Channel Gain Estimation from Sounding Reference Signal in LTE

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***Abstract*—3G Long Term Evolution (LTE) technology aims at addressing the increasing demand for mobile multimedia services in high user density areas. Channel aware scheduling across wide system bandwidth is one of the key techniques enabling this goal. The Sounding Reference Signal (SRS) in uplink comes in support of this feature as its main purpose is to allow the LTE Base Station (eNodeB) estimating the UL channel of the users across the scheduling bandwidth. Therefore, SRS channel and channel gain estimators are important functions that can drive the overall performance of the system. In this paper, we study such estimation algorithms and show that SRS noise variance estimation is necessary to deliver a non-biased channel gain estimation. We suggest to provision a cyclic shift of the SRS for this purpose and show simulation results validating the performance of the proposed scheme.**

***Index Terms*—LTE, E-UTRAN, channel estimation, channel gain, sounding, OFDM.**

I. INTRODUCTION

The Long Term Evolution (LTE) wireless network, also known as Evolved Universal Terrestrial Radio Access Network (E-UTRAN), has been standardized by the 3GPP working groups (WG) as part of the Release 8 specifications. OFDMA and SC-FDMA (single carrier FDMA) access schemes were chosen for the down-link (DL) and up-link (UL) of E-UTRAN, respectively [1]. User Equipments (UE's) are time and frequency multiplexed on a physical uplink shared channel (PUSCH), and time and frequency synchronization between UE's guarantees optimal intra-cell orthogonality. An important UL reference signal, the Sounding Reference Signal (SRS) is defined in support of frequency dependent scheduling, link adaptation, power control and UL synchronization maintenance, which are functions handled above the Physical Layer, mainly at layer 2. Indeed, the main purpose of this signal is to allow the Base Station, also referred to as eNodeB, estimating a UE's radio channel information on time and frequency resources possibly different from those where it is scheduled. SRS processing is handled at the Physical Layer and may deliver to upper layers the following measurements:

- Channel and channel gain estimates across the system bandwidth;
- Noise variance;
- Timing and frequency offsets.

Both UL MU-MIMO/SIMO and DL eigen-beamforming based schedulers rely on the SRS to get the UE's channel estimates and derive the relevant scheduling metric. In particular, for the SIMO UL scheduler [7] [8], eNodeB makes use of the UE's signal to interference plus noise ratio (SINR) information to compute the scheduling metric and perform link adaptation. The UE's SINR can be derived from the first two above metrics but would preferably use interference estimates from other reference signals such as the Demodulation Reference Signal (DMRS), more representative of the interference experienced by the data symbols. Therefore, the channel gain estimates delivered by the SRS estimation should be noise and interference-free. In this paper, we propose unbiased algorithms to estimate the channel and channel gains, and assess their performance. The paper is organized as follows: Section II first summarizes the design choices for the SRS in LTE. Section III revisits the state of art time-domain method [3] for estimating the SRS channel and proposes a reduced-complexity extension allowing de-multiplexing multiple UEs in one operation. Section IV provides the performances of the proposed channel estimator. Section V demonstrates that direct channel gain estimation from the channel estimates is biased and that estimating and removing the channel estimation variance error by means of a reserved cyclic shift fixes this issue. Finally, the resulting channel gain estimation performances measured from simulations are presented before conclusions are drawn in Section VI.

II. SOUNDING REFERENCE SIGNAL DESIGN FOR LTE

The LTE sub-frame is depicted in Figure 1: it is made of two 0.5 ms slots, each made of six DFT-SOFDM data symbols (normal cyclic prefix configuration) and one central demodulation reference symbol (DMRS). When the sub-frame is configured for SRS transmission, the last symbol is reserved for the SRS [1].

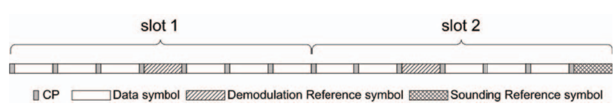

Figure 1. SRS symbol in the LTE subframe.

