The Voracity Effect

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We analyze an economy that lacks a strong legal-political institutional infrastructure and is populated by multiple powerful groups. Powerful groups dynamically interact via a fiscal process that effectively allows open access to the aggregate capital stock. In equilibrium, this leads to slow economic growth and a "voracity effect," by which a shock, such as a terms of trade windfall, perversely generates a more-than-proportionate increase in fiscal redistribution and reduces growth. We also show that a dilution in the concentration of power leads to faster growth and a less procyclical response to shocks. (JEL F43, O10, O23, O40)

Two common characteristics of developing countries that have grown slowly in the last several decades are the absence of strong legal and political institutions and the presence of multiple powerful groups in society. In this paper, we analyze a dynamic model of the economic growth process that contains these features. We employ the model to ask three questions. First, why does the combination of a weak institutional structure and fractionalization inside the governing elite generate slow growth? Second, what is the relationship between the concentration of power (the number of powerful groups) and growth? Third, why do such countries not only grow slowly but also frequently respond in a perverse fashion to favorable shocks, by increasing more than proportionally fiscal redistribution and investing in inefficient capital projects?

The importance of weak institutions and fractionalization in explaining poor growth performance has been highlighted in the empirical literature. Furthermore, country studies have recently emphasized these features in explaining procyclical fiscal policies and a decline in the quality of investment in response to favorable shocks. However, a theoretical analysis that explicitly jointly links these perverse responses and the prevalence of low growth to the primitive characteristics of an economy has been lacking. This provides the motivation for our paper.

We focus on the fiscal process as an important arena in which powerful groups interact in a society with a weak legal-political infrastructure and emphasize discretionary fiscal redistribution as a key mechanism by which such groups appropriate national resources for themselves. Examples of powerful groups are provincial governments that extract transfers from the center, strong unions and industrial conglomerates that seek protection, and patronage networks that obtain kickbacks from public works.

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2 See Max Royko (1971), Ernesto Ezakis (1994), and Eduardo Martin de Pozuelo et al. (1994), for examples of
We consider a two-sector economy. The formal sector employs the efficient production technology but is subject to taxation; the shadow sector enjoys a less productive technology but is nontaxable. For instance, the shadow sector may represent the domestic informal sector, sectors sheltered from international competition or, if capital is mobile, secret overseas bank accounts that are out of the reach of the domestic fiscal authorities. In each case, it should be clear that the raw rate of return in the inefficient sector is lower than in the formal one, especially in the case of less-developed countries (LDCs).

We model the extent of discretionary fiscal redistribution as a variable that is endogenously determined by three primitives: the existence of powerful groups, raw rates of return, and institutional barriers that limit the power of groups to extract transfers from the rest of society. Countries where the extent of fiscal redistribution is limited are those in which powerful groups do not exist, groups coordinate and act as one agent, or where there are well-developed legal and other institutional structures that make it impossible for powerful groups to extract transfers arbitrarily from the rest of society. These limits do not exist in countries with powerful groups and a lack of institutional barriers. Contrasting the behavior of the economy across these different regimes constitutes a major part of the paper.

The idea is that when groups have the power to extract fiscal transfers, capital stocks in the formal sector of the economy are not truly private. Since transfers must be financed by some form of taxation, higher transfers to one group result in higher taxes for the entire formal sector of the economy. In order to protect their profits from arbitrary taxation, agents transfer part of their resources to where they are free from this taxation. Agents can do this by investing in the shadow sector which is beyond the reach of the fiscal authorities. Typically these investments yield a lower raw rate of return.

We model the interaction of the powerful groups as an infinite-horizon dynamic game. In this game each group has open access, via the fiscal process, to the capital stocks that other groups have in the formal sector. In contrast, capital in the shadow sector is truly private. The solution concept we use is Markov perfect equilibrium, which restricts strategies to be functions of payoff-relevant state variables only. History-dependent strategies, such as trigger strategies, are not permitted.

Our first point is that if there do not exist institutional barriers to discretionary redistribution, the existence of powerful groups reduces the growth rate relative to an economy in which society is composed of a single group or where groups can coordinate. This is because the existence of noncooperative powerful groups generates a redistributive struggle and, as a result, a greater share of resources ends up in nontaxable inefficient activities.

Second, we show that if there exist multiple powerful groups, a reduction in power concentration (an increase in the number of groups) leads to better economic performance. This result is reminiscent of the result that in a market in which firms play Cournot, the outcome approaches the competitive one as the number of firms increases.

Our third point is that if there do not exist institutional barriers to discretionary redistribution, an increase in the raw rate of return in the formal sector reduces growth. The intuition is as follows. An increase in the raw rate of return in the formal sector unleashes two conflicting effects: a direct effect that increases the profitability of investment in the formal sector, and a voracity effect that leads each group to attempt to grab a greater share of national wealth by demanding more transfers. This is reflected in a higher tax rate in the formal sector, which induces a reallocation of capital to the informal sector, where it is safe from taxation. This shift reduces the growth

3 Of course, there are other mechanisms by which an increase in the terms of trade can depress growth (e.g., taking more leisure when a windfall occurs). We want to focus on a mechanism that emphasizes the role of the security of property rights because it is the mechanism most clearly identified in the case-study literature.
rate in the economy, counteracting the direct positive effect of an increase in the raw rate of return. We prove that redistribution increases more than proportionally to the windfall, and that the direct effect of the windfall is dominated by the voracity effect, so that the growth rate declines as the raw rate of return increases. Note that, in the absence of powerful groups, no such endogenous increase in discretionary transfers and taxation occurs. In this case, the rate of return improvement stimulates investment and hence growth.

Finally, we want to note that our approach is very different from the “Dutch disease” analysis of country adjustment to terms of trade windfalls (see J. Peter Neary and Sweder Van Wijnbergen, 1986). According to that literature, a positive terms of trade windfall leads to a contraction of the nonresource tradeable sector, either due to the crowding-out effect of an expansion in the natural resource sector or a positive wealth effect that raises demand for nontradeables. In contrast, according to our model, the sector experiencing the positive price shock actually shrinks and there is no positive wealth effect; the decline in growth arises from the endogenous increase in distortionary redistributive activity.

Section I provides an overview of the model. Section II contains the model. In section III, we analyze the relationship between power concentration and growth, the voracity effect and welfare. Section IV discusses some related empirical evidence in the context of our model. Finally, in Section V, we present our conclusions.

I. Overview of the Model

In this section we make an intuitive presentation of the model. We consider an economy populated by infinitely lived groups and formed by two sectors: a high-return formal sector and a less efficient shadow economy. The shadow economy can be identified with a foreign tax haven or with the domestic informal sector. Taxes can only be levied in the formal sector. If powerful groups exist, each group is able to extract fiscal transfers. The government in turn must finance such transfers by levying taxes on the formal sector. This interaction is repeated for an infinite horizon. We want to stress that this dynamic game is a minimal model to address the issues discussed in the introduction. First, we need a dynamic setup because otherwise groups would just try to appropriate as much as they can and there would be no role for productivity shocks to affect the intensity of rent-seeking activity. Second, we need more than one group to analyze redistribution.

It is straightforward to see that if there is only one powerful group, all powerful groups can coordinate, or there are institutions that prevent discretionary fiscal redistribution, then all capital will be allocated in the efficient formal sector and first-best outcome achieved.

If the above is not the case, there are two types of Markov perfect equilibria in the dynamic game among the powerful groups: extreme and interior. In an extreme equilibrium, groups transfer as much capital as possible from the formal to the shadow sector. This is a socially inefficient outcome because the return in the shadow sector is lower. In contrast, along the interior equilibrium, groups limit their demand for transfers. To illustrate the mechanism that underlies the anomalous response of growth to windfalls observed in the data (the voracity effect) we will use equations (1) and (2) below.

Suppose that the rates of return in the formal and shadow sectors are \( \alpha \) and \( \beta \) respectively (\( \alpha > \beta \)), that there are only two groups and that each group extracts \( x_i \) out of the aggregate formal capital stock. It follows that the aggregate capital in the formal sector evolves according to

\[
\dot{K}(t) = (\alpha - x_1 - x_2)K(t).
\]  

Consider the case in which, along the interior equilibrium, capital is transferred from the formal to the shadow sector (the conditions under which this occurs are derived in Section II). In this case each group demands transfers up to the point where the other group is indifferent between investing in the two sectors. That is, each group sets \( x_i^* \) so that

\[
\alpha - x_i^* = \beta, \quad i = 1, 2.
\]
To illustrate the voracity effect consider an increase in the rate of return in the formal sector equal to $\Delta \alpha$. In the interior equilibrium each group increases the transfer it demands up to the point where the net rate of return available to the other group is equal to $\beta$. That is, $\Delta x_i = \Delta \alpha$ [by equation (2)]. Since both groups behave the same way, the increased redistribution induced by the increase in the raw rate of return is $2\Delta \alpha$. This is the voracity effect. Since the voracity effect dominates the direct effect of the windfall, the rate of accumulation in the formal sector falls. From (1) we can see that $\Delta K/K = \Delta \alpha - 2\Delta \alpha < 0$. The counterpart of higher voracity is a shift of capital to the inefficient shadow economy.

The argument we have made is loose. First, we did not prove that agents will choose linear transfer policies as assumed in equation (1). Second, we did not show that the voracity effect is present when agents anticipate shocks. Also, we should note that equation (2) is valid only when capital flows from the formal to the informal sector. To determine when this is the case we need to solve consumption-savings problems of the $n$ groups and fully characterize the interior equilibrium. We do this in Section II. In that section we embed the argument just made in a two-sector growth model, and let the number of groups be arbitrary. We compute the consumption policies and the accumulation paths for both types of capital. We prove the following results. First, if there initially exists multiple powerful groups, a reduction in power concentration (an increase in $n$) reduces discretionary redistribution and raises the average rate of return in the economy. As the number of powerful groups increases, there are two conflicting effects. On the one hand, there are more groups with the ability to extract subsidies. On the other hand, each group knows that it must ask for a smaller subsidy if the formal sector is to offer a satisfactory after-tax rate of return to the other groups. In equilibrium, the second effect dominates.

This result is analogous to the result in industrial organization that the Cournot equilibrium converges to the perfect competition outcome as the number of sellers increases. Second, when there are groups with the power to extract subsidies and there do not exist institutional barriers to discretionary redistribution, a positive shock to the terms of trade induces an increase in redistribution and a reduction in the growth rate. Moreover, we show that this anomalous response is also operational in the presence of anticipated future shocks.

II. The Model

We consider a two-sector growth model. There is an efficient formal sector, and an inefficient shadow sector. Resources in the formal sector are susceptible to taxation whereas, although productivity is lower, resources in the shadow economy are free from taxation. This is a caricature of what goes on in many economies. In the real world, both sectors may be subject to some form of taxation but the formal sector is subject to higher rates and is less able to evade taxation.

