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Biomechanics of Swimming

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Introduction

In competitive swimming, world-level performances require years of hard training. Training that is devoted to improvement of determinants of performance like technique and coordination, strength and aerobic capacity. It may be argued that training-time will be especially efficient when devoted to the enhancement of those performance factors that are weak links in the performance chain. That is, they represent the phase of the process where the performance system first becomes insufficient. These are known as limiting or determining factors, because they are the first to reduce, and hence determine, performance (1). This could lead to the conclusion that the most specific form of training is to swim races, because then the greatest stress is put on the weakest factor needing the most improvement. However, this will only stress one, albeit dominant, factor sufficiently to produce a maximum training effect. The other factors may not be stressed optimally and will, therefore, not improve as much as they might with another form of training (2). Therefore, it might make more sense to train several factors in a somewhat isolated manner and overloading them maximally without interference from other processes, such that each would improve separately to a greater extent and then contribute more to performance when integrated with the other factors during a race. It seems therefore necessary to identify the relevant performance factors and design the optimal training programs to improve them. Competitive swimming events differ in stroke employed (breaststroke, backstroke, butterfly and front crawl) and distance swum (50 m - 1500 m). Swimming a 50 m sprint lasts ± 23 s and will require for instance considerable strength, power and technique, while the 1500 m takes at least 14 min 40 s to complete and thus calls for a high endurance capacity. It is obvious that these different events will place different demands on the swimmer's body and that performance factors will depend on the specific event the swimmer is training for. Still, some important performance factors are involved in all competitive swimming events. An analysis of these factors would be useful in designing training programs, taking into consideration whatever deficiencies there may be in the swimmers resources or capabilities.

We will start with an analysis of the resistance encountered during swimming, as resistance is thought to be a major performance factor. The hydrodynamic backgrounds of this velocity dependent force are discussed and the different attempts to measure *drag* will be presented. This is followed by an overview of the different theories proposed to relate the kinematics of the propelling surfaces to the produced propulsive forces. A common characteristic of swimming propulsion is that, apart from the start and turns, it can not be generated by pushing-off from a fixed object. Therefore, a thorough analysis of the mechanics of the swimmers' body and of the surrounding water, involved in the generation of propulsion will be presented with special emphasis on fluid dynamics. It will then become clear that the mechanics (and especially the propulsion technique) of swimming is intimately tied to the energetics of swimming. And, thus, finally, the link between the biomechanical and physiological bases of performance will be treated. This review does not dissect each competitive stroke or explain techniques. It only covers the theoretical bases of the mechanics and energetics of swimming (with an emphasis on swimming the front crawl). Special attention will be given to the role of the arms, since it is generally agreed that the arms provide more than 85% of the total thrust in the crawl stroke (3-6). More detailed explanations of the other swimming strokes will have to remain for another forum.

Drag

When swimming through the water the body will undergo a retarding force due to resistance, or drag. This force is due to the viscosity of the water and also, at high speeds, to turbulence behind the swimmer. Furthermore, when movement occurs at the water surface, additional resistance will occur due to gravitational forces on the waves set up by the motion. Each of these three components will be discussed from a basic hydrodynamic point of view.

Friction drag

Viscosity is the property which makes syrup difficult to stir. It is essentially a frictional force between different layers of water as they move past one another. The viscosity of water (or any fluid) can be expressed by the coefficient of viscosity η ($0.897 \cdot 10^{-3}$ N•s•m^{-2} for water at 26°C). The water-layer directly in contact with the swimmer is held to the skin by adhesive forces (this is known as the no-slip condition). Thus this fluid layer moves with the same speed v as the swimmer. The fluid velocity will diminish with distance from the body. Fluid far in front of the body, far at either side of it and far behind it is at rest. Thus the layer of water close to the body will be retarded by the layer just beside it; this layer is retarded by the next layer and so on. It is found that this friction or viscous force is proportional to the total surface area of the swimmer (the wetted surface) and to the speed v. As long as the water around the swimmer is neatly arranged in layers (the flow is then called *laminar*), the total drag force exerted by the water will equal this viscous force.

