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Fuzzy Set Based Consensus Schemes for Multicriteria Group Decision making Applied to Strategic Planning

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Abstract This paper studies three consensus schemes based on fuzzy models for dealing with the input of multiple experts in multicriteria decision making. The consensus schemes are based on different aggregation procedures for constructing a collective decision. In the paper, we propose a methodology that makes use of the three consensus schemes implemented by a coordination mode that creates an efficient manner of exploiting the capabilities of each member of the group in a cooperative work. The applicability and efficiency of the proposed methodology is demonstrated through an application related to strategic planning.

Keywords Group decision · Consensus · Fuzzy preference relations · Strategic planning

1 Introduction

The process of deciding in advance what kind of effort is to be undertaken, when it is to be done, who is going to do it, and what will be done with the results is what is

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called strategic planning (Steiner 1979). In other words, it is the process of deciding on the objectives of the organization, on changes in those objectives, on the resources used to obtain these objectives. This type of decision process is the most important for the future of organization and is applicable to any situation as long as it is directly related to overall organizational purposes; it is future-oriented; it significantly involves uncontrollable environmental forces that affect organizational performance (Kaplan and Norton 1996).

Decision making processes in organizational strategic planning are usually very complex and involve multiple factors. Several elements characterize these complex problems: (1) numerous complicated linkages among organizational and environmental elements, (2) dynamic and uncertain environments, (3) ambiguity of available information, (4) lack of complete information, and (5) conflicts concerning the outcomes of decisions among interested parties (Black and Boal 1994).

According to Ackoff (1969), the longer the effect of a plan, the more difficult it is to reverse, the more strategic it is. In general, strategic planning is concerned with the longest period of time worth considering. In Harrison (1995), it is stated that strategic planning focuses on decision situations concerned with both internal and external environments of the organization, recognizes the concept and importance of positive acceptance from stakeholder groups, and accepts the inevitability of rapid change in a complex external environment.

Strategies actions can be proposed either by experts who work individually or by a group of experts who analyze a problem collectively. They are based on multiple and complex interactions between the actors of the process and usually count on the participation of various departments in the process of defining as well as executing a plan of actions. One of the main problems is to meet a consensus on the strategy to be followed. Thus, cooperation and collaboration within groups is critical to an organization's effectiveness.

According to Phillips and Phillips (1993), group work offers a multitude of advantages to an organization through sharing information, generating ideas, making decisions, and reviewing the effects of decisions. Ideally, the group will reach a "better" decision than an individual because the collective knowledge is typically greater than an individual's knowledge.

In real situations, when a set of experts takes part in the decision process, it is quite natural that, initially, their opinions disagree. In such a case, a consensual opinion must be constructed, taking into account the information provided by each decision-maker (DM). The literature on fuzzy methods and models for decision making under group settings presents procedures named consensus schemes, which consist of systematic discussion processes within the group (along with their appropriate aggregation procedures for generating collective decisions), implemented under the supervision of a moderator, with the intention of reducing the discordance among opinions (Herrera-Viedma et al. 2002; Lu et al. 2007; Parreiras et al. 2010). The moderator is supported by indices that reflect the current level of consensus among the experts and the level of concordance among any pair of opinions to supervise the discussion. The consensus index permits verifying whether the group has met an acceptable level of concordance. The concordance index can be used to identify the expert that is least concordant with the group so that he/she can be invited to review his/her opinion or to clarify it to the remaining experts (Herrera-Viedma et al. 2002).

Multicriteria decision problems under group settings involve diverse types of uncertainty, including (Lu et al. 2007):

- the preference for alternatives: it is frequently impossible to evaluate precisely the preference level of one alternative over another or to precisely evaluate the performance of each alternative;
- the importance of each criterion: it is always complicated to evaluate the importance level of each criterion for the final decision.
- the decision-makers' role in the enterprise: it is difficult to assign a weight to the opinion of each expert;

The consideration of the uncertainty factor in constructing mathematical models serves as a means for increasing their adequacy and, as a result, the credibility and factual efficiency of the decisions based on their analysis. Fuzzy set theory (Zimmermann 1990; Pedrycz and Gomide 1998) provides an environment suitable to deal with diverse types of uncertainty. Particularly, the fuzzy preference relations have proved to be an expedient instrument for portraying uncertain preference judgments in the context of individual, as well as of group decision making (Herrera-Viedma et al. 2002).

In the present paper, a multicriteria decision method, based on fuzzy preference relations is extended in order to handle the input of multiple experts involved in a strategic planning. This extension is implemented with the use of three consensus schemes (being each one based on a different aggregation approach for constructing collective opinions) synergistically combined by a coordination mode that allows a more efficient manner of exploiting the capabilities of each expert. Here, the proposed methodology is applied for solving an enterprise strategic planning problems generated with the use of the Balanced Scorecard methodology (Kaplan and Norton 1996) under a group environment.

The paper is organized as follows. A method of multicriteria analysis based on fuzzy preference relations is briefly explained in Sect. 2. Section 3 presents three aggregation procedures for the analysis of group decision making problems. Section 4 shows how to construct collective opinions on the basis of fuzzy models. Fuzzy concordance and fuzzy consensus measures which are utilized within the framework of different consensus schemes are shown in Sect. 5. In Sect. 6, we introduce a procedure to support the selection of aggregation strategies to implement the coordination mode appropriate to real-world problems and, in Sect. 7, a strategic planning problem is solved with the use of the proposed methodology. Finally, in Sect. 8 we draw our conclusions.

2 Multicriteria Analysis Based on Fuzzy Preference Relations

Assume we are given a finite and discrete set $X = \{X_1, X_2, \ldots, X_n\}$ of alternatives, which are to be evaluated, compared, and/or ordered under the consideration of the criteria $F_1, F_2, \ldots, F_q$. In a fuzzy environment, the problem of decision making may be modeled as a pair $< X, R >$, where $R = \{R_1, R_2, \ldots, R_q\}$is a set of fuzzy nonstrict preference relations (Orlovsky 1981; Fodor and Roubens 1994).

A fuzzy nonstrict preference relation R_p is a fuzzy set with bi-dimensional membership function $R_p(X_k, X_l) : X \times X \to [0, 1]$. In essence, such relation associates with each ordered pair of elements (X_k, X_l) a number $R_p(X_k, X_l)$ coming from the unit interval that reflects a degree to which X_k weakly dominates (or is at least as good as) X_l, when the criterion F_p is considered by a DM. A possible manner of representing R_p is by means of a square matrix of dimensions $n \times n$.

Here, we describe techniques for constructing and analyzing $< X, R >$ models as well as producing a ranking of all alternatives (from the most important to the least important) on their basis. The ranking is based on the level of membership of each alternative to a fuzzy nondominance set with an one-dimensional membership function $ND(X_k) : X \to [0, 1]$ (which can be represented by means of a vector of dimension equal to n). The fuzzy nondominance set may be derived from a set R of fuzzy nonstrict preference relations, as it is described below.

When we consider the use of a fuzzy nonstrict preference relation to represent the preferences of a DM, the entries of each pairwise comparison matrix R_p, $p = 1, \ldots, q$, may be directly or indirectly defined by a DM. For instance, in Herrera-Viedma et al. (2002), a DM can directly specify these entries or can provide the preferences using certain formats (multiplicative preference relations, utility values, and ordering of the alternatives) and, then, the supplied information is transformed to fuzzy preference relations, under adequate transformation functions. Here, as in Ekel et al. (1998), a natural way of constructing these matrices on the basis of fuzzy or linguistic estimates $\tilde{F}_p(X_k)$, specified by the experts, is utilized.

In particular, let us consider a pair of alternatives X_k and X_l along with the fuzzy estimates $\tilde{F}_p(X_k)$ and $\tilde{F}_p(X_l)$, corresponding to the evaluation of these alternatives from the point of view of the criterion F_p. Then, if the pth criterion is associated with maximization, the following elements of the matrix R_p can be constructed:

$$R_p(X_k, X_l) = \sup_{\substack{X_k, X_l \in X \\ F_p(X_k) \geq F_p(X_l)}} \min \left\{ \tilde{F}_p(X_k), \tilde{F}_p(X_l) \right\}, \tag{1}$$

$$R_p(X_l, X_k) = \sup_{\substack{X_k, X_l \in X \\ F_p(X_l) \geq F_p(X_k)}} \min \left\{ \tilde{F}_p(X_k), \tilde{F}_p(X_l) \right\}. \tag{2}$$

The application of (1) and (2) is associated with constructing the Cartesian product $\tilde{F}_p(X_k) \times \tilde{F}_p(X_l)$ whose elements are defined as $\min \left\{ \tilde{F}_p(X_k), \tilde{F}_p(X_l) \right\}$ (Zimmermann 1990; Pedrycz and Gomide 1998). The availability of $\tilde{F}_p(X_k) \times \tilde{F}_p(X_l)$ permits us to carry out the operation sup for the region $F_p(X_k) \geq F_p(X_l)$ if we use (1) or for the region $F_p(X_l) \geq F_p(X_k)$ if we use (2) (Orlovsky 1981). It is natural that $F_p(X_k) = \text{supp}\tilde{F}_p(X_k)$ and $F_p(X_l) = \text{supp}\ \tilde{F}_p(X_l)$.

If the pth criterion is associated with minimization, then (1) and (2) are to be written for $F_p(X_k) \leq F_p(X_l)$ and $F_p(X_l) \leq F_p(X_k)$, respectively.

Simple examples of applying (1) and (2) are given in Pedrycz et al. (2010).

