Gravitational Search Algorithm for Optimal Economic Dispatch

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Abstract

This paper presents a novel optimization approach to constrained economic load dispatch (ELD) problems using gravitational search algorithm (GSA). Economic dispatch determines the electrical power to be generated by the committed generating units in a power system so that the generation cost to be minimized while satisfying the constraints. This paper presents a new algorithm based on law of gravity and mass interaction to solve economic load dispatch problem (ELD) by a new optimization algorithm called as Gravitational Search Algorithm (GSA). The simulation results reveal that the developed technique is easy to implement, converged with less execution time and highly optimal solution for economic dispatch with minimum generation cost can be achieved. Simulations results were performed over various systems with different number of generating units and comparisons are performed with other prevalent approaches. The findings affirmed the robustness, fast convergence and proficiency of proposed methodology over other existing technique.

1. Introduction

The economic dispatch problem (EDP) is related to the optimum generation scheduling of available generators in a power system to minimize total fuel cost while satisfying the load demand and operational constraints. EDP plays an important role in operation planning and control of modern power systems [1]. Over the past few years, a number of approaches have been developed for solving EDP using classical mathematical programming methods [2-4]. Meanwhile, classical optimization methods are highly sensitive to starting point and frequently converge to local optimization solution or diverge altogether. Linear programming methods are fast and reliable but main disadvantages associated with the piecewise linear cost approximation. Nonlinear programming methods have a problem of convergence and algorithmic complexity. Recently, in order to make numerical methods more convenient for solving EDPs modern optimization techniques have been successfully employed to solve the EDPs as a non smooth
optimization problem. A number of conventional approaches have been developed for solving EDPs such as gradient method, linear programming algorithm[5], lambd iteration method, quadratic programming, non-linear programming algorithm[6], lagrangian relaxation algorithm [7], and the artificial intelligence technology has been successfully used to solve EDPs such as genetic algorithm [8-9], neural networks[10], simulated annealing and tabu search[11], evolutionary programming[12-13], particle swarm optimization [14], ant colony optimization [15] and so on. Recently a new heuristic search algorithm, namely gravitational search algorithm (GSA) motivated by gravitational law and law of motion has been proposed by Rashedi et al [16-18]. They have been applied successfully in solving various non linear functions. The obtained results confirm the high performance and efficient of the proposed method. GSA has a flexible and well-balanced mechanism to enhance exploration ability. Main objective of this study to present the use of GSA optimization technique for economic operation of power system.

In this paper GSA method has been proposed to solve economic dispatch problem with valve point effect for 3 and 13 unit test system. The results obtained with the proposed GSA approach were analyzed and compared with other optimization results reported in the literature [17]. This paper is organized as fellows. In section 2, the problem formulation is presented. In section 3 the concept and application of GSA is explained. The parameter settings for the test system to evaluate the performance of GSA and results are discussed in section 4. The conclusion are given in section 5

2. Problem formulation

the economic load dispatch problem can be described as an optimization (minimization) process with the following objective function

$$\text{Min} \sum_{i=1}^{n} FC_j \left( P_j \right)$$

(1)

Where $FC_j \left( P_j \right)$ is the total cost function of the $j^{th}$ unit and $P_j$ is the power generated by the $j^{th}$ unit.

Subject to power balance equation:

$$D = \sum P_j - P_L$$

(2)

Where $D$ is the system demand and $P_L$ is the transmission loss, and generating capacity constraints:

$$P_{j_{\min}} \leq P_j \leq P_{j_{\max}}$$

for $j=1, 2, \ldots n$

(3)

Where $P_{j_{\min}}$ and $P_{j_{\max}}$ are the minimum and maximum power output of $j^{th}$ unit.

The fuel cost function without valve-point loading of the generating unit by

$$f(P_j) = a_j + b_j P_j + c_j P_j^2$$

(4)

And the fuel cost function considering valve point loading of the generating unit are given as

$$f(P_j) = a_j + b_j P_j + c_j P_j^2 + \left| e_j \times \sin(f_j \times (P_{j_{\min}} - P_j)) \right|$$

(5)
Where $a_j, b_j,$ and $c_j$ are the fuel cost coefficients of the $j$th unit and $e_j$ and $f_j$ are the fuel cost coefficient of the $j$th unit with valve-point effects. The generating units with multi-valve steam turbine exhibit a greater variation in the fuel cost functions. The valve-point effects introduce ripples in the heat rate curves.

