The retailer’s procurement policy with credit-linked demand under inflationary conditions

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Abstract: The one-stage credit policy is a situation that arises, under a permissible delay in payments, when the supplier offers a credit period to the retailer, but the latter does not offer any credit period to his/her customers. However, this type of credit policy is debatable in most business transactions. In reality, the retailer also adopts the credit policy to stimulate his/her own demand. Such a situation, where both the supplier and the retailer offer the credit period to their respective customers, is termed two-stage credit policy. Moreover, nowadays it is a well-known fact that the credit period offered by the retailer to the customers has a positive impact on the demand of an item. Keeping this in mind, a credit-linked demand function has been considered. Further, the inflation and time value of money also play a very vital role in determining the procurement policy of the retailer, specifically in developing countries.

Based upon these arguments, the present paper addresses the retailer’s procurement policy, where the decision is influenced by the inflation and time value of money under a permissible delay in payments, for a credit-linked demand function. The main objective is to maximise the retailer’s profit by jointly optimising his credit as well as the procurement period. Results have been illustrated with the help of a numerical example. Computational results provide some interesting policy implications.

Keywords: inventory; credit-linked demand; two-stage credit policy; delay in payments; inflation.


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1 Introduction

In the recent past, many researchers have studied inventory models with permissible delays in payments, with the assumption that the supplier would allow a certain fixed period for the retailer to settle the account. During this period the retailer can sell the goods and earn interest on them. After the expiration of this period, an interest is charged by the supplier, as has been agreed upon. This type of inventory model falls under the one-stage trade credit policy category. A good amount of literature exists in this area: Haley and Higgins (1973) introduced the first model to determine Economic Order Quantity (EOQ) under the condition of a permissible delay in payment with deterministic demand, without shortages. Goyal (1985) considered a model similar to that of Haley and Higgins with the exclusion of the penalty cost due to a late payment. Inventory models for deteriorating items under a permissible delay in payment have been developed by Jaggi and Aggarwal (1994), Aggarwal and Jaggi (1995), Jamal et al. (1997), Sarker et al. (2000a), Liao et al. (2000) and Dye (2002). Apart from this, Chang et al. (2004) considered an inventory model for deteriorating items with instantaneous stock-dependent demand and time value of money when a credit period is provided. Teng et al. (2005) determined the optimal pricing and ordering policy under a permissible delay in payments. Goyal et al. (2007) formulated optimal ordering policies for when the supplier provides a progressive interest scheme. Chang et al. (2008) presented a complete and up-to-date survey of published inventory literature under trade credits.

Basically, in the one-stage trade credit policy, the retailer gets a fixed period from the supplier but does not offer any fixed period to his/her customers in turn. In reality this seems unrealistic now, since the supplier offers a credit period – a powerful promotional tool – to the retailer in order to stimulate his/her demand. On similar lines, the retailer also offers, certain fixed period to his/her customers so as to attract new customers. This phenomenon, i.e., when the supplier as well as the retailer offers a certain fixed period to their respective customers, is termed the two-stage trade credit policy. Huang (2003)
developed an inventory model assuming that the retailer also offers a credit period to its customer, which is shorter than the credit period offered by the supplier. Added to this, recently Huang (2007) has presented an optimal retailer’s replenishment decisions in the Economic Production Quantity (EPQ) model, assuming two levels of trade credit policy. Further, Chung and Huang (2007) also formulated a two-warehouse inventory model for deteriorating items under two-stage trade credit financing. In all the mentioned articles, although the presence of a credit period has been incorporated in the mathematical formulation, the impact of the credit period on demand has not been considered; whereas, in reality, it has been observed that the demand for an item does get influenced by the length of the credit period offered by the retailer. Jaggi et al. (2007) presented the joint optimisation of the retailer’s unit selling price and cycle length under the two-stage credit policy when the end demand is price as well as credit period sensitive. In 2008, Jaggi et al. determined a retailer’s optimal replenishment decisions with credit-linked demand under a permissible delay in payments.