The fuzzy nonstrict preference relation R_p can be represented by an indifference relation I_p and a fuzzy strict preference relation P_p as given by the following expression

$$R_p = I_p \cup P_p. \tag{3}$$

The indifference relation is constituted by all pairs of alternatives that simultaneously satisfy the conditions: $(X_k, X_l) \in R_p$ and $(X_l, X_k) \in R_p$. If $(X_k, X_l) \in I_p$, it can be said that X_k is indifferent to X_l. On the other hand, if $(X_k, X_l) \in P_p$, it can be said that X_k is strictly better than X_l (or X_k dominates X_l). The strict preference relation P_p is constituted by all pairs of alternatives that satisfy the conditions: $(X_k, X_l) \in R_p$ and $(X_l, X_k) \notin R_p$. With the use of the operations on fuzzy sets, it is possible to define P_p in terms of R_p as follows:

$$P_p = R_p \cap R_p^d, \tag{4}$$

where R_p^d corresponds to the dual relation of R_p, that is $R_p^d(X_k, X_l) = 1 - R_p(X_l, X_k)$ (Fodor and Roubens 1994). Accordingly, the membership function of $P_p(X_k, X_l)$ may be obtained by means of a t-norm operator. Unfortunately, as it has been discussed, for instance, in De Baets and Fodor (1997), it is not that simple to select a t-norm to implement (4), as neither all t-norms are capable of preserving certain desirable properties of a fuzzy preference structure. Among the admissible t-norms to be utilized in this context, we selected the Lukasiewicz t-norm, which yields a rather simple expression for P_p (Orlovsky 1978):

$$P_p(X_k, X_l) = \max\{R_p(X_k, X_l) - R_p(X_l, X_k), 0\}. \tag{5}$$

The use of (5) permits one to carry out the choice of alternatives. In particular, as $P_p(X_l, X_k)$ describes the set of all alternatives X_k that are strictly dominated by X_l, its compliment $\bar{P}_p(X_l, X_k)$ corresponds to the set of alternatives X_k that are not dominated by X_l. Therefore, in order to meet the set of alternatives from X that are not dominated by any other alternative, it suffices to obtain the intersection of all $\bar{P}_p(X_l, X_k)$. This intersection is the set of nondominated alternatives with the membership function

$$ND_p(X_k) = \inf_{X_l \in X} \left\{1 - P_p(X_l, X_k)\right\} = 1 - \sup_{X_l \in X} P_p(X_l, X_k), \tag{6}$$

which reflects the level of nondominance of each alternative X_k.

A natural choice for a monocriteria problem based on this model should be the alternatives providing:

$$X_{R_p}^{ND} = \left\{ X_k^{ND} \in X \,\middle|\, R_p^{ND}\left(X_k^{ND}\right) = \sup_{X_k \in X} ND_p(X_k) \right\}. \tag{7}$$

It is worth emphasizing that the alternatives satisfying $X_{R_p}^{ND} = \left\{X_{R_p}^{ND} \in X \,\middle|ND_p \left(X_k^{ND}\right) = 1\right\}$ are actually nonfuzzy nondominated and can be considered as the nonfuzzy solution for the choice problem (Orlovsky 1981).

Expressions (5–7) may be used to solve choice or ranking problems not only with a single criterion, but also with multiple criteria. In particular, several procedures that allow one to include multiple criteria in the analysis of a decision making problems are discussed in Ekel and Schuffner Neto (2006). Let us consider one of them.

Having at hand nonstrict preference relations for each criterion, one possible procedure for solving multicriteria problems consists in obtaining a global relation through the intersection of those relations as follows:

$$G = R_1 \cap \ldots \cap R_q. \tag{8}$$

The use of intersection to aggregate all criteria is suitable, when it is a necessary condition that a good alternative X_k must simultaneously satisfy F_1 and F_2 and …and F_q. Among the t-norm operators, the use of the min operator, as proposed in Orlovsky (1981), allows one to construct the global fuzzy nonstrict preference relation

$$G(X_k, X_l) = \min\{R_1(X_k, X_l), \ldots, R_q(X_k, X_l)\}, \tag{9}$$

under a completely non-compensatory approach for multicriteria decision making, in the sense that the high satisfaction of some criteria does not relieve the remaining ones from the requirement of being satisfied (there is no compensation among the criteria). Such approach is also considered pessimistic, since it gives emphasis to the worst evaluations of each alternative (such aspect may be particularly advantageous to strategic planning, when it is possible to permit the professionals in charge of each project to review and improve them). On the other hand, the use of the union operation is also admissible if each alternative is supposed to satisfy at least one criterion. In other words, when it is sufficient to require that X_k satisfies F_1 or F_2 or … or F_q. For instance, the use of max operator to implement the union operation results in an extremely compensatory approach, in the sense that the high level of satisfaction of any criterion is sufficient (independently of which criterion is highly satisfied). Finally, it can be useful to apply the so-called ordered weighted aggregation operator (Grabisch et al. 1998), which can produce a result that is more compensatory than min or that is less compensatory than max, under a proper adjustment of its weights.

Having at hand the global fuzzy nonstrict preference relation, Eqs. (5–7) can be subsequently applied directly to G and the result corresponds to a fuzzy set of nondominated alternatives fulfilling the role of a Pareto set (Orlovsky 1981). When two or more alternatives are considered undistinguishable, this set can be contracted, in a subsequent analysis, where the importance of each relation R_p, $p = 1, \ldots, q$ is differentiated through their weighted aggregation

$$T(X_k, X_l) = \sum_{p=1}^{q} \lambda_p R_p(X_k, X_l). \tag{10}$$

In (10), the weights (or importance factors) of each criterion must satisfy the conditions: $\lambda_p \geq 0$, $p = 1, \ldots, q$; $\sum_{p=1}^{q} \lambda_p = 1$. With the construction of $T(X_k, X_l)$, it is possible to obtain the membership function $ND_T(X_k)$ of the set of nondominated alternatives, following a procedure similar to the one described above for the global relation $G(X_k, X_l)$, which involves Eqs. (5–7). Finally, the nondominance level can be obtained on the basis of the intersection

$$ND_W(X_k) = \min\{ND_G(X_k), ND_T(X_k)\}, \tag{11}$$

which allows one to obtain the following set of fuzzy nondominated alternatives:

$$X^{ND} = \left\{ X_k^{ND} \in X \left| ND_W\left(X_k^{ND}\right) = \sup_{X_k \in X} ND_W(X_k) \right. \right\}. \tag{12}$$

3 Aggregation Procedures for the Analysis of Group Decision Making Problems

When the multicriteria analysis of a decision making problem is carried out under group settings, a team of experts $E_1, E_2, \ldots, E_v$ is supposed to cooperate within the decision process. Next, we present three different aggregation approaches for dealing with the input of multiple experts under these circumstances (Ekel and Parreiras 2009).

- *Aggregation of individual evaluations* (AIE). As represented in a diagram of Fig. 1, the experts are supposed to evaluate each alternative by forming fuzzy or linguistic estimates. Afterwards, the estimates provided by each expert for each alternative, and taking into account each criterion, are aggregated into some collective estimates. Having at hand an evaluation matrix of the alternatives, it is possible to construct fuzzy preference relations per criterion by means of Eqs. (1) and (2), then, one of methods for analyzing $< X, R >$ models can be utilized.
- *Aggregation of individual preferences per criterion* (AIC). As can be seen in the diagram of Fig. 2, the experts can provide their preferences for each criterion, using any preference format that can be transformed into fuzzy preference relations (Herrera-Viedma et al. 2002; Pedrycz et al. 2010). Then, the information is made uniform, in terms of fuzzy preference relations. The resulting matrices, obtained for each expert, are aggregated into collective preferences per criterion. Having a collective fuzzy preference relation per criterion at hand, it is possible to apply a method for multicriteria analysis of $< X, R >$ models to solve the problem and obtain a ranking of the alternatives.
- *Aggregation of individual results* (AIR). The multicriteria decision making problem is solved by each member of the group and, then, the individual results are combined into a collective result, as it is illustrated in a diagram of Fig. 3. When AIR is used, *a priori*, each DM is allowed to select a different multicriteria decision making method to solve the problem. However, as each method has its own fundaments and underlying principles for addressing the problem, it is expected that each of them may produce different results for the same problem and the same input of preferences. In this context, the use of different methods may increase the dissimilarities among results and, consequently, can make it harder to construct satisfactory results of aggregation.

