Novel Design, Characterization, and Control Method for Large Motion Range Magnetic Levitation

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Abstract—This letter describes a method to quickly calculate the coil current to force and torque transformations required for levitation of cylindrical magnets above a given array of cylindrical coils, and evaluate the feasibility, efficiency, dynamic performance, and stability of the resulting design. This method has been used to develop levitation systems for one or two magnets with translation ranges greater than the dimensions of the levitated body in all directions and potential for large rotation ranges. Results of the evaluation method are applied to three cases, and experimental results are given for single magnet levitation in horizontal and vertical orientations, and for a double magnet platform in the horizontal orientation.

Index Terms—Electromagnetics, magnetoacoustic emission, levitation.

I. INTRODUCTION

The principal disadvantage of magnetic levitation systems has been their limited motion ranges. Our method uses measured forces and torques between a single magnet and coil to find current to force and torque transformations over a range of magnet positions and orientations, and uses the condition number of the resulting matrices to evaluate the feasibility and performance of levitation for a given magnet and coil array. Using this design method, we have levitated single magnets with 5, 10, and 16 coil arrays, and two-magnet platforms with 10 and 16 coil arrays, over translation ranges larger than the dimensions of the levitated body in all directions, up to $30 \times 90 \times 120$ mm. Adding more coils to the array reduces the average coil current required for levitation, increasing vertical range, and increases the horizontal range by extending the area of the array. Fig. 1 shows levitation of a single magnet and a two-magnet platform above a ten-coil array.

Prior suspension and repulsion levitation systems [Wang and Busch-Vishniac 1994; Lai 2007; Robertson 2005] typically have ranges of motion which are limited to a fraction of the dimensions of the levitated body in most or all directions, and to rotation angles of a few degrees. A large range of motion levitation system for small magnets with uncontrolled orientation is described in Khamesee and Shameli [2005]. The work mostly related to our current research was by Groom and Britcher [1992] and Britcher and Ghofrani [1993], who carried out extensive analysis of electromagnetic actuation, rigid body dynamics, and feedback-control methods for levitation with large rotations, however, implementation was limited to small motions. Koumboulis and Skarpetis [1996] have also carried out related work on magnetic suspension and balance systems for models in wind tunnels. An initial small range-of-motion proof-of-concept setup of our method is described in Berkelman and Dzadovsky [2008a], and a redundant control setup with a larger translational range of motion in Berkelman and Dzadovsky [2008a], with application to haptic interaction described in Berkelman and Dzadovsky [2009]. In this letter, we show new results from two-magnet levitated platforms and vertical orientation of single magnets.

II. SINGLE COIL FORCE AND TORQUE MODELING

We considered three approaches to modeling the forces and torques generated as a function of coil currents and magnet position and orientation: 1) derivation from electromagnetism equations; 2) electromagnetic finite element analysis (FEA); and 3) direct measurement of forces and torques.

A magnetized body surrounded by a static magnetic field generates force $F$ dependent on the gradient of the magnetic field and torque $\tau$ to align its magnetization axis with that of the field as

$$F = \int (M \cdot \nabla) B dV, \quad \tau = \int (M \times B) dV$$

for magnetization $M$ and external flux density $B$, integrated over the magnet volume $V$. If the coil dimensions are much larger than the magnet size, then $B$ and its gradient are nearly uniform over the volume of the magnet and it is fairly straightforward to calculate $B$ surrounding a cylindrical coil with constant current density according to the Biot–Savart law

$$B = \int \frac{\mu_0 l dI \times r}{4\pi r^2}$$

Fig. 1. Single and double disk magnet levitation.
integrated over wire length $l$, coil radius $r$, with current $I$ and magnetic constant $\mu_0$. In our setup, however, the coil dimensions are comparable to the motion range and magnet dimensions, and the variations in the $B$ field are significant and complex, so that the resulting forces and torques are much more difficult to calculate to sufficient precision for levitation.

We found that calculation of actuation forces and torques by typical FEA software (ANSYS Emag) was not sufficiently smoother fast to be practical for large range of motion levitation, owing to the coarseness of the finite element mesh attainable with general-purpose computing hardware and the remeshing and recalculation required for each discrete sampled position and orientation of the magnet. Numerical integral-based software methods using (1) and (2) are promising, as they are much faster and do not require remeshing as the magnet position and orientation are varied, yet the magnetization of the magnets used must be uniform and known to a high precision to obtain results which closely match the physical system. In our method, we directly measure forces and torques experimentally, as described in the following.

Linear motion stages and an ATI Industrial Automation Mini40 six-axis force–torque sensor were used to determine the force and torque actuation model for a single coil and magnet experimentally. The stages were controlled by a PC to move the magnet through all combinations of the following coordinates as measured in $r$, $z$, and $\theta$, using four different magnet mounting fixtures to provide $0^\circ$, $30^\circ$, $60^\circ$, and $90^\circ$ magnet inclination angles $\phi$ (see Table 1).

