Outage Analysis of Full Duplex Coded Cooperation

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Abstract—Achieving high data rates is the key to the future generation wireless systems. Full duplexity in transmission enables the cooperative communication system to double its transmission rate compared to that of half duplex systems. However the detrimental effects of the echo-interference existing between transmitting and receiving antennas of relay, are among the primary challenges of full duplex relaying (FDR). In this paper we investigate the outage performance of the FDR cooperative system employing coded cooperation diversity. We find the exact outage probability expression for two user system model. Throughout the analysis we consider all the channels to be independent but not necessarily identically distributed (i.n.i.d.) as generalized Nakagami-m fading.

I. INTRODUCTION

The coded cooperation [1]-[2] is a relaying paradigm in which concept of channel coding is integrated with cooperative signalling. In coded cooperation each user transmit some incremental redundancy for partner’s data instead of just repeating the received bits. Whenever that is not possible, user transmits additional parity bits for its own data. The transmission of redundant bits facilitate the coded cooperation to outperform the repetition based cooperative protocols like amplify-forward (AF) and decode-forward (DF).

In the earlier works on coded cooperation [1]-[6], the performances of different system models have been analyzed with the relays or partners operating in half duplex mode. In half duplex relaying (HDR), the relay node either listens to sources transmission or transmits its own data to destination, which limits the overall transmission rate. Further to meet the high data rate demand of future generation wireless systems, relays must be equipped with full duplex relaying (FDR) capabilities to facilitate frequency reuse and thereby improving spectral efficiency. For analyzing the performance and evaluating the possibility of deployment of FDR in cooperative communication systems, in recent literature [7]-[11], the research trend has been shifted from HDR to FDR. However all the FDR analysis till date are focussed on repetition based cooperative protocols like AF [7]-[8] and DF [9]-[11].

To the best of authors’ knowledge none of the earlier works analyzed the performance of coded cooperation along with FDR. In this paper we extend the analytical framework presented in [2] to FDR and analyze the outage behavior of a two user system model under generalized Nakagami-m fading environment.

II. SYSTEM MODEL

We have taken a simple model which consists of two users \((u_1, u_2)\) and a destination \((d)\) as shown in Fig.1. Both the users are equipped with FDR capabilities i.e. each of them can receive partner’s data and transmit its own data simultaneously. The total information of each user is first partitioned into several blocks, each of \(N\) bits encoded for a rate \(R\). The users cooperate by dividing the each \(N\) bits block into two sub-blocks and transmitting them in two successive time instants. At any instant each user transmits its \(N_1\) of \(N\) bits to other user and to destination. If the receiving user successfully decodes the partner’s information, it computes and transmits \(N_2\) additional parity bits for its partner in successive instant. In case user does not successfully decode the partner’s information, it transmits \(N_2\) additional parity bits for itself in successive instant. In a long run, at any instant each user transmits its \(N_1\) of \(N\) bits to other user and to destination. If the receiving user successfully decodes the partner’s information, it computes and transmits \(N_2\) additional parity bits for its partner in successive time instant. In case user does not successfully decode the partner’s information, it transmits \(N_2\) additional parity bits for itself in successive instant. In a long run, at any instant each user is transmitting \(N_2\) bits (of partner or of itself) through its transmitting antenna and simultaneously receiving \(N_1\) bits of partner through its receiving antenna. These two transmissions thus interfere as they are taking place in a common frequency band. Here \(N_1\) bits are obtained from length \(N\) codeword by partitioning it with rate \(R_1\) using a rate compatible punctured convolutional (RCPC) code [12], and \(N_1 + N_2 = N\). The level of
cooperation i.e. the cooperation ratio \( \alpha \) is defined as
\[
\alpha = \frac{N_1}{N} = \frac{R}{R_1} \quad (1)
\]
Further we consider that there is no feedback between both the users, hence there are four possible cooperative cases, case 1: when both users \( u_1 \) and \( u_2 \) decode correctly, case 2: when neither user \( u_1 \) nor user \( u_2 \) decode correctly, case 3: only user \( u_1 \) decodes correctly, and case 4: only user \( u_2 \) decodes correctly. The correctness of reception, which depends on the channel capacity conditioned on instantaneous signal to noise ratio (SNR) \( \gamma \), is validated using cyclic redundancy checks (CRCs). The fading characteristics of all the channels, including echo-interference channels are assumed to be flat and distributed as Nakagami-\( m \).