An important difference between our model and conventional growth models is that in our model the economy is populated by groups that have power to extract subsidies from the government rather than by atomistic agents that behave competitively. This captures the fact that fiscal policies in many countries are determined by powerful interest groups.

Since we want to analyze the effects of shocks that change the productivity of the formal sector relative to the informal sector, we shall consider two goods: an exportable and an importable. The exportable is produced in the efficient formal sector and the importable in the inefficient shadow economy. The importable will be the numeraire. In this section we will solve the model for a given price of the exportable. In Section III, subsection D, we will consider anticipated future shocks.

The objective function of each group is the present value of utility derived from consumption of the importable good:

$$\int_t^\infty \frac{\sigma}{\sigma - 1} c_i(s)^{\sigma - 1/\sigma} \times e^{-\delta(s-t)} \, ds = \delta > 0.$$
Within each instant \( t \) the timing is as follows. Each group \( i \) enters period \( t \) with a stock of capital in the formal sector \( k_i(t) \) and a stock of capital in the shadow economy \( b_i(t) \). The formal sector capital stock \( k_i(t) \) is used to produce the exportable good with a constant returns technology that is sold at a price \( p \) in terms of the importable good. The shadow economy capital stock \( b_i(t) \) is used to produce the importable good in the shadow economy, again using a linear technology. Next, group \( i \) requests a fiscal transfer \( r_i(t) \). Lastly, group \( i \) pays a tax \( T_i(t) \) from its income in the efficient sector, and consumes \( c_i(t) \). It follows that the accumulation equations for efficient-taxable capital and for inefficient-nontaxable capital are given by

\[
\begin{align*}
\dot{k}_i(t) &= p\alpha k_i(t) - T_i(t) \\
\dot{b}_i(t) &= \beta b_i(t) + r_i(t) - c_i(t).
\end{align*}
\]

Next, we describe the fiscal-redistribution process that links the \( r_i \)’s and the \( T_i \)’s. We assume that the fiscal authority has no objective of its own. It acts solely as the agent of powerful groups. Using the terminology in Per Krusell et al. (1997), the “fiscal constitution” of this economy is the following:

**Fiscal Constitution.**—Transfers and taxes must satisfy the following conditions.

- The fiscal budget has to be balanced during each instant: \( \sum_{j=1}^{n} r_j(t) = \sum_{j=1}^{n} T_j(t) \).
- Only proportional taxes on income in the formal sector can be levied, and the tax rate must be equal across groups.
- The fiscal transfer that a group can obtain is bounded by

\[
(6) \quad r_i(t) \leq \bar{x} \sum_{j=1}^{n} k_j(t), \quad \frac{\alpha p - \beta}{n - 1} < \bar{x} < \infty.
\]

The last restriction precludes each group from appropriating the aggregate capital stock at once. The lower bound on \( \bar{x} \) is equal to the appropriation rate in the interior equilibrium [see (18)]. The fiscal constitution implies that the tax rate \( \tau(t) \) must be adjusted continuously to ensure a balanced budget

\[
(7) \quad T_i(t) = \tau(t)\alpha p k_i(t), \quad \tau(t) = \frac{\sum_{j=1}^{n} r_j(t)}{\alpha p K(t)},
\]

\[
K = \sum_{j=1}^{n} k_j.
\]

This tax rule implies that if group \( i \) Increases its subsidy by an amount \( \Delta r_i \), its tax burden increases by only \( \Delta r_i [k_i(t)/K(t)] \). In effect, the subsidy is financed largely by other groups in the economy. In this way, each group’s ability to extract subsidies grants it “open access” to the other groups’ capital stocks in the formal sector. This implies that the capital held in the formal sector is not truly private. Only capital in the informal sector is truly private in that it enjoys “closed access.” Using the terminology of the introduction, we may say that there is no possibility of discretionary fiscal redistribution if there is no open access to the capital stocks in the formal sector. In contrast, if there is open access, fiscal redistribution can occur. To finalize the description of the economy, we list the initial conditions and the restrictions we impose. First, initial conditions are \( b_i(0) = 0 \) and \( k_i(0) = k_{i0} > 0 \) for all \( i \).

Second, we restrict all capital stocks to be nonnegative

\[
(8) \quad k_i(t) \geq 0, \quad b_i(t) \geq 0, \quad i = 1, \ldots, n; \quad t \geq 0.
\]

Third, the rate of return in the inefficient sector is lower than in the efficient sector

\[
(9) \quad 0 < \beta < \alpha p.
\]

We would like to point out that (3)–(9) is in fact a minimal model to analyze the problem at hand. First, multiple agents that interact strategically are necessary to analyze redistribution. Second, a dynamic setup is needed in order to analyze investment and saving decisions. Third, we assume that agents can costlessly move resources between sectors. Including appropriation or adjustment costs would add nothing to the insights provided by the model.
A. The First Best

In this case the setup we described reverts to the standard one-sector representative agent growth model. The solution to this case will be a useful benchmark. The first best allocation obtains in the following cases: (i) if powerful groups can coordinate and act cooperatively; (ii) if there is just one group; or (iii) if there are several groups, but institutional barriers do not permit them to extract any fiscal transfers. The first two cases cover what Mancur Olson (1982) labels encompassing groups.

The allocation in case (i) is the solution to the problem in which a central planner maximizes (3) subject to the accumulation equations in (4), the fiscal constitution (6)–(7), and nonnegativity constraint (8). Since fiscal transfers do not generate any externality, net transfers to each group should be zero. Moreover, since the rate of return in the formal sector is higher than in the shadow economy, the central planner would allocate all resources in the formal sector. In terms of our setup this entails setting the consumption of each group \( c_i(t) \) equal to the transfer \( r_i(t) \) it receives, and making \( r_i(t) \) equal to the tax paid by each group \( T_i(t) \). This implies that accumulation equations (4) and (12) can be rewritten as

\[
\dot{k}_i(t) = \alpha p k_i(t) - c_i(t),
\]

\[
\dot{b}_i(t) = 0.
\]

It follows that the optimization problem solved by the central planner is the standard consumption-savings Ramsey problem (see Robert J. Barro and Xavier Sala-i-Martin, 1995). The solution is

\[
(10) \quad \dot{k}_i(t) = \alpha p k_i(t) - c_i(t),
\]

\[
\dot{b}_i(t) = 0.
\]

The consumptions of each group are given by

\[
(11) \quad r_i^{fb}(t) = c_i^{fb}(t)
\]

\[
= [\alpha p (1 - \sigma) + \delta \sigma] k_i(t)
\]

\[
= z(\alpha p) k_i(t)
\]

\[
k_i^{fb}(t) = k_i(s) e^{\omega(t - s)}, \quad b_i^{fb}(t) = 0,
\]

where the superscript \( fb \) stands for first best. In this case the transversality condition is satisfied if and only if \( z(\alpha p) > 0 \). The consumption of each group is proportional to its own capital, and in the case of logarithmic utility consumption is equal to the familiar \( \delta k_i(t) \).

In the second case, in which there is only one group it is straightforward to see that the optimal allocation is given by (11), replacing \( k_i(t) \) by aggregate capital. Lastly, in case (iii) when groups cannot extract transfers, the individual capital that each group owns in the formal sector is truly private. Thus, we may replace the fiscal constitution by the condition \( r_i(t) = T_i(t) \). Therefore, in this case we may reinterpret \( r_i(t) \) as the amount that group \( i \) takes out from the formal sector to either consume or invest in the informal sector. Of course, as in the central planner’s case since \( \beta < \alpha p \) each group will set \( b_i(t) = 0 \) and \( c_i(t) = r_i(t) \).

Hence, group \( i \)’s accumulation equations will be given by (10) and the equilibrium allocation will be given by (11).

B. Solution Concept

In the case where powerful groups do not coordinate we will use Markov perfect equilibrium (MPE) as the solution concept. This is the natural extension of the first-best solution, which is standard in representative-agent macroeconomic models. In these models control variables such as consumption and investment are typically functions of the state variables: the capital stock, wealth, etc. Similarly, in a MPE strategies are just functions of payoff-relevant state variables, not of history (see Eric Maskin and Jean Tirole, 1994; Tamer Basar and Jan Olsder, 1995). This restriction captures the notion that bygones are bygones. In particular, MPE rules out history-dependent strategies, such as trigger strategies. We consider that MPE is a more appropriate concept than trigger strategies to study the problem at hand for the following reason. Countries with procyclical government spending or voracity effects are not countries with well-established institutional arrangements, such as the congressional committee system in the United States, that allow powerful agents to coordinate on specific agreements and to design the threats that support these agreements. Another reason why we consider MPE more appropriate is that it reduces considerably the
multiplicity of equilibria in dynamic games. As is well known, if the discount rate is sufficiently low, trigger strategies can support virtually any outcome as an equilibrium.

In this model the payoff-relevant variables for group $i$ are the aggregate capital stock in the formal sector $K(t)$ and group $i$’s closed-access capital stock $b_i$. To see why $K(t)$, and not $k_i(t)$, is payoff-relevant for group $i$, note that although the efficient capital stocks of the other $n-1$ groups are nominally private, group $i$ has open access to them via the fiscal process. Since the transfer appropriated by group $i$, $r_i$, is financed by taxing income in the formal sector of all groups (not only that of $i$), it follows that by demanding $r_i$, group $i$ appropriates $r_i(1-k_i/K)$ from the formal capital stocks of the other groups. This is because the fiscal constitution implies that the tax paid by $i$ only has to finance a proportion $k_i/K$ of the transfer it receives. Hence, the consumption possibilities of group $i$ (and its payoff) depend on $K(t)$ and not on $k_i(t)$. To obtain the accumulation equation for aggregate capital we substitute (7) in (4).

\begin{equation}
\dot{K}(t) = pαK(t) - \sum_{j=1}^{n} r_j(t).
\end{equation}

To see why any $b_j(t) j \neq i$ is not payoff-relevant for group $i$, note that since none of the capital stocks groups hold in the shadow economy are subject to taxation, they are truly private. Therefore group $i$ does not have open access to them.

