Global Optimization of Electromagnetic Devices Using an Exponential Quantum-Behaved Particle Swarm Optimizer

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Particle swarm optimization is a population-based swarm intelligence algorithm based on the simulation of a social psychological metaphor instead of the survival of the fittest individual paradigm. Inspired by the classical particle swarm method and quantum mechanics theories, this work presents a new quantum-behaved approach using a mutation operator with exponential probability distribution. The simulation results demonstrate good performance of the proposed algorithm in solving a significant benchmark problem in electromagnetics, namely the TEAM workshop benchmark problem 22.

\textit{Index Terms}—Electromagnetic optimization, particle swarm optimization, quantum mechanics, TEAM 22 problem.

I. INTRODUCTION

IN THE area of electromagnetic design optimization, many problems can be described by nonlinear relationships, which often give rise to multiple local minima. In this context, a relevant benchmark problem to verify the robustness and performance of different optimization techniques is the TEAM workshop problem 22 \cite{1}.

TEAM workshop problem 22 consists in determining the optimal design of a superconducting magnetic energy storage (SMES) device \cite{2}, \cite{3} in order to store a significant amount of energy in the magnetic field with a fairly simple and economical coil arrangement which can be rather easily scaled up in size. The literature contains references to several optimization algorithms, for solving TEAM problem 22, e.g., \cite{3}--\cite{6}.

Particle swarm optimization (PSO) is a population-based optimization method in which each member is seen as a particle, and each particle is a potential solution to the problem under analysis. In PSO each particle which moves through the space of the problem has a randomized velocity associated to it.

Similarly to genetic algorithms, the PSO is an optimization method based on a population. However, unlike genetic algorithms, PSO does not have operators, such as crossover and mutation. Furthermore, PSO does not implement the “survival of the fittest individuals” concept; rather, it implements the simulation of social behavior.

Recently, concepts of quantum mechanics and physics have inspired the development of some new optimization methods (see e.g., \cite{7} and \cite{8}). Inspired by PSO and quantum mechanics theories, this work presents new quantum-behaved PSO (QPSO) approaches with mutation based on exponential probability distribution for the optimization of TEAM workshop problem 22.

The remainder of this paper is organized as follows. Sections II and III describe the fundamentals of QPSO and G-QPSO approaches. Numerical simulation and comparisons are provided in Section IV. Finally, Section V outlines the conclusion and future research.
cates the iterations, \( w \) is a parameter called the inertial weight; \( v_i = [v_{i1}, v_{i2}, \ldots, v_{in}]^T \) stands for the velocity of the \( i \)th particle, \( x_i = [x_{i1}, x_{i2}, \ldots, x_{in}]^T \) stands for the position of the \( i \)th particle of population, and \( p_i = [p_{i1}, p_{i2}, \ldots, p_{in}]^T \) represents the best previous position of the \( i \)th particle.

Positive constants \( c_1 \) and \( c_2 \) are the cognitive and social components, respectively, which are the acceleration constants responsible for varying the particle velocity towards \( p_{best} \) and \( g_{best} \), respectively. Index \( g \) represents the index of the best particle among all the particles in the swarm. Variables \( u_d \) and \( U_d \) are two random functions in the range \([0,1]\).

The first part in (1) is the momentum part of the particle. The inertia weight \( w \) controls the influence of the previous history of velocities on current velocity. The second part is the ‘cognition’ part, which represents the independent thinking of the particle itself.

The inertia weight \( w \) can be set to a fixed value or adapted during iteration (for example linearly between 0.9 and 0.4 as suggested in [18]). Equation (2) represents the position update, according to its previous position and its velocity, considering \( \Delta t = 1 \).

6) Return to Step 2) until a stop criterion is met, usually a sufficiently good fitness or a maximum number of iterations (generations).

Particle velocities in each dimension are clamped to a maximum velocity \( V_{\text{max}} \). If the sum of accelerations causes the velocity in that dimension to exceed \( V_{\text{max}} \), which is a parameter specified by the user, then the velocity in that dimension is limited to \( V_{\text{max}} \).

