Compensation of Observability problem in a Multi-Robot Localization Scenario using CEKF

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Abstract— Many localization techniques today rely on absolute landmark measurements to efficiently track the robot’s position in space. Although absolute landmarks are essential for correctly estimating its position, they might be rare in an unknown environment. This means that the robot will have to traverse long distances without any outside reference point resulting in system degradation. In case of a robot team though, each robot can rely on both absolute or relative position measurements between robots. This paper describes an approach for a multi-robot localization system, based on a single centralized extended Kalman filter (CEKF) to track the position and orientation changes of a group of robots. Moreover, it is shown that if the robots in the group collect only relative measurements the system suffers from observability problem. It is proven that an increasing number of mobile robots capable of relative measurements only, reduces the observability problem and compensates for the need of external absolute landmarks thus providing efficient localization.

Keywords— Extended Kalman Filtering (EKF), Mobile Robot, Motion-Planning, Multi-Robot Localization

I. INTRODUCTION

For distributed robotic applications that require robots to share sensor information (e.g. mapping, surveillance, etc.) and efficiently coordinate any actions taken, it is imperative to know the robots’ position and orientation in space. Most of the previous approaches have dealt with the single robot localization problem by using stationary strategically placed beacons [1] or using pre-constructed world model of their surroundings [2]. In both methods the robot’s environment has to be manipulated or known before any localization can take place. In a multi-robot localization scenario, the robots can rely on both absolute (landmarks) and relative (measurements between robots) observations providing alternative options to the designer. Some representative examples from early approaches are discussed in [3]. These approaches describe a cooperative localization scheme based on Kalman filters and introduce the concept of “portable landmarks”. At each time instant, at least one robot in the group remains stationary and acts as a reference point or “portable landmark”. An EKF estimates the robots’ position and orientation using a set of virtually absolute measurements collected from the stationary robot. Conducted experiments suggest an average error of 0.4% for the position estimate and 1° for the orientation over distances of approximately 20m [4]. A similar approach can be found in [5] where the authors present an alternative form of cooperative robot localization. Their system does not rely on fixed beacons. Instead, a minimum of three robots called “Millibots” remains stationary and serves as ultrasonic beacons at any time. Methods like this restrict the robots’ movement, while line of sight has to be maintained throughout the mission. Another example can be found in [6] where a catadioptric camera system is developed to obtain direction measurements amongst a number of robots and localize the robots with respect to each other. This system fails to collect any absolute observations, resulting in accumulation of position and orientation errors and degradation of the system. In [7], the authors develop a probabilistic approach that uses a sample-based version of Markov localization to synchronize each robot’s belief of its position in space, whenever one robot detects another. This approach assigns a small false-positive detection rate, which describes the chance of erroneously detecting a robot. A drawback of this method is that in large environments, where the probability of one robot meeting another is rare, this small false-positive detection rate dramatically decreases the robustness of the relative (between robots) measurements to estimate the system.

The proposed localization scheme provides a modular solution, which relies on either absolute (landmarks) or relative (between robots) measurements to estimate the actual robots’ position using a CEKF. This technique eliminates the need for constantly monitoring stationary or “portable landmarks” and allows the robot group to dynamically explore large areas of space without any movement restrictions. Furthermore, the localization algorithm is not based on pre-constructed world model, allowing the team to operate in different and unknown environments.

II. STATE SPACE MODEL

In this scenario, the group of robots is considered as a single centralized system composed of every individual robot moving in an area. A number of assumptions are made regarding the physical configuration and sensor capabilities of the mobile robot team: (i) A group of $I$ independent robots move in a $M$-dimensional space. (ii)
Each robot carries proprioceptive and exteroceptive [8], [9] sensing devices in order to predict and update its own position estimate. (iii) Each robot carries exteroceptive sensors to detect and identify other robots moving in its vicinity and measure their respective displacement. (iv) All the robots are equipped with communication devices that allow exchange of information within the group.

The motion of each robot is described by a differential drive configuration employing two passive wheels [10]. In this design incremental encoders are mounted onto the drive motors to count the wheel revolutions. The instantaneous left and right wheel speeds are given from the equation, \( s_p \) is the angular velocity of left and right wheel (scalar) respectively at discrete time index \( k \). The radius of left and right wheels is denoted by \( r \). The linear speed \( SP(k) \) and the rotational velocity \( \theta(k) \) of the robot at time index \( k \) are defined by equations \( 1 \) and \( 2 \) respectively:

\[
SP(k) = \frac{\omega_{l}(k) + \omega_{r}(k)}{2} \times r
\]

\[
\dot{\theta}(k) = \frac{\omega_{l}(k) - \omega_{r}(k)}{d} \times r
\]

where \( d \) is the distance from the centre of the robot to the wheels.