In the game we are considering a Markov strategy of group $i$ is composed of a consumption policy and a transfer policy that are only functions of the payoff-relevant state variables $K(t)$ and $b_i(t)$: $\{ \phi_u(K(t), b_i(t)) \}_{i=s}^{\infty}$ and $\{ c_u(K(t), b_i(t)) \}_{i=s}^{\infty}, r_u(K(t), b_i(t)) \}_{i=s}^{\infty}$. An $n$-tuple of Markov strategies $\{ \phi^*_i(K(t), b_i(t)) \}_{i=s}^{\infty}$ is payoff-relevant for group $i$ if it is a subgame-perfect equilibrium for every realization of the state $(K(t), b_1(t), ..., b_n(t))$. That is, if for all $i, s$

\begin{equation}
J(\{ \phi^*_u(K(t), b_i(t)) \}_{i=s}^{\infty}) = J(\{ \phi_u(K(t), b_i(t)) \}_{i=s}^{\infty})
\end{equation}

where $*$ denotes equilibrium value, $\Phi^*_{-i}(K(t), b_{-i}(t)) \}_{i=s}^{\infty}$, and $J(\cdot, \cdot)$ is the value taken by payoff function (3). In order to be able to apply the techniques used to solve differential games we will allow groups to choose transfer policies from the class of continuously differentiable functions of the payoff-relevant state variables. That is, we assume that

\begin{equation}
r_u = r_u(K(t), b_i(t))
\end{equation}

\begin{equation}
c_u = c_u(K(t), b_i(t))
\end{equation}

We will derive the equilibria of this game in two steps. First, we find a set of $n$ transfer policies that are best responses to each other. Second, using the equilibrium transfer policies we derive the equilibrium paths of the capital stocks in the formal and informal sectors and the consumption policies. During each instant $s$, group $i$ solves the following problem.

**PROBLEM** $(P_i(s))$: Choose a consumption policy $\{ c_u(K(t), b_i(t)) \}_{i=s}^{\infty}$ and a transfer policy $\{ r_u(K(t), b_i(t)) \}_{i=s}^{\infty}$ in order to maximize payoff function (3) subject to accumulation equations (4) and (12), restrictions (6) and (8), and the transfer policies of the other groups (14).

The present-value Hamiltonian associated with $i$’s problem is

\begin{equation}
H_i = ν_i U(c_i)
\end{equation}

\begin{equation}
+ λ_i [ pαK - r_i - \sum_{j \neq i} r_j(K, b_j) ]
\end{equation}

\begin{equation}
+ ζ_i [ βb_i + r_i - c_i ]
\end{equation}

\begin{equation}
+ ξ_i [ k_iK - r_i ] + \mu_i b_i,
\end{equation}

where $U(c_i) = ((σ - 1)/σ)c_i^{(σ-1)/σ}$. The second and third terms correspond to accumulation equations (12) and (5). The fourth and fifth terms to restriction (6), and the last
term corresponds to the second constraint in (8). We have disregarded the first constraint in (8). It turns that it is not binding in equilibrium. Notice that in deriving the first-order conditions for group \( i \), \( r_i \), and \( c_i \), are treated as control variables, while the other \( n - 1 \) transfers \( r_j^*(K, b_j) \) are treated as functions of the state. In fact, these functions are the equilibrium policies derived from analogous control problems. To find an MPE, it is necessary to find \( n \) transfer policies \( r_j^*(K, b_j), j = 1, \ldots, n \) that simultaneously solve \( n \) Hamiltonian problems like (15). There are two types of Markov perfect equilibria in the game we are considering: interior and extreme. In an interior equilibrium, the transfers demanded by all groups are within the bounds defined in (6) at all times. This is not true in an extreme equilibrium.

C. Extreme Equilibrium

In the extreme equilibrium all groups set their appropriations equal to the upper bound \( \bar{\lambda}K(t) \) defined in (6). It is straightforward to check that if \( n - 1 \) groups set \( r_j(t) = \bar{\lambda}K(t) \), the best response of group \( i \) is to set \( r_i(t) = \bar{\lambda}K(t) \). To do this we will define the post-redistribution rate of return perceived by \( i \) in the efficient sector as the raw rate of return minus the transfer rates of the other groups. If this rate is lower than the private rate of return that \( i \) gets in the informal sector (\( \beta \)), then \( i \)'s best response is to transfer as much as possible from the formal to the informal sector. Since \( r_j(t) = \bar{\lambda}K(t) \) for \( j \neq i \), group \( i \)'s post-appropriation rate of return in the formal sector is \( \alpha p - (n - 1)\bar{\lambda} \). Furthermore, since \( \bar{\lambda} > \alpha p - \beta/n - 1 \) by definition [see (6)], \( \alpha p - (n - 1)\bar{\lambda} < \beta \). Therefore, \( i \)'s best response is to shift as many resources as possible away from the efficient formal sector.

In an extreme equilibrium any productivity shock does not have any effect on the transfer policies because groups extract the highest possible transfer \( \bar{\lambda}K \) regardless of the returns in the formal and informal sectors. Therefore, in order to make interesting the analysis of extreme equilibria one would require a theory that explains the level of the appropriation bound (\( \bar{\lambda} \)). In this paper, instead, we will focus on the interior equilibrium.

D. Interior Equilibrium

In this subsection we will characterize the interior equilibrium and show that it is stable against unilateral deviations for a wide range of parameter values. There are two cases to consider depending on the number of powerful groups \( n \). Define \( \bar{n} \) as

\[
(16) \quad \bar{n} = 1 + \frac{\alpha p - \beta}{z(\beta)},
\]

where \( z(\beta) \) is defined in (17) below. If \( 1 < n < \bar{n} \), capital is continuously transferred from the efficient to the inefficient sector and the stock of inefficient capital \( b_i(t) \) is increasing. Meanwhile, if \( n > \bar{n} \), all the capital stock is allocated to the efficient sector and \( b_i(t) \) is always zero. The following proposition characterizes the strategies that support the interior equilibrium for the case \( n < \bar{n} \). Proposition 2 covers the case \( n > \bar{n} \).

PROPOSITION 1: In the case \( 1 < n < \bar{n} \) there exists an interior MPE if and only if

\[
(17) \quad z(\beta) = \beta(1 - \sigma) + \delta \sigma > 0.
\]

This equilibrium is unique within the class of differentiable strategies defined in (14). The transfer and consumption policies are, respectively,

\[
(18) \quad r_i^*(K, b_i; n < \bar{n}) = \frac{\alpha p - \beta}{n - 1} K,
\]

\[
(19) \quad c_i^*(K, b_i; n < \bar{n}) = z(\beta)[K + b_i].
\]

The proof is in the Appendix. Here we present a heuristic derivation and provide some

\[\text{footnote} \]

These results are not specific to the continuous-time specification of the model. Tornell (1999) considers the discrete-time version of the model where the accumulation equations are: \( K(t + 1) = [1 + \alpha p]K(t) - \Sigma_{i=1}^n r_i(t) \) and
intuition. Condition (17) is necessary for the transversality condition (A6) to be satisfied. It also insures that in each case consumption is positive and the value function converges. Note that in the logarithmic utility case (\(\sigma = 1\)) the condition \(z(\beta) > 0\) is equivalent to the familiar condition that the discount rate \(\delta\) be positive. Note also that if \(z(\beta) < 0\), \(\tilde{n} < 1\). In that case, the interior equilibrium would be characterized by Proposition 2 below.

The intuition behind (18) is as follows. During each instant, group \(i\) must decide how to allocate its capital between the efficient and inefficient sectors \(K(t)\) and \(b_i(t)\). Given the tax rule (7), it can do this by choosing its desired transfer \(r_i(t)\). To illustrate suppose that the transfer to each of the other groups is \(r_j(t) = x_j(t)K(t)\), with \(x_j(t)\) being an undetermined coefficient. It follows that \(i\)'s post-appropriation rate of return in the formal sector is \(\alpha p - \sum_j z_i x_j(t)\). For \(i\) to find it optimal to set \(r_i(t)\) within the admissible bounds given by (6), it is necessary that the rate of return on \(i\)'s closed-access capital \(\beta\) be equal to its rate of return on the open-access capital after redistribution to other groups has taken place. This implies that the following condition must hold for any group \(i\) in an interior equilibrium: \(\beta = \alpha p - \sum_j z_i x_j(t)\). The unique solution of this system of \(n\) simultaneous linear equations is that all \(x_j(t)\)’s be equal to \(\alpha p - \beta / n - 1\) as shown in (18).

Next, note that the consumption policy in (18) has the same form as in the standard representative agent models. That is, at all times \(i\)'s consumption is a fixed proportion of \(i\)'s wealth, which in this case consists of \(i\)'s private capital in the informal sector \(b_i\) plus aggregate capital in the formal sector \(K\), not only \(k_i(t)\). Note also that the consumption policy can be rewritten as \(c_i^*(t) = z(\beta)[K(s) + b_i(s)]e^{(\beta - \delta)(t - s)}\). This implies that regardless of the value of \(\alpha p\), consumption grows at the constant rate \(\sigma[\beta - \delta]\) as in the standard representative agent model with constant relative risk aversion (CARRA) utility, elasticity of intertemporal substitution \(\sigma\), discount rate \(\delta\), and rate of return \(\beta\) (see Barro and Sala-i-Martin, 1995). The reason for this is that regardless of how each group distributes its resources between the efficient and the inefficient sectors, it faces a rate of return \(\beta\) in equilibrium. Substituting (18) in accumulation equations (5) and (12) it follows that (see the Appendix)

\[
K^*(t; n = \tilde{n}) = K(s)\exp\left(\frac{n\beta - \alpha p}{n - 1} (t - s)\right)
\]

\[
b_i^*(t; n = \tilde{n}) = [K(s) + b_i(s)]e^{(\beta - \delta)(t - s)} - K^*(t).
\]

A comparison with the first-best allocation reveals the inefficiencies introduced by having groups with power to extract transfers from the rest of society. First, when groups are powerful and do not coordinate, they consume “too much” in the sense that consumption is a function of aggregate capital in the formal sector, not only of individual capital as in the first best. That is \(i\)'s consumption is proportional to \(K(t) + b_i(t)\), not just to the capital nominally owned by group \(i\): \(k_i(t) + b_i(t)\). Second, with the existence of powerful groups, capital may be invested in a socially inefficient way. That is, a low-return technology in the shadow economy may be used. As a result of these inefficiencies, the growth rate of efficient capital is lower than under the first best.

Next, we consider the case \(n > \tilde{n}\). In this case all capital is allocated to the efficient sector and the stock of inefficient capital \(b_i(t)\) is always zero. The following proposition characterizes the interior equilibrium in this case.

**PROPOSITION 2:** In the case \(n > \tilde{n}\) there exists an interior MPE if and only if

\[
\sigma > \frac{n}{n - 1}
\]

and

\[
z(\alpha p) = \alpha p(1 - \sigma) + \delta\sigma < 0.
\]
The transfer and consumption policies coincide. They are given by

\[ r_i^*(K, b_i; n > \bar{n}) = c_i^*(K, b_i; n > \bar{n}) = \frac{z(\alpha p)}{n - \sigma(n - 1)} K. \]

The proof is in the Appendix. Here we just present some intuition. First, note that in equilibrium the post-redistribution rate of return in the formal sector is greater than that in the inefficient sector \( \beta \). Thus, groups do not allocate capital to the inefficient sector. This implies that each group sets the transfer it demands equal to its desired consumption as shown in (22). To show that the post-redistribution rate of return obtained by group \( i \) in the formal sector \( \alpha p - \sum_j r_j^*(K, b_j)/K \) is greater than \( \beta \), we replace \( r_j^*(K, b_j) \) by (22)\(^6\)

\[ D(n) = \alpha p - (n - 1) \frac{z(\alpha p)}{n - \sigma(n - 1)} - \beta \]

\[ = \frac{(n - 1)z(\beta) + \beta - \alpha p}{n - \sigma(n - 1)} > 0 \]

for all \( n > \bar{n} \).

