DOWNLINK BEAMFORMING AVOIDING DOA ESTIMATION FOR CELLULAR MOBILE COMMUNICATIONS

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ABSTRACT

A new technique to overcome induced difficulties of FDD (Frequency Division Duplex) for the design of a forward link beamformer in cellular mobile communications systems is presented. It takes advantage of the array topology at the base station, used to transpose second order statistics of the propagation channel from uplink frequency to downlink frequency, thus enabling to optimize directly any beamforming criterion based on these statistics at downlink frequency, without feedback nor DOA (Direction Of Arrival) estimation. It can be applied whatever the criterion used to design the beamformer is. Effectiveness is verified by the mean of simulation results.

1. INTRODUCTION

Due to an always growing demand, some solutions are studied in order to improve spectral efficiency of current mobile communication systems, amongst which spatial diversity. Implementing some antenna array at the base station theoretically allows to reduce CCI (Co Channel Interference) in both uplink and downlink, thus enhancing system capacity. A set of weights has to be choosen and applied specifically to each user, by optimizing some criterion that involves the propagation channels that links the desired and undesired sources to the base station.

Moreover, for FDD systems, these propagation channels can be completely different, in a numerical sense, between uplink and downlink. If the choice of a set of weights for uplink can be grounded on the disponibility of direct information about the channel (by using a training sequence or blind methods), this is not the case for downlink processing. The use of feedback makes knowledge of the downlink channel available at the base station [1], but wastes bandwidth. For designing a downlink beamformer from uplink data, only one assumption seems valid : the geometry of the propagation channel remains the same between uplink and downlink, in terms of DOAs, paths, and average power.

This assumption already grounded many developments. Detection and estimation of DOA and power of the paths from uplink, can be used to design a beamformer at downlink frequency, but is computationally exhaustive and may not be adapted to angular spreading and distributed sources. [2, 3] addressed that problem, also with parametric methods, by modeling this distributed character.

We suggest a new way to overcome the problem, derived from wide-band array processing [4], which does not make any assumption about spatial distribution of sources. Focusing on array topology, it allows to transpose second order statistics of the channel from uplink to downlink frequency, then any criterion based on these statistics. It will be referred as frequency transposition, and its effectiveness will be verified with simulations, without loss of generality, when coupled to a given beamformer in a GSM context.

2. FREQUENCY TRANPOSITION

In this paper, we focus our attention on circular arrays since they seem appropriate for base stations. The method can also be used for other topologies, but performance will be highly dependant on this parameter. The number of sensors will be denoted $M$, the downlink and uplink frequencies $f_d$ and $f_u$ (wavelengths $\lambda_d$ and $\lambda_u$), the radius $R$.

2.1. Heuristic Approach

As will be discussed in section 3.3, one can deduce second order statistics at $f_d$ from these statistics at $f_u$ if an $M \times M$ mathematical operator $T_{u \rightarrow d}$ is available, that turns any steering vector at $f_u$ into its equivalent at $f_d$. Considering two array manifolds $A_u$ and $A_d$ corresponding to $f_u$ and $f_d$, which dimensions are $M \times P$ (supposing the array manifold is sampled every $\frac{\lambda_u}{2}$ degrees), $T_{u \rightarrow d}$ should verify ($\cdot^*$ denotes transconjugate):

$$T_{u \rightarrow d}^* A_u = A_d$$
Minimizing the error in a Least Square sense yields the estimate:

$$\hat{T}_{u\rightarrow d} = (A_u^* A_u)^{-1} A_u A_d^*$$  \hfill (1)

**2.2. Theoretical Development for Circular Arrays**

Let us consider the case of a plane wave, with carrier frequency $f_u$, impinging from direction $\theta_0$ on the circular topology (see Fig. 1). The signal at location $\theta$ is a delayed version of the one at the center of the array (chosen as reference point), and this delay leads to a phase shift:

$$x(\theta, \theta_0) = e^{j2\pi \frac{d}{\lambda_u} \cos(\theta - \theta_0)}$$

$x(\theta, \theta_0)$ phase shift

![impinging wave](image)

