paper. Due to bandwidth constraint, each sensor is restricted to send only one bit of information to a fusion center. We propose a non-parametric estimator that employs a recently introduced adaptive quantization (AQ) scheme. Specifically, the position parameter is estimated as the sample mean of the quantization thresholds used in AQ. The proposed non-parametric estimator is based on the fact that the AQ thresholds asymptotically converge (in mean) to the unknown position parameter, under the condition that the position parameter is an integer multiple of the stepsize used in AQ. When the condition is not met, there is a bias which can, however, be made negligible by choosing the stepsize to be small (compared with the position parameter). Numerical results are provided to demonstrate the effectiveness of the proposed non-parametric estimator.**

***Index Terms*—Wireless sensor network, distributed estimation, non-parametric estimation.**

I. Introduction

Wireless sensor networks (WSNs) have been receiving considerable attention. It can be applied for military surveillance, communications and seismic analysis [1]–[3]. Bandwidth consumption is a critical issue in WSNs, as limited communication bandwidth is shared across the entire network [4]–[7]. As such, a major challenge of the WSN research is to design bandwidth efficient signal processing techniques. A number of studies have appeared recently in the context of distributed detection [8]–[10], distributed estimation [6], [11]–[13], distributed compression [14], [15], etc.

When the probability density function (PDF) of the sensor noise is known, parametric estimators for bandwidth-constrained WSNs were proposed in [6], [11]–[13], [15], [16]. However, in many practical applications, e.g. large-scale sensor networks operating in time-varying environments, an accurate parametric noise model is often difficult to obtain. This motivates distributed non-parametric estimation method without the knowledge of the PDF of the sensor noise. In [5], a non-parametric estimation scheme was proposed by letting a half of the sensors send the first most significant bit (MSB) of their observations to the fusion center, one fourth of the sensors send their second MSB, and so on and so forth. In [7], a non-parametric estimator was introduced by utilizing the fact that numerically integrating the complementary cumulative density function (CCDF) using the trapezoidal rule yields an approximation of the mean.

In this paper, we propose a new distributed non-parametric estimator for position parameter estimation by utilizing an adaptive quantization (AQ) scheme recently introduced in [16]. In AQ, the thresholds are adaptively adjusted from one sensor to another, in a way such that the thresholds converge (in mean) to the unknown position parameter. By noticing that the AQ scheme of [16] is a non-parametric quantization scheme, a non-parametric estimator is introduced by averaging the AQ thresholds after convergence. When the unknown position parameter is an integer multiple of the stepsize of AQ, the proposed estimator is shown to be asymptotically unbiased. When the position parameter is not an integer multiple of the stepsize, the proposed estimator has a bias, which can be made negligible by choosing the stepsize to be small (compared with the position parameter).

The rest of this paper is organized as follows. In Section II, the distributed non-parametric estimation problem is formulated. In Section III, we first briefly review the AQ scheme and then introduce our AQ-based non-parametric estimator. Numerical results are provided in Section IV, followed by concluding remarks in Section V.

II. Problem Formulation

Consider a WSN consisting of N sensors, where each sensor makes a noisy observation of an unknown position parameter θ as

$$x_n = \theta + w_n, \qquad n = 1, 2, \cdots, N, \tag{1}$$

where w_n denotes the sensor noise with zero-mean and variance σ_w^2, assumed independent and identically distributed (i.i.d.) across sensors. The PDF of w_n is assumed symmetric with respect to zero but is otherwise unknown to the estimator.

If local sensors have sufficient power and no bandwidth constraint, they can send their unquantized observations to the fusion center. Then, the fusion center can simply use the sample mean estimator

$$\hat{\theta} = \frac{1}{N}\sum_{n=1}^{N} x_n, \tag{2}$$

which has a MSE of

$$E\left(|\hat{\theta} - \theta|^2\right) = \frac{\sigma_w^2}{N}. \tag{3}$$

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However, due to bandwidth constraint, the sensor observations have to be quantized and the estimation can only be performed using the quantized data. Specifically, we consider the case where each sensor uses a 1-bit quantizer:

$$b_n = Q_n(x_n), \qquad n = 1, 2, \cdots, N, \tag{4}$$

where Q_n denotes a 1-bit quantizer for the nth sensor. The problem of interest is to determine the binary quantizers and estimate the unknown parameter θ based on the binary data $\{b_n\}$.