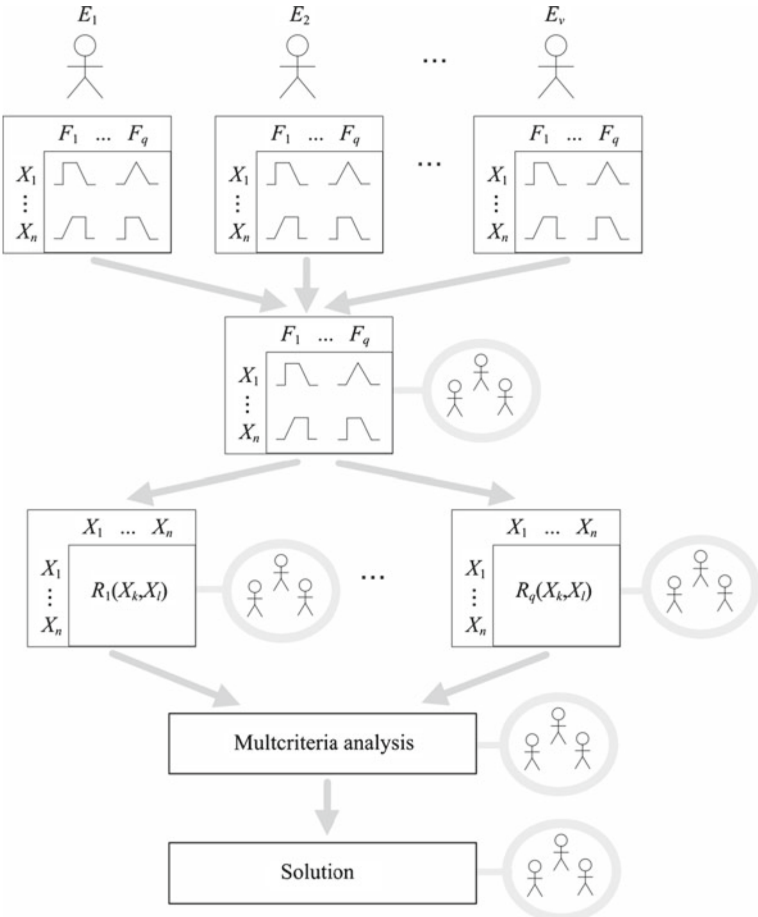

Fig. 1 Aggregation of individual evaluations (adapted from Pedrycz et al. (2010))

When comparing the three strategies, it is possible to observe that their main differences are associated with: the different points along the process of the multicriteria analysis in which the aggregation of the opinions of the multiple experts is realized, the character of numerical values being aggregated (fuzzy sets, fuzzy preference relations, or fuzzy nondominance degrees), and the way the group members are handled (as a synergetic unique individual or a collection of individuals). Refer to Table 1 for a summary of the main differences among the three strategies, taking into account these three aspects.

4 Constructing Collective Opinions on the Basis of Fuzzy Models

Depending on the selected strategy, AIE, AIC, or AIR, the aggregation of individual contributions is performed on the basis of different fuzzy models namely fuzzy

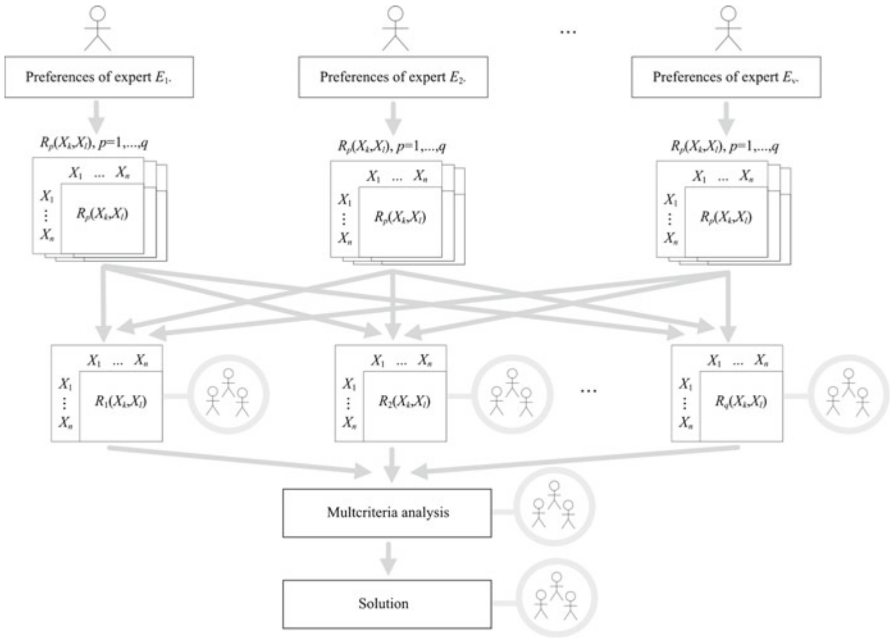

Fig. 2 Aggregation of individual preferences per criterion (a modified version of a schema shown in Pedrycz et al. (2010))

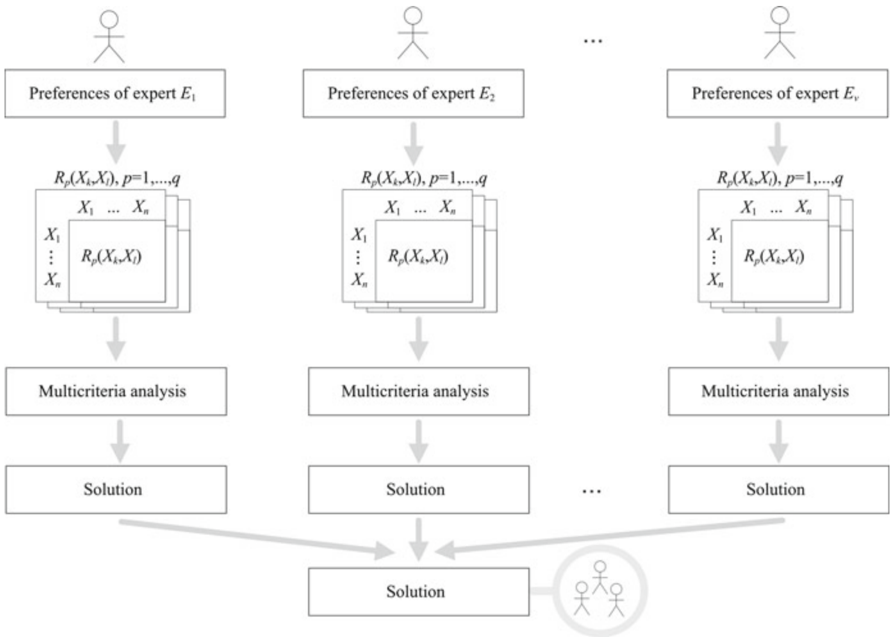

Fig. 3 Aggregation of individual results (adapted from Pedrycz et al. (2010))

Table 1 Summary of the main differences between AIE, AIC, and AIR

	Moment of aggregation	Aggregated units	Group management
AIE	Subsequent to the input of the opinions of all experts.	Fuzzy sets	Unique individual
AIC	Subsequent to the transformation of the opinions or preferences provided by all experts to fuzzy preference relations.	Fuzzy preference relations	Unique individual
AIR	After the problem is solved by each expert.	Fuzzy nondominance degrees	Collection of individuals

estimates, fuzzy preference relations, or the cardinal rating of each alternative, reflected by their respective fuzzy nondominance levels.

First, let us consider strategy AIE, which requires each DM to express his/her opinion using fuzzy or linguistic estimates. Given a particular alternative $X_k \in X$, all evaluations for a specific criterion F_p are represented as fuzzy sets $\tilde{F}_p^y(X_k)$, $y = 1, \ldots, v$, in the same universe of discourse, but possibly having different membership functions. The collective opinion $\tilde{F}_p^C(X_k)$ is commonly obtained by applying the weighted arithmetic mean to combine the estimates provided by each expert $\tilde{F}_p^y(X_k)$, y $= 1, \ldots,$ v, into a collective estimate

$$\tilde{F}_p^C(X_k) = \sum_{y=1}^{v} w_y \otimes \tilde{F}_p^y(X_k), \tag{13}$$

where $\otimes$ is the fuzzy multiplication operator, $X_k \in X, 0 \leq w_y \leq 1, y = 1, \ldots, v$, and $\sum_{y=1}^{v} w_y = 1$ (Kauffman and Gupta 1985).

In AIC, the operation allowing one to generate collective information is performed over the fuzzy preference relations per criterion (before the aggregation across all criteria is performed). Hence, given the pth criterion, the operation of aggregation makes use of a function that maps a set of fuzzy nonstrict preference relations $R_p^= \left\{R_p^1, \ldots, R_p^v\right\}$, to another fuzzy nonstrict preference relation R_p^C. When the weighted arithmetic mean is utilized, we have

$$R_p^C(X_k, X_l) = \sum_{y=1}^{v} w_y R_p^y(X_k, X_l), \tag{14}$$

where $X_k, X_l \in X, 0 \leq w_y \leq 1, y = 1, \ldots, v$, and $\sum_{y=1}^{v} w_y = 1$.

In AIR, regardless of the multicriteria decision method selected by each expert to solve the problem, the individual results invariably correspond to the fuzzy nondominance degree associated with each alternative, which can be taken as cardinal ratings of each alternative. The use of the weighted arithmetic mean in this context leads to

$$ND^C(X_k) = \sum_{y=1}^{v} w_y ND^y(X_k), \tag{15}$$

where $X_k \in X$, $0 \leq w_y \leq 1$, for $y = 1, \ldots, v$, and $\sum_{y=1}^{v} w_y = 1$.

5 Fuzzy Concordance and Fuzzy Consensus Measures

Even in a friendly environment, the occurrence of significant disagreements across the group is inevitable and aggregating highly discordant opinions may result in an intermediate opinion with which none expert agree. Taking this into consideration, it would be a very simplistic approach to just combine the individual preferences into a collective preference, without first trying to minimize the divergence among opinions. Indeed, the recognized importance of achieving a satisfactory level of concordance among the experts has motivated the development of consensus schemes for increasing the efficiency of the discussion among experts. As already mentioned, within the framework of consensus schemes, certain indices are made available in order to support the moderator in the coordination of the discussion process. Next, we consider different indices of consensus and indices of concordance that can be utilized for comparing the preferences of each expert expressed in terms of fuzzy estimates, fuzzy preference relations, and fuzzy nondominance degrees.

The concordance index is a function that quantifies the level of similarity or correspondence between any pair of opinions. For practical purposes, it is supposed to satisfy some conditions (García-Lapresta 2008):

- it achieves its maximum value, only if both opinions are identical;
- the value of the index depends on the agreement between two opinions, regardless of which expert is the person responsible for each opinion.

The main use of a concordance index within the framework of a consensus scheme is associated with the identification of the least concordant expert in each cycle of the discussion process. As already mentioned, this expert is supposed to review his/her opinion or to explain his/her opinion to the group. With the use of such an index, it is possible to calculate the level of concordance between the current opinion of each expert and the group temporary opinion, in order to identify who is the expert with the most discrepant opinion within the group.