Multiple UEs can be multiplexed in the same SRS symbol in a combination of FDM and CDM, as illustrated in Figure 2. The sounding signal is built from a pilot root sequence, which is an extended Zadoff-Chu (EZC) sequence constructed by extending the closest prime-length Zadoff-Chu

(ZC) [2] sequence to the SRS sequence length N providing the configured SRS bandwidth. Such sequence has Constant Amplitude Zero Autocorrelation (CAZAC) properties. The latter guarantees discrete periodic autocorrelations are zero for all non-zero lags, allowing orthogonal code multiplexing by duplicating and cyclic shifting the same root sequence. The constant amplitude property allows controlling the PAPR and generates bounded and time-flat interference to other users. In a given sub-frame, all UEs in the same cell and with the same SRS bandwidth share the same root EZC sequence $X = (X_0, X_1, ..., X_{N-1})^T$, defined in frequency domain. Then, the sequence is modified per (1) to produce a cyclic shift $C_u = Nm_u/8$ in time-domain, configured for user u, resulting in a CDM multiplexing capacity of 8 users:

$$X_{u,k} = X_k e^{j2\pi k m_u/8};\ m_u \in \{0, ..., 7\} \tag{1}$$

The sequence is further mapped to the N sub-carriers allocated to SRS out of M, where M is the total amount of sub-carriers of the system bandwidth, e.g. $M = 2048$ for a 20 MHz LTE system bandwidth. The tone mapping also reflects the Single Carrier Interleaved Frequency Division Multiple Access (SC-IFDMA) transmission scheme of the SRS: within its allocated bandwidth, a UE's SRS sequence is mapped on every other tone, leaving in-between tones to zero, thus producing two combs per SRS bandwidth (Figure 2). As a result, the total SRS multiplexing capacity for a given SRS bandwidth is 16 users. With the IFDM multiplexing scheme, the sequence duration equals half the OFDM symbol duration T. Hence, in LTE where $T = 66.67\mu s$, the minimum cyclic shift increment between two CDM users is $T/16 = 4.17\mu s$.

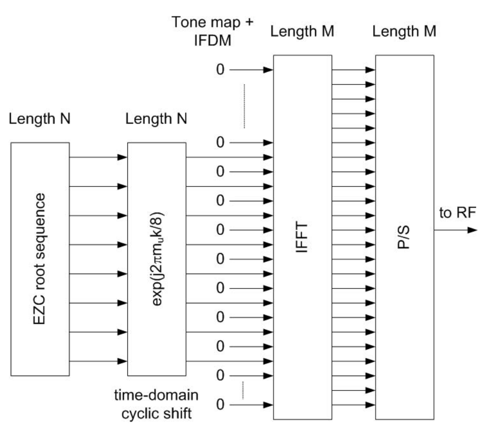

Figure 2. SRS transmission scheme.

III. Channel estimation from SRS

In [3], an SRS receiver was proposed where the received time sample sequence r is first converted in frequency domain through an M-length FFT, from which SRS sub-carriers are demapped to produce a frequency-domain sequence Y, carrying all CDM users. Y is then converted back to time domain sequence y through N-length IDFT, where cyclic-shift de-multiplexing is performed for each of the 8 CDM users. This method takes profit of the SRS OFDM symbol structure and CAZAC sequence to calculate each multiplexed UE's channel impulse response (CIR) through a frequency-domain computed periodic correlation (matched filter). Frequency-domain channel estimates are then obtained by extracting each user's relevant samples from the total CIR samples and converting them back to frequency-domain through N-length DFT. This method is referred to as *time-domain based channel estimation*. The SRS receiver considered in the present paper follows the same principle with an additional complexity reduction achieved from *group-UE cyclic shift de-multiplexing*: instead of correlating y with each UE's sequence, the received frequency-domain sequence Y is element-wise multiplied with the complex conjugate of the expected root sequence X before the IDFT, as illustrated in Figure 3. This provides in one shot the concatenated CIRs of all UEs multiplexed on the same root sequence (Figure 6). Cyclic-shift de-multiplexing reduces to selecting the relevant samples for each UE. This method can be expressed as:

$$Y = \mathbf{F}_{NM} r \tag{2}$$

$$y = \mathbf{F}_N^{-1} diag\left(X^* Y^T\right) \tag{3}$$

$$y_u = \left(0, ..., 0, y_{n_1(u)}, y_{n_2(u)}, ..., y_{n_L(u)}, 0, ..., 0\right)^T \tag{4}$$

$$\hat{H}_u = \mathbf{F}_N y_u \tag{5}$$

where $N \times M$ matrix $\mathbf{F}_{NM}$ corresponds to M-point FFT and N subcarrier demapping, $N \times N$ matrix $\mathbf{F}_N$ and $\mathbf{F}_N^{-1}$ correspond to N-point DFT and IDFT respectively, $n_1(u), ..., n_L(u)$ are the sample indexes defining the cyclic shift window of user u, and L is the number of time samples corresponding to the maximum expected delay spread among users. Both user CIR extraction and cyclic shift de-multiplexing are performed simultaneously in (4) by selecting the appropriate user's cyclic shift window from the concatenated time-domain CIRs sequence y of all multiplexed UEs.