Furthermore, inflation as well as the time value of money are playing a very vital role on the economic front, especially in developing countries. Their effects were studied by Buzacott (1975) and Misra (1975) by developing inventory models with constant demand and a single inflation rate for all associated costs. Misra (1979) introduced different inflation rates for various costs associated with an inventory system. Bose et al. (1995) developed the EOQ inventory model under inflation and time discounting. Aggarwal et al. (1997) investigated the economic ordering policies in the presence of trade credit with inflation for nondeteriorating items. Yang et al. (2001) provided inventory models with time-varying demand patterns under inflation and Yang (2006) developed a two-warehouse partial backlogging inventory models for deteriorating items under inflation.

Also, Sarker et al. (2000b) discussed the effects of inflation, time value of money, deterioration and a permissible delay in payment in their model. Chang (2004) presented an EOQ model for deteriorating items under inflation when the supplier credits are linked to the order quantity. Manna and Chaudhri (2005) developed an EOQ model for deteriorating items with nonlinear demand under inflation and a trade credit policy. Not only this, Jaggi and Goel (2005) investigated the economic ordering policies of deteriorating items with trade credit under inflationary conditions. Further, Jaggi and Khanna (2009) formulated a retailer’s ordering policy for deteriorating items with inflation-induced demand under the trade credit policy \( \frac{dD}{dt} = \text{Net} D \).

However, none of these models has considered the effects of inflation and the time value of money for the two-stage trade credit policy; therefore, in this paper, an inventory model is formulated to investigate the effects of inflation and the time value of money on the retailer’s procurement policy, for a credit-linked demand function, under the two-stage trade credit policy. The main motive of this paper is to explore how the trade credit offered by the retailer to his/her customer serves as a powerful tool to attract more and more customers in order to capture unrealised demand. An algorithm is provided which jointly optimises the credit as well as the procurement period. Finally, a numerical example is presented along with the sensitivity analysis on some parameters. Some interesting policy insights are obtained.
2 Assumptions and notations

The following assumptions were made to develop the mathematical model:

- The supplier provides a fixed credit period $M$ to settle the accounts to the retailer and the retailer, in turn, also offers a credit period $N$ to each of his/her customers to settle the accounts.
- The demand rate is a function of the customer’s credit period offered by the retailer ($N$).
- The replenishment rate is instantaneous.
- The effects of inflation and time value are considered.
- Shortages are not allowed.
- The lead time is negligible.

In addition, the following notation is used:

- $Q$ order quantity
- $T$ inventory cycle length
- $q(t)$ inventory level at time $t$
- $A_0$ ordering cost per order at time $t = 0$
- $C_0$ unit purchase cost of the item at time $t = 0$
- $P_0$ unit selling price of the item at time $t = 0$
- $A(t)$ ordering cost per order at time $t$
- $C(t)$ unit purchase cost of the item at time $t$
- $P(t)$ unit selling price of the item at time $t$
- $I$ out-of-pocket inventory carrying charge per $ per unit time
- $I_e$ interest that can be earned per $ per unit time
- $I_p$ interest payable per $ per unit time
- $d$ discount rate, representing the time value of money
- $\alpha$ inflation rate
- $K_{d - \alpha}$ representing the net discount rate of inflation, is constant
- $M$ retailer’s credit period offered by the supplier for settling the accounts
- $N$ customer’s credit period offered by the retailer for settling the accounts
- $Z(T, N)$ retailer’s profit which is a function of two variables, $T$ and $N$, where $Z(T, N) = (a)$ revenue from sales – (b) cost of purchasing units – (c) cost of placing orders – (d) cost of carrying inventory (excluding interest charges) + (e) interest earned from the sales during the permissible period – (f) cost of interest charges for the unsold items after the permissible delay.
3 Mathematical formulation

As a matter of fact, the credit period has been proved to be a powerful tool for promoting the sales of an organisation. Keeping this in mind, a flexible credit-linked demand function has been considered, which is given by the following differential difference equation:

\[ D(N + 1) - D(N) = r[S - D(N)] \]

where:

- \( D = D(N) \) = demand rate for any \( N \) per unit time
- \( N \) = customer’s credit period offered by the retailer for settling the accounts
- \( S \) = maximum demand
- \( R \) = rate of saturation of demand (which can be estimated using past data).