The 3-D forces and torques at the center of the magnet were measured for a 2.0 A coil current. Due to the radial symmetry of the magnet and coil, measurements in these 4-DOF are sufficient to model the variations in actuation forces and torques with respect to 6-DOF of magnet translation and rotation. The horizontal magnet position is sampled in cylindrical $r$, $\theta$ rather than Cartesian $x$, $y$ coordinates, so that points are sampled more densely as the magnet approaches the coil and the variations in the actuation forces and torques increase. Subsets of measured data are shown for an untilted magnet moved in the vertical and radial offset directions, and for a $90^\circ$-tilted magnet moved in horizontal circles in Fig. 2. All forces and torques were confirmed to be closely proportional to the coil current.

### III. COMBINED COIL CURRENT TO FORCE AND TORQUE TRANSFORMATIONS

The total force and torque generated by all the coils together can be represented as matrix transformation $\mathbf{F} = \mathbf{A}_1$, where $\mathbf{F}$ is the six-element vector $(F_x, F_y, F_z, T_x, T_y, T_z)$ of the total force and torque on the magnet, and $I$ is a vector of coil currents. To simplify the transformation calculations, forces and torques on the magnet from each coil are measured in the coordinate frame with $\hat{x}$ as the horizontal direction from the coil center to the magnet center. In these coordinates, the forces generated on a tilted magnet do not depend on the coil-to-magnet angle but only on the direction of the inclination $\theta$ of the coil measured with respect to $\hat{x}$. Forces and torques from each coil in its rotated frame must be transformed back to Cartesian coordinates before being combined in matrix $\mathbf{A}$. By defining the angle from each coil center $j$ to the magnet center in the horizontal plane as $\psi_j$, the transform from current vector $\mathbf{I}$ to total force–torque vector $\mathbf{F}_x, \mathbf{F}_y, \mathbf{F}_z, \mathbf{T}_x, \mathbf{T}_y, \mathbf{T}_z$ is

$$\mathbf{A} = [\mathbf{A}_1 \mathbf{A}_2 \ldots \mathbf{A}_N]$$

where each column for a single levitated magnet is

$$\mathbf{A}_j = \begin{bmatrix}
\cos(\psi_j)f_z(r_j, z, \phi_j) - \sin(\psi_j)f_z(r_j, z, \phi_j) \\
\sin(\psi_j)f_z(r_j, z, \phi_j) + \cos(\psi_j)f_z(r_j, z, \phi_j) \\
f_x(r_j, z, \phi_j) \\
\cos(\psi_j)\tau_z(r_j, z, \phi_j) - \sin(\psi_j)\tau_z(r_j, z, \phi_j) \\
\sin(\psi_j)\tau_z(r_j, z, \phi_j) + \cos(\psi_j)\tau_z(r_j, z, \phi_j) \\
\tau_z(r_j, z, \phi_j)
\end{bmatrix}$$

for matrix column $j$, with $z$ as the levitation height of the magnet center above the coil plane, and $r_j$ as the horizontal distance from the center coil $j$ to the center of the magnet. Since the coil forces and torques are measured at discrete values of $\theta$, cubic interpolation is used to estimate the continuous functions. For a platform with multiple magnets ($k = 1, \ldots, m$)

$$\mathbf{A}_j = \sum_{k=1}^{m} \begin{bmatrix}
\cos(\psi_{jk})f_z(r_{jk}, z_k, \phi_k, \theta_{jk}) - \sin(\psi_{jk})f_z(r_{jk}, z_k, \phi_k, \theta_{jk}) \\
\sin(\psi_{jk})f_z(r_{jk}, z_k, \phi_k, \theta_{jk}) + \cos(\psi_{jk})f_z(r_{jk}, z_k, \phi_k, \theta_{jk}) \\
f_z(r_{jk}, z_k, \phi_k, \theta_{jk}) \\
\cos(\psi_{jk})\tau_z(r_{jk}, z_k, \phi_k, \theta_{jk}) - \sin(\psi_{jk})\tau_z(r_{jk}, z_k, \phi_k, \theta_{jk}) \\
\sin(\psi_{jk})\tau_z(r_{jk}, z_k, \phi_k, \theta_{jk}) + \cos(\psi_{jk})\tau_z(r_{jk}, z_k, \phi_k, \theta_{jk}) \\
\tau_z(r_{jk}, z_k, \phi_k, \theta_{jk}) + \Sigma_{jkz}
\end{bmatrix}$$