3. Gravitational search Algorithm

The gravitational search algorithm (GSA), is one of the newest heuristic search algorithm was developed by Rashedi et al. in 2009[16]. GSA is followed the physical law of gravity and the law of motion. The gravitational force between two particles is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. In the proposed algorithm, agents are considered as objects and their performance is measured by their masses. 

\[ P_i = (p_{i1}, \ldots, p_{id}, \ldots, p_{in}) \quad i = 1, 2, \ldots, m \]

where $p_{id}$ is the position of the $i$th mass in the $d$th dimension and $n$ is the dimension of the search space. At specific time $t$, a gravitational force form mass $j$ act on mass $i$ and is defined as follows.

\[ F^d_{ij}(t) = G(t) \frac{M_j(t) \times M_i(t)}{R_{ij}(t) + \epsilon} \left( p^d_j(t) - p^d_i(t) \right) \]

where $M_j$ is the mass of the object $j$, $M_i$ is the mass of the object $i$, $G(t)$ is the gravitational constant at time $t$, $R_{ij}$ is the Euclidian distance between the two objects $i$ and $j$, and $\epsilon$ is a small constant.

\[ R_{ij}(t) = \left\| p_i(t), p_j(t) \right\|_2 \]

The total force acting on the agent $i$ in the dimension $d$ is calculated as follows.

\[ F^d_i(t) = \sum_{j \neq i} \text{rand}_j F^d_{ij}(t) \quad (9) \]

where $\text{rand}_j$ is a random number in the interval $[0, 1]$.

According to the law of motion, the acceleration of the agent $i$, at time $t$, in the $d$th dimension, $\alpha^d_i(t)$ is given as follows;

\[ \alpha^d_i(t) = \frac{F^d_i(t)}{M_i(t)} \quad (10) \]

To find the velocity of a particle is a function of its current velocity added to its current acceleration. Therefore, the next position and next velocity of an agent can be calculated as follows.

\[ \begin{align*}
   v_{i}^d(t+1) &= \text{rand}_d \left( p_{i}^d(t) + \alpha_{i}^d(t) \right) \\
   p_{i}^d(t+1) &= p_{i}^d(t) + v_{i}^d(t+1)
\end{align*} \]

where $\text{rand}_d$ is a uniform random variable in the interval $[0, 1]$.

The gravitational constant, $G$, is initialized at the beginning and will be decreased with the time to control the search accuracy. In other words, $G$ is function of the initial value ($G_0$) and time ($t$):

\[ G(t) = G(G_0, t) \]

(13)
The masses of the agents are calculated using fitness evaluation. A heavier mass means a more efficient agent. This means that better agents have higher attractions and moves more slowly. Supposing the equality of the gravitational and inertia mass, the values of masses is calculated using the map of fitness. The gravitational and inertial masses are updating by the following equations.

\[ m_i(t) = \frac{\text{fit}_i(t) - \text{worst}(t)}{\text{best}(t) - \text{worst}(t)} \]  

\[ M_i(t) = \frac{m_i(t)}{\sum_{j=1}^{m} m_j(t)} \]

where \( \text{fit}_i(t) \) represents the fitness value of the agent \( i \) at time \( t \), and the \( \text{best}(t) \) and \( \text{worst}(t) \) in the population respectively indicate the strongest and the weakness agent according to their fitness value. For a minimization problem:

\[ \text{best}(t) = \min_{j \in \{1, \ldots, m\}} \text{fit}_j(t) \]  

\[ \text{worst}(t) = \max_{j \in \{1, \ldots, m\}} \text{fit}_j(t) \]  

For a Maximization problem:

\[ \text{best}(t) = \max_{j \in \{1, \ldots, m\}} \text{fit}_j(t) \]  

\[ \text{worst}(t) = \min_{j \in \{1, \ldots, m\}} \text{fit}_j(t) \]  

### 4. Gravitational search algorithm based economic load dispatch

In order to handle the constraints conveniently, the structure of solutions for ED problem has solved by the proposed method is composed of a set of real power output decision variables for each unit in all over the scheduling periods. The section provides the solution methodology to the above-mentioned economic dispatch problems through gravitational search algorithm.

#### 4.1. Initialization

In the initialization procedure, the candidate solution of each individual (generating unit’s power output) is randomly initialized within the feasible range in such a way that it should satisfy the constraint given by Eq. (4). A component of a candidate is initialized as \( P_i - U \left( P_{i_{\min}}, P_{i_{\max}} \right) \), where \( U \) is the uniform distribution of the variables ranging in the interval of \( \left( P_{i_{\min}}, P_{i_{\max}} \right) \).
4.2. **Fitness evaluation**: (objective function)

The fitness evaluation in each agent in the population set is evaluated using the equation (4). Iteration count from this step, \( t=1 \). Update \( G(t) \), best \( (t) \), worst \( (t) \) and \( M_i(t) \) for \( i=1, 2...m \)

4.3. **Agent force calculation**

The total force acting on the agent \( i \) in the dimension \( d \) is calculated in equation (7).