The solution to the above difference equation, under the condition that at \( N = 0 \), \( D(0) = s \) (initial demand), keeping other attributes like price, quantity, etc., at a constant level, is given by:

\[ D = s(1 - r)^N + S(1 - (1 - r)^N) \]

i.e., \( D = S - (S - s)(1 - r)^N \). \hspace{1cm} (1)

The behaviour of the demand function given by Equation (1) is an exponential curve (Figure 1), which takes into account the impact of the credit period on the demand function.

Figure 1 Impact of credit period on demand (see online version for colours)
The order quantity for the replenishment cycle is:

$$Q = \int_{0}^{T} D(t) \, dt = DT.$$  \hfill (2)

Now, let $q(t)$ be the inventory level at time $t$. A batch of $Q$ units enters the inventory system at the beginning of the cycle. As time passes, the inventory level decreases due to demand.

Thus, the inventory level at any time $t$ during the cycle is:

$$q(t) = Q - \int_{0}^{t} D(t) \, dt = D(T - t) \quad 0 \leq t \leq T.$$  \hfill (3)

Since we are investigating the impact of inflation on the retailer’s procurement policy, its effect on the cost parameters can be modelled as follows:

Let $C(t)$ be the unit purchase cost of an item at time $t$, which becomes, through inflation, a cost $C(t + \delta t)$ at time $t + \delta t$, and let $\alpha$ be the constant rate of inflation.

Then, $C(t + \delta t) = C(t) + \alpha C(t) \delta t$

$$\Rightarrow \frac{C(t + \delta t) - C(t)}{\delta t} = \alpha C(t).$$

As $\delta t \to 0$, we get $\frac{dC(t)}{dt} = \alpha C(t)$.

The solution to the above differential equation along with the boundary condition at time $t = 0, C(t) = C_0$ is:

$$C(t) = C_0 e^{\alpha t}$$

where $C_0$ is the cost at time zero.

Similarly, the ordering costs and selling price at any time $t$ are:

$$A(t) = A_0 e^{\alpha t}$$

$$P(t) = P_0 e^{\alpha t}.$$  \hfill (4)

Now the retailer’s net profit consists of the following components:

(a) revenue from sales
(b) cost of purchasing units
(c) cost of placing orders
(d) cost of carrying inventory (excluding interest charges)
(e) interest earned from the sales during the permissible period
(f) cost of interest charges for the unsold items after the permissible delay.

By using the DCF approach, the present worth of the various costs for the first replenishment cycle is evaluated as follows:
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(a) The present worth of sales revenue for the first cycle is:

\[ S_r = P_0 \int_{N}^{N+T} D e^{-K(t-N)} \, dt \]

\[ = \frac{P_0 D}{K} \left( e^{-K2N} - e^{-K(2N+T)} \right) \quad (5) \]

(b) The present worth of ordering cost during the first cycle is:

\[ C_r = A \quad (6) \]

(Since ordering is made at time \( t = 0 \), the inflation does not affect the ordering cost).

(c) The present worth of purchase cost during the first cycle is:

\[ C_p = CQ \quad (7) \]

(d) The present worth of inventory holding cost during the first cycle is:

\[ C_h = I C_0 \int_{N}^{T} q(t) e^{-Kt} \, dt \]

\[ = I C_0 \int_{N}^{T} (T-t) e^{-Kt} \, dt \]

\[ = I C_0 D \frac{T}{K^2} (Kt - 1 + e^{-KT}) \quad (8) \]

The computations for interest earned and interest payable (i.e., (e) and (f)) will depend on the following three possible cases based on the lengths of \( T, N \) and \( M \).

3.1 Case 1: \( N \leq M \leq T + N \) (Figure 2)

In this case, the retailer starts getting actual sales revenues from time \( N \) to \( M \) and earns interest on the average sales revenue for the time period \( (M - N) \). At \( M \) accounts are settled; if all the credit sales are not realised, the finances are to be arranged to make the payment to the supplier.

