Single Magnet Levitation by Repulsion using a Planar Coil Array

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Abstract—This paper describes the controlled levitation of a single disk magnet above an array of cylindrical coils, using electromagnetic repulsion to generate actuation forces and torques from coil currents, and an optical motion tracking system to provide realtime position and orientation measurements for feedback control. Our method is to measure the electromagnetic forces and torques generated from a single cylindrical disk magnet on a disk magnet as a function of their relative positions beforehand, then combine the forces and torques from multiple identical coils in a planar array to derive matrix transformations from coil currents to forces and torques, given the position of the magnet during levitation. If the transformation matrix is invertible, then the coil currents necessary to generate forces and torques for feedback control can be calculated. With our setup, proportional-derivative feedback control applied in each direction of motion is sufficient for stable magnetic levitation. Experimental results for levitation of a single magnet are given for a simplified case with 3 coils with the magnet motions partially constrained by an attachment to a ball joint, and for 5 coils with the rotation of the magnet about its vertical axis left uncontrolled to rotate freely.

I. INTRODUCTION

To achieve stable controlled magnetic levitation of a rigid body, 6 degrees of freedom (DOF) of electromagnetic actuation, noncontact position and orientation sensing, and feedback control are required to produce a full 6-element vector of forces and torques on the levitated body. Earnshaw's theorem of 1842 [1] regarding electrostatic levitation of rigid bodies is applicable to the electromagnetic case as well; no static arrangement of permanent magnetic fields can stably levitate a rigid body.

In this work we use a planar array of cylindrical coils to levitate one or more magnets. This arrangement is distinct from magnetic levitation systems such as magnetic bearings and levitation systems for transportation in that multiple coils generate forces on each magnet in all directions, rather than using separate magnet and coil assemblies for actuation and control of each degree of freedom of the levitated body. In our system each coil generates forces and torques in all directions and their combined actuation provides the stabilizing forces and torques for levitation using feedback from a motion tracking position sensor.

Magnetic levitation in general provides considerable benefits to practical applications such as precision motion control, vibration isolation, and haptic interfaces, as friction and other dynamic nonlinearities such as hysteresis and cogging typical of other actuation methods are eliminated.

This work was supported by NSF grant CNS-05515, a donation from Real Data Inc., and the University of Hawaii-Manoa College of Engineering. Peter Berkelman and Michael Dzadovsky are with the Department of Mechanical Engineering, University of Hawaii-Manoa, 2540 Dole St, Honolulu HI USA. Email: peterb@hawaii.edu

II. BACKGROUND

Research in magnetic levitation with multiple DOF can be classified into the areas of Lorentz force systems for fine positioning, teleoperation, and haptic interaction; magnetic bearings; planar motor systems for fine positioning; and systems for transportation.

Planar magnetic levitation systems for silicon wafer photolithography have been developed by Kim and Trumper [2], [3], [4] based on planar Sawyer motors [5]. Their devices provide up to 30 mm position repeatability and up to 50 mm motion in the horizontal plane, yet less than 1 mm of motion in the vertical direction and milliradians-range rotation. Hollis and Salcudean developed magnetic levitation using Lorentz forces [6] rather than electromagnetic attraction or repulsion. Lorentz magnetic levitation devices have been applied to compliant robot wrists [7], force-feedback teleoperation [8], and haptic interaction [9]. Magnetic bearings levitate rotating shafts using electromagnetic attraction to eliminate friction. Various systems for magnetic levitation trains have been developed [10] including electromagnetic systems based on attraction, electrodynamic systems with superconducting coils to generate stronger fields, and the Inductrack system [11] which uses induced currents in passive coils to generate levitation and guidance forces.

Non-contact position feedback for magnetic levitation control systems can be provided by Hall effect sensors, laser interferometry, or combinations of LED markers and position sensing photodiodes. It has also been shown to be possible to levitate magnets without feedback control by using highly diamagnetic [12] or superconducting materials, or by using spin to stabilize the motion of the magnet [13], [14].

A maglev stage developed by Lai, Lee, and Yen [15] also uses a planar array of cylindrical coils to levitate disk magnets, however their system requires 15 magnets and 37 solenoid coils for stable levitation, as separate sets of coils and magnets are used to stabilize each axis of translation and rotation.

III. GENERAL LEVITATION APPROACH

Our current system levitates a single magnet using a limited number of coils, but the method can be extended without difficulty to arbitrarily large coil arrays and levitated platforms with multiple magnets to increase the planar motion range and lifting capacity, such as pictured in Fig. 1.