In most of the earlier works [7] the echo-interference channel has been modeled with Rayleigh fading distribution after being characterized as the residual interference after the employment of some interference mitigation technique. However, the Rayleigh fading distribution is not sufficient to model the echo-interference channel in practical scenarios of no mitigation or imperfect mitigation (which may result into LOS). So in this paper the echo-interference channel is also modeled with Nakagami-\( m \) fading distribution as higher values of fading parameter \( m \) (generally for \( m > 2 \)) incorporate the LOS component.

III. OUTAGE ANALYSIS

Outage event for any channel is defined as the event when its capacity conditioned on instantaneous SNR, \( \{C(\gamma) = \log_2(1+\gamma)\} \) falls below the target threshold rate \( R \), i.e. \( \{C(\gamma) = \log_2(1+\gamma) < R\} \) or \( \{\gamma < 2^R - 1\} \). For Nakagami-\( m \) distribution, instantaneous SNR has a Gamma distributed p.d.f. \( f_\gamma(\gamma) \), hence outage probability can be written as
\[
P_{out} = Pr\{\gamma < 2^R - 1\} = \int_0^{2^R-1} f_\gamma(\gamma) d\gamma
\]
\[
= \int_0^{2^R-1} \frac{1}{\Gamma(m)} \frac{m^m}{\Omega^m} \gamma^{m-1} \exp\left(-\frac{m\gamma}{\Omega}\right) d\gamma
\]
\[
= 1 - \frac{\Gamma\left(m, \frac{m(2^R-1)}{\Omega}\right)}{\Gamma(m)} \quad , \quad (2)
\]
where \( \Omega \) denotes the average value of SNR over the fading and shadowing effects, \( \Gamma(m) \) is Gamma function defined as \( \Gamma(m) = \int_0^\infty x^{m-1} \exp(-x) dx \). \( (m, \tau) \) is upper incomplete Gamma function defined as \( \Gamma(m, \tau) = \int_\tau^\infty x^{m-1} \exp(-x) dx \).

We denote the instantaneous SNR, average SNR, and fading parameter of the channel from node \( p \) to node \( q \) as \( \gamma_{p,q}, \Omega_{p,q} \) and \( m_{p,q} \) respectively. Here \( p, q \in \{1, 2, d\} \), \( p \neq q \). Node 1 represents \( u_1 \), and node 2 represents \( u_2 \). The instantaneous interference to noise ratio (INR), average INR, and fading parameter of the echo-interference channel between the transmitting and receiving antennas of \( u_1 \) are denoted as \( \gamma_{1,eq}, \Omega_{1,1} \) and \( m_{1,1} \) respectively. Similarly the instantaneous INR, average INR, and fading parameter of the echo-interference channel between the transmitting and receiving antennas of user \( u_2 \) are denoted as \( \gamma_{2,eq}, \Omega_{2,2} \) and \( m_{2,2} \) respectively. The equivalent SNRs (which actually are the signal to interference and noise ratio (SINRs)) of the inter-user channels are
\[
\gamma_{1,2eq} = \frac{\gamma_{2,1eq}}{\gamma_{2,2eq} + 1}
\]
\[
\gamma_{2,1eq} = \frac{\gamma_{2,2eq}}{\gamma_{1,1eq} + 1} \quad \quad (3)
\]
We consider that the inter-user channels are independent i.e. \( \gamma_{1,2eq} \) and \( \gamma_{2,1eq} \) are independent. Now the outage events corresponding to each of four cooperative cases described in section II can be written as,