Above a certain speed, the flow may become turbulent. Turbulent flow is characterized by erratic movement of fluid elements, compared to the orderly behavior in the laminar domain. At what speed and where on the body turbulence first becomes apparent depends on the shape and size of the swimmer. The onset of turbulence is often abrupt and occurs at a critical value of the so-called Reynolds number, *Re* (a dimensionless scaling number):

$$Re = \frac{vL\rho}{\eta} \qquad 1$$

where ρ and η are the density and viscosity of water, v is the swimming velocity, and L is a characteristic length of the swimmer. Depending on the shape of the object, the critical value of *Re* will be in the order of 500,000 (7). For a competitive swimmer, with v = 2m•s^{-1}, L =2 m, ρ = 1000 kg•m^{-3}, and $\eta = 0.897 \cdot 10^{-3}$ N•s•m^{-2}, *Re* will be about 4.5×10^6 for the swimmer body. This implies that in competitive swimming turbulence will probably always play a role.

Pressure drag

The orderly flow over the swimmers' body may separate at a certain point, depending on the shape, size and velocity of the swimmer. Behind the separation point, the flow reverses and may roll up into distinct eddies (vortices). As a result, a pressure differential arises between the front and the rear of the swimmer, resulting in 'pressure drag', which is proportional to the pressure differential times the cross sectional area of the swimmer. In general, pressure drag D_p will be dependent on the square of the swimming speed v, a dimensionless drag coefficient C_D accounting for form effects (e.g. bluff versus torpedo-shaped), the cross-sectional area A_p, and ρ, the density of water:

$$D_p = \tfrac{1}{2}\rho A_p v^2 C_D \qquad 2$$

Wave drag

For swimming near the water surface, a third component of the total resistance is due to the so-called 'wave-making resistance'. When swimming near the surface, water tends to pile up in front of the swimmer and to form 'hollows' behind, thus creating a wave system. With increasing velocity both the wave-length (the crest to crest distance) and the wave amplitude increase. At a certain speed the wave-length will equal the "water-line length" of the swimmer, which is presumably proportional to the height of the swimmer. This swimming speed is called the "hull speed", a term from shipbuilding introduced into competitive swimming by Miller (8). At that velocity the swimmer is trapped in a self-created hollow between crests of waves. More effort will lead to a higher wave amplitude leading to a deeper hollow and thus any further attempts to increase swimming speed will be extremely as most energy is used to "climb out of the hollow" (9). In practice, this means that it is impossible to swim faster than the hull speed. The relative speed that, together with the 'form of the swimmer' determines the magnitude of the wave-drag is defined as the Froude-number (*Fr*, another dimensionless number):

$$Fr = \frac{v}{\sqrt{gL}} \qquad 3$$

where v and L have the same meaning as before, and g is the acceleration of free fall (9.81 m•s^{-2}). Swimmers with equal body form, but differing in height, will create an identical wave system (relative to body dimensions) when their Froude number is equal. For the shorter swimmer this corresponds to a lower velocity (see below).

Measurement of drag

A considerable part of the energy expenditure in swimming is utilized to overcome drag (10). It is therefore one of the factors that may limit swimming performance. Throughout the history of swimming research attempts have been made to measure this resistance. As early as 1905, Dubois-Reymond (11) towed people behind a rowing boat, measuring resistance using a dynamometer. Liljestrand (12) towed swimmers by means of a windlass on shore. Amar (13) was the first to assume that the resistance is related to the square of the swimming velocity (see also 14, 15) according to (compare equation 2):

$$D = K \cdot v^2 \qquad 4$$

in which D denotes drag force, K is a constant incorporating ρ, C_D and A_p, while v is the swimming velocity. Karpovich (15) used a 'natograph' to register drag dependent on velocity. Both Amar (13) and Karpovich (15) used measurement techniques determining the resistance of swimmers gliding passively through the water. The relation between resistance (N) and velocity (m•s^{-1}) based on their experiments was approximately D = 29•v^2. However, the body is of course never in a stable prone position when swimming, since propulsive forces need to be generated. It was conjectured that the movements necessary to create propulsion could induce additional resistance (16-18). This led to attempts to determine the drag of an active swimming person.