\( V_{\text{max}} \) is a parameter which determines the resolution with which the regions around the current solutions are searched. If \( V_{\text{max}} \) is too high, the PSO facilitates a global search, and particles might fly past good solutions. Conversely, if \( V_{\text{max}} \) is too small, the PSO facilitates a local search and particles may not explore sufficiently beyond locally good regions. Previous experience with PSO (trial and error, mostly) led us to set the \( V_{\text{max}} \) to 20% of the dynamic range of the variable in each dimension.

It can be shown that each particle will converge (under mild hypothesis) to following coordinates [19]:

\[
p = (c_1 p_i + c_2 p_g)/(c_1 + c_2) \tag{3}
\]

where \( c_1 \) and \( c_2 \) are two random numbers in \([0,1]\).

III. QUANTUM-BEHAVED PARTICLE SWARM OPTIMIZATION

In terms of classical mechanics, a particle is depicted by its position vector \( x_i \) and velocity vector \( v_i \), which determine the trajectory of the particle. The particle moves along a determined trajectory in Newtonian mechanics, but this is not the case in quantum mechanics. In the quantum setting, the term ‘trajectory’ is meaningless, because \( x_i \) and \( v_i \) of a particle can not be determined simultaneously according to the uncertainty principle. Therefore, if individual particles in a PSO system have quantum behavior, the PSO algorithm is bound to work in a different fashion [9].

In this work, the philosophy of QPSO with delta potential, proposed in [9]–[11], is retained, but in the new QPSO approach presented here mutation is based on an exponential probability distribution.

In the quantum model of PSO, the state of a particle is depicted by the wave function \( \psi(x,t) \) (Schrödinger equation) [12], instead of position and velocity. The dynamic behavior of the particle is widely divergent form that of the particles in classical PSO systems in that the exact values of \( x_i \) and \( v_i \) cannot be determined simultaneously. In this context, the probability of the particle appearing in position \( x_i \) is given by the probability density function \( |\psi(x,t)|^2 \), the form of which depends on the potential field the particle lies in [11].

Employing the Monte Carlo method, particles move according to the following iterative equation:

\[
\begin{align*}
x_i(t+1) & = p + \beta \cdot (M_{\text{best}}_i - x_i(t)) \cdot \ln(1/u), \quad \text{if } k \geq 0.5 \\
x_i(t+1) & = p - \beta \cdot (M_{\text{best}}_i - x_i(t)) \cdot \ln(1/u), \quad \text{if } k < 0.5
\end{align*} \tag{4}
\]

where \( \beta \) is a design parameter called contraction-expansion coefficient (linearly varying during iteration), and \( u \) and \( k \) are values generated according to a uniform probability distribution in the range \([0,1]\).

In very simple terms the new position is a random \( \ln(1/\beta) \) perturbation around an equilibrium position which varies during iteration.

The global point called Mainstream Thought or Mean Best \( (M_{\text{best}}) \) of the population is defined as the randomized weighted mean of the \( p_{best} \) and \( g_{best} \) positions of all particles and is given by

\[
M_{\text{best}} = \frac{1}{N} \sum_{d=1}^{N} \alpha p_{d} + (1 - \alpha)p_g \tag{5}
\]

where \( \alpha \) is a random number in \([0,1]\) and \( N \) is the size of the population. The procedure for implementing QPSO is given by the following steps.

1) Initialize a population (array) of particles with random positions in the \( n \) dimensional problem space using a uniform probability distribution function.
2) Evaluate the objective function value of each particle.
3) Compare each particle’s fitness with the particle’s \( p_{best} \).
   If the current value is better than \( p_{best} \), then set the \( p_{best} \) value equal to the current value and the \( p_{best} \) location equal to the current location in \( n \)-dimensional space.
4) Compare the fitness with the population’s overall previous best. If the current value is better than \( g_{best} \), then set \( g_{best} \) to the current particle’s array index and value.
5) Calculate \( M_{\text{best}} \) using (5).
6) Change the position of the particles of the population according to (4).
7) Loop to Step 2) until a stop criterion is met, usually a sufficiently good fitness or a maximum number of iterations (generations).