The consensus index is another valuable tool, which can be utilized by a moderator for controlling the discussion process. It is modeled as a function that quantifies, in the unit interval, how far a group of experts is from perfect unanimity. A Boolean (two-valued) notion of consensus does not allow one to distinguish different levels of concordance among experts. Such discrimination is important due to the fact that, in practice, the discussion process towards a consensus is frequently interrupted before a perfect concordance among experts is achieved, considering that the perfect consensus is almost unachievable. In this way, with the use of a consensus index, it becomes possible to interrupt the discussion process at an earlier moment.

Here, 1 corresponds to full and unanimous concordance; 0 corresponds to nonexistence of concordance. Intermediate values, between 0 and 1, are also possible to reflect levels of partial agreement among all experts.

First, let us consider indices for comparing preferences expressed in terms of fuzzy estimates. In this case, we can utilize a concordance index which reflects the level of similarity between a pair of fuzzy estimates and a consensus index that reflects the mean level of similarity among a collection of fuzzy estimates. The concordance index proposed in Lu et al. (2006), which combines both fuzzy distance and fuzzy similarity concepts, allows a fair comparison between a pair of fuzzy estimates.

The weighted similarity between the fuzzy estimate $\tilde{F}_p^y(X_k)$ provided by the yth expert, and the collective fuzzy estimate $\tilde{F}_p^C(X_k)$ is given in Hsu and Chen (1996) as follows:

$$S_w^y\left(\tilde{F}_p^y(X_k), \tilde{F}_p^C(X_k)\right) = \frac{\int_x \left(\min\left\{F_p^y(X_k), F_p^C(X_k)\right\}\right) dX}{\int_x \left(\max\left\{F_p^y(X_k), F_p^C(X_k)\right\}\right) dX}, \tag{16}$$

where the ratio between two integrals reflects the proportion of the concordant area $\int_x \left(\min\left\{F_p^y(X_k), F_p^C(X_k)\right\}\right) dX$ to the total area $\int_x \left(\max\left\{F_p^y(X_k), F_p^C(X_k)\right\}\right) dX$. In Lu et al. (2006), an improved version of the similarity index (16), is proposed in order to allow differentiating the level of similarity between certain fuzzy numbers. Here we utilize this improved similarity index, which is given by

$$S_w^y\left(\tilde{F}_p^y(X_k), \tilde{F}_p^C(X_k)\right) = \frac{\int_x \left(\min\left\{F_p^y(X_k), F_p^C(X_k)\right\}\right)^2 dX}{\int_x \left(\max\left\{F_p^y(X_k), F_p^C(X_k)\right\}\right)^2 dX}. \tag{17}$$

The distance between $\tilde{F}_p^y(X_k)$ and $\tilde{F}_p^C(X_k)$ can be calculated with the following expression (Lu et al. 2006):

$$D_h\left(\tilde{F}_p^y(X_k), \tilde{F}_p^C(X_k)\right) = \frac{1}{2}\left[\int_x^{|} F_p^y(X_k) - F_p^C(X_k)| dX + d_{\text{inf}}\left(\tilde{F}_p^y(X_k), \tilde{F}_p^C(X_k)\right)\right]. \tag{18}$$

In (18), the integral corresponds to the Hamming distance between $\tilde{F}_p^y(X_k)$ and $\tilde{F}_p^C(X_k)$ and the term d_{inf} is calculated as follows:

$$d_{\text{inf}} = \inf\left\{d(a,b), a \in \tilde{F}_p^y(X_k), b \in \tilde{F}_p^C(X_k)\right\}, \tag{19}$$

where $d(a,b)$ is the absolute value of the difference between a and b, which are scalar values. Supposing that the membership functions of $\tilde{F}_p^y(X_k)$ and $\tilde{F}_p^C(X_k)$ correspond to the trapezoidal fuzzy numbers $\{a_1, a_2, a_3, a_4\}$ and $\{b_1, b_2, b_3, b_4\}$

(Pedrycz and Gomide 1998), then $d_{\inf} = \inf\{d(a,b), a \in [a_1, a_4], b \in [b_1, b_4]\}$. The inclusion of the term $d_{\inf}$ in the distance metric from (18) is important to adequately handle the cases where the fuzzy estimates have no intersection.

Finally, the concordance level between $\tilde{F}_p^y(X_k)$ and $\tilde{F}_p^C(X_k)$ consists (Lu et al. 2006) in a linear aggregation of the distance and the weighted similarity metrics

$$S_{FE}^y\left(\tilde{F}_p^y(X_k), \tilde{F}_p^C(X_k)\right) = \beta S_w\left(\tilde{F}_p^y(X_k), \tilde{F}_p^C(X_k)\right) + (1-\beta)\left(1 - \bar{D}_h\left(\tilde{F}_p^y(X_k), \tilde{F}_p^C(X_k)\right)\right), \tag{20}$$

where the parameter β, defined in the range $0 \leq \beta \leq 1$, allows S_w to have a certain level of influence on the concordance value.

In (20), $\bar{D}_h$ is the normalized distance calculated as follows:

$$\bar{D}_h = \frac{D_h\left(\tilde{F}_p^y(X_k), \tilde{F}_p^C(X_k)\right)}{\max\{D_h\}}, \tag{21}$$

being $\max\{D_h\}$ the maximum possible distance between two extreme fuzzy estimates as proposed in Bernardes et al. (2009). This maximum distance will depend on the universe of discourse being considered. It is worth mentioning that, this normalization usually facilitates to empirically fix β as it guarantees that $0 \leq \bar{D}_h \leq 1$.

The consensus level across the group per alternative can be calculated as the arithmetic average

$$C(X_k) = \frac{\sum_{y=1}^{v} S_{FE}^y\left(\tilde{F}_p^y(X_k), \tilde{F}_p^C(X_k)\right)}{v}. \tag{22}$$

Now, let us consider indices for comparing preferences expressed in terms of fuzzy preference relations. A simple manner of measuring the concordance level between the collective preferences and the preferences provided by an expert E_y consists (Alonso et al. 2007) in calculating, for each pair of alternatives X_k and X_l, the differences

$$S^y(X_k, X_l) = 1 - \left|R^y(X_k, X_l) - R^C(X_k, X_l)\right|, \tag{23}$$

where R^y corresponds to the fuzzy preference relation supplied by E_y and R^C corresponds to the aggregated (or collective) fuzzy preference relation.

The concordance level per alternative can be calculated for the yth expert with the use of the following expression:

$$Sa^y(X_k) = \frac{\sum_{l=1;l\neq k}^{n} S^y(X_k, X_l) + S^y(X_l, X_k)}{2(n-1)}. \tag{24}$$

The mean level of concordance per relation can be calculated for the yth expert as follows:

$$Sp^y = \frac{\sum_{k=1}^{n} Sa^y(X_k)}{n}. \tag{25}$$

Equation (23) can be utilized to measure the concordance level between the preferences of two experts, namely E_y and E_z, as follows:

$$S^{yz}(X_k, X_l) = 1 - \left| R^y(X_k, X_l) - R^z(X_k, X_l) \right|. \tag{26}$$

By aggregating the similarity matrices S^{yz}, $z = 1, \ldots, v$, $y < z$, for every pair of experts, with the use of the arithmetic mean operator, one determines the consensus level for a pair of alternatives

$$Ca(X_k, X_l) = \frac{\sum_{z=1}^{v} \sum_{y=1 \wedge y>z}^{v} S^{yz}(X_k, X_l)}{(v-1)(v-2)} \tag{27}$$

or the consensus level for a unique alternative

$$Ca(X_k) = \frac{\sum_{l=1, l \neq k}^{n} Ca(X_k, X_l) + Ca(X_l, X_k)}{2(n-1)} \tag{28}$$

or the consensus level (Alonso et al. 2007) for a single relation

$$Cr = \frac{\sum_{k=1}^{n} Ca(X_k)}{n}. \tag{29}$$

Now, let us consider indices of concordance and of consensus adequate for comparing the opinions of different experts, expressed in terms of the degree of fuzzy nondominance of each alternative. As discussed in Herrera-Viedma et al. (2002), different cardinal results (which correspond to the degree of fuzzy nondominance of each alternative) may lead to the same ranking of the alternatives. For instance, both $ND^1(X_k) = [0.8\ 1\ 0.6\ 0.7]$ and $ND^2(X_k) = [0.81\ 0.9\ 0.3\ 0.57]$ result in $X_2 \succ X_1 \succ X_4 \succ X_3$. Although there is no a perfect concordance of the cardinal rating of each alternative, there may be a perfect consensus on their ranking. For this reason, it is more reasonable to compare the final results of the multicriteria analysis by taking into account the final ranking of the alternatives and not their respective fuzzy nondominance degrees.

Therefore, now, let us assume that we have at hand the ranking of all alternatives, obtained in accordance with the opinion of the group as a whole, as well as with the opinion of each expert individually. One way of measuring the concordance between the rankings of all alternatives is by using a decreasing function as the one below (Herrera-Viedma et al. 2002):

$$Sr^y(X_k) = 1 - \left(\frac{\left| B_k^C - B_k^y \right|}{n-1} \right)^b. \tag{30}$$

In expression (30), B_k^C and B_k^y correspond to the position of the kth alternative, taking into account the collective results (which should be previously calculated with the use of (15)) and the results obtained by expert E_y, respectively. It is important to indicate that, when two or more alternatives are considered undistinguishable because they present exactly the same level of fuzzy nondominance, the value of B_k for each of them is given by the mean value of the positions that they would have if they were ranked in the descending order of their values. For instance, if alternatives X_4 and X_5 are in the second position in the ranking, then $B_4 = B_5 = (2+3)/2 = 2.5$. Besides, it should be indicated that the constant b can assume any value in the interval [0, 1]. Particularly, when b is close to 1, the concordance measure is less rigorous than when b is close to 0. It should be empirically specified considering that a lower value of b corresponds to a higher number of rounds required to achieve a consensus. In Herrera-Viedma et al. (2002), it is suggested the use of 0.5, 0.7, 0.9, or 1.