Figure 2 When \( N \leq M \leq T + N \)
(e-Case 1) Consequently the present worth of the interest earned for the first cycle is:

\[
C_r = p_k I_r \int_0^M \frac{D_te^{k(N+n)}}{K^n} dt \\
= \frac{p_k I_r D}{K^n} e^{-KN} \left[ (1 + KN)e^{-KM} - (1 + KM)e^{-KN} \right]
\] (9)

(f-Case 1) The present worth of the interest payable for the first cycle is:

\[
C_p = c_0 I_p \int_M^{T+N} \frac{D_te^{-K(M+n)}}{K^n} dt \\
= \frac{c_0 I_p D}{K^n} e^{-KM} \left[ (1 + KM)e^{-KM} - (1 + K(T + N))e^{-K(T+N)} \right]
\] (10)

Using Equations (5) to (10), the present worth of the retailer’s profit for the first cycle \(Z_1(T,N)\) can be expressed as:

\[
Z_1(T,N) = S_r - C_r - C_p - C_h + C_\mu - C_\nu \\
= \left( \frac{p_k D}{K} \left( e^{-K2N} - e^{-K(T+N)} \right) - A_\mu - C_\mu DT - \frac{IC_\mu D}{K^2} (KT - 1 + e^{-KT}) \right) \\
+ \left( \frac{p_k I_r D}{K^n} e^{-KN} \left[ (1 + KN)e^{-KN} - (1 + KM)e^{-KM} \right] \right) \\
- \left( \frac{c_0 I_p D}{K^n} e^{-KM} \left[ (1 + KM)e^{-KM} - (1 + K(T + N))e^{-K(T+N)} \right] \right)
\] (11)

3.2 Case 2: \(N \leq T + N \leq M\) (Figure 3)

In this case, the retailer earns interest on the average sales revenues received during the period \((N, T + N)\) and on the full sales revenue \((PQ)\) for the period \((M - T - N)\), but there is no interest payable from the retailer.

Figure 3 When \(N \leq T + N \leq M\)
Consequently the present worth of interest earned for the first cycle is:

\[ C_{\mu} = P_0 I_e \left[ \int_{T+M}^{T+N} Dte^{-K(T+M+1)} dt + \int_{T+N}^{M} Dte^{-K(T+N+1)} dt \right] \]

\[ = P_0 I_e D \frac{e^{-KM} \{ (1 + KN)e^{-KN} - (1 + K(T + N))e^{-K(T+N)} \}}{K^2} \]

As a result, using Equations (5), (6), (7), (8) and (12), the present worth of the retailer’s profit for the first cycle \( Z_2(T,N) \) in this case is:

\[ Z_2(T,N) = S_r - C_r - C_p - C_h + C_{\mu} \]

\[ = \frac{P_0 D}{K} \left( e^{-K2N} - e^{-K2(N+T)} \right) - A_0 - C_p DT - \frac{IC_0 D}{K^2} (KT - 1 + e^{-KT}) \]

\[ + \frac{P_0 I_e D}{K^2} \left[ e^{-KN} \{ (1 + KN)e^{-KN} - (1 + K(T + N))e^{-K(T+N)} \} \right] \] \hspace{1cm} (13)

3.3 Case 3: \( M \leq N \leq T + N \) (Figure 4)

In this case, the retailer earns no interest but pays interest on full order quantity \((Q)\) for the period \((N - M)\) and on average stock held during the cycle length \((T)\).

(f-Case 3) Consequently the present worth of the interest payable for the first cycle is:

\[ C_{\mu} = C_0 I_p \left[ \int_{T+M}^{T+N} Dte^{-K(T+M+1)} dt + \int_{T+N}^{M} Dte^{-K(T+N+1)} dt \right] \]

\[ = C_0 I_p D \frac{e^{-KM} KT \{ e^{-KM} - e^{-KN} \}}{K^2} \]

\[ + e^{-KN} \left\{ (1 + KN)e^{-KN} - (1 + K(T + N))e^{-K(T+N)} \right\} \] \hspace{1cm} (14)
As a result, using Equations (5), (6), (7), (8) and (14), the present worth of the retailer’s profit for the first cycle $Z(T, N)$ in this case is:

$$
Z(T, N) = S_r - C_r - C_p - C_b - C_h
$$

$$
Z_1(T, N) = \frac{P_D}{K} \left( e^{-K2N} - e^{-K(2N+T)} \right) - A_h - C_r DT - \frac{IC_r D}{K^2} (KT - 1 + e^{-KT})
- \frac{C_b I_r D}{K^2} e^{-KM} \left[ e^{-KM} KT (e^{-KM} - e^{-KT}) + e^{-KN} \left( (1 + KN)(e^{-KN} - (1 + K(T + N)) e^{-K(T+N)} \right) \right].
$$

