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Large Eddy Simulation for Incompressible Flows

An Introduction

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With a Foreword by Charles Meneveau

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