Each robot’s state vector is composed of position \((x, y)\) and orientation \( \theta \) components, \( X_{\text{ij}}(k) = [x_{\text{ij}}(k), y_{\text{ij}}(k), \theta_{\text{ij}}(k)]^T \) with \( k \in \mathbb{N} \). The recursive state model is governed by non-linear stochastic difference equations, which describe the trajectory \( X_{\text{ij}}(k+1) \) of each robot \((i, j = 1...I)\), \( i \neq j \) in the group and is given by:

\[
X_{\text{ij}}(k+1) = f(X_{\text{ij}}(k), u(k)) + w_{\text{ij}}(k), \quad i = 1...I
\]

\[
z_{\text{ij}}(k+1) = h_{\text{ij}}(X_{\text{ij}}(k+1)) + v_{\text{ij}}(k+1), \quad n = 1, 2...N
\]

where \( z_{\text{ij}}(k+1) = [z_{x_{\text{ij}}}(k+1), z_{y_{\text{ij}}}(k+1)]^T \), \( n = 1, 2...N \), is the \( n \)-th measurement vector (relative or absolute distance, in \( x \) and \( y \) directions, between two robots or between a robot and a landmark point respectively) available to the CEKF at every update cycle.

Utilizing the differential drive configuration in the robot design, the non-linear state transition function \( f \) is given by:

\[
f(X_{\text{ij}}(k), u(k)) = \begin{bmatrix} x_{\text{ij}}(k) + SP(k) \cos \theta(k) T \\ y_{\text{ij}}(k) + SP(k) \sin \theta(k) T \\ \theta(k) + \dot{\theta}(k) T \end{bmatrix}
\]

where \( T \) is the sample time interval (constant). Furthermore, the linear speed of each robot is assumed to be constant, \( SP(k+1) = SP(k) \). The measurement components of the non-linear function \( h_{\text{ij}}(X_{\text{ij}}(k+1)) \) express the relative distance in \( x \) and \( y \) directions between robots \( i \) and \( j \), as follows:

\[
k_{\text{ij}}(X_{\text{ij}}(k+1)) = \begin{bmatrix} z_{x_{\text{ij}}}(k+1) \\ z_{y_{\text{ij}}}(k+1) \end{bmatrix} = \begin{bmatrix} \sqrt{(x_{\text{ij}}(k) - x_{\text{ij}}(k))^2 + (y_{\text{ij}}(k) - y_{\text{ij}}(k))^2} \cos \arctan(y_{\text{ij}}(k) - y_{\text{ij}}(k)) \\ \sqrt{(x_{\text{ij}}(k) - x_{\text{ij}}(k))^2 + (y_{\text{ij}}(k) - y_{\text{ij}}(k))^2} \sin \arctan(y_{\text{ij}}(k) - y_{\text{ij}}(k)) \end{bmatrix}
\]

If during an update cycle a robot \( i \) in the group detects a landmark point in space, then it uses equation \( 6 \) by substituting robot \( j \) with landmark \( l \), \( l = 1...L \) to collect an absolute measurement vector.

The state noise \( w_{\text{ij}}(k) \) and measurement noise \( v_{\text{ij}}(k+1) \) are assumed to be independent with each other, independent of the state components, white (uncorrelated in time), and Gaussian distributed with zero-mean and covariances \( Q_{\text{ij}}(k) = E\{w_{\text{ij}}(k)w_{\text{ij}}^T(k)\} \), \( i = 1...3 \) and \( R_{\text{ij}}(k+1) = E\{v_{\text{ij}}(k+1)v_{\text{ij}}^T(k+1)\} \), \( n = 1, 2...N \) respectively. The deterministic forcing function \( u(k) \) represents optional external control input.