(15)

Therefore, the present worth of the retailer’s profit for the first cycle $Z(T, N)$ is:

$$
Z(T, N) = \begin{cases} 
Z_1(T, N) & \text{if } N \leq M \leq T + N \\
Z_2(T, N) & \text{if } N \leq T + N \leq M \\
Z_3(T, N) & \text{if } M \leq N \leq T + N 
\end{cases}
$$

(16)

which is a function of the two variables $T$ and $N$, where $T$ is continuous and $N$ is discrete.

Further, the present worth of all the future cash flows over the finite planning horizon, $H = nT$ (= 1 year), where $n$ is the complete number of cycles over $H$, is given by:

$$
Z_{II}(T, N) = Z(T, N) \left[ \left( 1 + e^{-K} \right) + e^{-2KT} + \ldots + e^{-(n-2)KT} + e^{-(n-1)KT} \right]
$$

$$
= Z_2(T, N) \left\{ \frac{1 - e^{-(n-1)KT}}{1 - e^{-K}} \right\}
$$

$$
Z_{III}(T, N) = Z(T, N) \left[ \left( 1 + e^{-K} \right) + e^{-2KT} + \ldots + e^{-(n-2)KT} + e^{-(n-1)KT} \right]
$$

$$
= Z_3(T, N) \left\{ \frac{1 - e^{-(n-1)KT}}{1 - e^{-K}} \right\}
$$

$$
Z_{IV}(T, N) = Z(T, N) \left[ \left( 1 + e^{-K} \right) + e^{-2KT} + \ldots + e^{-(n-2)KT} + e^{-(n-1)KT} \right]
$$

$$
= Z_3(T, N) \left\{ \frac{1 - e^{-(n-1)KT}}{1 - e^{-K}} \right\}
$$

(17)

where:

$$
Z_{II}(T, N) = \text{the present worth of total profit over the planning horizon for Case 1: } N \leq M \leq T + N
$$
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\[ Z_{H2}(T,N) = \text{the present worth of total profit over the planning horizon for} \]

Case 2: \[ N \leq T + N \leq M \]

\[ Z_{H3}(T,N) = \text{the present worth of total profit over the planning horizon for} \]

Case 3: \[ M \leq N \leq T + N. \]

4 Solution procedure

Since the profit function \( Z(T,N) \) is a function of two variables, one discrete, \( i.e., N \), and one continuous, \( i.e., T \), our problem is to determine the optimum value of \( T \) and \( N \) which maximises \( Z(T,N) \). For a fixed value of \( N \), the necessary and sufficient conditions for \( Z(T,N) \) to be concave are:

\[ Z'(T,N) = 0 \]

and \( Z''(T,N) \leq 0 \) respectively.

Therefore, taking the first- and second-order derivatives of \( Z_1(T,N) \), \( Z_2(T,N) \) and \( Z_3(T,N) \) with respect to \( T \), we get:

\[
Z_1'(T,N) = \frac{P_0D}{K}e^{-Kn}Ke^{-KT} - C_0D - \frac{IC_0D}{K^2}(K - Ke^{-KT})
\]
\[ - \frac{C_0I_1D}{K^2}e^{-KM}[(1 + K(T + N))Ke^{-K(T+N)} - Ke^{-K(T+N)}] \] \hspace{1cm} (18)

\[
Z_1''(T,N) = -\frac{P_0De^{-Kn}}{K}Ke^{-KT} - IC_0De^{-KT}
\]
\[ - \frac{C_0I_1De^{-KM}}{K^2}e^{-K(T+N)}[1 - K(T + N)] \leq 0. \] \hspace{1cm} (19)

Thus, \( Z(T,N) \) is concave on \( T > 0 \). Hence, there exists a unique value of \( T \) (say \( T_1^* \)) which maximises \( Z_1(T) \) and can be obtained by solving equation \( Z_1'(T,N) = 0 \).