The average level of concordance for each expert is calculated as follows:

$$\bar{S}r^y = \frac{\sum_{k=1}^{n} Sr^y(X_k)}{n}. \tag{31}$$

The consensus level of all experts on each alternative can be determined (Herrera-Viedma et al. 2002) as

$$Cr(X_k) = \frac{\sum_{y=1}^{v} Sr^y(X_k)}{v}. \tag{32}$$

The mean level of consensus taken over all alternatives is given by

$$\bar{C}r = \frac{\sum_{k=1}^{n} Cr(X_k)}{n}. \tag{33}$$

6 The Selection of Aggregation Strategies for a Coordination Mode

In the current literature, we can find several works where one of the aggregation procedures AIE, AIC, or AIR is applied to solve group decision making problems on the basis of analyzing $< X, R >$ models. In Table 2, the works dedicated to the application of the procedures AIE, AIC, and AIR are classified. The results of their comparative study for their separate (or individual) utilization (in different implementations) for solving strategic planning problems are given in Ekel and Parreiras (2009); Pedrycz et al. (2010). It is necessary to indicate that AIC is the most common approach when we consider methods based on applying $< X, R >$ models. At the same time, although AIR is a well-established aggregation procedure (Forman and Peniwati 1998), we could not find many examples of its applications within the framework of $< X, R >$ models.

In this paper, we propose a methodology that makes use of the three aggregation procedures (and three consensus schemes, each one based on the corresponding aggregation procedure) in different rational combinations that creates an efficient manner of exploiting the capabilities of each member of the group in a cooperative work.

Table 2 Examples of papers based on the different aggregation procedures

Aggregation procedures	References
AIE	Li (1999), Bernardes et al. (2009), Ekel and Parreiras (2009), Parreiras et al. (2010), Pedrycz et al. (2010)
AIC	Herrera-Viedma et al. (2002), Herrera-Viedma et al. (2007), Wang and Fan (2007), Wang and Parkan (2008), Ekel and Parreiras (2009), Ekel et al. (2009), Pedrycz et al. (2010)
AIR	Ekel and Parreiras (2009), Pedrycz et al. (2010)

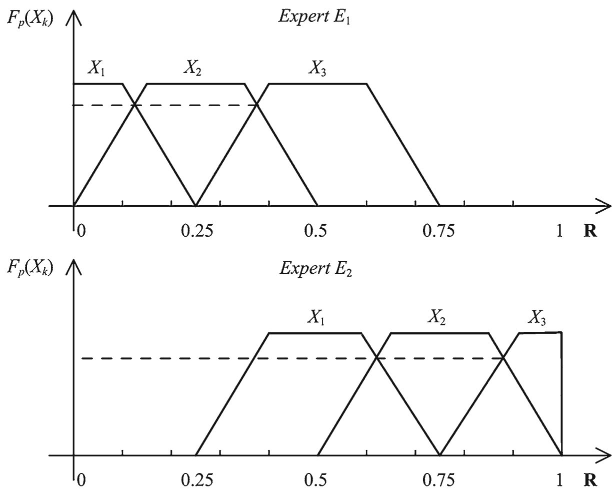

Fig. 4 Example of evaluations through fuzzy estimates that results in similar fuzzy preference relations

Three fundamental aspects should be analyzed in the selection of those aggregation strategies that are adequate for a specific real-world problem.

The first one to be considered here is associated with the need of considering a standardized level of exigency for a criterion. AIE should not be utilized when the experts can have and do have very different levels of exigency. As it is shown in Fig. 4, two experts, namely E_1 and E_2, being the former more exigent than the latter, evaluated the alternatives X_1, X_2, and X_3, using fuzzy estimates. In AIE, the concordance index (20) may indicate a substantial difference between their opinions. In AIC, if the concordance index given by (23) is utilized to compare the fuzzy preference relations derived from these fuzzy estimates, it will indicate a high concordance among them. It is a consequence of the fact that although the fuzzy estimates provided by each expert for evaluating each alternative are significantly different, they result in similar preference strengths of one alternative over another, when they are utilized to construct fuzzy preference relations with the use of expressions (1) and (2).

Considering the fact that it is not always simple to define a standardized level of exigency for every criterion of a multicriteria decision making problem and that, for practical purposes, it may be sufficient to achieve a consensus on the ranking of the alternatives, in some cases, it may not be worthy to use AIE.

The second aspect concerns the observed fact that each expert frequently prefers a different format to express his/her opinions. None DM should be pressed to use a specific preference format against his/her will; otherwise, the input of preference information can become a very critical step in the group decision analysis. Due to the difficulty in assessing preference levels or in understanding particularities of a preference format, a DM may provide data that does not correspond to the true state of affair, which, obviously, reduces the soundness of results. The selection of an adequate format should be considered subjective and personal, being made taking into account several aspects, such as easy assessment, intuitive appeal, and acceptable precision level.

Does any expert want to use a different preference structure to analyze each criterion? Do any of them disagree at using fuzzy or linguistic estimates to evaluate each alternative? An affirmative answer for any one of those questions suggests discarding AIE as an admissible strategy, but does not provide enough reasons or incentives to decide between AIC and AIR.

The third aspect can be briefly summarized as the following question: can the group be considered as a synergistic unit or can it be managed as a collection of independent individuals (Forman and Peniwati 1998)? In practice, if it can be assumed that each member has perfect knowledge of the problem, it may be interesting to allow each member to completely solve the problem using AIR. But, if each member has only partial knowledge about the problem, it is more reasonable to manage the group as a unique individual, using AIE or AIC. If any member of the group is incapable of evaluating all the alternatives for a criterion, the use of AIE is recommended. Otherwise, AIC may be the most natural choice when a different expert (or a different set of experts) is supposed to analyze each criterion, as AIC allows a more flexible input of preferences (with the use of different preference formats) than AIE.

The main aspects discussed above can be summarized in the procedure represented in Fig. 5, which may help one in the selection of the aggregation strategies to be applied to each criterion, in order to implement a suitable coordination mode among the experts involved in the decision making process.

Having selected an aggregation strategy to deal with each criterion (in accordance with the procedure shown in Fig. 5), the discussion process among the experts can be initiated. The diagrams shown in Figs. 6, 7, and 8 represent three consensus schemes (in their respective simplified versions), which are implemented on the basis of strategies AIE, AIC, and AIR, respectively.

It is worth noting that the output data of the discussion process based on AIE (refer to Fig. 6) is a set of collective fuzzy estimates which can be utilized to produce a collective fuzzy preference relation with the use of expressions (1) and (2). The output data of the discussion process based on AIC (refer to Fig. 7) is a collective fuzzy preference relation constructed by aggregating the fuzzy preference relation provided by each expert. Therefore, those two consensus schemes, which are based on the strategies AIE and AIC, allow the construction of collective fuzzy preference relations, and

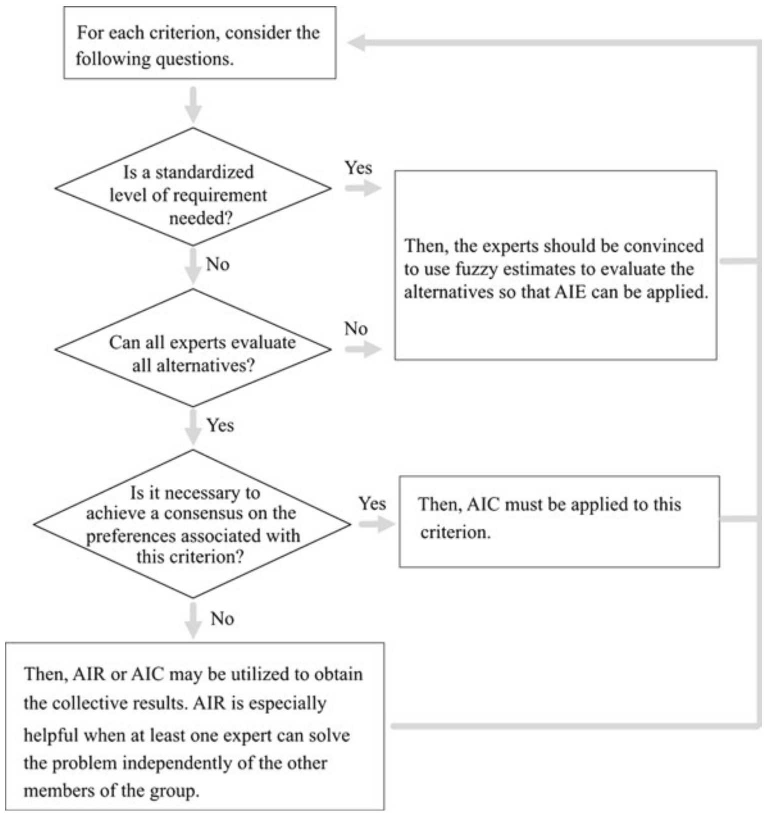

Fig. 5 Selection of aggregation strategies to implement a coordination mode

leave the multicriteria analysis to be accomplished later. On the other hand, the output data of the discussion process based on AIR (refer to Fig. 8) is a collective fuzzy set of nondominated alternatives from which a recommendation for the decision problem is directly assessed. Taking the above into account, when AIR is to be utilized together with AIE or/and AIC, it is valuable to begin the group decision making process by considering the criteria associated with AIE and AIC and leave to the end of the group decision making process the criteria that are to be dealt with AIR. In this manner, as it is shown in the application example presented in Sect. 7, it is possible to make use of the collective fuzzy preference relations previously obtained by means of AIE or by means of AIC to generate the final results of AIR (this idea is clarified through the application example). Besides, by doing in this manner, AIR can be utilized to guarantee a consensus on the final recommendation for the decision problem.