III. LOCALIZATION FOR A GROUP OF THREE MOBILE ROBOTS

This centralized approach estimates the motion of the robot group in a \( M \times I \)-dimensional space by applying CEKF. To linearize equation \( 1 \), the first order Taylor expansion is applied about the current estimated state vector \( \hat{X}_{\text{ij}}(k|k) \) for each robot \( i \) resulting in:

\[
\Delta \dot{X}_{\text{ij}}(k+1) = \Phi_{\text{ij}}(\hat{X}_{\text{ij}}(k|k))\Delta X_{\text{ij}}(k|k) + w_{\text{ij}}(k), i = 1...3
\]

where the matrix \( \Phi_{\text{ij}}(\hat{X}_{\text{ij}}(k|k)) \) is the state transition matrix of robot \( i \) and \( \Delta \hat{X}_{\text{ij}}(k+1) \) is its predicted state error vector.

For a group of three robots, the linearized state equations of the centralized system can be written in a block matrix form as follows:

\[
\begin{bmatrix}
\Delta X_{x_{\text{11}}}(k+1) \\
\Delta X_{y_{\text{11}}}(k+1) \\
\Delta X_{\theta_{\text{11}}}(k+1)
\end{bmatrix} =
\begin{bmatrix}
\Phi_{x_{\text{11}}}(\hat{X}_{\text{11}}(k|k)) & 0 & 0 \\
0 & \Phi_{y_{\text{11}}}(\hat{X}_{\text{11}}(k|k)) & 0 \\
0 & 0 & \Phi_{\theta_{\text{11}}}(\hat{X}_{\text{11}}(k|k))
\end{bmatrix}
\begin{bmatrix}
\Delta X_{x_{\text{11}}}(k) \\
\Delta X_{y_{\text{11}}}(k) \\
\Delta X_{\theta_{\text{11}}}(k)
\end{bmatrix}
+ \begin{bmatrix}
w_{x_{\text{11}}}(k+1) \\
w_{y_{\text{11}}}(k+1) \\
w_{\theta_{\text{11}}}(k+1)
\end{bmatrix}
\]

or

\[
\Delta \hat{X}_{\text{ij}}(k+1) = \Phi_{\text{ij}}(\hat{X}_{\text{ij}}(k|k))\Delta \hat{X}_{\text{ij}}(k|k) + w_{\text{ij}}(k), \quad i = 1...3
\]

where \( \Phi_{\text{ij}}(\hat{X}_{\text{ij}}(k|k)) \) is the centralized state transition matrix with main diagonal, the third individual state transition matrices \( \Phi_{\text{ij}}(\hat{X}_{\text{ij}}(k|k)) \), of the three robots \( i = 1...3 \). The non-linear measurement model \( 4 \) is
linearized about the current predicted state vector \( \hat{X}_i(k|k) \) with respect to robot \( i \) resulting in:

\[
\Delta \hat{x}_i(k+1) = H_i \left( \hat{X}_i(k+1|k) \right) \Delta \hat{x}_i(k+1|k) + v_i(k+1), \quad n = 1, 2, \ldots, N
\]

where the matrix \( H_i(\hat{X}_i(k+1|k)) \) is the measurement matrix of the \( n \)-th measurement vector. The linearized measurement model for a complete update cycle of the CEKF uses \( N = 6 \) measurement vectors:

\[
\Delta \hat{x}_i(k+1) = H_i \left( \hat{X}_i(k+1|k) \right) \Delta \hat{x}_i(k+1|k) + v_i(k+1)
\]

The \( 2 \times 3 \) matrix \( H_{ij} \) contains the Jacobians of the non-linear relative measurement vector between robots \( i \) and \( j \). More precisely, the contents of \( H_{ij} \) and \( -H_{ij} \) matrices are as follows:

\[
H_{ij}(k+1) = \begin{bmatrix}
\frac{\partial h_{ij}}{\partial x_j}(z_i) & \frac{\partial h_{ij}}{\partial y_i}(z_i) & 0 \\
- \frac{\partial h_{ij}}{\partial x_j}(z_i) & - \frac{\partial h_{ij}}{\partial y_i}(z_i) & 0 \\
\end{bmatrix}
\]

and

\[
-H_{ij}(k+1) = \begin{bmatrix}
\frac{\partial h_{ij}}{\partial x_j}(z_i) & \frac{\partial h_{ij}}{\partial y_i}(z_i) & 0 \\
\frac{\partial h_{ij}}{\partial x_j}(z_i) & - \frac{\partial h_{ij}}{\partial y_i}(z_i) & 0 \\
\end{bmatrix}
\]

Furthermore, it can be shown that: \( H_{ij}(k+1) = -H_{ij}(k+1) \).