We derive the intuition for (22) in three steps. First, since at this equilibrium \( b_i(t) = 0 \), it is natural to conjecture that \( i \)'s consumption is proportional to aggregate capital in the efficient sector: \( c_i(K(t), b_i(t)) = x_i K(t) \), where \( x_i \) is an undetermined coefficient. This implies that \( c_i/c_j = x_i / x_j + K/K \). Second, as in any standard savings problem the Euler condition for group \( i \) is \( \dot{c}_i/c_i = \sigma[R_i - \delta] \), where \( R_i \) is the post-redistribution rate of return obtained by group \( i \) in the formal sector, i.e., \( R_i = \alpha p - \sum_j x_j \). This Euler equation is simply saying that optimal consumption must grow (fall) when the rate of return is greater (smaller) than the subjective discount rate.

Third, combining these two differential equations we obtain \( n \) linear simultaneous equations in the \( x_i \)'s. The solution is given by (22).

Lastly, substituting \( c_i^*(K(t), b_i(t)) = r_i^*(K(t), b_i(t)) \) in accumulation equations (5) and (12), it follows that the capital stocks are given by

\[ K^*(t; n > \bar{n}) = K(s)\exp\left(\frac{\sigma[\alpha p - n\delta]}{n - \sigma(n - 1)} [t - s]\right), \]

\[ b_i^*(t; n > \bar{n}) = 0. \]

Summing up, when there are multiple powerful groups equilibrium allocation depends on the number of groups. If \( n \) is no greater than a certain threshold \( \bar{n} \), capital is allocated to the efficient and inefficient sectors. In contrast, if \( n > \bar{n} \), all capital is allocated to the efficient sector. We will show in Section III that despite these differences in equilibrium allocations, in both cases the voracity effect is present and power concentration is negatively related to the growth rate of the efficient sector.

E. Stability of the Interior Equilibrium

Here we show that the interior equilibrium is stable in the sense that if one group deviates by setting its appropriation policy different from \( r^*(K, b_i) \) in (18) the other \( n - 1 \) groups will not respond by changing their appropriation rates in the same direction as the deviant. Thus, a deviation by one group will not induce convergence to an extreme equilibrium. The results of this subsection are summarized in the following proposition.

**PROPOSITION 3:** The interior MPE is stable against unilateral deviations if and only if

- \( n > 2 \) when \( n \leq \bar{n} \).
- \( n \geq 2 \) and \( \sigma > (n - 1/n - 2) \) when \( n > \bar{n} \).

The threshold \( \bar{n} \) is defined by (16).

To prove this proposition consider first the case \( 1 < n < \bar{n} \). Let \( r_j(K, b_j) = x_j K \) and suppose that the \( n^{th} \) group deviates by setting its
appropriation rate equal to \( x^d \neq x^* = \alpha p - \beta / n - 1 \), and that the other \( n - 1 \) groups play the game defined by (3)–(12) taking as given that \( x^d \neq x^* \). Following the same steps as before we have that the best responses to this deviation \( \hat{\xi}(x^d) \) must satisfy the following conditions: \( \beta = \alpha p(t) - x^d - \sum_{i = 1}^n \hat{\xi}(x^d) \), for \( i = 1, \ldots, n - 1 \). The unique solution to this system of linear simultaneous equations is

\[
\hat{x}(x^d) = \frac{\alpha p - x^d - \beta}{n - 2} \quad \text{if } 1 < n \leq \bar{n}.
\]

This expression is analogous to (18). To determine under which circumstances the interior equilibrium is stable note that \( \hat{x}(x^d) = x^* \) (where \( x^* \) is the interior equilibrium appropriation rate) and \( \partial \hat{x}(x^d)/\partial x^d = -1/n - 2 \). Therefore, the interior equilibrium is unstable if \( n \in (1, \min(2, \bar{n})] \), while it is stable if \( \bar{n} > 2 \) and \( n \in (2, \bar{n}] \).

Now we consider the case \( n > \bar{n} \). Following the same steps as before we have that the best response of each of the \( n - 1 \) groups that did not deviate originally is given by (22) replacing the rate of return \( \alpha p \) by \( \alpha p - x^d \) and the number of groups \( n \) by \( n - 1 \):

\[
\hat{x}(x^d) = \frac{[\alpha p - x^d][1 - \sigma] + \delta \sigma}{n - 1 - \sigma[n - 2]}
\]

if \( n > \bar{n} \).

It follows that \( \hat{x}(x^d = x^*) = x^* \) and \( \partial \hat{x}(x^d)/\partial x^d = \sigma - 1/n - 1 - \sigma[n - 2] \). Since in the case \( n > \bar{n} \) an interior equilibrium exists only if \( \sigma > n/n - 1 \) [see (21)], the numerator of the derivative is positive. Thus, the interior equilibrium is stable if and only if the denominator is negative. That is, when \( n > 2 \) and \( \sigma > n - 1/n - 2 \). This completes the proof of Proposition 3.

Note that since when the interior equilibrium exists \( \sigma \) must be greater than \( n/n - 1 \), the region of instability is \( \sigma \in (n/n - 1, n - 1/n - 2] \). This instability interval is quite small and it shrinks very fast as \( n \) grows. For \( n = 3 \) it is \( \sigma \in (1.5, 2) \) and for \( n = 4 \) it is \( \sigma \in (1.3, 1.5) \).

Lastly, we note that the deviant does not gain by deviating even if it can appropriate the entire aggregate capital stock. To see this, let us make the extreme assumption that the upper bound on the appropriation rate \( x^d \) is infinity, so that the deviant can appropriate the entire aggregate capital stock in the efficient sector and invest it in the inefficient sector (\( b_d(0) = K(0) \)). The deviant would then maximize (3) subject to accumulation equation (5). As in any standard representative agent model its consumption would be \( c_d(t) = z(\beta) b_d(t) \), and \( b_d(t) = K(0) e^{\sigma(\beta - \delta)t} \). Therefore, its payoff would be \( U_d = (\sigma/\sigma - 1) K(0) \sigma - 1/\sigma z(\beta)^{-1/\sigma} \), which is the same as the payoff it gets in the interior equilibrium if \( n \leq \bar{n} \), while it is lower if \( n > \bar{n} \) [see (30)].

### III. Power Concentration and Voracity

In this section we use the model of Section II to analyze both the relationship between power concentration and growth and the voracity effect in response to productivity or terms of trade shocks.

#### A. Power Concentration and Growth

When there exist powerful groups with the capacity to extract fiscal subsidies, we can view an increase in the number of powerful groups \( n \) as a dilution of power concentration. In our model there is a nonmonotonic relationship between power concentration and economic performance. Given that groups do not cooperate, a shift away from the case of a single group \( (n = 1) \) leads to a deterioration in economic performance. However, if there exist multiple groups \( (n \geq 2) \), an increase in the number of powerful groups leads to better economic performance. This negative relationship between power concentration and economic performance shows up in two ways in the model: as a smaller growth rate of the formal sector, and as a more negative response of the growth rate to a terms of trade improvement.

In this subsection, we consider the former effect (in the next subsection we consider the latter effect). To check that the growth rate in the formal sector falls as an economy moves away from perfect power concentration, we compare the path of efficient capital in the first best (11) with the path along the
interior equilibrium (19). It is easy to show that
\[ g_k^f - g_k^*(\bar{n} \leq n \geq 2) = \sigma(\alpha p - \delta) - \frac{n\beta - \alpha p}{n - 1} > 0. \]

Next, we show that starting with less-than-perfect power concentration \((n \geq 2)\), the growth rate increases as power becomes less concentrated. First, note that within each of the regions \(n \leq \bar{n}\) and \(n > \bar{n}\) the growth rate is increasing in \(n\). Treating \(n\) as a continuous variable, we have from (19) and (23) that
\[ \frac{\partial g_k^*(n; n \leq \bar{n})}{\partial n} = \frac{\alpha p - \beta}{(n - 1)^2} > 0, \]
\[ \frac{\partial g_k^*(n; n > \bar{n})}{\partial n} = -\frac{\sigma z(\alpha p)}{(n - 1)^2} > 0. \]

The signs follow from (9) and (21). Second, note that the growth rate is higher for any \(n \geq \bar{n}\) than for any \(1 < n < \bar{n}\). This is because: (i) at \(n = \bar{n}\) the growth rates in both regions coincide,\(^8\) and (ii) within each region \(n \leq \bar{n}\) and \(n > \bar{n}\) the growth rate is increasing in \(n\).

As \(n\) increases, each group, in the interior equilibrium, has to reduce the subsidy it demands. On the other hand, there are more groups whose demands for subsidies must be satisfied. In equilibrium, the subsidy each group demands falls at a faster rate than \(n\) increases for \(n \geq 2\). As a result, the growth rate of the formal sector increases.

**PROPOSITION 4**: Consider an economy in which groups do not act in a coordinated manner and institutional barriers to discretionary redistribution are absent, then there is a non-monotonic relationship between power concentration and the growth rate of the efficient sector:

(i) A shift away from the \(n = 1\) case to \(n \geq 2\) reduces the growth rate. (ii) Starting at \(n \geq 2\), a further reduction in power concentration increases the growth rate.

We can draw an analogy between result (ii) and the Industrial Organization literature. Consider an oligopolistic market where firms engage in Cournot competition. As the number of firms grows, the quantity supplied by the industry and the price converge to those that would prevail under perfect competition. In our case, lower power concentration induces groups to appropriate less from the others’ profits. This leads to a socially more efficient allocation. Result (ii) stands in contrast to the standard argument that with a common pool or in the presence of externalities, the free-rider problem is exacerbated by an increase in the number of groups \(n\), leading to worse outcomes. In a static setup the argument commonly made is that the greater the number of groups, the smaller is the share of the costs of a “bad action” that are imposed on any individual group, and thus the more of the bad action any group undertakes. Conversely, in the small-\(n\) case, each group would internalize

---

\(^7\) Since \(n\beta - \alpha p/n - 1\) attains its maximum at \(n = 1 + \alpha p - \beta/\sigma(\beta)\), it follows that \(\sigma(\alpha p - \delta) - n\beta - \alpha p/n - 1 > 0\) if and only if \(\alpha p > \beta\), which is condition (9).