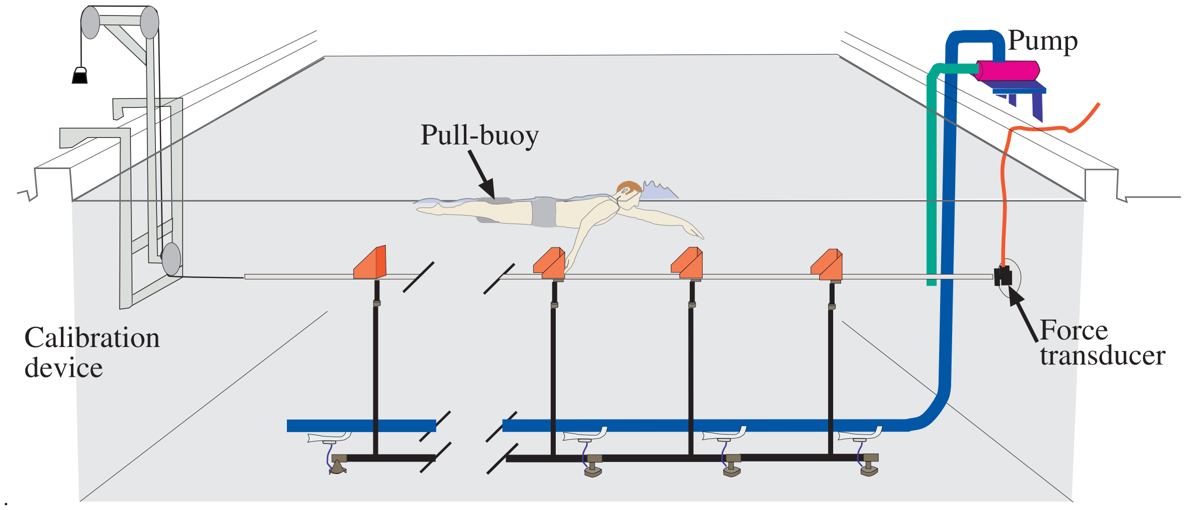

Figure 1: Schematic drawing of the MAD-system mounted in a 25 meter pool. The MAD-system allows the swimmer to push off from fixed pads with each stroke. These push-off pads are attached to a 22 meter long rod. The distance between the push-off pads can be adjusted (normally 1.35 m. The rod is mounted ± 0.8 m below the water surface. The rod is connected to a force transducer enabling direct measurement of push-off forces for each stroke. Subjects use their arms only for propulsion; their legs are floated with a small buoy. If a constant swimming velocity is maintained, the mean propelling force equals the mean drag force. Hence, swimming one lap on the system yields one data-point for the velocity-drag-curve. (note: the cord leading to the calibration device is detached during drag-measurement)

Determination of Active Drag

Techniques to determine this *active* drag were developed by several groups in the 70's (19-23). In the method used by Holmér (21), Prampero (22), and Rennie (23), the variation in oxygen consumption as a result of small additional forces applied to the swimmer is extrapolated. In the method introduced by Clarys (19), variations in external forces applied on a moving carriage as a function of imposed speed variations are extrapolated. Both methods yielded comparable results and, as expected, higher drag values (150-300%) than the previously reported values for *passive* drag.