If additional absolute measurements are collected between any of the three robots and a landmark during an update cycle, supplementary rows are inserted in the centralized measurement matrix \( H_i(\hat{X}_i(k+1|k)) \) associated with the robot responsible for the absolute measurement vector. Now the CEKF uses \( N = 7 \) measurement vectors resulting in:

\[
H_i(\hat{X}_i(k+1|k)) = \begin{bmatrix}
H_{x_i}(k+1) & 0 & 0 \\
-H_{x_i}(k+1) & H_{y_i}(k+1) & 0 \\
-H_{y_i}(k+1) & -H_{x_i}(k+1) & H_{y_i}(k+1) \\
\end{bmatrix}
\]

where the additional matrix \( H_{x_i} \) contains the Jacobians of the non-linear relative measurement vector between robot \( 1 \) and landmark point \( l \).

The state prediction equation for the CEKF is given by:

\[
\hat{X}_i(k+1) = \begin{bmatrix}
\hat{X}_1(k+1) \\
\hat{X}_2(k+1) \\
\hat{X}_3(k+1) \\
\end{bmatrix} = \begin{bmatrix}
\hat{X}_1(k+1|k) \\
\hat{X}_2(k+1|k) \\
\hat{X}_3(k+1|k) \\
\end{bmatrix} + \begin{bmatrix}
f_i(\hat{X}_1(k|k), u_i(k)) \\
f_i(\hat{X}_2(k|k), u_i(k)) \\
f_i(\hat{X}_3(k|k), u_i(k)) \\
\end{bmatrix}
\]

The prediction error covariance for the centralized system is given by:

\[
P_i(k+1|k) = \Phi_i(k+1|k)P_i(k|k)\Phi_i^T(k+1|k) + Q_i(k+1) \quad i = 1, 2, 3
\]

while no update has occurred, i.e. no relative distance and bearing has been measured, the initial prediction of the centralized system position and orientation uncertainty is written as a diagonal block matrix with components the individual robots’ predicted error covariances:

\[
P_i(k+1|k) = \begin{bmatrix}
\Phi_i(k+1|k)P_i(k|k)\Phi_i^T(k+1|k) + Q_i(k+1) \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

To update the multi-robot predicted position, the CEKF uses the conventional Kalman update equations:

\[
S_i(\hat{X}_i(k+1|k)) = H_i(\hat{X}_i(k+1|k))P_i(k+1|k)H_i^T(\hat{X}_i(k+1|k)) + R_i(k+1)
\]

\[
K_i(\hat{X}_i(k+1|k)) = P_i(k+1|k)H_i^T(\hat{X}_i(k+1|k))S_i^{-1}(\hat{X}_i(k+1|k))
\]

\[
\hat{X}_i(k+1|k) = \hat{X}_i(k+1|k) + K_i(\hat{X}_i(k+1|k))H_i^T(\hat{X}_i(k+1|k))
\]

where \( S_i(\hat{X}_i(k+1|k)) \) is the centralized innovation covariance, \( K_i(\hat{X}_i(k+1|k)) \) is the centralized Kalman gain, \( \hat{X}_i(k+1|k) \) is the centralized updated state estimate and \( P_i(k+1|k) \) is the centralized updated state error covariance.

IV. OBSERVABILITY STUDY

A given linear dynamic state model with a given linear input/output model is considered observable if and only if its state is uniquely determinable from the model definition, its inputs, and its outputs. In any other case, the system model is considered unobservable [11]. In a dynamic process, an observability analysis determines the convergence capabilities of a Kalman filter designed to estimate the states of that process. The KF’s estimation
efficiency of a completely observable process depends
only on the process noise and measurement noise [12],
[13]. In case of the multi-robot CEKF, observability
indicates how much the designed centralized system can
exclude the errors and estimate the states such as robots’
position and orientation. If the system is not completely
observable, we cannot estimate some states even if the
noise level is negligible. We can only reduce the error rate.
The observability sets a lower limit of the estimation error.