The coils currently in use in the planar array are approximately 28 mm in diameter and 28 mm high, with 1000 turns of wire and copper cores for effective heat dissipation. Neodymium-iron-boron disk magnets 37.5 mm in diameter and 12.5 mm high with a mass of 125 g and a maximum
energy product approaching 50 MGOe are used for levitation. Each coil produces a combination of forces and torques in different directions on each magnet in its vicinity. The rotational symmetry of the coils and magnets simplifies the model of the forces and torques generated between them.

A. Control Hardware

An optical motion tracking sensor\(^1\) is used with infrared LED markers fixed to a rigid body to provide position feedback with a resolution of approximately 0.01 mm, at a sample rate of 500 Hz with our current software interface. This sensor is mounted to a rigid frame 1.5 m above the plane of the levitation coils so that its range is approximately 1 m in each direction in the plane. The controller is implemented on a standard PC equipped with a 32-channel PCI analog output card\(^2\). A power supply and PWM current amplifiers\(^3\) generate the required coil currents from the PC analog output signals.

B. Actuation Force & Torque Measurements

The forces and torques generated generated between a single coil and magnet over a range of vertical separation distances and horizontal offsets for a given actuation current were measured using the setup shown in Fig. 2, where a 6-axis force/torque sensor\(^4\) is mounted between the magnet and an assembly of vertical and horizontal motion stages. The vertical force, radial horizontal force, and torque generated on the magnet were measured at 2 mm intervals of vertical separation and radial offset and the resulting data are shown in Fig. 3. The forces and torques generated as the roll

\[ F = AI, \]  

where the number of columns of \( A \) is equal to the number of active actuator coils and the number of rows is equal to the number of controlled DOF of the magnet. For the feedback control necessary for stable levitation, the inverse transform matrix \( A^{-1} \) is used to calculate the coil currents needed to generate the forces and torques to satisfy the control laws. Therefore, the \( A \) matrix must be invertable, at least as many coils are needed as controlled DOF of the magnet motion, and stable levitation is impossible for the magnet positions which produce singular \( A \) transformation matrices. Furthermore, increasingly high coil currents must

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\(^1\)OptoTrak, Northern Digital Inc.
\(^2\)PD2-AO-32/16, Ueidaq Inc.
\(^3\)4212Z, Copley Controls Inc.
\(^4\)Mini40, ATI Industrial Automation Inc.
be produced as the magnet position approaches these singular configurations, which are undesirable to avoid overheating the coils and/or saturating the current amplifiers. As coil currents of over 2.5 A were found to cause rapid overheating of the coils, the coil currents generated by the feedback control system were limited to ±2.5 A by the control software.

The condition number of the $A$ matrix may be used as a quantitative measure of how close the magnet is to a position which would generate a noninvertible, singular transform matrix. Analysis of the matrix condition number across ranges of possible magnet positions allows the best magnet positions for stable control and the effectiveness of various coil configurations to be evaluated. The matrix determinant or the maximum element in the inverse matrix could also be used as similar quantitative metrics in a manner similar to manipulability and controllability measures in robotics [16].

D. Feedback Control

Decoupled proportional-derivative feedback control laws are implemented for each degree of freedom of motion of the levitated magnet. A force feedforward term equal to the weight of the magnet is added to the vertical magnet position control law to cancel the effect of gravity.

IV. MAGNETIC LEVITATION WITH BALL JOINT CONSTRAINT

As a preliminary step towards unconstrained, spatial magnetic levitation, the magnet was connected to a low-friction ball joint by a short, lightweight rod to partially constrain its motion. This arrangement constrains the magnet freedom of motion to the 3 DOF consisting of the vertical motion $z$, horizontal motion perpendicular to the connecting rod $x$, and rotation around the connecting rod axis $L_y$. The magnet is thus restricted to planar rigid-body motion with two translational and one rotational DOF, rather than spatial rigid-body motion with 3 translational and 3 rotational DOF. The derivation of the transform matrix and the analysis of the coil configuration for this simplified planar rigid-body motion case is presented here to illustrate the methods used and the relationship between the arrangement of the coils, the position of the magnet, and the performance and stability of magnetic levitation in general.

A. Setup

A single row of 3 actuation coils with centers spaced 35 mm apart is used to control the 3 DOF of the magnet motion. A thin disk of clear plastic with 4 infrared LED position markers is attached to the top of the magnet for motion tracking position feedback, as shown in Fig. 4.