\(^8\) That is \(\lim_{\gamma \to \bar{n}} g_k^*(n; n > \bar{n}) = g_k^*(\bar{n}; n \leq \bar{n})\). To see this note that

\[ \lim_{\gamma \to \bar{n}} g_k^*(n; n > \bar{n}) - g_k^*(\bar{n}; n > \bar{n}) \]

\[ \frac{\sigma(n\beta - \alpha p)}{\bar{n}(\sigma - 1) - \bar{n}} \frac{\bar{n} - \alpha p}{\bar{n} - 1} \]

\[ = \frac{\sigma(n\beta - \alpha p)(n - 1) - (n\beta - \alpha p)(\bar{n}(\sigma - 1) - \bar{n})}{\bar{n}(\sigma - 1) - \bar{n}} \frac{\bar{n} - \alpha p}{\bar{n} - 1} \]

\[ = \frac{\bar{n}(\beta + \alpha p - \beta - \alpha p)}{\bar{n}(\sigma - 1) - \bar{n}} \frac{\bar{n} - \alpha p}{\bar{n} - 1} \]

\[ = \frac{\beta + \alpha p - \beta - \alpha p}{\bar{n}(\sigma - 1) - \bar{n}} \frac{\bar{n} - \alpha p}{\bar{n} - 1} = 0. \]
more of the costs of its bad action and hence better outcomes would be generated.9

In a dynamic setup the argument that higher \( n \) leads to bad outcomes is based on the idea that the smaller \( n \), the easier is for groups to cooperate and implement a low-appropriation high-growth equilibrium (Olson, 1982, 1993). Jakob Svensson (1996) considers a similar setup to ours, but analyzes trigger strategy equilibria. He finds that higher \( n \) is likely to reduce economic performance. He considers equilibria where groups agree to have low transfers. These equilibria are supported by the threat of a reversion to high transfers in case someone deviates. As \( n \) grows it becomes more difficult to support low-appropriation equilibria because the temptation to deviate increases faster than the punishment. Therefore, the greater \( n \), the more rent seeking and the lower growth. This holds true in our model when we go from \( n = 1 \) to \( n = 2 \), but is not true starting at starting any \( n \geq 2 \).10

Why this difference in predictions? The literature addresses the issue of when is it more likely that cooperation will emerge. We address a different issue: given that groups do not cooperate, what happens when \( n \) goes up? Since in our model each group has an outside option, in the interior equilibrium every group must receive a rate of return which is no lower than that of the outside option. As with Cournot competition, when \( n \) grows each group must reduce its appropriation rate to make sure the preceding condition is satisfied. As a result, the aggregate growth rate increases.

B. The Voracity Effect

In this subsection we rationalize the phenomenon that countries with powerful groups respond to a positive productivity or terms of trade shock by an increase in discretionary redistribution and slower growth. We will argue that this response is caused by the voracity effect, which we define next.

**Definition 1:** The “voracity effect” is a more-than-proportional increase in discretionary redistribution in response to an increase in the raw rate of return in the efficient sector.

To unveil the voracity effect we will compute the effect of a change in the terms of trade on the equilibrium growth rate. We can interpret this exercise in two ways. As an unanticipated permanent shock, or as a comparison of economies with different productivities. In subsection D we will consider the case of a fully anticipated future shock, and show that the voracity effect is still operative.

First, we consider the case where power is concentrated (\( 1 < n \leq \bar{n} \)). Defining the growth rate of the efficient sector as \( g_K = \bar{K}/K \) and using (19) we have that

\[
\frac{\partial g_k^*(\bar{n} \geq n > 1)}{\partial p} = -\frac{\alpha}{n - 1} < 0.
\]

This surprising result is caused by the *voracity effect*, which counteracts the standard effect that an increase in the raw rate of return increases the return on investment and the growth rate. The intuition for (26) is the following. The higher \( p \) leads to an increase in the pre-tax rate of return in the formal sector. Recall that, along the interior equilibrium, each group must perceive a post-redistribution rate of return on capital in the formal sector which is not lower than \( \beta \), the rate of return in the nontaxable informal sector. Thus, with higher \( p \), each group can afford to demand a higher transfer. How much higher? To answer this, note that a particular group (call it \( i \)) will still be willing to participate in the interior equilibrium, if the other \( n - 1 \) groups, as a whole, increase their appropriation rate by the same amount as the increase in \( p \). Since, by an analogous argument, group \( i \) also increases its transfer rate, it must be true that the increase in the aggregate transfer rate of the \( n \) groups must
be greater than the increase in the raw rate of return along the interior MPE. Thus, \textit{ex post}, the higher terms of trade reduces the growth rate of the efficient sector.

The mechanism by which this perverse outcome occurs is as follows. Note that the raw [i.e., pre-redistribution] rate of return goes up by \( \alpha \Delta p \). This increase in the raw rate of return represents an opportunity for some group to increase redistribution to itself without reducing below \( \beta \) the post-redistribution rate of return perceived by other groups. In equilibrium, every group increases the redistribution rate to itself by an amount \( \frac{1}{n-1} \alpha \Delta p \) following this reasoning. This lack of coordination implies that the aggregate redistribution rate increases by \( \frac{n}{n-1} \alpha \Delta p \), which is greater than \( \alpha \Delta p \). As a result, the growth rate of the efficient sector falls:

\[
\frac{\Delta K}{K} = \alpha \Delta p - \frac{\alpha \Delta p}{n-1} .
\]

The counterpart of this is an increase in the rate of growth of the shadow economy. This reallocation of resources toward more inefficient activities is the cause of lower growth.

In the case in which power is diffused among a large number of groups \( n > \bar{n} \), the growth rate of the formal sector also responds negatively to a productivity shock. From (23) we have that

\[
\frac{\partial g^*_K(n > \bar{n})}{\partial p} = \frac{\alpha \sigma}{n - \sigma(n-1)} < 0.
\]

The sign comes from the fact that an interior equilibrium exists only if \( \sigma > \frac{n}{n-1} \). Finally, in the first-best case (i.e., when groups have no effective power to extract transfers or when their behavior is coordinated by a central planner) the voracity effect is not present and the standard result that growth responds positively to an improvement in the terms of trade is generated. From (11) it follows that

\[
\frac{\partial g^b_K}{\partial p} = \alpha \sigma > 0.
\]

This is because the growth in the terms of trade increases the rate of return to investment, as in standard models. For future reference, we state these results in the following proposition.

**PROPOSITION 5:** In the presence of multiple powerful groups, along the interior equilibrium a positive shock to the productivity of the efficient sector leads to:

- A more-than-proportional increase in the fiscal transfers demanded, a fall in the growth rate of the (taxable) efficient sector, and a reallocation of resources toward the (nontaxable) inefficient sector, if there are no institutional barriers to discretionary redistribution.
- An improvement in the growth rate of the efficient sector if there are barriers to discretionary redistribution, or groups act in a coordinated manner.

Note that the squeezing of the sector that experiences the terms of trade improvement is in fact opposite to the predictions of the Dutch disease literature and is explained by endogenously higher redistribution.

Lastly, we analyze the relation between the degree of power concentration and the strength of the voracity effect. Within the regions \( 1 < n \leq \bar{n} \) and \( n > \bar{n} \) the voracity effect is decreasing in \( n \). That is, higher \( n \) diminishes the negative effect on the growth rate of a positive shock to the terms of trade. Using (26) and (27) we have that [the sign of the second equation follows from (21)]

\[
\frac{\partial^2 g^*_K(n \geq \bar{n})}{\partial p \partial n} = \frac{\alpha}{(n-1)^2} > 0,
\]

\[
\frac{\partial^2 g^*_K(n \geq \bar{n})}{\partial p \partial n} = \frac{\alpha \sigma(n-1)}{[n - (n-1)\sigma]^2} > 0.
\]

**C. Welfare**

We again analyze the two cases of uncoordinated powerful groups and the first best. In the first case, the improvement in the terms of trade does not generate any welfare gains for the powerful groups. By substituting (18) and (22) in (3), we have that for any level of the terms of trade that satisfies (9), the payoff of group \( i \) along the interior equilibrium path is given by
These integrals are bounded only if conditions (17) and (21) are satisfied. The first equation in (30) implies that if power is concentrated, the welfare of each group is not affected by the path of the terms of trade. The second equation implies that if power is diffused among many powerful groups, the payoff of each group falls as the terms of trade go up.\footnote{To check this, recall that if \( n > \bar{n} \), necessary conditions for the existence of an interior equilibrium are \( \sigma > \frac{n}{n-1} > 1 \) and \( z(\alpha p) < 0 \) [see (21)]. The derivative of (30) with respect to \( p \) is}

\[
\frac{1}{\sigma - 1} \left( K(0) + b_i(0) \right)^{\sigma - 1/\alpha} z(\alpha p)^{-1/\alpha} \left( \frac{\alpha(1 - \sigma)}{n - \sigma(n - 1)} \right) < 0.
\]

The negative sign follows directly from the conditions listed above.

\[
J_i(K(0) + b_i(0)) = \begin{cases} \\
\frac{\sigma}{\sigma - 1} \left( K(0) + b_i(0) \right)^{\sigma - 1/\alpha} z(\beta)^{-1/\alpha} & \text{if } 1 < n \leq \bar{n} \\
\frac{\sigma}{\sigma - 1} \left( K(0) + b_i(0) \right)^{\sigma - 1/\alpha} \left[ \frac{z(\alpha p)}{n - \sigma(n - 1)} \right]^{-1/\alpha} & \text{if } n > \bar{n}.
\end{cases}
\]

result, there is a reduction of average productivity in the economy. Along the interior equilibrium both effects cancel out. To see this, add up (19) and (20) to get

\[
K_e(t) + b^*_e(t) = \left[ K(0) + b_i(0) \right] e^{\alpha(t-\delta)},
\]

and note that it is independent of \( p \).

In the case \( n > \bar{n} \) resources are not allocated to the informal sector in equilibrium. Thus, the more-than-proportional increase in fiscal redistribution induced by an increase in \( p \) is reflected one for one in higher consumption and a lower growth rate of capital in the formal sector. As a result, the growth rate of consumption falls as well as the welfare of each group.

In the case of no discretionary redistribution, the payoff of each group is obtained by substituting the first-best consumption policy (11) into (3):

\[
J_{fb}(k_i(0) + b_i(0)) = \frac{\sigma}{\sigma - 1} \left[ k_i(0) + b_i(0) \right]^{\sigma - 1/\alpha} z(\alpha p)^{-1/\alpha}.
\]

This expression is unambiguously increasing in \( p \). Thus, an improvement in the terms of trade is sure to raise welfare in this case. We summarize the results of this subsection in the following proposition.