III. Proposed Non-Parametric Estimator

Our non-parametric estimator is based on the AQ scheme introduced in [16]. In the following, we first provide a brief review of AQ and then discuss our AQ-based non-parametric estimator.

A. AQ

In the AQ scheme, each sensor accumulates earlier transmissions from other sensors, and uses the accumulated value as the threshold for its 1-bit quantizer. Specifically, it is assumed that the sensors share the communication channel on a time-sharing basis (e.g., each sensor is polled by a fusion center), so that the first sensor transmits first, followed by the second sensor, and so on and so forth. The 1-bit quantizer at the first sensor uses a zero-threshold (i.e., $\tau_1 = 0$) to generate a binary data b_1:

$$b_1 = \text{sgn}\{x_1\}. \tag{5}$$

where sgn$(\cdot)$ denotes the sign function:

$$\text{sgn}(x) = \begin{cases} 1, & \text{if } x \geq 0, \\ -1, & \text{if } x < 0. \end{cases} \tag{6}$$

Then, b_1 is broadcasted to the fusion center as well as the other $N-1$ sensors. After receiving b_1, the second sensor computes the threshold $\tau_2 = \Delta b_1$, where Δ is a positive *stepsize* parameter, and generates b_2:

$$b_2 = \text{sgn}\{x_2 - \tau_2\}. \tag{7}$$

In general, for the nth sensor, it first forms a cumulative sum:

$$\tau_n = \tau_{n-1} + \Delta b_{n-1} = \Delta \sum_{i=1}^{n-1} b_i, \tag{8}$$

and then it uses τ_n as a threshold for quantization:

$$b_n = \text{sgn}\{x_n - \tau_n\}. \tag{9}$$

One can easily recognize that the above process is reminiscent of the Delta Modulation (DM), but is implemented in a distributed fashion.

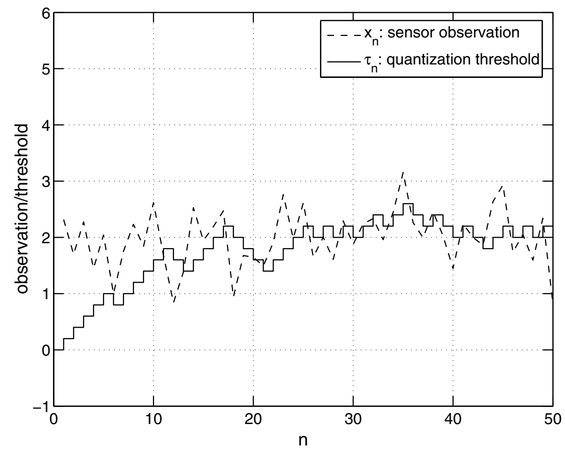

Fig. 1. Illustration of the convergence of the AQ thresholds.

B. Threshold Analysis

One interesting property of the AQ scheme is that the thresholds in (8) move towards the unknown parameter θ as n increases. Fig. 1 depicts the convergence of the thresholds towards the unknown parameter $\theta = 2$ corrupted by i.i.d. Gaussian noise with zero-mean and variance $\sigma_w^2 = 0.5$. From Fig. 1, it is clear that there are two types of states related to the thresholds: a *transient* state that moves the thresholds towards θ and a *convergent* state where the thresholds oscillate around θ.