- **Case 1:** If \( \gamma_{1,2eq} > 2^{\frac{R}{2}} - 1 \), \( \gamma_{2,1eq} > 2^{\frac{R}{2}} - 1 \), the outage event for \( u_1 \) is,
  \[
  \alpha \log_2(1 + \gamma_{1,d}) + (1 - \alpha) \log_2(1 + \gamma_{2,d}) < R
  \]
  and the outage event for \( u_2 \) is,
  \[
  \alpha \log_2(1 + \gamma_{2,d}) + (1 - \alpha) \log_2(1 + \gamma_{1,d}) < R
  \]
- **Case 2:** If \( \gamma_{1,2eq} < 2^{\frac{R}{2}} - 1 \), \( \gamma_{2,1eq} < 2^{\frac{R}{2}} - 1 \), the outage event for \( u_1 \) is,
  \[
  \log_2(1 + \gamma_{1,d}) < \frac{R}{2}
  \]
  and the outage event for \( u_2 \) is,
  \[
  \log_2(1 + \gamma_{2,d}) < \frac{R}{2}
  \]
- **Case 3:** If \( \gamma_{1,2eq} > 2^{\frac{R}{2}} - 1 \), \( \gamma_{2,1eq} < 2^{\frac{R}{2}} - 1 \), the outage event for \( u_1 \) is,
  \[
  \alpha \log_2(1 + \gamma_{1,d}) + (1 - \alpha) \log_2(1 + \gamma_{1,d} + \gamma_{2,d}) < R
  \]
and the outage event for $u_2$ is, 
\[
\log_2(1 + \gamma_{1,d}) < \frac{R}{\alpha}
\]
- Case 4: If $\gamma_{1,eq} < 2^\frac{R}{\alpha} - 1$, the outage event for $u_1$ is, 
\[
\log_2(1 + \gamma_{1,d}) < \frac{R}{\alpha}
\]
and the outage event for $u_2$ is, 
\[
\alpha \log_2(1 + \gamma_{2,d}) + (1 - \alpha) \log_2(1 + \gamma_{1,d} + \gamma_{2,d}) < R
\]
Assuming all the links to be mutually independent, we can write the overall outage probability for user $u_1$ considering these four disjoint cases as
\[
P_{\text{out},u_1} = \text{Pr}\{\gamma_{1,eq} > 2^\frac{R}{\alpha} - 1\} \text{Pr}\{\gamma_{2,eq} > 2^\frac{R}{\alpha} - 1\} \\
\times \text{Pr}\{(1 + \gamma_{1,d})^\alpha (1 + \gamma_{2,d})^{1-\alpha} < 2^R\} \\
+ \text{Pr}\{\gamma_{1,eq} < 2^\frac{R}{\alpha} - 1\} \text{Pr}\{\gamma_{2,eq} < 2^\frac{R}{\alpha} - 1\} \\
\times \text{Pr}\{\gamma_{1,d} < 2^R - R\} \\
+ \text{Pr}\{\gamma_{2,eq} > 2^\frac{R}{\alpha} - 1\} \text{Pr}\{\gamma_{2,eq} < 2^\frac{R}{\alpha} - 1\} \\
\times \text{Pr}\{\gamma_{2,d} < 2^R - 1\} \\
+ \text{Pr}\{\gamma_{1,eq} > 2^\frac{R}{\alpha} - 1\} \text{Pr}\{\gamma_{2,eq} > 2^\frac{R}{\alpha} - 1\} \\
\times \text{Pr}\{\gamma_{1,d} < 2^R - 2^\frac{R}{\alpha} - 1\}
\]
(4)

Now to obtain the exact expression of outage probability, we evaluate the each probability term of (4) for Nakagami-$m$ channel fading. To evaluate $\text{Pr}\{\gamma_{1,eq} < 2^\frac{R}{\alpha} - 1\}$ using (2), we write the p.d.f. of $\gamma_{1,eq}$ as
\[
f_{\gamma_{1,eq}}(\gamma_{1,eq}) = \frac{1}{B(m_{1,2},\lambda_1)} \left( \frac{\Omega_{1,2}}{m_{1,2}v_1} \right)^{m_{1,2}-1} \gamma_{1,eq}^{m_{1,2}-1} \\
\times \left( 1 + \frac{\gamma_{1,eq} \Omega_{1,2}}{m_{1,2}v_1} \right)^{-m_{1,2} - \lambda_1}
\]
where $B(\cdot,\cdot)$ is beta function [13], $\lambda_1 = m_{1,2} \frac{\Omega_{1,2}}{m_{1,2}v_1}$, and $v_1 = \frac{\Omega_{1,2}^2}{m_{1,2}^2 (\Omega_{1,2} + 1)}$. With some simple mathematical manipulations $\text{Pr}\{\gamma_{1,eq} < 2^\frac{R}{\alpha} - 1\}$ can easily be obtained as,
\[
\text{Pr}\{\gamma_{1,eq} < 2^\frac{R}{\alpha} - 1\} = 2F_1 \left[ m_{1,2}, m_{1,2} + 1, 1 + \frac{2^\frac{R}{\alpha} - 1}{\Omega_{1,2}}, \frac{\Omega_{1,2}}{m_{1,2}v_1} \right] \\
\times \frac{1}{m_{1,2}B(m_{1,2},\lambda_1)} \left( \frac{2^\frac{R}{\alpha} - 1}{\Omega_{1,2}} \right)^{m_{1,2}}
\]
(6)