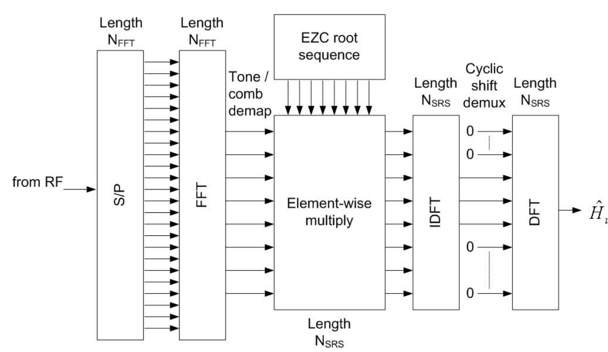

Figure 3. SRS receiver structure for time-domain based channel estimation with group-UE cyclic shift de-multiplexing.

Compared with a conventional frequency-domain approach, where the cyclic shift de-multiplexing is performed by de-spreading the de-mapped frequency-domain sequence Y with the user's EZC sequence, zeroing-out samples outside the user's energy window in the proposed method allows spreading, in the last stage N-length DFT, the energy of the AWGN

Parameter	Value or range
System Bandwidth	5 MHz
Number of antennas	2
Number of SRS users	2 - 16
SRS bandwidth	4 / 8 / 20 PRBs
SRS sequences	EZC [1] with random selection of ZC index and cyclic shift every subframe
Channel	TU6, PA [4]
UE speed	3 km/h

TABLE I
SIMULATION PARAMETERS.

samples in the user's window across the N sub-carriers. Since the user's energy is all contained in its cyclic shift window, this represents a reduction factor $G_{\sigma_H^2}$ on the channel estimation mean square error (MSE) σ_H^2 of N/L corresponding to the ratio of half the OFDM symbol duration $T/2$ over the maximum expected delay spread τ among users: $G_{\sigma_H^2} = \frac{T}{2\tau}$. For example, with LTE symbol duration of $66.67\mu s$ and TU channel delay spread of $5\mu s$ [4], an MSE improvement close to 8 dB is achieved for the channel estimation.

IV. CHANNEL ESTIMATION PERFORMANCE

We evaluated the performance of the proposed estimator in a realistic multi-user SC-FDMA multiplex. The simulator models a number of UEs multiplexed on a configurable SRS bandwidth within the total bandwidth available in 5 MHz spectrum: 25 physical resource blocks (PRB). The root sequence, cyclic shift and frequency mapping of the UEs are re-selected randomly every sub-frame. The SNR is measured in time domain, i.e. is representative of the average signal power across the SRS bandwidth, not in the user's comb only. Table I provides all parameters of the simulation. We use as performance criterions the normalized mean square error σ_H^2 of the channel estimates $\hat{H}$ per sub-carrier per antenna:

$$\sigma_H^2 = \frac{E\left\{\left|\hat{H} - H\right|^2\right\}}{a^2} \quad (6)$$

where $a^2 = E\left\{|H|^2\right\}$ is the averaged received power from the user.

A. Channel estimator distortions

We first assess the performance of the proposed estimator in absence of noise. Indeed, the time-domain approach requires that the channel be first down-sampled to time domain (3) and then interpolated to frequency domain (4). The former acts as a *sinc* band-pass filter on the channel, which has two consequences: 1) The narrower the SRS bandwidth, the coarser the CIR and therefore the channel estimates. 2) Some spill-over effects should be accounted for when designing the user window for cyclic shift de-multiplexing. In particular, this leads to non-perfect orthogonality between cyclic shifts. The latter unavoidably creates interpolation errors at both ends of the interpolation, i.e. at SRS bandwidth edges. As a result, it is recommended to reduce the scope of the channel estimation to the inner SRS bandwidth only. As can be observed from Figure 4, the MSE due to these distortions remains below -20 dB when applying 10% shrink to the SRS bandwidth, which is the baseline assumption in the rest of this paper. Another observation is the high error floor when 16 SRS users are multiplexed with TU channel. This is due to the delay profile truncation since, in this configuration, the cyclic shift increment is $4.17\mu s$ (Section II) but the delay spread of the channel is $5\mu s$.