**PROPOSITION 6:** A productivity improvement in the efficient sector fails to lead to an increase in welfare when there are powerful groups and no institutional barriers to discretionary redistribution. In contrast, when groups are powerless, act in a coordinated manner, or when there are barriers to redis-
distribution, a productivity improvement raises welfare.

D. Anticipated Shocks

To show that the voracity effect is operative when the presence of shocks is explicitly taken into account, we consider the case where at time 0 there is an announcement that the terms of trade will increase from $p_t = p$ on $[0, T)$ to $p_t = p + \epsilon$ on $[T, \infty)$.\(^\text{12}\) The optimality conditions are the ones we derived for the case of no shocks (A1)-(A7), replacing $p$ by $p_t$, plus the following transversality condition. At time $T$ the change in utility generated by a marginal change in consumption should equal the induced change in the value of the continuation game (30). Again, we will focus on the interior equilibrium. We will first consider the case $1 < n \leq \bar{n}$, and then the case $n > \bar{n}$. In the former case the transversality condition at $T$ is

\[
(32) \quad c_i(T^-)^{-1/\sigma} = z(\beta)^{-1/\sigma} [b_i(T) + K(T)]^{-1/\sigma}.
\]

This condition just determines consumption at $T$. As in the no-shock case, transfer policies are determined independently of consumption, and they must equalize, for each group, the post-redistribution rate of return in the formal sector to the one in the informal sector: $ap_i - \sum_{j \neq i} r_{ij} \beta = \beta$. The unique solution to this system of $n$ equations is

\[
(33) \quad r^*_u(K(t); n \leq \bar{n}) = \begin{cases} 
\alpha p_i - \beta / n - 1 \quad & \text{for all } t \geq 0.
\end{cases}
\]

\[
(35) \quad r^*_u(K(t); n > \bar{n}) = \begin{cases} 
\left[ \frac{e^{z(\alpha p)|t-T|}}{z(\alpha p) + \epsilon \alpha [1 - \sigma]} + \frac{1 - e^{z(\alpha p)|t-T|}}{z(\alpha p)} \right]^{-1} K(t) & \text{if } t < T \\
z(\alpha p + \epsilon) K(t) & \text{if } t \geq T.
\end{cases}
\]

Surprisingly, this transfer policy is the same as the one we derived in the previous case replacing $p$ by $p_t$. Replacing (33) in accumulation equation (12) we have that the growth rate of the formal sector is $g_i(n < \bar{n}) = n \beta - \alpha p_i / n - 1$. Therefore, the voracity effect is still operative: $\partial g_i / \partial p_t = -1/n - 1 < 0$. That is, a productivity improvement reduces the growth rate contemporaneously. Note that the fact that a shock is fully anticipated does not smooth the effects of the shock on the growth rate of the formal sector. We show in the Appendix that on $[0, \infty)$ the consumption policy is given by

\[
(34) \quad c^*_i(K(t), b_i(t); n \leq \bar{n}) = z(\beta)[K(t) + b_i(t)]
\]

for all $t \geq 0$.

A remarkable property of (34) is that consumption is not affected by an anticipated future shock to $p$. This can be seen more clearly by rewriting it as $c^*_i(t) = z(\beta)[K(0) + b_i(0)]e^{\alpha(\beta - \beta)\epsilon}$. Moreover, comparing (34) with (18) we can see that the consumption policy is identical to the one in the no-shock case if $1 < n \leq \bar{n}$.

Now we consider the case $n > \bar{n}$. In this case consumption and transfers are equal. Thus, we should expect that through the consumption-smoothing channel anticipated shocks will have an effect on the consumption path. As in Proposition 2, in this case the interior equilibrium exists if and only if $\sigma > n/[n - 1]$ and $0 > z(\alpha p + \epsilon) = z(\alpha p) + \alpha \epsilon [1 - \sigma] = \alpha [p + \epsilon [1 - \sigma] + \delta \sigma$. We show in the Appendix that the transfer policies are given by

\[
12 \text{ The solution method we use in this subsection is the same as the one in Tornell (1997).}
\]
Substituting the consumption policies in accumulation equation (12) we have that the growth rate is given by

\[
g(t, n > \bar{n})
\]

\[
= \begin{cases} 
\alpha p - n \left[ n - \sigma(n - 1) \right] \left[ \frac{e^{z(\alpha p)(t-T)}}{z(\alpha p) + \epsilon\alpha[1 - \sigma]} + \frac{1 - e^{z(\alpha p)(t-T)}}{z(\alpha p)} \right]^{-1} & \text{for } t < T \\
\alpha[p + \epsilon] - n \frac{z(\alpha p)(1 - \sigma)}{n - \sigma(n - 1)} & \text{for } t \geq T.
\end{cases}
\]

In contrast to the \( n \leq \bar{n} \) case, the voracity effect manifests itself immediately after the announcement of a future positive shock is made. At \( t = 0 \) the growth rate falls relative to a no-shock economy. Afterwards, it follows an increasing path and when the shock occurs, at time \( T \), it experiences an upward jump. This is because although transfer rates evolve smoothly, the raw return on the formal sector jumps at \( T \). Note that the fact that the growth rate increases when the positive shock occurs does not mean that the voracity effect is not operative. Since \( \partial g / \partial \epsilon = \alpha\sigma/n - \sigma(n - 1) < 0 \) (because \( \sigma > n/n - 1 \)), we have that (i) the post-shock growth rate is smaller than the growth rate before the announcement takes place, and (ii) at a given point in time economies with greater shocks have lower growth rates. We summarize the results of this subsection in the following proposition.

**PROPOSITION 7:** The voracity effect is operational in the presence of anticipated productivity shocks. When a positive future shock is announced:

- If \( 1 < n \leq \bar{n} \), the growth rate of the formal sector remains unchanged until the time of the shock. At the time of the shock it falls.
- If \( n > \bar{n} \), the growth rate falls when the announcement is made, and follows an increasing path until the shock takes place. At that time it experiences an upward jump, but remains below its preannouncement level.

**IV. Empirical Discussion**

In this section, we discuss some recent empirical evidence on country responses to windfalls such as terms of trade shocks, foreign-aid transfers, and natural resource endowments, as well as on the relationship between fractionalization and economic growth. The model we have presented can be used to rationalize how, in some of these episodes, apparently perverse collective economic behavior can be generated by the rational actions of individual groups, in the absence of countervailing institutions. As such, it can help us understand the accumulating evidence that “windfalls” are often dissipated, with no gain in welfare or growth, and that divided societies with weak institutional structures suffer from chronically low growth. In this section, we discuss some of this evidence in the context of our theoretical framework. The aim of this section is simply to present a selective summary of evidence that we think is relevant for our theory.

In Table 1, terms of trade and fiscal data are presented for three countries (Nigeria, Venezuela, and Mexico) that enjoyed significant oil windfalls but dissipated the revenues. \[14\] The 1974 shock permanently raised oil prices, which

\[13\] To see this, note that \( \lim_{\epsilon \to 0} g(t < T) - g(t \geq T) = -\epsilon \alpha < 0.\]

The table presents ratios of different components of government spending to GDP. Thus an increase in a ratio indicates that category of government spending rose by more than the increase in GDP during the windfall period. For capital expenditure one could give the countries the benefit of the doubt and argue that if the shocks were considered permanent, an increase in public investment might be justified. However, it is more difficult to rationalize a more-than-proportional increase in government consumption and transfers.

In each case, as can be seen in Table 1, government spending rose sharply in response to the improvement in the terms of trade and peaked at the crest of the oil boom in 1980–1982. A startling example is Nigeria: the average ratio of government expenditure (net of interest payments) to GDP doubled from 0.2 in 1970/73 to 0.399 in 1980/82 before reverting to 0.198 after 1986. Especially interesting is the increase in transfer payments, with central-government resources being distributed to state-owned and private enterprises, local governments, and the banking sector. Moreover, these data seriously underestimate the increase in public expenditure in Nigeria, as much of the oil revenues were diverted into extrabudgetary secret accounts (see Abdul-Ganiya Garba, 1996). In Figure 1, we plot the terms of trade and total government expenditure (net of interest payments) for Mexico:

### Table 1—Fiscal Policy Responses

<table>
<thead>
<tr>
<th></th>
<th>TT</th>
<th>GTOTY</th>
<th>CONY</th>
<th>CAPY</th>
<th>TRANY</th>
<th>INTY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nigeria</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1970</td>
<td>34.2</td>
<td>0.217</td>
<td>0.056</td>
<td>0.037</td>
<td>0.062</td>
<td>0.023</td>
</tr>
<tr>
<td>1981/82</td>
<td>195.7</td>
<td>0.359</td>
<td>0.103</td>
<td>0.106</td>
<td>0.13</td>
<td>0.045</td>
</tr>
<tr>
<td>1970/90</td>
<td>111.5</td>
<td>0.27</td>
<td>0.072</td>
<td>0.074</td>
<td>0.101</td>
<td>0.03</td>
</tr>
<tr>
<td>Venezuela</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1970</td>
<td>41.9</td>
<td>0.194</td>
<td>0.099</td>
<td>0.027</td>
<td>0.044</td>
<td>0.004</td>
</tr>
<tr>
<td>1981/82</td>
<td>196.6</td>
<td>0.257</td>
<td>0.104</td>
<td>0.03</td>
<td>0.093</td>
<td>0.019</td>
</tr>
<tr>
<td>1970/90</td>
<td>111.6</td>
<td>0.218</td>
<td>0.091</td>
<td>0.025</td>
<td>0.086</td>
<td>0.016</td>
</tr>
<tr>
<td>Mexico</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1970</td>
<td>78.1</td>
<td>0.114</td>
<td>0.046</td>
<td>0.017</td>
<td>0.041</td>
<td>0.01</td>
</tr>
<tr>
<td>1981/82</td>
<td>161.4</td>
<td>0.262</td>
<td>0.079</td>
<td>0.043</td>
<td>0.102</td>
<td>0.038</td>
</tr>
<tr>
<td>1970/90</td>
<td>114.7</td>
<td>0.203</td>
<td>0.061</td>
<td>0.023</td>
<td>0.061</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Notes: TT is terms of trade index (1987 = 100); GTOTY is total government expenditure; CONY is government consumption; CAPY is government investment; TRANY is government transfers; INTY is government debt interest payments. All are expressed as ratios to GDP.

Sources: TT and GDP data for Venezuela and Mexico are from World Development Indicators (World Bank, 1997); fiscal data for Venezuela and Mexico are from Roberto Perotti (1997); GDP and fiscal data for Nigeria are from the Central Bank of Nigeria’s Annual Report (1994).