For a group of three robots, the dynamic movement of
each of them is modeled by three states
\[ \mathbf{X}(k) = [x(k), y(k), \theta(k)]^T. \]
The centralized system composed of three robots has maximum 9 states or by
considering every robot as a single centralized state the
system can be assumed to have 3 individual states.
Therefore, the observability matrix of the centralized time
invariant system is described by:

\[
\mathbf{M}_c = \begin{bmatrix} \mathbf{H}^T & \Phi \mathbf{F}^T \mathbf{H}^T \end{bmatrix} \]  \hspace{1cm} (20)

Assuming that we have small estimated errors in
orientation \( \Delta \hat{\theta}(k) \rightarrow 0 \), the state transition matrix for
each robot can be approximated by \( \Phi \mathbf{F}(\mathbf{k}|k) = \mathbf{I} \), where
\( \mathbf{I} \) is a 3 × 3 identity matrix. The centralized state
transition matrix in (8) can now be written as:

\[
\Phi_c = \begin{bmatrix} \mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\
\mathbf{0}_{3 \times 3} & \mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\
\mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{I}_{3 \times 3} \end{bmatrix} \]  \hspace{1cm} (21)

For a system with only relative measurement
capabilities, the CEKF uses \( N = 6 \) measurement vectors.
The centralized measurement matrix of the system is listed
below in an alternative form:

\[
\mathbf{H}(\mathbf{X}(k)|k) = \begin{bmatrix} \mathbf{H}_x \mathbf{0}_{3 \times 3} \mathbf{0}_{3 \times 3} \mathbf{0}_{3 \times 3} \\
\mathbf{0}_{3 \times 3} \mathbf{H}_y \mathbf{0}_{3 \times 3} \mathbf{0}_{3 \times 3} \\
\mathbf{0}_{3 \times 3} \mathbf{0}_{3 \times 3} \mathbf{H}_\theta \mathbf{0}_{3 \times 3} \\
\mathbf{0}_{3 \times 3} \mathbf{0}_{3 \times 3} \mathbf{0}_{3 \times 3} \mathbf{H}_\omega \end{bmatrix} \begin{bmatrix} \mathbf{I}_{3 \times 3} & -\mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\
-\mathbf{I}_{3 \times 3} & \mathbf{I}_{3 \times 3} & -\mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\
-\mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{I}_{3 \times 3} & -\mathbf{I}_{3 \times 3} \\
\mathbf{0}_{3 \times 3} & -\mathbf{I}_{3 \times 3} & -\mathbf{I}_{3 \times 3} & \mathbf{I}_{3 \times 3} \end{bmatrix} \] \hspace{1cm} (22)

By definition, a system is assumed to be observable if
the dimension of the process state vector is the same as the
rank \( n \) of the centralized observability matrix \( \mathbf{M}_c \). In this
case, the rank of the matrix \( \mathbf{M}_c \) is 6 less than 9, which is
the number of states of the centralized process. Thus the
system is not observable. This is expected since none of
the robots has access to absolute measurements. By
measuring their relative positions, the three robots
improve their position tracking accuracy but they are not
able to bind the overall uncertainty.

In case one of the robots (e.g. Robot 1) has access to
absolute landmark measurements, the CEKF uses \( N = 7 \)
measurement vectors and thus the measurement matrix
becomes:

\[
\mathbf{h}(\mathbf{x}(k)|k) = \begin{bmatrix} \mathbf{H}_x \mathbf{0}_{3 \times 3} \mathbf{0}_{3 \times 3} \mathbf{0}_{3 \times 3} \\
\mathbf{0}_{3 \times 3} \mathbf{H}_y \mathbf{0}_{3 \times 3} \mathbf{0}_{3 \times 3} \\
\mathbf{0}_{3 \times 3} \mathbf{0}_{3 \times 3} \mathbf{H}_\theta \mathbf{0}_{3 \times 3} \\
\mathbf{0}_{3 \times 3} \mathbf{0}_{3 \times 3} \mathbf{0}_{3 \times 3} \mathbf{H}_\omega \end{bmatrix} \begin{bmatrix} \mathbf{I}_{3 \times 3} & -\mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\
-\mathbf{I}_{3 \times 3} & \mathbf{I}_{3 \times 3} & -\mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\
-\mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{I}_{3 \times 3} & -\mathbf{I}_{3 \times 3} \\
\mathbf{0}_{3 \times 3} & -\mathbf{I}_{3 \times 3} & -\mathbf{I}_{3 \times 3} & \mathbf{I}_{3 \times 3} \end{bmatrix} \] \hspace{1cm} (23)

Here the rank of the rank of the observability matrix \( \mathbf{M}_c \) is
9 equal to the number of process states and thus the
system is observable. That happens when at least one of
the robots has access to absolute measurement information.