### Table 1. Sampled magnet position and orientation parameters for force and torque measurements.

<table>
<thead>
<tr>
<th>Motion direction</th>
<th>Variance</th>
<th>Range</th>
<th>Step</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of magnet center above coil top</td>
<td>$z$</td>
<td>25–35 mm</td>
<td>1 mm</td>
</tr>
<tr>
<td>Radial offset to magnet center from coil axis</td>
<td>$r$</td>
<td>0–75 mm</td>
<td>1 mm</td>
</tr>
<tr>
<td>Tilt of magnet disk</td>
<td>$\phi$</td>
<td>0–90°</td>
<td>30°</td>
</tr>
<tr>
<td>Direction of tilt</td>
<td>$\theta$</td>
<td>0–360°</td>
<td>30°</td>
</tr>
</tbody>
</table>
since each coil produces a force and a torque on each magnet, and additional $\sum R \times F$ torque terms $\sum_{i,j,k}$, $\sum_{i,j}$, $\sum_{j,k}$ are produced because the forces are not generated at the center of mass of the platform.

A. Inverting Current to Force–Torque Transformation

This rectangular matrix is kinematically redundant if the number of actuators is greater than the degrees of freedom to be controlled. The Moore–Penrose pseudoinverse of $A$ is used to calculate actuation currents with the lowest sum of squared currents for levitation control, similarly to control methods developed for redundant actuation velocity control and execution of subspace tasks as described in Nenchev [1992] and Baillieul [1987].

In single magnet levitation, the pseudoinverse of the transformation matrix cannot be directly inverted to produce the coil currents to produce a desired set of forces and torques, as no combination of coil currents can produce any torque on the magnet about its principal axis of symmetry. For 5-DOF levitation control at arbitrary orientations, the torque vectors in the transformation matrices can be rotated so that one of the torque directions is aligned with the magnet axis, and the row corresponding to these torques is reduced to approximately zero. This row can then be eliminated from the transformation matrix, and the pseudoinverse of the resulting reduced five-row transform matrix can then be used to calculate coil currents to generate torques perpendicular to the axis of the magnet to control its orientation while leaving the rotation of the magnet about its principal axis uncontrolled. The force–torque to current transforms are precalculated for magnet positions to the closest 1.0 mm in translation and orientations to the closest 30°, stored in a lookup table, and interpolated during levitation control.

B. Evaluation Using Condition Number

The condition number of each transformation matrix provides a measure of levitation stability and motion control performance, as the ratio of eigenvalues indicates the relative magnitudes of coil currents required to correct disturbances in different directions in translation and rotation. If the reduced transformation matrix has no pseudoinverse for the desired magnet position and orientation, then the magnet cannot be levitated, as it is impossible to independently control all required degrees of freedom of the rigid body. In general, higher condition numbers correspond to higher required coil currents for levitation, and with our current system levitation is feasible for condition numbers less than approximately 12–15. Torques are measured in N·cm, so that torque and force data are approximately of the same magnitude in the transformation matrices.

Evaluation of transform matrix condition numbers throughout the motion range of the magnet allows different magnet and coil configurations to be compared. For example, by comparing the condition numbers for levitation of a 25-mm diameter magnet and a 37.5-mm diameter magnet, with 0° of tilt at a 25 mm height as shown in Figs. 3 and 4, respectively, with coil centers indicated by *, one can conclude that levitation of the 37.5 mm magnet is much more stable and robust, and the 25 mm magnet would require much higher coil currents for levitation and would be more sensitive to small disturbances. Fig. 5 shows the transformation condition numbers for a 37.5-mm diameter magnet tilted by 90°. Levitation of this magnet in the vertical orientation should be marginally feasible with our setup without overheating the coils, as condition numbers are less than 15 over most of the area of the coil array.
Neodymium–iron–boron (Nd-Fe-B) cylindrical permanent magnets with an energy product of 50 MGOe, 37.5 mm in diameter, 12.5 mm thick, and 120 g were used. Each coil is 30 mm in height with inner and outer diameters of 12.5 and 25 mm, a copper core for heat dissipation, and 1000 windings. The 16 coils are arranged in four rows of four coils each in hexagonal spacing with 35 mm spacing between centers, as in Figs. 3–5.

Coil currents were limited to 2.5 A each to prevent overheating.

The position sensing LEDs, magnets, and coils, variations in the properties of the magnets and fabricated coils, the coarseness of the sampled forces and torques, coil current saturation effects, and disturbances from the LED position markers, which have some ferrous content.

V. CONCLUSION

Our system demonstrates the feasibility of magnetic levitation of magnets in any spatial orientation using a planar array of coils. Its novel features are that its actuation model is based on direct experimental force and torque measurements rather than being dependent on the accuracy of analytic or finite element analysis methods, its horizontal motion range is extendable to any area, and redundant control analysis methods can be used to evaluate the effectiveness of levitation based on magnet position and orientation, the actuation model, and the coil configuration.

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REFERENCES