Substituting the optimal value of \( T \) into Equations (2) and (11), we can get the optimal values of \( Q \) and \( Z_1(T) \) (say, \( Q_1^* \) and \( Z_1^* \)).

Now:

\[
Z_1'(T,N) = \frac{P_0D}{K}e^{-Kn}Ke^{-KT} - C_0D - \frac{IC_0D}{K^2}(K - Ke^{-KT})
\]
\[ + \frac{P_0I_1D}{K^2}e^{-Kn} \left[ (1 + K(T + N))Ke^{-K(T+N)} - Ke^{-K(T+N)} \right]
\]
\[ + (Ke^{-KT} - K^2Te^{-KT})[e^{-K(T+N)} - e^{-KM}] \] \[ - K^2Te^{-KT}e^{-K(T+N)} \] \hspace{1cm} (20)

\[
Z_1''(T,N) = -\frac{P_0De^{-Kn}}{K}Ke^{-KT} - IC_0De^{-KT}
\]
\[ + \frac{P_0I_1De^{-Kn}}{K^2} \left[ (KTe^{-KT} - 2e^{-KT})e^{-KM} \right]
\]
\[ - e^{-K(T+N)}[1 - K(T + N) - 4e^{-KT} + 4KTe^{-KT}] \]. \hspace{1cm} (21)
For a fixed value of $N$, $Z_2(T,N)$ is concave on $T > 0$:

\[
\begin{align*}
\text{if} & \quad (KTe^{-KT} - 2e^{-KT})e^{-KM} \\ & \quad - e^{-K(T+N)}(1 - K(T+N) - 4e^{-KT} + 4KT e^{-KT}) \geq 0 \\
\text{or if} & \quad 4e^{-K(T+N)}e^{-KT}(1 - KT) \geq e^{-K(T+M)}(2 - KT) + e^{-K(T+N)}(1 - K(T+N)) \\
i.e., If & \quad 4e^{-KT}(1 - KT) \geq e^{-K(M-N)}(2 - KT) + (1 - K(T+N)) \\
& \quad 4e^{-KT}(1 - KT) - (1 - K(T+N)) \geq e^{-K(M-N)}(2 - KT)
\end{align*}
\]

or

\[
\begin{align*}
e^{K(M-N)}[4e^{-KT}(1 - KT) - (1 - K(T+N))] \geq (2 - KT) \\
\text{Now} & \quad e^{K(M-N)} \geq 1 \\
\therefore & \quad [4e^{-KT}(1 - KT) - (1 - K(T+N))] \geq (2 - KT), \text{then } Z_2(T,N) \text{ is concave.}
\end{align*}
\]

Now using the approximation $e^{-KT} = (1 - KT + ....)$ and ignoring the higher-order terms, we have:

\[
\begin{align*}
4(1 - KT)(1 - KT) - (1 - KT - KN) \geq (2 - KT) \\
\Rightarrow 4(1 - 2KT) - 1 + KT + KN \geq (2 - KT) \\
\Rightarrow 3 + KN \geq 2 + 6KT \\
\Rightarrow 6KT \leq 1 + KN \\
\Rightarrow T \leq \frac{1 + KN}{6K}.
\end{align*}
\]

Thus, $Z_2(T,N)$ is concave on $T > 0$ if $T \leq \frac{1 + KN}{6K}$.

Hence, there exists a unique value of $T$ (say $T' \leq \frac{1 + KN}{6K}$) which maximises $Z_2(T)$ and can be obtained by solving the equation $Z_2(T,N) = 0$.

Following the approach described above, we can obtain the optimal value of $T$ (say, $T''$) which maximises $Z_2(T)$ by solving the equation $Z_2'(T,N) = 0$.
Substituting the optimal value of $T$ into Equations (2) and (15), we can get the optimal values of $Q$ and $Z_0(T)$ (say, $Q_1^*$ and $Z_1^*$).