V. SIMULATED RESULTS

This section illustrates the simulated results of a 2D
multi-robot localization system based on the CEKF
formulae. Each experiment runs for 100 iterations and the
filter’s initial conditions assume that the robots’ \( x(0|0) \) and
\( y(0|0) \) starting positions are known, leading to zero initial
centralized error covariance matrix \( \mathbf{P}_c(0|0) = 0 \).
Additionally, the robots can move freely in an 10m \( \times \) 10m
empty rectangular environment (free of obstacles) as
shown in Figs. 1 and 2.

The centralized state noise covariance matrix \( \mathbf{Q}_c(k) \) is a
diagonal matrix, due to the state noise independence
conditions, with robots’ noise variances 4cm\(^2\) for the \( x \)
and \( y \) states and 1° for \( \theta \). The centralized measurement
noise matrix \( \mathbf{R}_c(k) \) is also diagonal with variances 25cm\(^2\)
for the relative measurements in \( x \) and \( y \) direction.

The first simulation tests the accuracy of the CEKF for
a group of three robots. All three robots are capable of
relative distance measurements in \( x \) and \( y \) directions as
long as they are in the vicinity of each other. One of these
robots (Robot 1) has access to absolute measurements
collected from a known and already identified landmark
located in the middle of the room at coordinates (0,0) as
shown in Fig. 1. The same figure displays the motion
trajectories of the three robots. The dots correspond to the
robots’ actual sequential position points and the side-
crosses correspond to the estimated positions supplied by
the CEKF. Section IV describes that due to the robot 1
absolute measurement capabilities, the centralized system
is fully observable resulting the convergence of the CEKF
and the efficient tracking of the actual robots’ trajectories,
Fig. 1. Here the average squared error of robot 1’s \( x_1 \) and
\( y_1 \) estimated states was calculated as

\[
\frac{\sum_{i=1}^{100} (x_i(k) - \hat{x}_i(k))^2}{100} = 25.4 \text{cm}^2 \quad \text{and} \quad \frac{\sum_{i=1}^{100} (y_i(k) - \hat{y}_i(k))^2}{100} = 61.1 \text{cm}^2
\]
respectively with a maximum squared error around 40cm² and 150cm² for \( x_i \) and \( y_i \).

The second simulation examines the accuracy of the CEKF for the same group of three robots, equipped with only relative measurement capabilities. Fig. 2 displays the motion trajectories of the robots. The dots correspond to the robots’ actual sequential position points and the side-crosses correspond to the estimated positions supplied by the CEKF. Figs. 3 and 4 give the \( x_i \) and \( y_i \) coordinate values of robot 1. The solid lines indicate the actual robot trajectory for both \( x_i \) and \( y_i \) positions, while the dashed ones show the estimates \( \hat{x}_i \) and \( \hat{y}_i \). Although these two figures indicate a highly accurate localization algorithm (no more than 25cm maximum error in \( x_i \) and \( y_i \)), section IV proves that if the robots are capable of only relative measurements, the system becomes unobservable. Thus, the major diagonal of the centralized error covariance matrix, \( P_{kk} \), increases to infinity resulting to filter divergence as time progresses. Because of the unobservability problem the average squared error of robot 1’s \( x_i \) and \( y_i \) estimated states, has been increased to 155.3cm² and 140.1cm² respectively, with maximum squared error reaching 400cm² and 300cm² for \( x_i \) and \( y_i \).

To solve the problem, a system requires additional measurements to become observable; these are provided when more robots are added in the group. Due to the modularity of the proposed localization method, robotic agents can be easily added in real-time, whenever necessary. In this case, the rate of error drops exponentially until the average values in the main diagonal of the error covariance matrix assume their steady state values. Figs. 5 and 6 display robot 1’s average squared error values of \( x \) and \( y \) components for 100-iterations recorded in twenty simulations. It is clear that in order to acquire average square error levels (below 60cm² for \( x_i \) and \( y_i \)) and to achieve reliable position and orientation estimation in a robotic team, the ideal number of robots should be six or more were the steady state error conditions start to apply. To the best of our knowledge, there exists only one case of similar quantitatively observability analysis for a multi-robot localization scenario in [14].
This research would easily be extended in application areas such as 3D localization of underwater or space vehicles networks, thus resulting in a modular dynamic and portable positioning system.

REFERENCES