4.4. **Evaluation of acceleration of an agent**

The acceleration of an agent in \( d^{th} \) dimension over \( T \) dispatch period has evaluated using equation (10).

4.5. **Update the agents’ position**

The next velocity of an agent is calculated by adding the acceleration of an agent to the current velocity and also position of an agent will updated.

4.6. **Stopping criterion**

Repeat the step from 4.3 to 4.7 until the stopping criteria is reached. There are various criteria available to stop a stochastic optimization algorithm. In this paper, to compare with the previous results, maximum number of iterations is chosen as the stopping criterion. If the stopping criterion is not satisfied, the above procedure is repeated from fitness evaluation with incremented iteration.

5. **Computational procedure**

The purposed GSA approach for economic load dispatch problem with valve-point effect can be summarized as follows. The computational methodology of GS algorithm has given in figure 1.

Step 1. Search space identification

Step 2. Generate initial population between minimum and maximum values.

Step 3. Fitness evaluation of agents.

Step 4. Update gravitational constant \( G(t) \), pbest \( (t) \) and worst \( (t) \) in the population and update the mass of the object \( M_i(t) \).

Step 5. Force calculation in different direction.

Step 6. Calculation of acceleration and velocity of an agent.

Step 7. Updating the position of an agent.

Step 8. Repeat step 3 to step 7 until the stop criteria is satisfied
Step 9. Stop.

6. Simulation results

Each proposed gravitational search algorithm has been implemented in command line MATLAB 7.0.1 for solution of two test cases of economic load dispatch. In this paper, to access the efficiency of the proposed algorithm has been applied to 3 and 13 thermal units of ED problems in which objective function is non smooth because the valve point effect are taken into account. The entire program is run Pentium-IV, 2.80GHZ with 506,604KB RAM PC.

6.1. Test case 1

The input data for three-generator system are given in [13] and test data has given in table I. The maximum total power output of the generator is 850 MW. These results give the minimum generation cost for each approach after 50 runs. The setup for the proposed algorithm is executed with the following parameter M=100, Where G₀ is set to 100, \( \alpha \) is set to 8, Maximum iteration numbers are 100 for this case study. The total generation cost obtained by this proposed method is 8234.07 $/h. The
execution time for this case is 0.45s. However, the prime emphasis in this work to have comparative performance of EP with respect to GSA. The result of this test case is shown in Table II. The convergence characteristic has been shown in Fig. 2.

Table 1. Unit data for test case I (three-unit systems)

<table>
<thead>
<tr>
<th>Generators</th>
<th>$P_{\text{min}}$</th>
<th>$P_{\text{max}}$</th>
<th>a</th>
<th>b</th>
<th>e</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>600</td>
<td>561</td>
<td>7.92</td>
<td>0.001562</td>
<td>300</td>
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<tr>
<td>2</td>
<td>100</td>
<td>400</td>
<td>310</td>
<td>7.85</td>
<td>0.001940</td>
<td>200</td>
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<tr>
<td>3</td>
<td>50</td>
<td>200</td>
<td>78</td>
<td>7.97</td>
<td>0.004820</td>
<td>150</td>
</tr>
</tbody>
</table>

Table 2 Result of test case I

<table>
<thead>
<tr>
<th>Evolution Method</th>
<th>Mean Time. in Secs</th>
<th>Best Time in Sec.</th>
<th>Mean Cost ($)</th>
<th>Max. Cost ($)</th>
<th>Min Cost ($)</th>
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<tr>
<td>CEP [13]</td>
<td>20.46</td>
<td>18.35</td>
<td>8235.97</td>
<td>8241.83</td>
<td>8234.07</td>
</tr>
<tr>
<td>FEP [13]</td>
<td>4.54</td>
<td>3.79</td>
<td>8234.24</td>
<td>8241.78</td>
<td>8234.07</td>
</tr>
<tr>
<td>CEP [13]</td>
<td>20.46</td>
<td>18.35</td>
<td>8235.97</td>
<td>8241.83</td>
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<tr>
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<td>3.79</td>
<td>8234.24</td>
<td>8241.78</td>
<td>8234.07</td>
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<tr>
<td>GSA</td>
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<td>0.45</td>
<td>8234.11</td>
<td>8241.95</td>
<td>8234.07</td>
</tr>
</tbody>
</table>

6.2. Test case II

The unit data of 13 generating units has taken from reference [13] which has shown in Table 3. In this case the load demand is 1800MW. The results from the simulation are summarized in Table 4. For obtaining the minimum generation cost, mean time and best time is achieved by the proposed method. The simulation parameters has been setup for the proposed algorithm is $M$ (mass) =100, Where $G_0$ is set
to 100, \( \alpha \) is set to 8, Maximum iteration numbers are 3000. The minimum generation cost obtained by the proposed method is 17963.84 \$/h. The convergence graph of the GSA method is shown in figure 3.