Now, in order to find the optimal values of $T$ and $N$, which maximise the retailer’s profit, we make use of the following algorithm:

Step 1  Put $N = 1$.
Step 2  Determine the optimal values of $T$ (i.e., $T_1^*$ or $T_2^*$ or $T_3^*$) using Equations (18) or (20) or (22).
Step 3  If $T \geq M - N \geq 0$ then calculate $Z_{hi}(T, N)$, else go to Step 5.
Step 4  If $Z_{hi}(T, N) > Z_{hi}(T, N - 1)$, increment the value of $N$ by 1 and go to Step 2, else current value of $N$ is optimal and the corresponding value of $T$ and $Z_{hi}(T, N)$ can be calculated.
Step 5  If $0 \leq T \leq M - N$ then calculate $Z_{hi}(T, N)$, else go to Step 7.
Step 6  If $Z_{hi}(T, N) > Z_{hi}(T, N - 1)$, increment the value of $N$ by 1 and go to Step 2, else current value of $N$ is optimal and the corresponding value of $T$ and $Z_{hi}(T, N)$ can be calculated.
Step 7  If $M - N \leq 0 \leq T$ then calculate $Z_{hi}(T, N)$.
Step 8  If $Z_{hi}(T, N) > Z_{hi}(T, N - 1)$, increment the value of $N$ by 1 and go to Step 2, else current value of $N$ is optimal and the corresponding value of $T$ and $Z_{hi}(T, N)$ can be calculated.

5 Numerical example and sensitivity analysis

In this section, the retailer’s optimal procurement policy for a credit-linked demand function under inflationary conditions is examined with the help of a numerical example.

Let max demand ($S$) = 100 units/day, min demand ($s$) = 30 units/day, rate of saturation of demand ($r$) = 0.12, $A$ = $1000/order, $M$ = 30 days, $C$ = $30/unit, $P$ = $40/unit, $d$ = 12%, $\alpha$ = 6%, $I_p$ = 15% per year, $I_e$ = 10% per year and $I$ = 15% per year.

Using the proposed algorithm, we obtained the optimal results as follows: optimal cycle length ($T^*$) = 19.6 days, optimal credit period offered by retailer ($N^*$) = 22 days and profit per day ($Z^*$) = $849.

Table 1 presents the retailer’s optimal procurement policy for different values of $M$.

<table>
<thead>
<tr>
<th>$M$</th>
<th>$N$</th>
<th>$T$</th>
<th>$Q$</th>
<th>Profit</th>
<th>Case applicable</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>24</td>
<td>26.4</td>
<td>2559.8</td>
<td>816</td>
<td>Case 3</td>
</tr>
<tr>
<td>30</td>
<td>22</td>
<td>19.6</td>
<td>1837.0</td>
<td>849</td>
<td>Case 1</td>
</tr>
<tr>
<td>70</td>
<td>33</td>
<td>26.8</td>
<td>2658.0</td>
<td>947</td>
<td>Case 2</td>
</tr>
</tbody>
</table>
The following are observed from Table 1:

- When the credit period offered by the supplier to the retailer, $M (= 10 \text{ days})$, is less than that given by the retailer to his/her customers $N (= 24 \text{ days})$, the retailer is undoubtedly able to generate more demand, as expected. But he is unable to generate profit according to his sales, as the retailer earns no interest but rather pays interest on the full order quantity.

- When the credit period offered by the supplier to the retailer, $M (= 30 \text{ days})$ is greater than that given by the retailer to his/her customers $N (= 22 \text{ days})$, there is a decrease in order quantity, because if $N$ decreases, demand also decreases. In such a case, the retailer’s profit increases as he/she starts earning actual sales revenues from time $N$ to $M$ and earns interest on average sales revenue for the time period $(M - N)$. Also, if all the credit sales are not realised, then the amount is financed to make the payment to the supplier.

- Lastly, when the credit period offered by the supplier to the retailer is $M (= 70 \text{ days})$ and the credit period offered by the retailer to his customer is $N (= 33 \text{ days})$, there is an increase in order quantity as demand goes up due to the large $N$. There is a significant increase in the retailer’s profit as well, because in this case the retailer does not have to get his/her inventory financed, but only earns interest on average sales revenues received during the period $(N, T + N)$ and on the full sales revenue for a period of $(M - T - N)$.