From (8), the thresholds τ_n can be expressed as

$$\tau_n = \tau_{n-1} + \Delta \text{sgn}\{x_{n-1} - \tau_{n-1}\}. \tag{10}$$

Clearly, the thresholds $\tau_n, n = 1, \cdots, N$, form a Markov chain, since

$$P\{\tau_n | \tau_1, \tau_2, \cdots, \tau_{n-1}\} = P\{\tau_n | \tau_{n-1}\}, n = 1, \cdots, N. \tag{11}$$

Furthermore, the transition probabilities are independent of n and the Markov chain is homogeneous. Since the thresholds are modified with an increment of Δ and $-\Delta$ at each step, the transition probability is given by

$$\begin{aligned} &P(\tau_{n+1} = j\Delta | \tau_n = i\Delta) \\ &= \begin{cases} 1 - F_w(i\Delta - \theta) & \text{if } i = j+1 \\ F_w(i\Delta - \theta) & \text{if } i = j-1 \\ 0 & \text{if } |i-j| > 1 \end{cases}, \end{aligned} \tag{12}$$

where F_w denotes the CCDF of w_n.

Based on the above observation, we have the following asymptotic result concerning the mean of τ_n:

Proposition 1: When θ is an integer multiple of Δ, the threshold τ_n asymptotically converges in mean to the unknown θ, i.e.,

$$\lim_{n \to \infty} E\{\tau_n\} = \theta, \text{ with probability } 1. \tag{13}$$

Proof: See Appendix. ■

In practice, since θ is unknown, we cannot ensure θ/Δ is an integer and, therefore, the above result does not hold exactly.

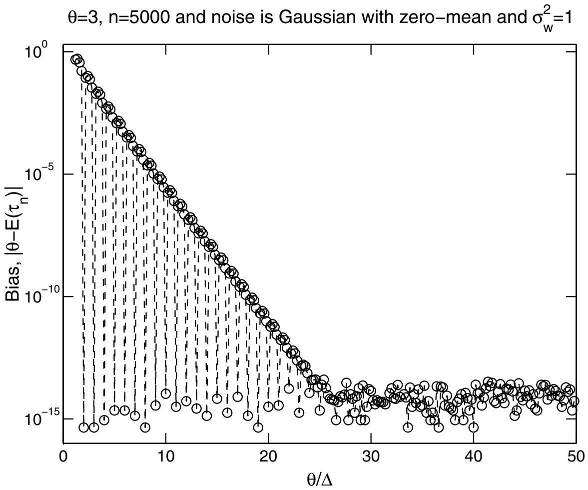

Fig. 2. Asymptotical bias of τ_n versus θ/Δ when $\theta = 3, n = 5000$, and the sensor noise is i.i.d. Gaussian with zero-mean and variance $\sigma_w^2 = 1$.

However, the bias, i.e., $|E\{\tau_n\} - \theta|$, for large n can be made negligible by choosing Δ sufficiently small such that $\Delta/\theta \ll 1$ (therefore θ/Δ is an approximate integer). To show this, Fig. 2 depicts the bias versus θ/Δ when $\theta = 3$ and $n = 5000$, which is chosen large enough to mimic an "asymptotic" scenario, when the sensor noise is i.i.d. Gaussian with zero-mean and variance $\sigma_w^2 = 1$. Note that the bias is computed numerically using the distribution of τ_n given in [16]. It is seen that when θ/Δ is an integer, the bias drops lower than 10^{-14}, suggesting τ_n is effectively unbiased in such a case. On the other hand, the bias is noticeable larger when θ/Δ is not an integer, and eventually becomes non-negligible when Δ is comparable to θ. Nevertheless, it is interesting to note that when Δ is small enough, say $\theta/\Delta \geq 25$ (i.e., $\Delta/\theta \leq 0.04$), the bias is less than 10^{-13} irrespective of whether θ is an integer multiple of Δ or not. Hence, by choosing Δ sufficiently small, the bias of τ_n for large n is negligible.

C. AQ-Based Non-Parametric Estimator

The above asymptotic result motivates us to use the mean of the thresholds, i.e., $E\{\tau_N\}$, as a non-parametric estimate of the unknown θ. A reasonable estimate of $E\{\tau_N\}$ is the sample mean of the thresholds after convergence, i.e.,

$$\hat{\theta} = \frac{1}{N-M} \sum_{n=M+1}^{N} \tau_n, \tag{14}$$

where M is the sensor index where the threshold hits on θ (i.e., $\tau_M = \theta$) or practical convergence to θ has been achieved. Since M is unknown, we need to find an estimate of M and replace it in (14)

$$\hat{\theta} = \frac{1}{N-\check{M}} \sum_{n=\check{M}+1}^{N} \tau_n, \tag{15}$$

where $\check{M}$ denotes an estimate of M. In Section III-D, we will discuss how to obtain $\check{M}$.

