Synthesis Circuit Correction for Speed Sensors of Physical Quantities and Current-Voltage Converters with Parasitic Capacitance

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Abstract

The paper presents the method of increasing the speed of sensors with high output resistance, for example, photodiode of optical systems for transmitting digital information. The features of method of correcting circuit of current-voltage converters, which compensate the influence of parasitic capacitance of sensor on its bandwidth and the settling time of the transient process are considered.

1. Introduction

In automatic control devices, computation and telecommunications systems are widely used sensors with high output resistance \( R_i \) and the parasitic capacitance \( C_0 \), which significantly increases the settling time of the transient process \( t_{\text{stab}} \). To reduce \( t_{\text{stab}} \) of photodiode receivers of digital optical systems [1], using any special techniques of measuring information [2] or a special correcting device and current-voltage converters, including into sensor [1, 3].

The paper discusses the compensation method of improving performance of a wide class of sensors of various physical quantities and current-voltage converters, which the output signal depend from current \( I_s(t) \), Fig. 1. The necessary correction circuit is synthesized by an analytical method.

2. Formulation of the problem

In general, the equivalent circuit of the sensor (Fig. 1) with a potential output is characterized by resistance \( Z_L(p) \) of the Laplace transform:

\[
Z_L(p) = \frac{U_{out}(p)}{I_L(p)}
\]

The compensation method of increasing speed is to connect the output of the sensor “Out” of correction circuit with the transfer function \( S_c(p) \). This circuit provides compensation for adverse effects of stray capacitance \( C_0 \) sensor on its dynamic characteristics.

![Figure 1. Functional diagram of the sensor to the circuit of correction](image)

The performance of sensors will be characterized the stabilization time transient \( t_{\text{stab}} \) to a step change of the output current \( I_s(t) \) of the sensor.

3. Synthesis of correction circuit

Turning to the solving of the problem of synthesis, the transfer function of the desired circuit correction, denoted as \( S_c(p) \):

\[
S_c(p) = \frac{I_c(p)}{U_{out}(p)},
\]

where \( U_{out}(p) \), \( I_s(p) \) – the Laplace transform sensor output voltage and output current circuit correction. In this case, we assume that the correction circuit \( S_c(p) \) has a high input and output impedance, i.e. its input and output cannot shunted \( Z_L(p) \).

We denote \( t_{\text{stab}}' \) the desired time of the stabilization time transient corrected sensor.

Confirmation. If a sensor with a potential output is characterized by the resistance...
the desired stabilization time of transient.

The corresponding mathematical model correction circuit that provides resistance to the consideration sensor without correction circuit (Fig. 1) characterized by resistance to currents in the output node of the sensor satisfy the output voltage is corrected sensor (Fig. 1), the output equation (5) takes the form

\[ \frac{Z_L(p)}{1-Z_L(p)S_c(p)} = \frac{K}{l + T_c p}, \]  

(2)

where \( T_c \) – the time constant of the corrected sensor, \( K \) – transfer coefficient. In this case, the corrected sensor has a settling time of the transition process equal \( \tau_{stab} = 3T_c \).

**Demonstration.** According to Kirchhoff's law, the currents in the output node of the sensor satisfy the equation

\[ I_L(p) = I_S(p) + I_c(p). \]  

(3)

After transformations, from (1) and (3) with regard equality \( U_{out}(p) = Z_L(p)I_L(p) \), we will have

\[ U_{out}(p) = \frac{Z_L(p)}{1-Z(p)S_c(p)} I_S(p). \]  

(4)

This equation describes the dependence of the output voltage is corrected sensor (Fig. 1), the output current sensor. From expressions (2) and (4), in particular, that the conditions of Theorem corrected sensor is stable.

It is known that if a link transfer function has the form \( W(p) = K/(1 + T_c p) \), the time of the transition process to a step change in its input influence of the same \( 3T_c \), regardless of the value of the coefficient \( K \) [9]. As for the condition of the theorem \( T_c = \frac{\tau_{stab}}{3} \), then from equations (2) and (4) should be proof of claim.

From (2) and (4) it also follows that, in this case, the transfer function of the required correction circuit is defined by the following expression

\[ S_c(p) = Z_L(p) - K^{-1}(1 + pT_c). \]  

(5)

With regard equality \( T_c = \frac{\tau_{stab}}{3} \), the expression (5) takes the form

\[ S_c(p) = Z_L(p) - K^{-1}\left(1 + \frac{\tau_{stab}}{3} \right) \]  

(6)

The resulting equation is a solution in the general case of the above problem of synthesis of a mathematical model correction circuit that provides the desired stabilization time of transient.