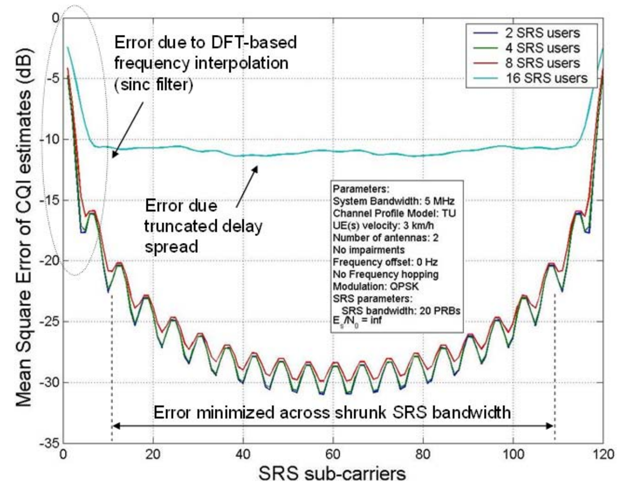

Figure 4. Channel estimates MSE without noise (TU channel).

B. Channel estimator performance with AWGN

The normalized mean square error σ_H^2 is plotted in Figure 5 when varying the number of multiplexed SRS users (at 20-PRB SRS bandwidth) and the SRS bandwidth (where 8 and 16 SRS users are multiplexed with TU and PA channels respectively). The distortion effects discussed in Section IV-A can be observed as error floors with TU channel: with 16 SRS users because of the delay profile truncation and with 4 and 8-PRBs SRS bandwidths due to the band-pass filter effect on the CIRs. The smaller delay spread of PA channel ($0.9\mu s$) allows multiplexing 16 SRS users and provides a larger reduction factor $G_{\sigma_H^2}$ on the channel estimation mean square error. Its low frequency selectivity makes it less sensitive to the SRS bandwidth.

V. CHANNEL GAIN ESTIMATION FROM SRS

A. Non-biased estimator

A broad use of the SRS is to allow predicting the UE's signal to interference plus noise ratio (SINR) information for the UL scheduler to derive appropriate scheduling metric and perform link adaptation [5]- [8]. This involves computing the channel gain estimate per sub-carrier per antenna $\hat{G} = \left|\hat{H}\right|^2$. In absence of other distortion but AWGN, channel estimates $\hat{H}(a) = \hat{H}_x(a) + j\hat{H}_y(a)$ are complex valued random variables which components follow a non-centered Normal distribution:

$$\begin{cases} \hat{H}_x(a) = a_x N\left(1, \frac{\sigma_H^2}{2}\right); \; \hat{H}_y(a) = a_y N\left(1, \frac{\sigma_H^2}{2}\right) \\ a_x^2 + a_y^2 = a^2 \end{cases} \quad (7)$$

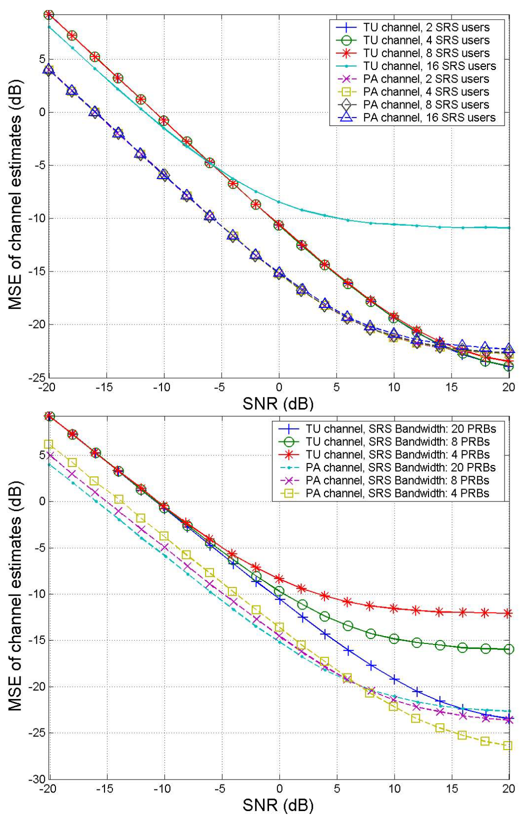

Figure 5. Channel estimation MSE performance.