Since we estimate M using the binary data b_n, $\check{M}$ is a random variable. To gain an insight into the behavior of the non-parametric estimator, we consider two cases when M is *overestimated* (i.e., $\check{M} > M$) and when it is *underestimated* ($\check{M} < M$), respectively. It is easy to verify that, when $\check{M} \geq M$, the proposed non-parametric estimator is asymptotically unbiased, i.e., $E\{\hat{\theta}|\check{M} \geq M\} = \theta$. On the other hand, when $\check{M} < M$, we take expectation on both sides of (15)

$$\begin{aligned} E\{\hat{\theta}|\check{M} < M\} =& \frac{1}{N-\check{M}} \sum_{n=\check{M}+1}^{N} E\{\tau_n\}, \\ =& \frac{1}{N-\check{M}} \left[\sum_{n=\check{M}+1}^{M} E\{\tau_n\} + (N-M)\theta \right] \\ =& \theta, \qquad N \to \infty \end{aligned} \tag{16}$$

where the last equation is based on the fact that $\{\tau_n\}_{n=\check{M}+1}^{M}$ are bounded and, thus, the corresponding summation is bounded. By combining both cases, we have

$$\begin{aligned} E\{\hat{\theta}\} =& \sum_{\check{M}<M} E\{\hat{\theta}|\check{M} < M\} P\{\check{M} < M\} \\ &+ \sum_{\check{M} \geq M} E\{\hat{\theta}|\check{M} \geq M\} P\{\check{M} \geq M\} \\ =& \theta, \qquad N \to \infty. \end{aligned} \tag{17}$$

Therefore, we conclude that the proposed non-parametric estimator is unbiased in the asymptotical sense.

Remark: Clearly, the choice of Δ is crucial to the performance of the proposed non-parametric estimator. While the above asymptotic analysis suggests choosing a small Δ such that τ_n is asymptotically unbiased, choosing Δ too small when the total number of sensors N is limited may cause τ_n not reaching the convergent state, in which case the asymptotic analysis is no longer valid. Hence, the choice of Δ should be made with a tradeoff by considering both effects. Meanwhile, the previous discussion may also suggest overestimation of M may be acceptable (e.g., by choosing M close to N to avoid underestimation) since it does not affect the asymptotic result. However, when N is finite as in practical applications, excessively overestimating M leads to inadequate averaging in (15) and, in turn, a large variance in the estimate $\hat{\theta}$.

D. Estimate of Convergence Index

The estimate of M is based on the following observation: during the transient phase, the binary data $\{b_n\}_{n=1}^{M}$ are more likely to be 1 since the thresholds $\{\tau_n\}_{n=1}^{M}$ are lower than θ (suppose $\theta > 0$), while, during the convergent state, $\{b_n\}_{n=M+1}^{N}$ tend to be 1 and -1 equally likely. A simple strategy to differentiate these two states is to employ windowed summation of these binary data. In the transient state, the windowed summation of the binary data is approximately equal to the window size, while it oscillates around zero after convergence. Hence, to determine the convergence index or estimate M, an algorithm counting the number of zero-crossings of the windowed summation is proposed as follows:

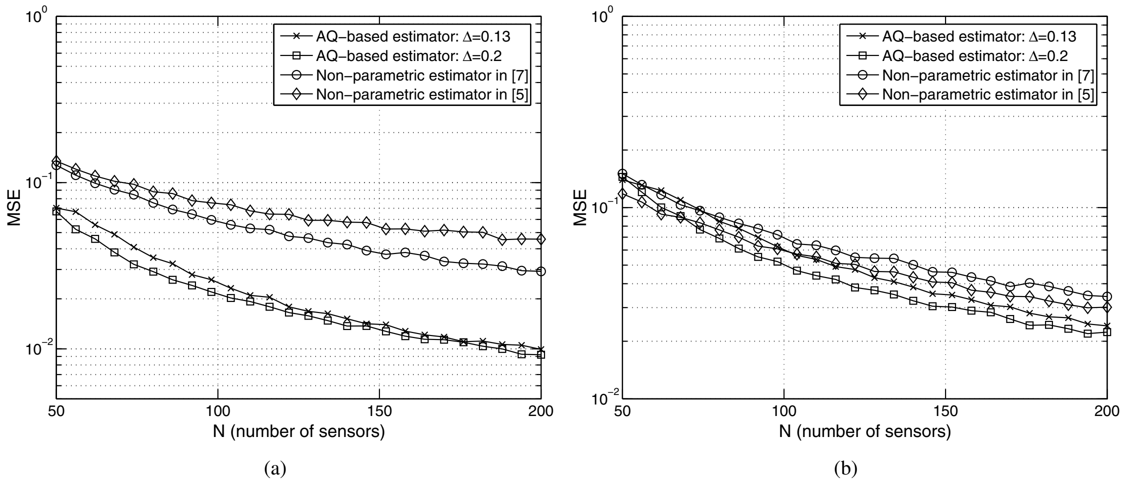

Fig. 3. MSEs of three non-parametric estimators in (a) i.i.d. Gaussian noise with $\sigma_w^2 = 1$ and (b) Uniform noise within [-2, 2].

Step 1. Choose the window size L_w and stopping threshold on the number of zero-crossings λ.

Step 2. Set $N_{\text{zc}} = 0$, set $m = 1$ and calculate $T(m) = \sum_{n=m}^{m+L_w-1} b(n)$.

Step 3. If $T(m) = 0$, then $N_{\text{zc}} = N_{\text{zc}}+1$. If $N_{\text{zc}} = \lambda$, stop the algorithm and set $\check{M} = m$; otherwise, set $N_{\text{zc}} = N_{\text{zc}}$, $m = m + 1$ and go to Step 2.

From our numerical study, the threshold λ is better to be a small number, e.g., $\lambda \in \{2, 3, 4, 5\}$.

IV. Numerical Results

In the following, we compare the proposed non-parametric estimator with the estimators in [5] and [7] in terms of the mean squared error (MSE). The number of sensors N is from 50 to 200, $\theta = 3$, and the noise is Gaussian with zero-mean and $\sigma_w^2 = 1$ and, respectively, Uniform noise within $[-2, 2]$. In both the Gaussian and uniform cases, the parameters used to estimate the convergence index are $L_w = 20$ and $\lambda = 2$. Fig. 3 (a) shows the MSEs of the three non-parametric estimators as a function of the number of sensors. It is observed that the AQ-based non-parametric estimators with $\Delta = 0.13$ and $\Delta = 0.2$ outperform the other two estimators for all N. Note that θ is not an integer multiple of $\Delta = 0.13$. Also, we observe that, when the WSN consists of a limited number of sensors, i.e., $N \leq 200$, a larger Δ gives better performance than that with a smaller Δ.

For the Uniform noise, Fig. 3 (b) shows the MSEs of these non-parametric estimators as a function of N. It is observed that, for a moderate number of sensors, the AQ-based non-parametric estimator with $\Delta = 0.13$ and $\Delta = 0.2$ provides a lower MSE, compared to the other two estimators. As far as the AQ-based non-parametric estimator, performance degradation from Fig. 3 (a) to Fig. 3 (b) can be observed. The reason behind this effect is that, the thresholds converge faster in the Gaussian noise case than in the Uniform noise case.

V. Conclusion

We have proposed a distributed non-parametric estimator, using the recently introduced adaptive quantization (AQ) scheme, for position (mean) parameter estimation in a bandwidth-constrained WSN. Based on an asymptotic analysis of the quantization threshold of AQ, the proposed estimator is approximately unbiased when the stepsize of AQ is small and the number of sensors is large. In practice when the number of sensors is limited, the choice of the stepsize cannot be made too small to avoid the threshold of AQ not achieving convergence. We have also examined how to estimate when the AQ threshold reaches practical convergence.