In order to obtain more specific results assume that the consideration sensor without correction circuit (Fig. 1) characterized by resistance to

\[ Z_L(p) = \frac{R}{1 + T_0 p}, \]  

where \( T_0 = R_c C_0 \). At the time of the stabilization time \( \tau_{stab} = 3T_0 \). Given this assumption, setting in (6) \( K = R_c \), we find that the desired transfer function correction circuit for this case has the form

\[ S_c(p) = C_0 - \frac{\tau_{stab}}{3R_c} p. \]  

(7)

4. The practical implementation of the correction circuit

In accordance with the expression (7) as a circuit of correction is possible to apply the scheme shown on Fig. 2, where

\[ K_{VF}(p) = \frac{U_{out}(p)}{U_{in}(p)}, \quad K_{CF}(p) = \frac{I_S(p)}{I_{12}(p)}, \]

\[ I_{12}(p) = \frac{U_{out}(p)}{Z_c(p)} = C_c p U_{out}(p). \]  

(8)

If a voltage follower (VF) and current follower (CF) are a noninertial, i.e. \( K_{VF}(p) = K_{CF} = 1 \) and \( K_{CG}(p) = K_{CG} = 1 \), then the transfer function of the correction circuit is described by

\[ S_c(p) = C_c p. \]

\[ \text{Figure 2. Practical scheme of correction a circuit} \]

So, by \( K_{VF}(p) = 1 \) and \( K_{CG}(p) = 1 \) the capacity of the correction capacitors in the circuit in Fig. 2 should be chosen from the condition of the

\[ C_c = C_0 - \frac{\tau_{stab}}{3R_c}. \]  

(10)

Hence it follows, that to obtain the stabilization time of transient \( (\tau_{stab} \approx 0) \) the implementation of the correction circuit in accordance with the scheme in Fig. 2 and noninertial amplifiers VF and CF, we must have \( C_c \approx C_0 \).
If you want to get the improved performance of the sensor in the $N_i$-times ($N_i \ll \infty$) compared with the scheme without correction, where $N_i = 3R_vC_0/t_{stab}^*$, the capacitance of a capacitor $C_v$ must meet the condition

$$C_v = \frac{N_i - 1}{N_i} C_0.$$  \hspace{1cm} (11)

As shown, the assumption that a voltage follower ($VF$) and a current follower ($CF$) be noninertial is greatly simplifies the correlation calculation. However, the time constants of the amplifier, even very small, can cause oscillatory transients.

5. Stability analysis

Stability of dynamic systems, which include the sensor, strictly speaking, is estimated at change of frequencies from zero to infinity [4]. At the same time, the model of the amplifiers $VF$ and $CF$ as it $W_i(p) = K_v$ is not entirely adequate for the specified frequency range. Hence for stability studies and analysis of the sensor (Fig.1) with the correction circuit shown in Fig. 2 where $VF$ and $CF$ followers are functioning inertial first-order, suppose

$$K_{VG}(p) = \frac{K_v}{1 + T_{VG} p}, \quad K_{CG}(p) = \frac{K_v}{1 + T_{CG} p}.$$  \hspace{1cm} (12)

With taking into account equations (12) the transfer function $S_i(p)$ of the circuit correction (Fig. 2) has in this case, the type of

$$S_i(p) = \frac{K_{VG}K_{CG}C_vp}{(1 + TX_{VG}p)(1 + T_{CG}p)}.  \hspace{1cm} (13)$$

As a consequence, this transfer function is corrected sensor $Z_L(p) = \frac{R_v}{1 + T_v p}$ will not be equal to the right side of expression (2). However, if is corrected sensor to be sustainable, then an appropriate choice of parameters of its transfer function, as shown below, will be close to the transition function of inertial link [4]. For this you can provide and the desired stabilization time of transient $t_{stab}^*$.

To determine the stability conditions of the corrected sensor will record it as the characteristic polynomial $D(p)$ of the denominator of the transfer function of equation (4) with (13) and the above equality $Z_L(p) = R_v/(1 + T_v p)$. As a result we will have

$$D(p) = T_v T_{CG} C_v p^3 + (T_v T_{CG} + T_v T_{VG} + T_{CG} T_{VG}) p^2 + (T_v + T_{VG} + T_{CG} - K_v K_{CG} R_v C_v) p + 1.$$  \hspace{1cm} (14)

By applying to this polynomial criteria Vyshnegradsky [4], we find that the conditions of this criterion is met, if:

$$T_v T_{CG} + T_v + T_{CG} > K_v K_{CG} R_v C_v,$$  \hspace{1cm} (15)

$$(T_v T_{CG} + T_v T_{VG} + T_{CG} T_{VG}) (T_v + T_{CG} + T_{VG} + T_0 - K_v K_{CG} R_v C_v) > T_v T_{VG} T_{CG}.  \hspace{1cm} (16)$$

Conditions (15) and (16) are the general terms of the stability of the corrected sensor (Fig. 1, Fig. 2) for the given parameters of the sensor and the transfer functions (12) of the voltage ($VF$) and current ($CF$) followers.