Although overall government spending fell in Venezuela after 1986, in line with our model, this was not the case for one subcomponent, government transfers.
again, the sensitivity of government spending to the terms of trade is clearly evident. A positive fiscal response to terms of trade shocks has also been recorded by Ludger Schuknecht (1996) who calculated, in a study of 17 beverage booms, an average increase in government spending of 2.2 percentage points of GDP.

Moreover, despite the windfall revenues, these countries had dismal growth performance during this period. In Table 2, we compare fitted average annual growth rates for these countries (generated from a "Barro-type" cross-sectional growth regression) to the actual growth outcomes: in each case, the residual is negative and large. The poor growth performance is suggestive that increases in public capital expenditure were not productively deployed and that appropriated resources were consumed, invested in safe but inefficient activities, or transferred overseas. A failure to grow among developing countries that have enjoyed positive terms of trade shocks, despite increases in measured investment, has also been found by Paul Collier and Jan Willem Gunning (1996) in a 19-country panel of annual observations over 1964–1991.

A more short-lived terms of trade shock was the temporary coffee boom of 1977/80 that was generated by a frost in Brazil that sharply reduced output in the most important coffee-producing nation. In Table 3, we present data that shows the coffee windfall similarly unleashed a more-than-proportionate expansion in government consumption in three major coffee exporters: Costa Rica, Côte d’Ivoire, and Kenya. When the boom was over, government spending fell back.

We would like to note that the evidence we have presented is not a test of our theory. We have only considered the most often-mentioned cases of squandered terms of trade windfalls. A test of our theory would entail considering all the countries that have experienced major terms of trade shocks, selecting those countries with divided fiscal control, and investigating whether the growth rate in these countries has responded inversely to terms of trade shocks. We plan to implement such a test when the detailed data on government expenditures and fiscal institutions that is relevant for our theory becomes available.

Other researchers have also investigated the apparently perverse responses of some countries to exogenous endowment shocks. Gelb (1988) and Little et al. (1993) present detailed country studies that show a recurrent pattern of developing countries failing to take advantage of the sharp terms of trade improvements of the 1970’s. Peter Boone (1996) and Svensson (1996) have recently studied coun-

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**Figure 1. Mexico: Plot of Government Spending (Net of Interest Payments) as a Ratio of GDP (MEGOVY) and the Terms of Trade (METT), 1970–1993**

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16 We present data only for government consumption, as other data for Côte d’Ivoire were not reported until the late 1970’s and some data for Kenya only began in the mid-1970’s.
tries that are recipients of foreign-aid transfers, which are another type of windfall income. Boone (1996) finds, in a panel of developing countries, that foreign aid fails to raise the investment rate in recipient countries, being mostly consumed. Svensson (1996) shows that, in countries suffering from ethno-lingual fractionalization and weak political institutions, injections of foreign aid generate increases in corruption, indicating that such windfall income is dissipated in rent seeking. This response would be predicted by our model: the receipt of foreign aid induces powerful groups to increase their appropriation rates, leading to a dissipation of the revenues and no gain in welfare.

Jeffrey D. Sachs and Andrew M. Warner (1995) have recently presented evidence that countries with high endowments of natural resources have had significantly worse growth performance than other countries. In line with our model, the explanation for this result may lie in the distributive struggle in these countries, as groups attempt to appropriate the rents generated by these natural resource endowments. Barro (1996) similarly suggests such an explanation of the findings of Sachs and Warner.

The other empirical regularity that can be rationalized by our model is the chronic low growth of countries that suffer from sociopolitical divisions and weak institutions. This evidence is comprehensively documented by Easterly and Levine (1997). These authors argue the root cause of the inferior growth performance of African countries is the combined effect of deep ethnic and tribal divisions, complemented with underdeveloped property rights and legal and political institutions that fail to induce rival groups to submit to a more socially optimal resource allocation process. Again using an index of ethno-linguistic fractionalization to capture the presence of multiple-interest groups within society and commercial indicators to measure institutional quality, they demonstrate the empirical importance of these concepts in explaining Africa’s poor growth performance, in the framework of panel cross-country growth regressions. In a similar vein, Robert Tamura (1995) shows that countries with greater linguistic heterogeneity typically grow more slowly. Our analysis provides a formal mechanism that explains why, in the absence of countervailing institutions, fractionalization can lead to lower growth and hence rationalizes this empirical evidence.

V. Conclusion

In this paper, we endogeneize the extent of discretionary fiscal redistribution to more fundamental characteristics of a country, namely

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17 Tamura explains his result by the restricted scope for human capital spillovers in a heterogeneous society, which is a complementary mechanism to ours.
the existence of powerful groups, physical rates of return, and institutional barriers to discretionary redistribution. We show that an economy in which there are powerful groups grows more slowly than one in which groups are powerless or act in a coordinated manner. Moreover, in the case of powerful groups, growth is lower when power is concentrated among only a few groups than when power is diffused across many groups.

We also explain the anomaly described in the case-study literature that a number of countries respond perversely to terms of trade windfalls by experiencing a decline in growth performance. The voracity effect—a more-than-proportional increase in redistribution in response to a windfall—generates in equilibrium a negative relationship between improvements in raw rates of return and growth, in the case of powerful groups.

Our findings are relevant in evaluating the growth prospects of developing nations that are undergoing democratization. According to our view, the effect on growth of a switch from autocracy to democracy will depend on the effect that the shift has on the ability of powerful groups to extract transfers. If the collapse of an autocracy relaxes restrictions on the behavior of the powerful groups in a society, democratization may actually intensify the redistribution struggle in these countries. From our analysis, this will lead to lower growth and poorer adjustment to windfalls. In contrast, if the shift to democracy brings with it the destruction of entrenched interest groups, and power becomes more diffused, then growth performance and adjustment to windfalls will improve. It also follows that procompetition policies—for example, making easier market entry or exposing domestic behemoths to foreign competition—may be as important in terms of altering a country’s propensity to arbitrarily appropriate private wealth as in their direct impact on efficiency.

APPENDIX

PROOF OF PROPOSITIONS 1 AND 2:

As discussed in the text, a strategy \( \phi_i(t) = \{ c_i(t), s_i(t) \} \) is optimal for group \( i \) only if it satisfies the first-order conditions associated with the present-value Hamiltonian (15)

\[
(A1) \quad 0 = \frac{\partial H_i}{\partial c_i} = \nu_i c_i(t)^* - \zeta_i(t)^{-\sigma}.
\]

\[
(A2) \quad 0 = \frac{\partial H_i}{\partial r_i} = \zeta_i(t) - \lambda_i(t) - \xi_i(t).
\]

\[
(A3) \quad \dot{\zeta}_i(t) = \delta \zeta_i(t) - \frac{\partial H_i}{\partial b_i} = \zeta_i(t) [\delta - \beta] - \mu_i(t).
\]

\[
(A4) \quad \dot{\lambda}_i(t) = \delta \lambda_i(t) - \frac{\partial H_i}{\partial K} = \lambda_i(t) \left[ \delta - p \alpha + \sum_{j \neq i} \frac{\partial r^*_j}{\partial K} \right] - \bar{x}_i(t).
\]

\[
(A5) \quad \mu_i(t) b^*_i(t) = 0, \quad \mu_i(t) \geq 0, \quad b^*_i(t) \geq 0.
\]

\[
(A6) \quad \lim_{t \to \infty} K(t)^* \lambda_i(t) e^{-\delta t} = 0,
\]

\[
\lim_{t \to \infty} b^*_i(t) \dot{\zeta}_i(t) e^{-\delta t} = 0.
\]

\[
(A7) \quad \nu_i = \{ 0, 1 \},
\]

\[
\{ \nu_i, \lambda_i(t), \zeta_i(t) \} \neq \{ 0, 0, 0 \} \text{ for all } t.
\]

To find an equilibrium candidate we need to find \( n \) pairs \( \{ c_i(t), r_i(t) \} \) that simultaneously solve \( n \) sets of equations (A1)–(A7), one for each group \( i \). Then we need to check that this equilibrium candidate is admissible in the sense that it satisfies constraints (6) and (8), and that it satisfies the second-order conditions. In what follows we will derive the interior equilibrium. First, since by definition \( r^*_i(t) < \bar{x} K(t) \) at all times, \( \xi_i(t) = 0. \) Second, the constant \( \nu_i \) cannot be zero. If \( \nu_i = 0, \) (A1) would imply \( \zeta_i(t) = \infty \) for all \( t \), and (A6) would be violated. Third, the multiplier \( \mu_i(t) \) will be equal to or different from zero depending on the value \( n \) takes.
Case i: \(1 < n \leq n = 1 + \alpha p - \beta / z(\beta)\).

We will set \(\mu_i(t) = 0\) and show that in this case \(b_i^\ast(t) \equiv 0\) for all \(t\), so that (A5) is satisfied. Since \(\mu_i(t) = 0\), it follows from (A2) – (A4) that along the interior equilibrium it is necessary that for each \(i\), \(\alpha p - \beta = \sum_{j \neq i} \partial r_i^\ast(K, b_i) / \partial K\). This set of \(n\) linear equations has a unique solution, which is given by \(\partial r_i^\ast(K, b_i) / \partial K = [\alpha p - \beta] / [n - 1]\). Integrating with respect to \(K\) we get \(r_i^\ast = A + K(t)[\alpha p - \beta] / [n - 1]\). Since \(r_i^\ast(0, b_i) = 0\), the constant \(A\) is zero, and we obtain equation (18) in the text. Note that \(r_i^\ast\) lies within the bounds given in (6). Consumption policy (18) is derived as follows. From (A1) and (A3) we have that consumption grows at the constant rate \(\sigma[\beta - \delta]\). Thus

\[
(A8) \quad c_i(t) = c_i(s) e^{\sigma[\beta - \delta]\{t - s\}}, \quad \forall t, s.
\]

To obtain initial consumption we first solve for the stock of inefficient capital \(b_i(t)\) by substituting (A8), (18), and (19) into accumulation equation (5), and solving the differential equation, it follows that for all \(t\) and \(s\)

\[
(A9) \quad b_i(t) = e^{\beta\{t - s\}} \{c_i(s) + K(s)[1 - e^{-\delta\{t - s\}}] - [1 - e^{-\beta\{t - s\}}]c_i(s) / z(\beta)\},
\]

where \(x = (\alpha p - \beta) / (n - 1) > 0\). Substituting (A9) into the second transversality condition in (A6) and setting \(s = 0\), we have that \(c_i(0)\) must satisfy

\[
(A10) \quad \lim_{t \to \infty} c_i(0)^{-1/\sigma} \left\{ b_i(0) + K(0) \times [1 - e^{-\delta t}] - [1 - e^{-\beta t}] c_i(0) / z(\beta) \right\} = c_i(0)^{-1/\sigma} \left\{ b_i(0) + K(0) - c_i(0) / z(\beta) \right\} + c_i(0)^{\sigma - 1/\sigma} \lim_{t \to \infty} e^{-\beta t} = 0.
\]