Appendix

Since θ is an integer multiple of Δ, it belongs to the set of possible thresholds (i.e., $\pm k\Delta$, where k is an integer) of AQ. Proposition 1 is proved in a two-step approach. The first step is to show that, conditioned on the event that there exists one sensor, say sensor m, whose threshold τ_m takes the value of θ, i.e., $\tau_m = \theta$, the subsequent thresholds $\tau_n, n \geq m$, converge in mean to θ. Then we complete the proof by showing that the probability of the event that at least one threshold hitting on θ is one, i.e., $\sum_{m=1}^{\infty} P(\tau_m = \theta) \to 1$. In other words, we can express the asymptotical mean of τ_n as

$$E\{\tau_n\} = \sum_{m=1}^{\infty} E\{\tau_n | \tau_m = \theta\} P\{\tau_m = \theta\}, \text{ as } n \to \infty. \quad (18)$$

Conditioned on the event $\tau_m = \theta$, we can determine the possible subsequent thresholds: $\tau_{m+1} \in \{\theta - \Delta, \theta + \Delta\}$, $\tau_{m+2} \in \{\theta - 2\Delta, \theta, \theta + 2\Delta\}$, and so on and so forth. The possible thresholds taken after the mth sensor can be expressed in a tree diagram, shown in Fig. 4, which is symmetric with respect to the initial state, i.e., $\tau_m = \theta$. In addition to the symmetric structure of the tree, we note that the branch probabilities (which are also the transition probabilities of the Markov chain (12)) are also symmetric, due to the assumption

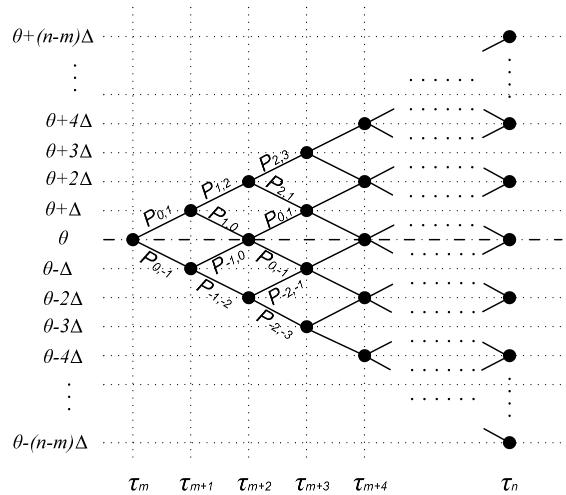

Fig. 4. Tree diagram for the AQ thresholds after the event $\tau_m = \theta$.

that the sensor noise w_n has a symmetric PDF. For example, we can easily verify

$$
\begin{aligned}
P_{0,1} &= P\{\tau_{m+1} = \theta + \Delta | \tau_m = \theta\} = F_w(0) = \frac{1}{2}, \\
P_{0,-1} &= P\{\tau_{m+1} = \theta - \Delta | \tau_m = \theta\} = 1 - F_w(0) = \frac{1}{2}, \\
&\Longrightarrow P_{0,1} = P_{0,-1}; \\
P_{1,2} &= P\{\tau_{m+2} = \theta + 2\Delta | \tau_{m+1} = \theta + \Delta\} = F_w(\Delta), \\
P_{-1,-2} &= P\{\tau_{m+2} = \theta - 2\Delta | \tau_{m+1} = \theta - \Delta\} = 1 - F_w(-\Delta), \\
&\Longrightarrow P_{1,2} = P_{-1,-2}; \\
P_{1,0} &= P\{\tau_{m+2} = \theta | \tau_{m+1} = \theta + \Delta\} = 1 - F_w(\Delta), \\
P_{-1,0} &= P\{\tau_{m+2} = \theta | \tau_{m+1} = \theta - \Delta\} = F_w(-\Delta), \\
&\Longrightarrow P_{1,0} = P_{-1,0}. \qquad (19)
\end{aligned}
$$