Under certain additional restrictions are various options to ensure the stability conditions. For example, if $T_{VG} \approx T_{CG} \ll T_0$ that the conditions (15), (16) with the account $T_0 = R_v C_0$ of the legend transferred in the following inequality

$$C_v > K_v K_{CG} C_0,$$  \hspace{1cm} (17)

or

$$\left(\frac{1}{T_{VG}} + \frac{1}{T_{CG}}\right) (T_0 - K_v K_{CG} R_v C_v) > 1.$$  \hspace{1cm} (18)

As $T_{VG} > 0$ and $T_{CG} > 0$, the condition (18) reduces to (17), which, on that $K_v = K_{CG} = 1$ follow, in particular, the expression (10). At the same time from the expressions (15) and (16) we see that as we approach the values $C_v$ to $C_v^*$ of the corrected sensor does not lose stability.

6. Simulation results

In the case of inertial followers $VF$ and $CF$ with transfer functions (12), an analytical expression for the stabilization time of transient, similar to (10) or (11) is obtained quite cumbersome. In this regard it is expedient to quantify the circuit parameters of correction, at which the corrected sensor has the required speed performance. For this purpose, we set: $T_{VG} = T_{CG} = \mu T_0$, $T_0 - K_v K_{CG} R_v C_0 = \beta T_0$, where is number $0 < \mu << 1$, $0 < \beta << 1$. Under these conditions, the transfer function of the sensor to the chain of correction (13) has the form

$$W(p) = \frac{U_{out}(p)}{I_s(p)} = \frac{\mu^2 T_0^2 p^2 + 2 \mu T_0 p + 1}{\mu^2 T_0^3 p^3 + (2 \mu + \mu^2) T_0^2 p^2 + (2 \mu + \beta) T_0 p + 1}.$$  \hspace{1cm} (19)

In this expression, there are three parameters, $T_0, \beta$ and $\mu$, which allows to $T_0 = R_v C_0$ find by predetermined numerical simulation parameters correction circuit $T_{CG} = T_{VG}$ and $K_{CG}, K_{VG}$, in which
the sensor is corrected desired transient nature and the desired performance.

For example, in Fig. 3a shows the transfer function of the corrected sensor when $T_0 = 0.1$, with the corresponding values found in MATLAB $\beta = 0.08$ and $\mu = 0.001$, as shown in Fig. 3b transition function when $- \beta = 0.008$ and $\mu = 0.00001$.

![Graph A](image1.png)

**Figure 3. Transient of the corrected sensor**

In this case, if uncorrected sensor has stabilization time of transient $t_{stab}^{ua} = 0.3$, when $\beta = 0.08$ and $\mu = 0.001$ the stabilization time is $t_{stab} = 0.0183$, i.e. in 16 times less, and when $\beta = 0.01$ and $\mu = 0.00001$ the stabilization time is $t_{stab} = 0.00252$, i.e. in the 119 times less.

It is easy to see that the possibilities of the compensation method are determined by the characteristics voltage ($VF$) and current ($CF$) followers, which can be quite broadband and implemented as a cascade with a common collector and a common base on the basis of the SiGe technological processes [5], ensuring that the working range of the data the functional units of the sensor frequencies up to a few tens of gigahertz [6].

7. Conclusion

1. Developed method of improving the performance of sensors and respective current-voltage converters, characterized by the output capacitance $C_0$ and the internal resistance $R_0$.

2. Received in article ratios allow on known parameters of the sensor to find the parameters of the chain of correction, providing stability and desired stabilization time of transient to a step change of the output current of the sensor, as well as expand the range of its operating frequency.

3. Correction circuit can be realized on the basis of classical repeater voltage and current with transmission ratios, close to unity. For this purpose may be used circuit of the transistor stages with a common collector and a common base, the inertia of which can be neglected to a frequency of several tens of gigahertz. For many sensors of physical quantities it allows to increase the speed from tens to hundreds of times.

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8. References

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