As a result, the channel gain estimates $\hat{G}(a)$ follow a non-central Chi-square distribution with 2 degrees of freedom and non-centrality parameter a^2, which normalized mean is given by [9]:

$$\frac{m_{\hat{G}(a)}}{a^2} = 1 + \sigma_H^2 \tag{8}$$

From (8) it is clear that this estimator is biased and the noise variance component $a^2\sigma_H^2$ should be removed from the gain estimate to produce a non-biased estimation $\hat{G}_0$:

$$\hat{G}_0 = \left|\hat{H}\right|^2 - \hat{\sigma}_N^2 \tag{9}$$

where $\hat{\sigma}_N^2$ is an estimate of the noise variance $a^2\sigma_H^2$.

However, $\left|\hat{H}\right|^2$ and $\hat{\sigma}_N^2$ are independent estimates which cumulative errors may lead to a negative value for $\hat{G}_0$. Therefore some additional adjustment is needed to prevent from negative gain estimates. We simply chose to clip the gain estimates below a clipping threshold G_{floor}:

$$\hat{G}_{clip} = max\left\{\hat{G}_0, G_{floor}\right\} \tag{10}$$

B. Noise variance estimation through cyclic shift reservation

The variance of the SRS noise is specific to the SRS signal which is expected to be interfered by other SRS signals from neighbour cells, and therefore reflects the cross-correlation characteristics of EZC sequences. The noise variance can be estimated from the areas where no signal energy is present in the concatenated delay profiles sequence y. However, for some channel types such as TU and when all multiplexing space is used, there is no such area available for noise variance estimation. Hence it is proposed to reserve one cyclic shift per comb for the purpose of noise variance estimation. As shown in Figure 6 the noise estimation window should be carefully designed to maximize the number of noise samples while not including samples carrying adjacent users' energy such as e.g. in spill-over regions. Then, the noise variance can be estimated as:

$$\hat{\sigma}_N^2 = \frac{1}{|I_N|}\sum_{i \in I_N} |y_i|^2 \tag{11}$$

where I_N is the noise estimation window and $|I_N|$ the number of samples in this window.

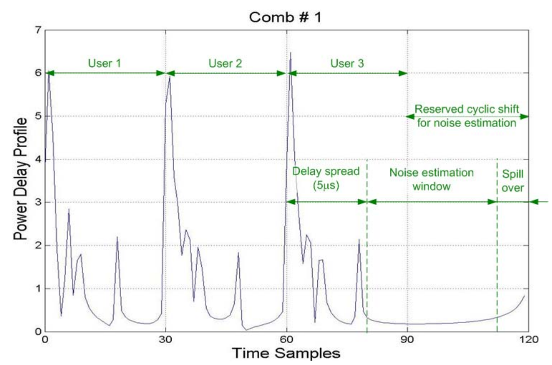

Figure 6. Noise variance estimation through cyclic shift reservation.

C. Channel gain estimator performance with AWGN

We first check that the modified channel gain estimator (9) is unbiased when using the noise variance estimator (11). Figure 7 confirms this, showing the channel gain estimation mean with and without noise variance estimation removal with 2 multiplexed SRS users per symbol and 20 PRB SRS bandwidth. Then, we assess the performance of the *positive* channel gain estimator (10). Given the channel gain is further used for SINR estimation, it is more convenient to express it in dB scale and use as performance metrics the mean m_{H^2dB} and standard deviation σ_{H^2dB} errors of the per-antenna per-subcarrier channel gain estimations $\hat{G}_{clip}$ expressed in dB:

$$m_{H^2dB} = E\left\{\left(\hat{G}_{clip}\right)_{dB} - \left(|H|^2\right)_{dB}\right\} \tag{12}$$

$$\sigma_{H^2dB} = \sqrt{E\left\{\left(\left(\hat{G}_{clip}\right)_{dB} - \left(|H|^2\right)_{dB} - m_{H^2dB}\right)^2\right\}} \tag{13}$$

Figure 8 shows the mean performance of the channel gain estimator with and without noise removal and for various negative gain avoidance clipping threshold values G_{floor}. Two SRS users are multiplexed per 20-PRB SRS bandwidth. As can be observed, the best performance compromise across the