The sensitivity analysis on $A$, $\alpha$, $r$ and $I_p$ is shown in Table 2, Table 3, Table 4 and Table 5 respectively.

**Table 2** Effects of changing $A$ on the optimal solution

<table>
<thead>
<tr>
<th>$A$</th>
<th>$T^*$ (days)</th>
<th>$N^*$ (days)</th>
<th>$Z(T^<em>, N^</em>)$ ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>19.6</td>
<td>22</td>
<td>849</td>
</tr>
<tr>
<td>1500</td>
<td>27.7</td>
<td>23</td>
<td>828</td>
</tr>
<tr>
<td>2000</td>
<td>33.9</td>
<td>24</td>
<td>812</td>
</tr>
</tbody>
</table>

**Table 3** Effects of changing $\alpha$ on the optimal solution

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$T^*$ (days)</th>
<th>$N^*$ (days)</th>
<th>$Z(T^<em>, N^</em>)$ ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.04</td>
<td>19.1</td>
<td>22</td>
<td>839</td>
</tr>
<tr>
<td>0.06</td>
<td>19.6</td>
<td>22</td>
<td>849</td>
</tr>
<tr>
<td>0.08</td>
<td>20.9</td>
<td>23</td>
<td>860</td>
</tr>
<tr>
<td>0.10</td>
<td>22.4</td>
<td>24</td>
<td>872</td>
</tr>
</tbody>
</table>

**Table 4** Effects of changing $r$ on the optimal solution

<table>
<thead>
<tr>
<th>$r$</th>
<th>$T^*$ (days)</th>
<th>$N^*$ (days)</th>
<th>$Z(T^<em>, N^</em>)$ ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.09</td>
<td>24.2</td>
<td>27</td>
<td>813</td>
</tr>
<tr>
<td>0.12</td>
<td>19.6</td>
<td>22</td>
<td>849</td>
</tr>
<tr>
<td>0.15</td>
<td>16.7</td>
<td>19</td>
<td>874</td>
</tr>
</tbody>
</table>
The retailer’s procurement policy with credit-linked demand

Table 5  Effects of changing $I_p$ on the optimal solution

<table>
<thead>
<tr>
<th>$I_p$</th>
<th>$T^*$ (days)</th>
<th>$N^*$ (days)</th>
<th>$Z(T^<em>, N^</em>)$ ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.12</td>
<td>22.0</td>
<td>23</td>
<td>855</td>
</tr>
<tr>
<td>0.15</td>
<td>19.6</td>
<td>22</td>
<td>849</td>
</tr>
<tr>
<td>0.18</td>
<td>17.0</td>
<td>21</td>
<td>845</td>
</tr>
</tbody>
</table>

From the above tables, the following inferences are made:

- Table 2 shows that a hike in ordering cost ($A$) causes higher cycle length and lower profit.
- Table 3 shows that for rising inflation, there is a marginal increase in cycle length as well as the credit period offered by the retailer, yet there is a significant increase in the retailer’s profit.
- Table 4 suggests that the retailer should offer a lower credit period to customers ($N$) as the rate of saturation of demand ($r$) increases.
- Finally, from Table 5 it is observed that an increase in the interest payable ($I_p$) causes a lower cycle length and profit.

6 Conclusion

In this article, an inventory model for retailers was developed when both the supplier and the retailer offer a credit period to their customers in order to stimulate their respective demand, under inflationary conditions. The demand was assumed to be a function of the credit period offered by the retailer to its customers, considering that it is an established fact now that the credit period has a positive impact on consumer demand, which ultimately helps the retailer to realise the unsaturated sales. An algorithm was proposed which jointly optimises the procurement as well as the credit period offered by the retailer to his/her customers. Finally, a numerical example was presented to illustrate the theoretical results obtained. The findings suggest that the retailer would be able to capture more sales when the credit period offered by him/her to his customers is large. Further, the retailer’s profit increases significantly when the credit period offered by the supplier to the retailer is greater than that from the retailer to his/her customers, as in this case, the retailer only earns interest on the sales revenue and does not have to get his inventory financed. The proposed model can be extended for different forms of credit-linked demand functions; for example, the rate of saturation of demand ($r$) can be taken as a function of $N$.

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