where $2F_1[\cdot,\cdot,\cdot]$ is Gauss hypergeometric function [13].
Similarly $\text{Pr}\{\gamma_{2,eq} < 2^\frac{R}{\alpha} - 1\}$ is
\[
\text{Pr}\{\gamma_{2,eq} < 2^\frac{R}{\alpha} - 1\} = 2F_1 \left[ m_{2,1}, m_{2,1} + 1, 1 + \frac{2^\frac{R}{\alpha} - 1}{\Omega_{2,1}}, \frac{\Omega_{2,1}}{m_{2,1}v_2} \right] \\
\times \frac{1}{m_{2,1}B(m_{2,1},\lambda_2)} \left( \frac{2^\frac{R}{\alpha} - 1}{\Omega_{2,1}} \right)^{m_{2,1}}
\]
(7)
where $\lambda_2 = \frac{m_{1,1}(\Omega_{1,1}+1)^2}{\Omega_{1,1}^2}$, and $v_2 = \frac{\Omega_{1,1}^2}{m_{1,1}(\Omega_{1,1}+1)}$.

The probabilities $\text{Pr}\{\gamma_{1,eq} > 2^\frac{R}{\alpha} - 1\}$ and $\text{Pr}\{\gamma_{2,eq} > 2^\frac{R}{\alpha} - 1\}$ can be obtained as
\[
\left( 1 - \text{Pr}\{\gamma_{1,eq} < 2^\frac{R}{\alpha} - 1\} \right)
\] and
\[
\left( 1 - \text{Pr}\{\gamma_{2,eq} < 2^\frac{R}{\alpha} - 1\} \right)
\]
respectively. Further the probabilities $\text{Pr}\{\gamma_{1,d} < 2^R - 2^\frac{R}{\alpha} - 1\}$ and $\text{Pr}\{\gamma_{2,d} < 2^R - 2^\frac{R}{\alpha} - 1\}$ can directly be written by using (2) as
\[
\text{Pr}\{\gamma_{1,d} < 2^R - 1\} = 1 - \frac{\Gamma(m_{1,1}, \frac{m_{1,1}(2^\frac{R}{\alpha} - 1)}{\Omega_{1,1}})}{\Gamma(m_{1,1})}
\]
\[
\text{Pr}\{\gamma_{2,d} < 2^R - 1\} = 1 - \frac{\Gamma(m_{1,1}, \frac{m_{1,1}(2^\frac{R}{\alpha} - 1)}{\Omega_{2,1}})}{\Gamma(m_{1,1})}
\]
(8)

To evaluate the probabilities $\text{Pr}\{(1 + \gamma_{1,d})^\alpha (1 + \gamma_{2,d})^{1-\alpha} < 2^R\}$ and $\text{Pr}\{(1 + \gamma_{1,d})^\alpha (1 + \gamma_{1,d} + \gamma_{2,d})^{1-\alpha} < 2^R\}$ we consider that the average SNR of $u_1$-d and $u_2$-d channels is same i.e. $\Omega_{1,d} = \Omega_{2,d}$, however without loss of generality, the depth of fading represented by fading parameter for both the channels is not necessarily same. The random variables $(1 + \gamma_{1,d})$, $(1 + \gamma_{2,d})$ and $(1 + \gamma_{1,d} + \gamma_{2,d})$ are gamma random variables with parameters which are easy to obtain. Owing to the fact that the cumulative distribution function $F_G(g)$ of a gamma random variable $g$ is zero at $g = 0$, the $\text{Pr}\{(1 + \gamma_{1,d})^\alpha (1 + \gamma_{2,d})^{1-\alpha} < 2^R\}$ can be given as [14, ch.5]
\[
\text{Pr}\{(1 + \gamma_{1,d})^\alpha (1 + \gamma_{2,d})^{1-\alpha} < 2^R\} = \left[ \frac{\lambda_{1,d}}{v_{1,d}} \right]^\frac{2}{\alpha} \left[ \frac{\lambda_{2,d}}{v_{2,d}} \right]^\alpha \left[ \frac{\Gamma(\lambda_{2,d} - \alpha)}{\Gamma(\lambda_{1,d})} \right] \\
\times \left[ \frac{\lambda_{1,d} - \alpha^2}{1 - \alpha} \right] - \Gamma\left( \frac{\lambda_{1,d} - \alpha^2}{1 - \alpha} \right)
\]
(9)

where $\lambda_{1,d} = \frac{m_{1,1}(\Omega_{1,1}+1)^2}{\Omega_{1,1}^2}$, $v_{1,d} = \frac{\Omega_{1,1}^2}{m_{1,1}(\Omega_{1,1}+1)}$, $\lambda_{2,d} = \frac{m_{2,1}(\Omega_{2,1}+1)^2}{\Omega_{2,1}^2}$, and $v_{2,d} = \frac{\Omega_{2,1}^2}{m_{2,1}(\Omega_{2,1}+1)}$.