7 Problem Description

As a case study to demonstrate the practicality of the proposed methodology based on the aggregation approaches AIE, AIC, and AIR, let us consider BYTE Electronics

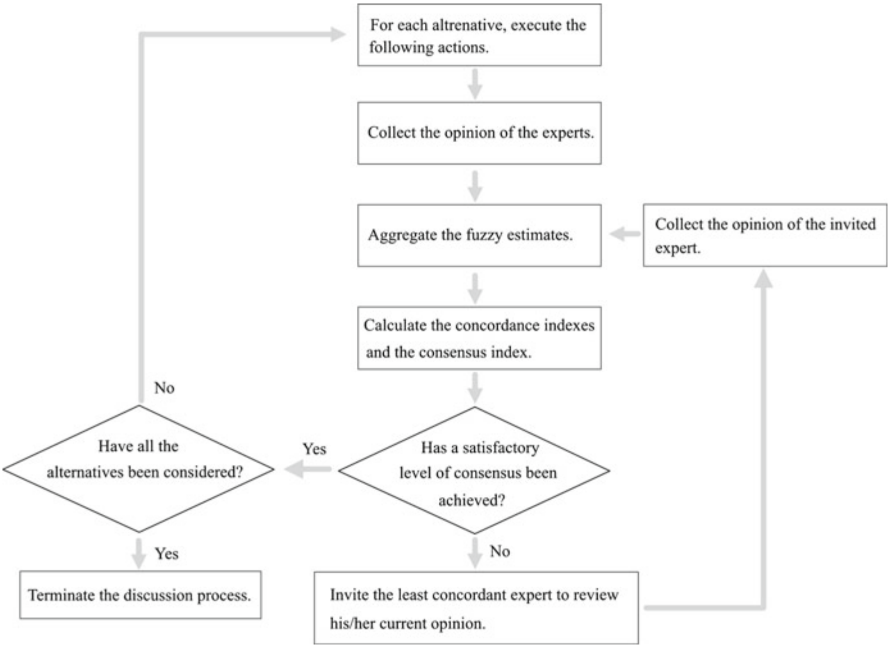

Fig. 6 A consensus scheme based on AIE

Technologies Corporation, which is the main producer of electronics devices in the southeast region of Brazil. It attains the market demand of four distribution centers with the production based at one plant in Belo Horizonte. In the present era of rapid technological change, technological aspects and customer satisfaction need to be explicitly considered in strategic planning, besides the evaluation of the strengths and weaknesses with respect to a firm's technological capability.

The enterprise's board of directors is to plan the development of large projects (strategy initiatives) for the following 5 years. This board of directors includes five members:

E_1 : financial manager;
E_2 : marketing manager;
E_3 : production manager;
E_4 : human resources manager;
E_5 : research and development manager.

They had a meeting for discussion about the future of the organization and have been marked four possible projects:

Strategy X_1 : to outsource certain activities traditionally done in-house;
Strategy X_2 : to redesign products in order to satisfy customer requirements;
Strategy X_3 : to adopt new technologies to be used in production phase, in order to improve the production process by solving some of the existing operational problems;

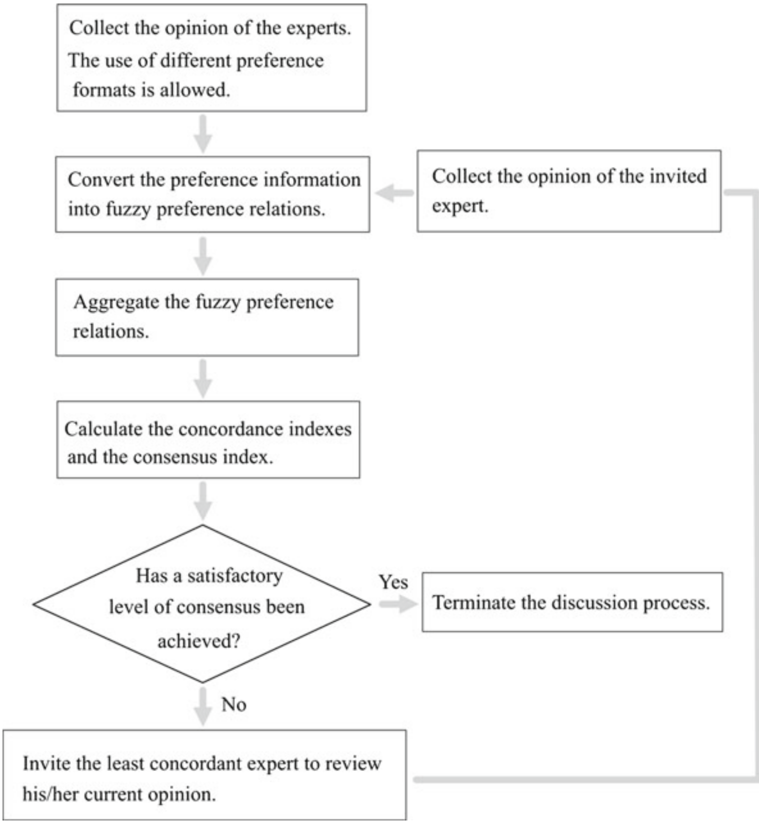

Fig. 7 A consensus scheme based on AIC

Strategy X_4 : to improve the quality of after-sales services by widening service network.

It is necessary to compare these projects to order them from the point of view of their importance, taking into account four criteria (categories) suggested by the Balanced Scorecard methodology (Kaplan and Norton 1996):

F_1 financial perspective,
F_2 the customer satisfaction,
F_3 internal business process perspective,
F_4 learning and growth perspective.

The group of experts involved in the strategic planning has some specific characteristics:

- In the consideration of F_1, expert E_2 does not feel capable of giving an opinion on alternatives X_1 and X_4. The other experts E_1, E_3, E_4, and E_5 can evaluate all alternatives. Besides, the group agrees that a standardized level of requirement should be utilized to make an economical evaluation of the alternatives.

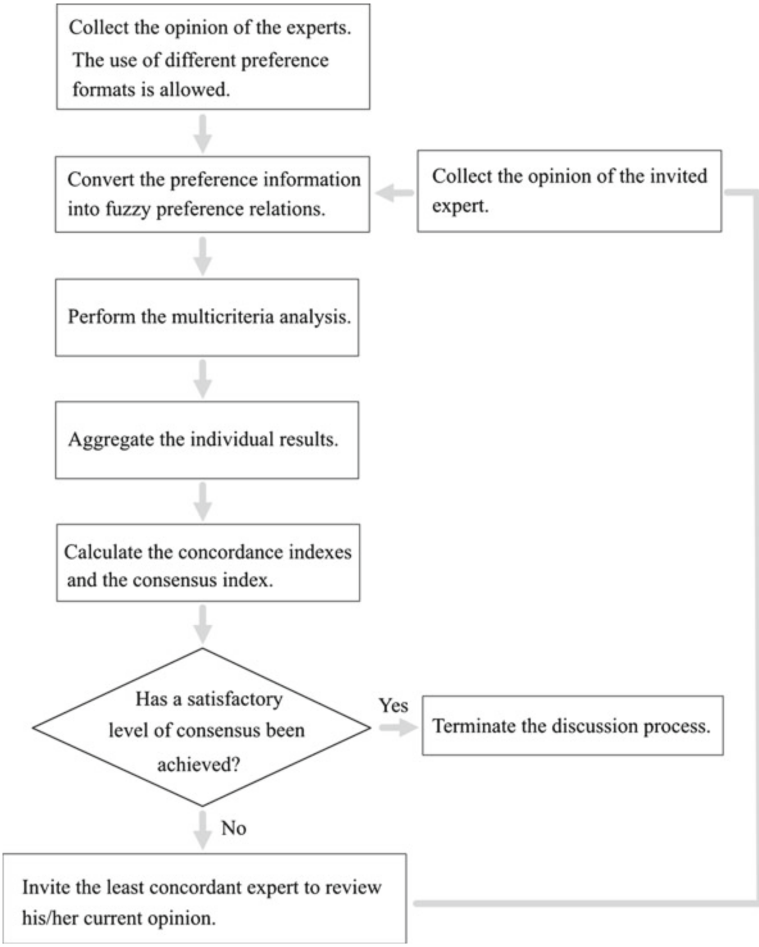

Fig. 8 A consensus scheme based on AIR

- Expert E_4 feels capable of considering only criterion F_1.
- Experts E_1, E_2, E_3, and E_5 are capable of evaluating all alternatives for the criteria F_2, F_3, and F_4.

Taking all those particularities into account, the following line of attack is utilized to obtain a decision for the strategic planning problem:

- In the consideration of F_1, a consensus scheme based on the AIE approach is utilized to obtain a collective fuzzy preference relation as follows: whereas experts E_1, E_3, E_4, and E_5 participate in the evaluation of all alternatives, E_2 just provides the evaluation of X_2 and X_3.
- In the consideration of F_2, as each expert wants to use a different format to articulate his/her preferences and it is not essential to consider a standardized level of requirement for F_2, a consensus scheme based on the AIC approach is

utilized. Besides, the group counts on only four members, as E_4 is not capable of considering criteria F_2, F_3, and F_4.