Various approaches using Gaussian, Cauchy and exponential probability distributions to generate random numbers to updating the velocity equation of PSO approaches have been proposed [13]–[16]. In this paper, following the same line of study,
we present new results for the mutation operator in QPSO using the exponential probability distribution called E-QPSO [17]. The QPSO approach proposed in this work combined with mutation operator uses a modification of parameters $c_1$ and $c_2$ of (3) with modification given by the following equation:

$$p = \left( |E_1| \cdot p_k + |E_2| \cdot p_0 \right) / (2 \cdot |E_3|)$$

(6)

where $E_1$, $E_2$, and $E_3$ are random numbers generated with exponential distribution.

Generating random numbers using exponential distribution sequences provides a good compromise between the probability of having a large number of small amplitudes around the current points (fine tuning) and a small probability of having higher amplitudes, which may allow particles to move away from the current point and escape from local minima.

### IV. Optimization Results

TEAM workshop problem 22 is a continuous, eight-parameter benchmark. Mathematically, this optimization problem has an objective function consisting of the weighted average of two conflicting goals (energy and stray field requirements). The optimization problem solved is the following:

$$\min \; OF = \frac{B_{\text{stray}}^2}{B_{\text{normal}}^2} + w \cdot \frac{|E_{\text{energy}} - E_{\text{ref}}|}{E_{\text{ref}}}$$

(7)

where $OF$ is the objective function to be minimized; the reference stored energy and stray field are $E_{\text{ref}} = 180$ MJ, $B_{\text{normal}} = 200 \mu T$, and $w$ is a penalty factor with value equals 100 (this is a deviation from the problem definition in which $w = 1.0$). This was done in order to make the stray field and energy terms error of roughly the same magnitude). $B_{\text{stray}}^2$ is defined as

$$B_{\text{stray}}^2 = \frac{\sum_{i=1}^{22} |B_{\text{stray},i}|^2}{22}$$

(8)

where $B_{\text{stray},i}$ is evaluated along 22 equidistant points along line $a$ and line $b$ in Fig. 1. Both the energy and the stray field are calculated using an integral formulation for the solution of the forward problem (Biot–Savart law) [4]. The bounds of the optimization parameters are shown in Table I.

Finding the optimal design is not an easy task because, besides usual geometrical constraints, there is a material related constraint: the given current density and the maximum magnetic flux density value on the coil must not violate the superconducting quench condition which can be well represented by a linear relationship shown in Fig. 2 [4].

TEAM 22 workshop problem is used to investigate the performance of the classical PSO optimization method, and its two quantum mechanics-inspired alternatives QPSO [9]–[11], and E-QPSO [17].

In the TEAM 22 workshop study, the population size $N$ was 30 and the stopping criterion $t_{\text{max}}$ was 200 generations for the PSO (using $c_1 = c_2 = 2.05$), QPSO and E-QPSO approaches. The rates of contraction-expansion coefficient ($\beta$) in QPSO and E-QPSO approaches and inertia weight ($w$) in PSO approach are decreased over time linearly (from 0.9 to 0.4).

Table II reveals that both E-QPSO and QPSO provide better solutions for the TEAM 22 workshop problem, particularly in terms of mean and best $OF$ values than the classical PSO. Furthermore, and maybe more interestingly in the case of problems with much more expensive to compute objective functions, QPSO and E-QPSO have lower standard deviation compared to the standard PSO meaning that good results can be obtained with a smaller number of runs. In Table III the
best results of each tested approach (mentioned with statistical details in Table II) are shown.

V. CONCLUSION AND FUTURE RESEARCH

In this paper, the efficiency of PSO, QPSO and E-QPSO approaches were tested on the well-known electromagnetic benchmark problem TEAM 22. The results indicate that QPSO and E-QPSO can tackle the chosen benchmark problem more efficiently than the classical PSO, providing superior solutions both in terms quality and reliability with which they can be obtained. Moreover, the tuning of QPSO and E-QPSO is much simpler than that of many of the classical versions of PSO with inertia weight and maximum velocity parameters.

The proposed method will shortly be applied to other optimization problems including Loney’s solenoid benchmark [20].

REFERENCES


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