B. Analysis

Given the horizontal and vertical position of the magnet and the coil positions, and using the single coil and magnet force and torque data of Fig. 3, it is straightforward to combine the force and torque contributions from each coil on the magnet to generate a 3x3 element transformation matrix which can be used to calculate a vector of forces and torques from a vector of coil currents. In this planar motion magnetic levitation case, the force/torque and current vectors from equation (1) are

$$ F = \begin{bmatrix} f_x \\ f_z \\ \tau_y \end{bmatrix}, \quad I = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}, $$

and the transformation matrix $A$ is

$$ A = \begin{bmatrix} u(m_x - c_1, m_z) & u(m_x - c_2, m_z) & u(m_x - c_3, m_z) \\ v(m_x - c_1, m_z) & v(m_x - c_2, m_z) & v(m_x - c_3, m_z) \\ w(m_x - c_1, m_z) & w(m_x - c_2, m_z) & w(m_x - c_3, m_z) \end{bmatrix}, $$

where the horizontal and vertical position of the magnet is given as $m_x$ and $m_z$, and the positions of the coil centers are $c_1$, $c_2$, and $c_3$. The horizontal force generated on the magnet by a single coil with a 1.0 A current as a function of the horizontal and vertical separation of the magnet and coil is $u(x, z)$, the vertical force is $v(x, z)$, and the torque is $w(x, z)$. The shapes of the $u$, $v$, and $w$ functions are shown in the top, middle, and bottom plots of Fig. 3 respectively, although the $u$, $v$, and $w$ functions are scaled to 1.0 A coil currents rather than 1.6 A as pictured. The torque function $w(x, z)$ is expressed in units of N-cm so that the elements of $A$ are of the same approximate order of magnitude on each row.

The condition number of the transformation matrix $A$, up to a maximum of 20, is shown in Fig. 5 as a function of the vertical and horizontal position of the magnet. This surface plot indicates that singular configurations are produced when the magnet is directly above any one of the three coils, and the magnet can be levitated with lower maximum coil currents and improved stability when it is approximately midway between two neighboring coils. When the magnet is directly above a coil the lifting force of the coil is maximized, but unfortunately these configurations lead to inability to control the horizontal position and tilt of the magnet unless additional coils and redundant actuation schemes are used.

Fig. 4. 3 DOF levitation setup with ball joint
C. Results

Fig. 6 shows the vertical, horizontal, and rotational motion of the magnet in response to a step input command from a levitated height of 3.0 mm to 4.0 mm. The desired horizontal position was -18.0 mm, which produces a transformation matrix with a condition number of 4.0.

The gains used for the position feedback control are:

\[
\begin{align*}
K_p & = 0.3 \text{ N/mm} & 5.25 \text{ N-mm/degree} \\
K_d & = 0.003 \text{ N-sec/mm} & 0.0525 \text{ N-mm-sec/degree}
\end{align*}
\]

where \( K_p \) is the proportional error and \( K_d \) is the derivative error feedback gain.

The steady-state errors of 0.26 mm in \( x \), 0.05 mm in \( z \), and -0.2 degrees in \( \theta_y \) in the magnetic levitation control results are due to small variations in the forces and torques generated by different coils and small misalignments of the individual coil positions and the motion tracking position sensor zero position. The addition of error integral control or adaptive control terms into the feedback control laws used would help to drive the steady-state levitated magnet position errors to approach zero.

V. FREE MAGNETIC LEVITATION

A single disk magnet can be levitated, but its rotation about its vertical axis cannot be controlled due to the symmetry of its magnetic field. We therefore leave this DOF uncontrolled. Accordingly, 5 coils are sufficient to levitate the magnet, provided that the magnet is allowed to rotate freely about its vertical axis. The transformation matrix \( A \) for this case can be obtained from a given 3D magnet position, the set of planar coil positions, and force/torque data of Fig. 3, by extending the methods of section IV.

Six different configurations for the 5 coils were evaluated by calculating the condition numbers of their coil current to force/torque transformation matrices across a range of magnet positions. The candidate configurations are shown in Fig. 7 and will be referred to as the ‘Cross’, ‘T’, ‘L’, ‘Pentagon’, ‘Hex’, and ‘C’ configurations. The coil center separation for adjoining coils in all configurations is 35 mm.

A. Coil Configuration Analysis

The force/torque and current vectors from equation (1) for the case of levitation leaving the yaw rotation \( \theta_z \) uncontrolled, are given by:

\[
F = \begin{bmatrix} f_x \\ f_y \\ f_z \\ \tau_x \\ \tau_y \end{bmatrix}, \quad I = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix}, \quad (4)
\]

and the transform matrix can be calculated as:

\[
A = \begin{bmatrix}
\cos(\theta_1)u(r_1, m_z) & \cos(\theta_2)u(r_2, m_z) & \cdots \\
\sin(\theta_1)u(r_1, m_z) & \sin(\theta_2)u(r_2, m_z) & \cdots \\
v(r_1, m_z) & v(r_2, m_z) & \cdots \\
-\sin(\theta_1)w(r_1, m_z) & -\sin(\theta_2)w(r_2, m_z) & \cdots \\
\cos(\theta_1)w(r_1, m_z) & \cos(\theta_2)w(r_2, m_z) & \cdots 
\end{bmatrix}, \quad (5)
\]
where the radial offset distances $r_i$ between the vertical axes of the magnet and each coil are given by:

$$r_i = \sqrt{(m_x - c_{ix})^2 + (m_y - c_{iy})^2}$$  \hspace{1cm} (6)

and the rotation angles $\theta$ to separate radial horizontal forces and torques into $x$ and $y$ components are:

$$\theta_i = \tan^{-1}\left(\frac{m_y - c_{iy}}{m_x - c_{ix}}\right),$$ \hspace{1cm} (7)

provided that the $\tan^{-1}$ function returns the correct angle $0 < \theta < 2\pi$ for all 4 possible quadrants determined by the signs of the numerator and denominator.

The rotational symmetry of the coils and magnets simplifies the force and torque generation models since only the vertical distance $z$ and the radial distance $r = \sqrt{x^2 + y^2}$ need to be measured, rather than measuring both $x$ and $y$.

The condition numbers of the $A$ transform matrices as a function of magnet $x$ and $y$ positions at a $z$ levitation height of 4 mm are shown in Fig. 8 for each coil configuration, for areas where the $A$ condition numbers are less than 16.0.

**B. Setup**

The ‘L’ configuration from Fig. 7 was used for the implementation of magnetic levitation experiments due to the large region where its transform matrix condition number is less than 10.0. The magnet and coil setup was similar to previous case of section IV, but with two additional coils aligned at a right angle to the previous three, and without the ball joint and rod connected to the levitated magnet. A 3.15 mm aluminum plate was placed on top of the levitation coils underneath the levitated magnet to provide additional passive damping of the motion of the magnet by generation of eddy currents, thereby stabilizing levitation for higher proportional control gains.

**C. Results**

The control gains used to obtain the levitation results shown in Fig. 9 are the same as those used in section IV. The magnet was levitated at the position of $x = -22$, $y = 13$, and $z = 4$, where the condition number of the current to force/torque transform matrix is 8.0. Static magnet levitation at this position for periods greater than approximately 30 seconds leads to overheating of the two coils farthest from the magnet however, as these coil currents are greater than 2.0 A. Small steady-state errors of up to 0.3 mm and 0.5 degrees are present in the previous results, due to small position and magnetic field variations in the experimental setup and the absence of integrating or adaptive control terms. The levitated magnet, actuator coils, and the plastic disk with LED position markers are shown in Fig. 10.

**VI. CONCLUSIONS AND FUTURE WORKS**

**A. Conclusions**

The experimental results have demonstrated the basic feasibility of the magnetic levitation control methods. These methods are applicable to any combination of actuation coils, in any planar configuration, levitating any number of magnets connected together, so that the present system can be easily reconfigured, or extended to an arbitrary size and motion range. The methods are first used to measure and compare the levitation effectiveness of different coil configurations and magnet positions, and then used to derive the force/torque and coil current transformations to be used in feedback control for levitation.

**B. Future Works**

The current and planned further research is to improve and extend these control methods for magnetic levitation for higher loads, larger ranges of motion, and more dynamic motion trajectories.

The actuation force and torque data of Fig. 3 will be extended to measure changes in forces in torques as the magnet tilt angles $\angle x$ and $\angle y$ or roll and pitch are varied by up to ±90 degrees. This actuation data will enable levitation of magnets at tilted angles, instead of with a flat orientation only. A spherical magnet will be levitated as well as the present disk magnet, and additional LED markers will be used to provide position feedback as the magnet rotation.

**Fig. 8.** Quality measures for 5 coil configurations

![Fig. 8. Quality measures for 5 coil configurations](image-url)
angles cause the present LED markers not to be visible to the overhead motion tracking sensor.

To realize smooth trajectory following over larger ranges of motion, the A transformation matrices and their inverses for each will be calculated from magnet position at each control update during motion. A linear interpolation of force/torque data points will be implemented instead of selecting the nearest data point. A model of the levitated rigid-body dynamics will be incorporated into the trajectory controller.

Finally, the coil array will be extended to a larger area by adding more coils in both directions, to increase the motion range of the levitation system. Multiple magnets will be attached together in a single levitated platform so that the yaw angle $\theta_z$ can be controlled and to increase the lifting capacity of the system. With more coils available than controlled motion DOF of the levitated platform, redundant actuation methods will be developed and a commutation system will be used to switch currents to different sets of coils as the levitated magnet moves over the coil array area.

ACKNOWLEDGMENTS

The authors thank Ji Ma for his assistance programming the software interfaces for the OptoTrak motion tracker.

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