Table 3. Unit data for test case II (thirteen-unit systems)

<table>
<thead>
<tr>
<th>Generators</th>
<th>( P_{\text{min}} )</th>
<th>( P_{\text{max}} )</th>
<th>( a )</th>
<th>( b )</th>
<th>( c )</th>
<th>( e )</th>
<th>( f )</th>
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<tr>
<td>1</td>
<td>0</td>
<td>680</td>
<td>550</td>
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<td>0.00028</td>
<td>300</td>
<td>0.035</td>
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<tr>
<td>2</td>
<td>0</td>
<td>360</td>
<td>309</td>
<td>8.10</td>
<td>0.00056</td>
<td>200</td>
<td>0.042</td>
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<tr>
<td>3</td>
<td>0</td>
<td>360</td>
<td>307</td>
<td>8.10</td>
<td>0.00056</td>
<td>200</td>
<td>0.042</td>
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<tr>
<td>4</td>
<td>60</td>
<td>180</td>
<td>240</td>
<td>7.74</td>
<td>0.00324</td>
<td>150</td>
<td>0.063</td>
</tr>
<tr>
<td>5</td>
<td>60</td>
<td>180</td>
<td>240</td>
<td>7.74</td>
<td>0.00324</td>
<td>150</td>
<td>0.063</td>
</tr>
<tr>
<td>6</td>
<td>60</td>
<td>180</td>
<td>240</td>
<td>7.74</td>
<td>0.00324</td>
<td>150</td>
<td>0.063</td>
</tr>
<tr>
<td>7</td>
<td>60</td>
<td>180</td>
<td>240</td>
<td>7.74</td>
<td>0.00324</td>
<td>150</td>
<td>0.063</td>
</tr>
<tr>
<td>8</td>
<td>60</td>
<td>180</td>
<td>240</td>
<td>7.74</td>
<td>0.00324</td>
<td>150</td>
<td>0.063</td>
</tr>
<tr>
<td>9</td>
<td>60</td>
<td>180</td>
<td>240</td>
<td>7.74</td>
<td>0.00324</td>
<td>150</td>
<td>0.063</td>
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<tr>
<td>10</td>
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<td>120</td>
<td>126</td>
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<td>0.084</td>
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<tr>
<td>11</td>
<td>40</td>
<td>120</td>
<td>126</td>
<td>8.6</td>
<td>0.00284</td>
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<td>0.084</td>
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<tr>
<td>12</td>
<td>55</td>
<td>120</td>
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<td>8.6</td>
<td>0.00284</td>
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<td>0.084</td>
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<tr>
<td>13</td>
<td>55</td>
<td>120</td>
<td>126</td>
<td>8.6</td>
<td>0.00284</td>
<td>100</td>
<td>0.084</td>
</tr>
</tbody>
</table>

Table 4. Simulation result II (thirteen-unit systems)

<table>
<thead>
<tr>
<th>Evolution Method</th>
<th>Mean Time</th>
<th>Best Time</th>
<th>Mean Cost ($ )</th>
<th>Maximum Cost ($ )</th>
<th>Minimum Cost ($ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEP [13]</td>
<td>294.96</td>
<td>293.41</td>
<td>18190.32</td>
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<td>FEP [13]</td>
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<td>IFEP [13]</td>
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<td>156.81</td>
<td>18127.06</td>
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<tr>
<td>GSA [13]</td>
<td>150.32</td>
<td>142.52</td>
<td>18041.21</td>
<td>18910.31</td>
<td>17963.84</td>
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</tbody>
</table>

7. Conclusion

New approaches of using gravitational search algorithm based to solve for economic load dispatch is presented. The presented evaluation function model and optimally selected the mass of an agents have enhanced the performance of the gravitational search algorithm. A comparative study was carried out between the proposed EP and GSA technique. The GSA method gives better results with reduced computational time. Hence, the study shows that GSA could be a promising technique for solving complicated optimization problems in power systems. The GSA has provided the global optimal solution
with a high probability for 3-generator systems and provided a set of quassioptimums for 13 generator system, which are better than other heuristic methods.

8. References

[17]. E. Rashedi, H. Nezamabadi-pour, S. Saryazdi, Filter modeling using gravitational search algorithm (Accepted for publication), Engineering Applications of Artificial Intelligence, to be published, 2010