Similarly $\text{Pr}\{(1 + \gamma_{1,d})^\alpha (1 + \gamma_{1,d} + \gamma_{2,d})^{1-\alpha} < 2^R\}$ is
\[
\text{Pr}\{(1 + \gamma_{1,d})^\alpha (1 + \gamma_{1,d} + \gamma_{2,d})^{1-\alpha} < 2^R\} = \left[ \frac{\lambda_{3,d}}{v_{1,d}} \right]^\frac{2}{\alpha} \left[ \frac{\lambda_{3,d}}{v_{3,d}} \right]^\alpha \left[ \frac{\Gamma(\lambda_{3,d} - \alpha)}{\Gamma(\lambda_{1,d})} \right] \\
\times \left[ \frac{\lambda_{1,d} - \alpha^2}{1 - \alpha} \right] - \Gamma\left( \frac{\lambda_{1,d} - \alpha^2}{1 - \alpha} \right)
\]
(10)

where $\lambda_{3,d} = \frac{m_{1,1}(\Omega_{1,1}+1)^2}{\Omega_{1,1}^2}$ and $v_{1,d} = \frac{\Omega_{1,1}^2}{m_{1,1}(\Omega_{1,1}+1)}$. 

Substituting (6), (7), (8), (9) and (10) in (4), we get an exact closed form expression outage probability for
user $u_1$. The overall outage probability for user $u_2$ can also be obtained similarly by replacing the role of $u_1$
with $u_2$ in (4) and thereafter.

IV. NUMERICAL RESULTS

We consider higher values of fading parameters $m_{1,1}$ and $m_{2,2}$ for the echo-interference channels existing between transmitting and receiving antennas of $u_1$ and $u_2$ considering that the interferers are not placed far apart. For the purpose of numerical analysis we assume that the inter-user channels i.e. $u_1 - u_2$ and $u_2 - u_1$ have the same fading statistics i.e. $m_{1,2} = m_{2,1}$ and $\Omega_{1,2} = \Omega_{2,1}$. In Fig. 2, we plot the outage probability for user $u_1$ with respect to (w.r.t) the average SNR of inter-user channels for different values of target rate $R$ and $\alpha = 0.25$. We can observe from this plot that improvement in the statistics of echo-interference channels degrades the outage performance. Further higher values of $m_{1,1}$ and $m_{2,2}$ diminish the dependency of outage probability over the inter-user channel statistics.

Fig. 3 plots the outage probability for user $u_1$ with respect to (w.r.t) the average SNR of inter-user channels for various values of target rate $R$ with $\Omega_{1,1} = \Omega_{2,2} = 8dB$, $\Omega_{1,d} = \Omega_{2,d} = 6dB$, $m_{1,2} = m_{2,2} = m_{1,d} = m_{2,d} = 1$, $m_{1,1} = m_{2,2} = 4$ and $\alpha = 0.25$. Here we can see that the higher values of target rate implies the higher outage probability and thus lower system performance. In Fig. 4 plots the outage probability for user $u_1$ with respect to (w.r.t) the cooperation ratio ($\alpha$) for various values of fading parameters $m_{1,1}$, $m_{1,2}$, $m_{1,2}$, and $m_{2,1}$ with $\Omega_{1,1} = \Omega_{2,2} = 8dB$, $\Omega_{1,d} = \Omega_{2,d} = 10dB$, $m_{1,d} = m_{2,d} = 1$, and $R = 1/2$. It is apparent from this figure that (i) for given inter-user fading parameters parameters high values of echo-interference fading parameter imply lower the performance, (ii) for given echo-interference fading parameters parameters high values of inter-user fading parameter imply improve the outage performance and (iii) improvement in inter-user channel reduces the value of critical cooperation ratio i.e the cooperation ratio at which outage probability is minimum.

V. CONCLUSIONS

The outage performance of the FDR cooperative system employing coded cooperation diversity is investigated. For a two user system model we derived the exact closed form expression of outage probability considering all the channels to be independent but not necessarily identically distributed (i.n.i.d.) as generalized Nakagami-m fading.

REFERENCES


Fig. 4. Outage Probability vs cooperation ratio.