- A consensus scheme based on the AIR approach is utilized in order to guarantee a satisfactory level of consensus on the results. It this way, a priori, the experts E_1, E_2, E_3, and E_5 are not supposed to meet a consensus on their opinions when criteria F_3 and F_4 are considered, unless the discordances associated with these criteria cause a low level of consensus on the ranking of the alternatives.

Here, a human moderator is necessary to guide the iterative discussion process among the experts, by determining in each cycle which expert is supposed to review his/her opinion and when the discussion process should be terminated. Besides, the moderator is also necessary to adjust the importance weights associated with the opinion of each expert. For the sake of simplicity, all weights are kept equal in the implementation of the three approaches.

7.1 Application of AIE for Considering the Financial Perspective

AIE permits the discussion among experts to be divided into sessions dedicated to each alternative separately. The moderator suggested experts E_1, E_3, E_4, and E_5 to begin by focusing on the evaluation of alternative X_1 from the perspective of criterion F_1 (financial perspective).

In this way, initially, the experts were invited to articulate their opinions about X_1 by means of the linguistic terms from the set shown in Fig. 9, which allows one to distinguish among five levels of performance in accordance with the following descriptions:

- Very low: a strategy with very low performance (from the financial perspective) involves a high investment with a low rate of return;
- Low: a strategy with low performance involves a small investment with a low rate of return;
- Average: a strategy with average performance involves a high investment with a high rate of return;
- High: a strategy with high performance involves a small investment with a high rate of return;

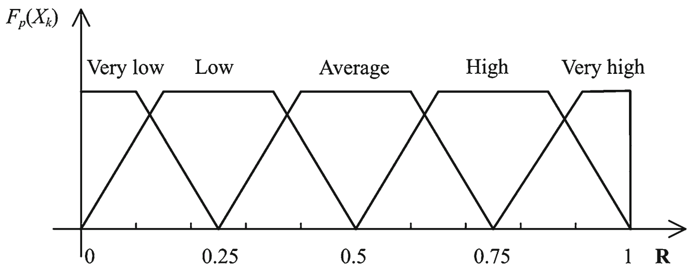

Fig. 9 Set of linguistic terms referring to the performance of each alternative, represented as fuzzy estimates

Table 3 Evaluation of alternative X_1 on criterion F_1, reported at the first iteration

	E_1	E_3	E_4	E_5	Group
X_1	Average	High	High	Low	(0.31, 0.46, 0.66, 0.81)

Table 4 Concordance level and consensus level, reported at the first iteration

	E_1	E_3	E_4	E_5	Consensus
Concordance level	0.79	0.52	0.52	0.37	0.55

Table 5 Evaluation of alternative X_1 on criterion F_1, reported at the second iteration

	E_1	E_3	E_4	E_5	Group
X_1	Average	High	High	Average	(0.37, 0.52, 0.72, 0.87)

Table 6 Concordance level and consensus level, reported at the second iteration

	E_1	E_3	E_4	E_5	Consensus
Concordance level	0.64	0.64	0.64	0.64	0.64

- Very high: a strategy with very high performance involves a small investment with a very high rate of return.

The preferences of each expert, as well as the collective preference, which is calculated with the use of (12), are shown in Table 3.

Table 4 shows the concordance level between the opinion of each expert and the collective opinion, calculated with the use of (19), as well as the consensus level estimated using (20). It should be indicated that here the β parameter in (19) was equal to 0.5. The moderator judged that the current level of consensus was unsatisfactory (he empirically defined that the level of consensus should be equal or higher than 0.6) and invited E_5 (the least concordant expert as it can be easily confirmed in Table 4) to review his/her position. After a new round of discussion, this expert changed his/her opinion about alternative X_1 from "low performance" to "average performance", which caused an alteration in the collective fuzzy estimate. Refer to Table 5 for the preferences of each expert and for the collective preferences in the second iteration of the discussion process. As it can be seen in Table 6, the index of consensus increased to 0.64, which the moderator considered an acceptable degree of consensus. In this way, the session was terminated.

In contrast with the situation described in this example, it should be indicated that the least concordant expert could also have moved his/her evaluation to a more divergent opinion or else could have kept his/her original position. In both cases, the moderator should invite this expert to try to persuade the other members of the group to modify their opinions and, in the subsequent round, the moderator should invite the second least concordant expert to review his/her opinion.

After the execution of a session like the one described above for each alternative, we obtain a satisfactory representation of the group preferences in terms of fuzzy estimates. It is important to notice that, for the alternatives X_2 and X_3, we evaluated the preferences from the five experts. Having at hand the collective evaluation of each alternative on criterion F_1 (refer to Fig. 10 for the obtained collective fuzzy estimates), the following fuzzy preference relation is constructed with the use of (1) and (2):

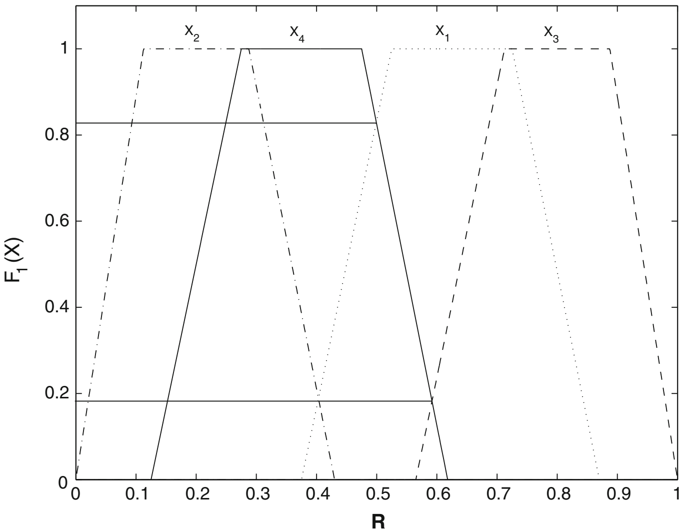

Fig. 10 Collective fuzzy estimates for criterion F_1

$$R_1^C = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.18 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0.83 & 1 & 0.18 & 1 \end{bmatrix} \tag{34}$$

7.2 Application of AIC for Considering the Consumer Satisfaction Perspective

Using the AIC approach, the experts are invited to evaluate all alternatives (in contrast with AIE, which allows different groups of experts to evaluate each alternative) but are allowed to evaluate only the criteria that they are capable to.

The moderator suggested experts E_1, E_2, E_3, and E_5 to select the preference format with which each of them feels more comfortable for expressing their preferences for criterion F_2. For the sake of brevity, we omit here the preference elicitation process and consider that the information provided by each expert has already been converted to the following fuzzy preference relations:

$$R_2^1 = \begin{bmatrix} 1 & 0.27 & 1 & 0.6 \\ 1 & 1 & 1 & 1 \\ 0.7 & 0 & 1 & 0.07 \\ 1 & 0.7 & 1 & 1 \end{bmatrix}, \quad R_2^2 = \begin{bmatrix} 1 & 0.07 & 0.7 & 0.33 \\ 1 & 1 & 1 & 1 \\ 1 & 0.6 & 1 & 0.83 \\ 1 & 0.78 & 1 & 1 \end{bmatrix},$$

$$R_2^3 = \begin{bmatrix} 1 & 0 & 0.17 & 0.5 \\ 1 & 1 & 1 & 1 \\ 1 & 0.6 & 1 & 1 \\ 1 & 0 & 0.6 & 1 \end{bmatrix}, \quad R_2^5 = \begin{bmatrix} 1 & 0 & 0.6 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0.6 \\ 1 & 0 & 1 & 1 \end{bmatrix}.$$

Table 7 Level of concordance per alternative calculated for each expert and level of consensus per alternative, calculated at the first iteration

	X_1	X_2	X_3	X_4
$Sa^1(X_k)$	0.83	0.87	0.74	0.80
$Sa^2(X_k)$	0.97	0.88	0.87	0.88
$Sa^3(X_k)$	0.88	0.87	0.75	0.80
$Sa^5(X_k)$	0.91	0.87	0.91	0.86
$Ca(X_k)$	0.82	0.82	0.71	0.74

Having at hand the fuzzy nonstrict preference relation per criterion provided by each expert, it is possible to obtain a representation of the group preferences per criterion by means of (14). The collective fuzzy nonstrict preference relation related to F_2 is given by:

$$R_2^C = \begin{bmatrix} 1 & 0.09 & 0.62 & 0.36 \\ 1 & 1 & 1 & 1 \\ 0.93 & 0.3 & 1 & 0.63 \\ 1 & 0.37 & 0.9 & 1 \end{bmatrix}.$$

Table 7 shows the level of concordance per alternative for each expert, which is calculated by means of (23) and (24), as well as the level of consensus per alternative, calculated with the use of Eqs. (26,27,28). The calculation of the current mean level of consensus by means of (29) yields $Cr = 0.78$. As the moderator empirically determined that the level of consensus per relation should be equal or higher than 0.8, the least concordant expert is invited to review his/her opinion. Having analyzed data from Table 7, the moderator decided to invite E_1 to review his/her preference judgments associated with X_3 and X_4. After a new round of discussion, E_1 improved his/her evaluation of X_3 as it is reflected in the following fuzzy preference relation:

$$R_2^1 = \begin{bmatrix} 1 & 0.27 & 1 & 0.6 \\ 1 & 1 & 1 & 1 \\ 1 & 0.27 & 1 & 0.6 \\ 1 & 0.7 & 1 & 1 \end{bmatrix}.$$

As a consequence, the collective fuzzy preference relation was updated to

$$R_2^C = \begin{bmatrix} 1 & 0.08 & 0.62 & 0.36 \\ 1 & 1 & 1 & 1 \\ 1 & 0.37 & 1 & 0.76 \\ 1 & 0.37 & 0.9 & 1 \end{bmatrix}. \quad (35)$$

The level of concordance and consensus per alternative reported at the second iteration are shown in Table 8. The calculation of the current level of consensus with the use of (28) reveals an acceptable mean level of consensus $Cr = 0.82$. In this way, the moderator terminated the discussion process at the second iteration.