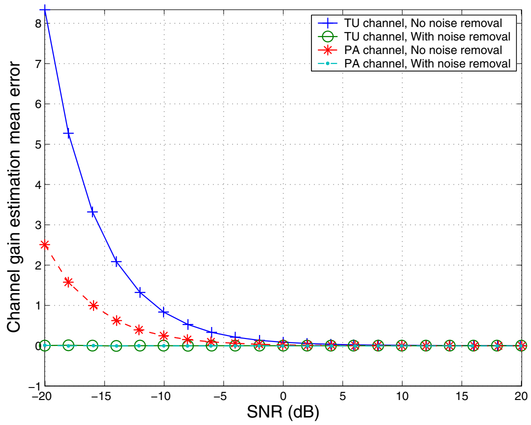

Figure 7. Channel gain estimation bias with and without noise removal.

SNR range is achieved with a clipping threshold of -20 dB, which we have used in the following simulations.

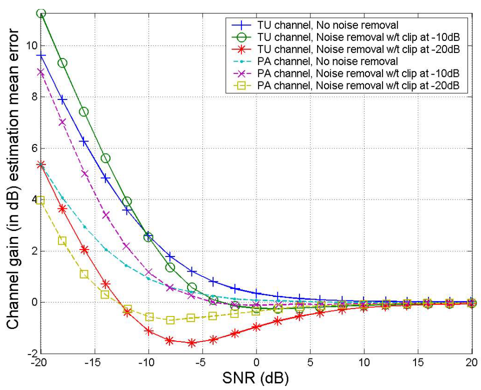

Figure 8. Channel Gain estimator (dB) mean error.

Figure 9 plots the standard deviation performance of the per-antenna per-subcarrier channel gain estimator $\left(\hat{G}_{clip}\right)_{dB}$ when varying the number of SRS users (at 20-PRB SRS bandwidth) and the SRS bandwidth (with 6 and 14 multiplexed SRS users for TU and PA channels respectively). Since one cyclic shift is reserved for noise variance estimation for each SRS comb, the remaining number of multiplexed users per SRS symbol is 2, 6 and 14 with 2, 4 and 8 cyclic shifts per comb respectively. Similar observations as in Section IV-B can be checked: with TU channel error floors occur with 14 SRS users and at small bandwidth due to truncated delay spread and channel band-pass filtering respectively, while with PA channel, the delay spread is small enough to prevent from strong co-cyclic-shift interference, even with 16 users per symbol and down to 4-PRB bandwidth.

VI. Conclusion

This paper describes in details the design choices for the LTE SRS channel, channel gain, and noise variance, from theoretical derivations and performance evaluations. In particular, it is shown that the proposed time-domain based channel estimation with group-UE cyclic shift de-multiplexing

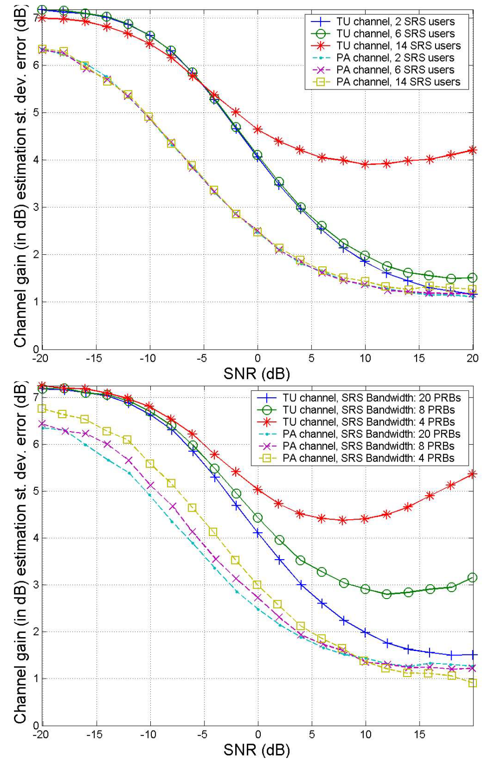

Figure 9. Channel Gain estimator (dB) standard deviation error.

is a low-complexity approach that allows sharing the same upfront computation for users' channels and noise variance estimation. In addition, it is shown that unbiased per-antenna per-subcarrier channel gain estimation requires estimating and removing the noise variance by means of one reserved cyclic shift per SRS comb. Performance results obtained from realistic multi-user link-level simulations over a wide SNR range are presented and can be used for further reference in system simulations to model the channel estimation error from SRS.

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