Figure 1: plane wave impinging on a circular topology

$x(\theta, \theta_0)$ is a $2\pi$ periodical signal that can be developed in Fourier series as follows:

$$x(\theta, \theta_0) = \sum_{n=-\infty}^{+\infty} C_n e^{j n (\theta - \theta_0)}$$

where:

$$C_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j2\pi \frac{d}{\lambda_u} \cos(\theta - \theta_0)} e^{-j n \theta} d\theta = j^n J_n(2\pi \frac{R}{\lambda_u})$$

$J_n(\cdot)$ is the Bessel function of order $n$. Using a circular array with $M$ elements corresponds to sampling $x(\theta)$ at a rate $\theta_s = \frac{2\pi}{M}$.

Let us consider the case of an odd $M = 2p + 1$, for sakes of simplicity. The Shannon theorem of sampling is here equivalent to: $C_n \approx 0$ for $|n| > p$. Assuming the fact that $J_n(x) \approx 0$ for $n > 2x$, one could easily deduce a theoretical relationship between $M$, $R$, and $\lambda_u$, to ensure that Shannon's theorem is respected:

$$M > \frac{8\pi R}{\lambda_u} + 1$$  \hfill (2)

Although this condition is sufficient, it may not be necessary, which means that it should be practically possible to override that limit, as will be discussed in section 2.3.

Suppose that condition (2) is verified for both frequencies $\lambda_u$ and $\lambda_d$. We get respectively for the two signals, at $f_u$ and $f_d$:

$$x_u(k\theta_s, \theta_0) \simeq \sum_{n=-p}^{+p} C_{n_u} e^{j n k (\theta_s - \theta_0)}, C_{n_u} = j^n J_n(2\pi \frac{R}{\lambda_u})$$

$$x_d(k\theta_s, \theta_0) \simeq \sum_{n=-p}^{+p} C_{n_d} e^{j n k (\theta_s - \theta_0)}, C_{n_d} = j^n J_n(2\pi \frac{R}{\lambda_d})$$

One can see that it is then possible to get $x_d(k\theta_s, \theta_0)$ from $x_u(k\theta_s, \theta_0)$, by filtering it with a filter of complex gain $G_n$ at frequencies $\frac{n}{M}$, where:

$$G_n = \frac{J_n(2\pi \frac{R}{\lambda_u})}{J_n(2\pi \frac{R}{\lambda_d})}$$  \hfill (3)

This result does not depend on $\theta_0$. Let $h_k$ be the impulse response of this filter. We have:

$$G_n = \sum_{k=-p}^{p} h_k e^{-j kn \theta_s} = h_0 + 2 \sum_{k=1}^{p} h_k \cos(kn \theta_s)$$  \hfill (4)

Now, filtering any steering vector in order to transpose it from frequency $f_u$ to $f_d$ clearly appears like multiplying it by a circulant $M \times M$ operator $T_{u\rightarrow d}$ with first line $[h_0 \ h_1 \ \ldots \ h_p \ h_p \ \ldots \ h_1]$.  

**2.3. Results**

We consider a circular array with $M = 9$ sensors ($p = 4$), $f_d = 900 MHz$ and $f_u = 945 MHz$. We first suppose that condition (2) is verified, which yields $R \simeq 10 cm$. Table 1 shows that the results obtained in a Least Square sense for the coefficients $\{h_k\}_{k=0..4}$ are the same as those given by equations (3) and (4).

<table>
<thead>
<tr>
<th>$\times 10$</th>
<th>$h_0$</th>
<th>$h_1$</th>
<th>$h_2$</th>
<th>$h_3$</th>
<th>$h_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>L S</td>
<td>9.513</td>
<td>0.713</td>
<td>0.240</td>
<td>0.246</td>
<td>0.204</td>
</tr>
<tr>
<td>Theoretical</td>
<td>9.517</td>
<td>0.700</td>
<td>0.243</td>
<td>0.244</td>
<td>0.204</td>
</tr>
</tbody>
</table>

Table 1: Theoretical and least square results for filtering coefficients, $f_u = 945 MHz$ and $f_d = 900 MHz$, $R = 0.1 m$, $M = 9$ sensors.