Table 8 Level of concordance per alternative calculated for each expert and level of consensus per alternative, calculated at the second iteration

	X_1	X_2	X_3	X_4
$Sa^1(X_k)$	0.87	0.90	0.88	0.87
$Sa^2(X_k)$	0.98	0.89	0.92	0.90
$Sa^3(X_k)$	0.89	0.89	0.80	0.82
$Sa^5(X_k)$	0.92	0.86	0.89	0.84
$Ca(X_k)$	0.88	0.83	0.80	0.79

After the execution of a session like the one described above for each criterion, it is possible to obtain a satisfactory representation of the group preferences in terms of fuzzy preference relations.

7.3 Application of AIR for Achieving a Consensus on the Ordering of the Alternatives

In AIR, the experts are invited to make a complete analysis of the problem and the group is supposed to achieve a consensus on the results. In this way, the discussion among the experts is based on the rank ordering of the alternatives.

Here, the AIR approach is applied as follows: experts E_1, E_2, E_3, and E_5 are supposed to adopt the collective fuzzy preference relations obtained for criteria F_1 and F_2, which are, respectively given by (34) and (35). Each expert is supposed to consider criteria F_3 and F_4 individually. For the sake of brevity, we omit the preference elicitation procedure. The individual preferences provided by each expert for criteria F_3 and F_4 are reflected by the following fuzzy preference relations:

- Expert E_1

$$R_3^1 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.6 & 1 & 0.83 & 1 \\ 0.77 & 1 & 1 & 1 \\ 0.07 & 0.7 & 0.33 & 1 \end{bmatrix}, \quad R_4^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0.8 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0.8 & 1 \end{bmatrix};$$

- Expert E_2

$$R_3^2 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0.8 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0.8 & 0 & 1 \end{bmatrix}, \quad R_4^2 = \begin{bmatrix} 1 & 0 & 0.5 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 0.5 & 1 & 0.5 \\ 1 & 1 & 1 & 1 \end{bmatrix};$$

- Expert E_3

$$R_3^3 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0.6 & 1 \\ 0.6 & 1 & 1 & 1 \\ 0 & 0.5 & 0.17 & 1 \end{bmatrix}, \quad R_4^3 = \begin{bmatrix} 1 & 0 & 0.8 & 0 \\ 1 & 1 & 1 & 0.8 \\ 1 & 0.8 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix};$$

Table 9 Importance weight of criteria

	E_1	E_2	E_3	E_4	E_5
λ_1	0.30	0.30	0.25	1.00	0.25
λ_2	0.20	0.30	0.25	–	0.20
λ_3	0.20	0.20	0.25	–	0.25
λ_4	0.30	0.20	0.25	–	0.30

- Expert E_5

$$R_3^5 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0.8 & 1 \\ 0.8 & 1 & 1 & 1 \\ 0 & 0.8 & 0 & 1 \end{bmatrix}, \quad R_4^5 = \begin{bmatrix} 1 & 0.17 & 0.5 & 0 \\ 1 & 1 & 1 & 0.6 \\ 1 & 0.6 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

The importance weights of each criterion determined by each expert are shown in Table 9. However, it should be indicated that they were not utilized in the multicriteria analysis, provided that it was not necessary to implement the subsequent analysis described in Sect. 2, as all alternatives were considered distinguishable in the analysis performed by each expert.

In order to illustrate the use of the method for multicriteria analysis described in Sect. 2, we present the multicriteria analysis performed by expert E_1. Initially, the global fuzzy preference relation is obtained by applying (9) to the collective fuzzy preference relations R_1^C and R_2^C and the fuzzy preference relations R_3^1 and R_4^1 provided by E_1 as follows:

$$G^C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.18 & 1 & 0 & 1 \\ 0.77 & 0.37 & 1 & 0.76 \\ 0.07 & 0.37 & 0.18 & 1 \end{bmatrix}. \tag{36}$$

Applying (5) to (36), we can construct the following fuzzy strict preference relation:

$$P^C = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.18 & 0 & 0 & 0.63 \\ 0.77 & 0.37 & 0 & 0.58 \\ 0.07 & 0 & 0 & 0 \end{bmatrix}. \tag{37}$$

Finally, the use of (6) to obtain the fuzzy nondominance level of each alternative yields

$$ND^1 = \{0.23\ 0.63\ 1\ 0.37\}. \tag{38}$$

At the same time, the other experts were invited to separately perform a multicriteria analysis of the decision making problem. For the sake of brevity, the individual analysis carried out by each expert is omitted here. The fuzzy degrees of nondominance obtained by each expert for each alternative are the following:

Table 10 Ranking of the alternatives, reported in the first iteration of AIR

	X_1	X_2	X_3	X_4	$\bar{S}r^y$
E_1	Fourth	Second	First	Third	0.67
E_2	First	Second	Third	Fourth	0.67
E_3	First	Third	Second	Fourth	0.83
E_5	Third	Fourth	First	Second	0.67
Group	Second	Third	First	Fourth	–

$$ND^2 = \{1\ 0.63\ 0.5\ 0.37\}; \tag{39}$$
$$ND^3 = \{1\ 0.63\ 0.83\ 0.57\}; \tag{40}$$
$$ND^5 = \{0.7\ 0.63\ 1\ 0.77\}. \tag{41}$$

The collective vector of fuzzy nondominance degrees of each alternative, calculated with the use of (15), is given by:

$$ND^C = \{0.73\ 0.63\ 0.83\ 0.52\}. \tag{42}$$

Table 10 shows the rank ordering of the alternatives associated with (38–42) as well as the level of concordance calculated for each expert with the use of (30) and (31). It should be indicated that the parameter b in (30) was set as 1. The mean level of consensus, which is obtained by means of (32) and (33), is given by $\bar{C}r = 0.71$. As the moderator determined empirically that the mean level of consensus should be equal or higher than 0.8, E_2 (one of the least concordant experts, as it can be confirmed in Table 10) was invited to reconsider his/her opinions associated with criteria F_3 and F_4.

In practice, one significant inconvenience in the implementation of AIR is associated with the difficulty of identifying the hidden information (i.e., information that a priori is not known by all experts), which may cause discordances in the group. Indeed, in AIR, differences among individual results may be associated with different preferences on the alternatives, the assignment of different importance weights to the criteria or even with the use of different decision making methods. Sometimes, there may be more than one factor causing discordances among the results. Fortunately, by using the concordance indices per relation and per alternative, given by (24) and (25), it may be possible to identify which is the most critical criterion and which is the most critical alternative.

For instance, by analyzing Table 11, which shows the concordance levels per criterion calculated for E_2 with the use of (25), it can be seen that the major discordances are associated with criterion F_3. The analysis of Table 12 indicates that, when considering F_4, the major discordances are associated with the evaluation of alternative X_1 and X_3. If we compare the ordering of the alternatives provided by the group with the ordering of the alternatives determined by expert E_2, we can see that the major discordance is also associated with alternative X_3. In this way, the moderator can call the attention of E_2 to this alternative.

Table 11 Mean level of concordance per relation

	F_3	F_4
Sp^3	0.92	0.94

Table 12 Mean level of concordance per alternative considering only F_4

	X_1	X_2	X_3	X_4
$Sa^2(X_k)$	0.88	0.95	0.88	0.96

Table 13 Ranking of the alternatives, reported at the second iteration of AIR

	X_1	X_2	X_3	X_4	$\bar{S}r^y$
E_1	Fourth	Second	First	Third	0.67
E_2	Second	Third	First	Fourth	1
E_3	First	Third	Second	Fourth	0.83
E_4	Third	Fourth	First	Second	0.67
Group	Second	Third	First	Fourth	–

After a round of discussion with the other experts, E_2 improved his/her evaluation on X_3 as it is reflected by the following fuzzy preference relation:

$$R_3^2 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0.8 & 1 \\ 0.8 & 1 & 1 & 1 \\ 0 & 0.8 & 0 & 1 \end{bmatrix}.$$

This change affected the fuzzy degrees of nondominance, as it can be seen next:

$$ND^2 = \{0.7\ 0.63\ 1\ 0.37\}.$$

As a consequence, the fuzzy degrees of nondominance calculated for the group changed to

$$ND^C = \{0.66\ 0.63\ 0.96\ 0.52\}.$$

Table 13 shows the ordering of the alternatives, as well as the level of concordance calculated for each expert at the second iteration of AIR. The mean level of consensus is given by $\bar{C}r = 0.79$. As it is very close to 0.8, the moderator decided to terminate the discussion process. Therefore, the final ranking of the alternatives for the board of directors involved in this strategic planning problem is as follows: $X_3 \succ X_1 \succ X_2 \succ X_4$.

8 Conclusions

In this paper, we introduced a methodology that consists in combining three consensus schemes, each one based on applying different aggregation approaches for constructing collective results, which allows each expert to cooperate with their respective capabilities. The methodology has a wide applicability, as it can be utilized to extend other decision making methods based on the analysis of fuzzy preference relations, such as the ones presented, for instance, in Herrera-Viedma et al. (2002); Ekel and Schuffner Neto (2006). The results of the paper can be utilized to solve multicriteria decision making problems from different areas; they can be adequately adapted to deal with very distinct group environments.

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