A *path* is a set of connecting branches that move the state variable (i.e., the threshold of AQ) from the initial state $\tau_m = \theta$ to a final state. For example, there exists a unique path that moves $\tau_m = \theta$ to $\tau_{m+3} = \theta + 3\Delta$:

$$\tau_m = \theta \rightarrow \tau_{m+1} = \theta{+}\Delta \rightarrow \tau_{m+2} = \theta{+}2\Delta \rightarrow \tau_{m+3} = \theta{+}3\Delta.$$

The path probability is the product of the corresponding branch probabilities. For example, the path probability of the above path is $P_{0,1}P_{1,2}P_{2,3}$. For a given path, there exists a unique *symmetric path* that moves from the initial state to the *symmetric final state* of the original path. For example, the symmetric path of the previous path is given by

$$\tau_m = \theta \rightarrow \tau_{m+1} = \theta{-}\Delta \rightarrow \tau_{m+2} = \theta{-}2\Delta \rightarrow \tau_{m+3} = \theta{-}3\Delta.$$

with path probability $P_{0,-1}P_{-1,-2}P_{-2,-3}$ which is identical to the previous path probability, since the branch probability is symmetric.

The number of paths that move from one initial state to a final state may be more than one. For example, there are 3 (and only 3) paths that travels from $\tau_m = \theta$ to $\tau_{m+3} = \theta + \Delta$, and the probability of $\tau_{m+3} = \theta + \Delta$ is given by the sum of the path probabilities of the 3 paths:

$$
\begin{aligned}
P(\tau_{m+3} = \theta + \Delta | \tau_m = \theta) = & P_{0,1}P_{1,2}P_{2,1} + P_{0,1}P_{1,0}P_{0,1} \\
& + P_{0,-1}P_{-1,0}P_{0,1}.
\end{aligned}
$$

Due to the symmetric structure of the tree, the number of the paths starting from the initial state $\tau_m = \theta$ to a final state $\tau_n = \theta + k\Delta$, $n > m$, is the same as that from $\tau_m = \theta$ to $\tau_n = \theta - k\Delta$ (i.e., the symmetric final state). Furthermore, every path in one group is associated with a symmetric path in the other group with the same path probability. As such, we can conclude that

$$P\{\tau_n = \theta + k\Delta | \tau_m = \theta\} = P\{\tau_n = \theta - k\Delta | \tau_m = \theta\}, \qquad (20)$$

where

$$k \in \begin{cases} \{0, \pm 2, \cdots, \pm(n-m)\}, & \text{if } (n-m) \text{ is even} \\ \{1, \pm 3, \cdots, \pm(n-m)\}, & \text{if } (n-m) \text{ is odd} \end{cases},$$

Therefore, the conditional mean of τ_n is

$$
\begin{aligned}
E\{\tau_n | \tau_m = \theta\} &= \sum_k (\theta + k\Delta) P\{\tau_n = \theta + k\Delta | \tau_m = \theta\} \\
&= \theta \sum_k P\{\tau_n = \theta + k\Delta | \tau_m = \theta\} = \theta, \quad (21)
\end{aligned}
$$

where the second equation is based on (20).

From (18), the asymptotical mean of τ_n can be rewritten as

$$E\{\tau_n\} = \theta \sum_{m=1}^{\infty} P(\tau_m = \theta), \quad n \rightarrow \infty. \qquad (22)$$

Note that $\sum_{m=1}^{\infty} P(\tau_m = \theta) = 1$ means that there exists one threshold hitting on θ when n goes to infinity. It is equivalent to say that the probability that no thresholds hit on θ is zero. In this case, the thresholds can only pass through the states that are less than θ. As $n \rightarrow \infty$, the probability that the thresholds pass through the states less than θ is zero due to the fact that path probability diminishes (each of such a non-hitting path has an infinite length with branch probability less than 1/2). Therefore, we conclude that

$$\sum_{m=1}^{\infty} P(\tau_m = \theta) \rightarrow 1, \quad n \rightarrow \infty. \qquad (23)$$

Taking (23) into (22) completes the proof.

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