Now let us examine the possibility to override condition (2). This is presented in Fig. 2, which shows the evolution of the LS error, computed on the array manifold, when $R$ and thus the ratio $\frac{R}{\lambda_u}$ increase.

In the remaining of this paper, we decide to choose for the radius of the antenna an $R$ that verifies $\frac{R}{\lambda_u} = 0.7$, in order to get a good compromise between the ability to transpose and the directivity of the array. For this antenna, the
Let us assume that the DOAs of the array manifold is close to $-20dB$, which yields good transposition results.

3. USING FREQUENCY TRANPOSITION FOR DOWNLINK BEAMFORMING

3.1. Channel Modelisation

Although frequency transposition may be useful with any beamforming procedure, we decided to use the one already introduced in [1] for the case of flat fading channels and feedback, extended to frequency selective fading. Consider a given source or user indexed by $k$. Its multidimensional uplink channel impulse response can be stored in an array $H_k$ of dimension $M \times N$, where $N$ corresponds to the maximum delay of multipath propagation for all users.

Let us denote as $\Theta_k(t)$ the vector of the DOAs of the $P_k$ paths that link the base station to that user, which can be time-varying, and as $D(\Theta_k(t), f)$ the $(M \times P_k)$ steering matrix corresponding to those DOAs at frequency $f$. Then the double varying nature of the channel can be explicited by the model:

$$H_k(f,t) = D(\Theta_k(t), f)B_k(f,t)$$

where the $(P_k \times N)$ matrix $B_k(f,t)$ summarizes all the phase shifts and attenuations encountered by the signal along the propagation paths, and an additional attenuation due to ISI introduced by the modulation into the channel. These coefficients appear to be highly dependent on time and frequency.

3.2. Temporal and Frequential Evolution of the Channel

Let us assume that the DOAs $\Theta_k$ are remaining the same over the processing interval. Of course this assumption cannot be valid for the matrix $B_k(f,t)$, as soon as the time considered exceeds one burst length, thus changing the fading state. But one can suppose that the phase of the coefficients of $B_k(f,t)$, considering time variations at constant frequency, are uniformly distributed between 0 and $2\pi$.

Moreover, considering frequency variations, $D(\Theta_k, f)$ remains completely dependent on frequency, as does $B_k(f,t)$. But one valid assumption is that the average power of the paths is the same at both frequencies.

Consequently, the matrix $B_k(f,t)$ can be modeled as a zero mean random process with diagonal covariance matrix independent from time and frequency:

$$\mathbf{E}[B_k(f,t)B_k^*(f,t)] = R_k$$

3.3. Beamforming Technique with Frequency Transposition

The average power transmitted on the downlink to desired user $k$ when applying a set of weights $w_k$ can be written, supposing transmitted symbols are uncorrelated and with unitary power:

$$P_{k \rightarrow k}^d = \mathbf{E}[w_k^*H_k(f_\delta,t)H_k^*(f_\delta,t)w_k]$$

$$= w_k^*D(\Theta_k, f_\delta)\mathbf{E}[B_k(f_\delta,t)B_k^*(f_\delta,t)]D^*(\Theta_k, f_\delta)w_k$$

$$= w_k^*D(\Theta_k, f_\delta)R_kD^*(\Theta_k, f_\delta)w_k$$

$$= w_k^*\Gamma_k^d w_k$$

(7)

In the same time, the average transmitted interfering power to users $j \neq k$ can be written:

$$P_{k \rightarrow j}^d = \sum_{j \neq k} w_k^*\Gamma_j^d w_k = w_k^*\left(\sum_{j \neq k} \Gamma_j^d\right) w_k$$

(8)