Note that (A10) is satisfied only if \(z(\beta) > 0\). If \(z(\beta)\) were negative, the last term in (A10) would be infinite unless \(c_i(0) = 0\). But \(c_i(0) = 0\) implies that the first term is infinite. Thus, (A10) would not be satisfied. Similarly, if \(z(\beta) = 0\) and \(c_i(0) > (\ast) 0\), the first term would be \(-\infty\) and the second \(+\infty\). Given that \(z(\beta) > 0\), (A10) is satisfied if and only if \(c_i(0) = z(\beta) [b_i(0) + K(0)]\), which corresponds to equation (18) in the text. To derive (20) we simply substitute this expression for \(c_i(0)\) in (A9) and replace \(K(s) e^{(\beta - \delta)(t - s)}\) by \(K(s) e^{(\beta - \delta)(t - s)}\). Finally, note that \(z(\beta) > 0\), (19) and (20) imply that \(b_i(t) > 0\) if and only if \(z(\beta) > (\ast) \{n - \alpha p / (n - 1)\} / [n - 1]\). Lastly, since both conditions are identical and since \(b_i(0) = 0\), it follows that \(b_i(t) > 0\) for all \(t\). Next, we check that the first transversality condition in (A6) is satisfied. Noting that (A2) and (A3) imply that \(\lambda(t) = \lambda(0) e^{(\beta - \delta) t}\), and using (19), it follows that

\[
\lim_{t \to \infty} K(s\lambda(0)) e^{-\delta t} = \lim_{t \to \infty} K(s\lambda(0)) e^{(\alpha p - \beta)(n - 1)} e^{(\beta - \delta) t} e^{-\delta t} = \lim_{t \to \infty} K(s\lambda(0)) e^{(\beta - \alpha p)(n - 1)} = 0.
\]

We have shown that the set of \(n\) strategies given by (18) and the associated paths of the state variables (19) and (20) satisfy the \(n\) sets of optimality conditions (A1) – (A7) and the constraints (6) and (8). To check that these strategies constitute a MPE [i.e., satisfy condition (13)], note that taking as given that the strategies of the other groups are \(\Phi_{-i}(t) = \{\Phi_i(K(t), b_i(t))\}_{i \neq i}\), by construction group \(i\) will find it optimal to set \(\phi_i(t) = \Phi_i(K(t), b_i(t))\). This is because \(\Phi_i(K(t), b_i(t))\) satisfies all the necessary conditions for an optimum of \(i\)’s control problem, and because the Hamiltonian of group \(i\) evaluated at the optimum is concave in \((K, b_i)\) (Theorem 2.3 of
Case ii: $n > \bar{n}$.

The solution derived in the previous case is not admissible because if we were to set $\mu_i(t) = 0$, then $b_i(t)$ would be negative [see (20)]. Since $b_i(t) = 0$, this would imply that $b_i(t) < 0$ for all $t$ and (8) would be violated. Thus, in this case we must have $\mu_i(t) > 0$, and $b_i^*(t) = 0$ for all $t$. It then follows from accumulation equations (5) and (12) that $r_i^*(t) = c_i^*(t)$. Since $b_i^*(t) = 0$, differentiating (14) we have that $c_i^* / c_i^* = \partial r_i^* / \partial K$ implies $c_i^* / c_i^* = -\sigma \lambda_i / \lambda_i = \sigma [ap - \delta + \sum_{j \neq i} \partial r_j^* / \partial K]$. Equalizing the two expressions for $c_i^* / c_i^*$ we have that the equilibrium transfer policies are the solution to the following $n$ simultaneous equations:

\begin{equation}
(11) \quad \frac{\partial r_i^*(K, 0)}{\partial K} \frac{\alpha p K - \sum_{j=1}^{n} r_j^*(K, 0)}{r_i^*(K, 0)} = \sigma \left[ ap - \delta + \sum_{j \neq i} \frac{\partial r_j^*(K, 0)}{\partial K} \right],
\end{equation}

$i = 1, ..., n$.

To find a solution to this set of equations we try $r_i^*(K, 0) = xK$, where $x$ is an undetermined coefficient. It follows that $x$ is given by (22). Next, we verify that condition (A5) is satisfied, i.e., that $\mu_i(t) \geq 0$. From (A2) – (A4) we have that $\mu_i / \zeta_i = \delta - \beta - \lambda_i / \lambda_i$. Replacing (22) in $\lambda_i / \lambda_i$, it follows that $\mu_i / \zeta_i = ap - \delta + \sigma (n - 1)(\beta - \delta)/n - \sigma (n - 1)$. The ratio $\mu_i / \zeta_i$ must be nonnegative because $\zeta_i$ is a costate variable and $\mu_i$ is the multiplier associated with the restriction $b_i \geq 0$. Since $n > \bar{n}$ implies that the numerator of $\mu_i / \zeta_i$ is negative, the denominator must be negative. Thus, we must impose $\sigma > n/(n - 1)$, which is condition (21). Now we verify that transversality conditions (A6) are satisfied. Since $b_i^*(t) = 0$, the second condition is trivially satisfied. The first condition is

\begin{equation}
(12) \quad 0 = \lim_{t \to \infty} K^*(t) \lambda_i(t) e^{-\beta t} = \lim_{t \to \infty} K^*(t) [K^*(t)x]^{-1/\sigma} e^{-\beta t} = \lim_{t \to \infty} [x]^{-1/\sigma} \left[ K(0) \times \exp \left( \frac{\sigma(n\delta - \alpha p)}{\sigma(n - 1) - n} t \right) \right]^{\sigma - 1/\sigma} e^{-\beta t} = \lim_{t \to \infty} [x]^{-1/\sigma} K(0)^{\sigma - 1/\sigma} \times \exp \left( \frac{z(\alpha p)}{\sigma(n - 1) - n} t \right)^{\sigma - 1/\sigma},
\end{equation}

where $x = z(\alpha p) / n - \sigma [n - 1]$. The second equality follows from (14), (A1) and (A2), the third equality follows from (23), and the last

\[ H_{kk} = H_{kb} = H_{bk} = -[K + b_i]^{-\sigma - 1/\sigma} z(\beta)^{-1/\sigma} < 0. \]

This implies that the associated Hessian is negative semidefinite. Therefore, the Hamiltonian is concave.

\footnote{Substituting (18) in (15) and taking derivatives we find that}

\[ H_{kk} = H_{kb} = H_{bk} = -[K + b_i]^{-\sigma - 1/\sigma} z(\beta)^{-1/\sigma} < 0. \]
equality uses \( z(\alpha p) = \alpha p [1 - \sigma] = \delta \sigma \). Since \( \sigma > n/[n - 1] \) implies that the denominator inside the exponential in (A12) is positive and since \( x \) is nonzero, this transversality condition is satisfied if and only if \( z(\alpha p) < 0 \), which is condition (21). Lastly, we verify that the restrictions on parameters in (21)—necessary for the first-order conditions to be satisfied—are mutually consistent in this case. Since \( n > \bar{n} \), we can be rewritten as \( \sigma < n\beta - \alpha/([n - 1])(\beta - \delta) \), it follows that \( \sigma > n/[n - 1] \) hold in this case if and only if

\[
\frac{n\beta - \alpha}{n - 1} < \frac{n\beta - \alpha}{(n - 1)(\beta - \delta)} \Rightarrow 0
\]

\[
< (n - 1)(n\delta - \alpha p)
\]

\[
> 0 \Leftrightarrow n\delta > \alpha p,
\]

and the condition \( z(\alpha p) < 0 \) can hold in this case if and only if

\[
\frac{\alpha p}{\alpha p - \beta} < \frac{n\beta - \alpha}{(n - 1)(\beta - \delta)}
\]

\[
\Leftrightarrow (\alpha p - \beta)(n\delta - \alpha p) > 0
\]

\[
\Leftrightarrow n\delta > \alpha p.
\]

Comparing (A13) and (A14) it follows that both conditions in (21) can be simultaneously satisfied in this case.

**Derivation of (34).**—Since the shock is permanent, for \( t \geq T \) consumption is given by (18) replacing \( p \) by \( p_t \). For \( t < T \) note that (33) implies that the rate of return faced by each group is \( \beta \) regardless of how it allocates its capital. Thus, \( c_i(T) = c_i(t) e^{\alpha(\beta - \delta)(T - t)} \). To determine initial consumption \( c_i(t) \) we use this equation and (32) to get

\[
c_i(t) e^{\alpha(\beta - \delta)(t - T)} = c_i(T) = z(\beta)[b_i(T) + K(T)].
\]

Replacing \( K(T) \) by (19) and using (A9) to obtain \( b_i(T) \), we have that

\[
\frac{c_i(t)}{z(\beta)} e^{\alpha(\beta - \delta)\Delta} = K(t) e^{\alpha(n\beta - \alpha p/n - 1)\Delta}
\]

\[
+ e^{\beta\Delta}\left[ b_i(t) + K(t)[1 - e^{-\Delta}] \right]
\]

\[
- [1 - e^{-z(\beta)\Delta}] \frac{c_i(t)}{z(\beta)}
\]

where \( \Delta = T - t \), \( x = \alpha p - \beta/n - 1 \) and \( z(\beta) = [(1 - \sigma) + \delta \sigma] \). Simplifying, we obtain

\[
c_i(t) = z(\beta)[K(t) + b_i(t)],
\]

which is equation (34).

**Derivation of (35).**—To obtain the equilibrium transfer functions we follow the same steps we used in Section II. However, since there is an anticipated shock, we need to consider nonstationary transfer policies. Let \( r_i(t) = c_i(t) = x(t)K(t) \), where \( x(t) \) is an undetermined function of time. This implies that \( \dot{r}_i/r_i = \dot{x}/x + K/K \). Since the rate of return perceived by \( i \) is \( \alpha p - [n - 1]x(t) \), its Euler equation is \( \dot{r}_i/r_i = \sigma[\alpha p - \beta - [n - 1]x(t)] \). Combining both equations for \( \dot{r}_i/r_i \) we have that consumption must satisfy \( x(t) = \int_{T}^{t} x(t)K(t) \) recall that \( z(\alpha p) = \alpha p [1 - \sigma] + \delta \sigma \). The general solution of this differential equation is

\[
x(t) = x(\alpha p) [n - \sigma(n - 1)] + A z(\alpha p) e^{z(\alpha p)\tau}.
\]

In order to determine the constant \( A \), we use the transversality condition

\[
c_i(T)^{-1/\sigma} = \left[ z(\alpha p + \sigma) K(T) \right]^{-1/\sigma} / n - \sigma(n - 1) \]

To obtain \( A \) we use (A15) and \( c_i(T) = x(T)K(T) \). Replacing the value for \( A \) in the equation for \( x(t) \) we get (35).

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