One can suggest to optimize the ratio between these two quantities, for each user, and thus to decide separately the sets of weights to apply by choosing:

$$w_k^{opt} = \arg\left(\max_{w_k} \frac{w_k^*\Gamma_k^d w_k}{w_k^*\Gamma_k^{bb} w_k}\right)$$

(9)

where $\Gamma_k^{bb} = \sum_{j \neq k} \Gamma_j^d$ denotes the covariance matrix of jammers related to user $k$ on the uplink, which yields the same result as:

$$w_k^{opt} = \arg\left(\max_{w_k} \frac{w_k^*\Gamma_k^{xx} w_k}{w_k^*\Gamma_k^{bb} w_k}\right)$$

(10)

where $\Gamma_k^{xx} = \sum_{j \neq k} \Gamma_j^d$ denotes the total covariance matrix of the array snapshots.

Of course, this criterion assumes knowledge of $\Gamma_k^{xx}$ and $\Gamma_k^{bb}$, which isn’t the case. But we can assume knowledge of...
of a good estimate of $\Gamma_{bb}$, resulting from the treatment of uplink data, as showed in [5]. Then let us introduce the frequency transposition and consider that:

$$
T_{u \rightarrow d}^* \Gamma_{uu}^T T_{u \rightarrow d} = T_{u \rightarrow d}^* D(\Theta_k, f_u) D^*(\Theta_k, f_u) T_{u \rightarrow d} = D(\Theta_k, f_d) R_k D^*(\Theta_k, f_d) = \Gamma_{dd}^d
$$

(11)

By linearity of the transposition we get:

$$
T_{u \rightarrow d}^* \Gamma_{bb}^u T_{u \rightarrow d} = \Gamma_{bb}^d
$$

$$
T_{u \rightarrow d}^* \Gamma_{xx}^u T_{u \rightarrow d} = \Gamma_{xx}^d
$$

(12)

(12) clearly show that any criterion using the downlink channel covariance matrices averaged on a sufficient time to assume (6) and stationary DOAs can be realised with frequency transposition on the basis of the knowledge of the uplink channel covariance matrices. (10) can be reformulated:

$$
W_{opt}^{dd} = arg \left( max_x \frac{W_k^T T_{u \rightarrow d}^* \Gamma_{uu}^x T_{u \rightarrow d} W_k}{W_k^T T_{u \rightarrow d}^* \Gamma_{uu}^u T_{u \rightarrow d} W_k} \right)
$$

(13)

Note that there is no power control here and that the weight vectors are defined up to a scale factor. In the simulations we shall constraint them with $W_k^T \Gamma_{dd}^d W_k = 1$, which may not be the best way to achieve performance, but our goal is to prove efficiency of the frequency transposition.

3.4. Results

Simulations are conducted with the $M = 9$ sensors circular array described in section 2.3, for the case of flat fading. Note that, up to minor modifications, this algorithm can be applied to frequency selective channels. 3 users are supposed to share the same channel, each of them having dispersive DOAs (10 degrees dispersion around a nominal DOA). Results are shown on figure 3 where the beam obtained at $f_d$ for user 1 after optimizing (13) is compared to the beam that would be obtained at frequency $f_u$ if frequency was remaining the same for uplink and downlink. 25 bursts are taken into account. Although these beams look slightly different (because of dB representation), they lead (for each mobile) to the same SINR at $f_u$, than the maximum theoretical one that may be expected at $f_u$, thus confirming the performance of the method.

4. CONCLUSION

A new technique has been introduced that can avoid DOA estimation or feedback in the downlink beamforming design for FDD mobile communication systems. It can be applied to any criterion based on second order statistics of the propagation channels of the desired user and the other ones (averaged over the fading states (i.e. some bursts)). This method has the peculiar advantage that it makes no assumption about the spatial distribution of the paths, avoiding all the intrinsic problems of parametric methods. Simulations and theory have been developed for the case of a circular topology of the array, but every topology has its own frequency transposability properties, so it may also work with some other array topologies, such as sectorized linear topologies.

5. REFERENCES