Schleihauf (24) developed a new approach (see below for more details) which did not rely on extrapolation. His technique was based on the balance of propulsive and resistive forces that, according to Newton's law, must exist when swimming at a constant speed. Hence by determining the propulsive forces, drag can be estimated. In the mid-80's, Hollander *et al.* (25) developed another approach to measure active drag (M.A.D. system, Figure 1). The technique relies on the *direct* measurement of the push-off forces while swimming the front crawl. Kolmogorov and Duplisheva designed yet another method to determine the active drag (26). In their so-called velocity perturbation method subjects are asked to swim a 30 m lap twice at maximal effort: once swimming free, and once swimming with a hydrodynamic body attached that created additional resistance. For both trials the average velocity is calculated. Under the assumption that in both swims the power output is maximal and constant, active drag can be calculated since power equals force times speed:

$$D_1 \cdot v_1 = D_2 \cdot v_2 \qquad 5$$

where the subscripts refer to the swims. Using equation 4 this can be expanded:

$$Kv_1^3 = K \cdot v_2^3 + F_b \cdot v_2 \qquad 6$$

where F_b represents the added drag due to the hydrodynamic body. Since the hydrodynamic properties of this added body were calibrated previously, it was possible to compute F_b at any velocity. Then, K can be solved and since $D_1 = K \cdot v_1^2$, D_1 will equal:

$$D_1 = K \cdot v_1^2 = \frac{F_b \cdot v_2 \cdot v_1^2}{v_1^3 - v_2^3} \qquad 7$$

The interesting aspect of this approach is that it can be applied to measure active drag in all four competitive strokes, while the MAD system and indirect methods are applicable only to front crawl. However, the approach will yield only one drag estimate at maximal speed.

When these more recent techniques were used to estimate *active* drag (18, 26-29) considerably lower values were found than values reported earlier (20). Except for the velocity perturbation method, the values were comparable to values reported earlier for passive drag (i.e. $D = 26 \bullet v^2$). Kolmogorovs approach yielded even lower values: $D = 16 \bullet v^2$. It was established that active drag is related to the square of the swimming velocity (v in m•s^{-1}) according to $D = K \bullet v^2$ (Equation 4). A complicating factor in this 'drag dispute' was the fact that methods were never compared using the same subjects (30). To finally resolve this issue, Hollander *et al.* (31) compared active and passive drag in 13 elite male swimmers. Active drag was determined using Schleihauf's technique and using the MAD-system, while passive drag was determined during towing experiments in a swimming flume. The two methods yielded similar active drag values (r = 0.76); again drag was related to velocity according to $D = 26.5 \bullet v^2$. Values for passive drag ($F_{passive}$) were much *lower* than the active drag values ($F_{passive} = 14.5 \bullet v^2$) and than older estimates for passive drag (13, 15). It was demonstrated that passive drag values are extremely dependent on body position during the measurements (see also 9). In particular, small variations of head position could induce drag values differing by ± 100%, a phenomenon previously observed by Miyashita (32). Given this discrepancy, active drag measurements probably result in the most reliable values for drag during swimming.

Factors determining active drag

Using the MAD-system, we found that mean values for K are about 30 for males and about 24 for females in front crawl swimming (28). These results are similar to the values previously reported by Karpovich (15). It is of course interesting to unravel what factors could determine the exact value of K. In the literature it was more than once suggested that body build could be such a factor. For instance, Cureton (33) reported that the "tall, slim type has been shown to glide better through the water". In addition several others noted that swimmers are taller than the mean population, suggesting a performance advantage related to height (34-38). In line with these suggestions Clarys proposed to use form indices derived from ship-building technology to study the relationship between body-build and drag (14, 19, 39). In collaboration with Clarys and with the help of the MAD-system the relation between morphology and active drag was evaluated (40). It was established that drag is determined to a great extent by the maximal body cross-section area (r^2 = 0.76). Furthermore, the difference in mean body cross sectional area (0.091 m^2 males versus 0.075 m^2 females) could explain the difference in mean drag between male and female swimmers (K = 30 versus K = 24 for females). The effect of height on drag became apparent from a longitudinal study (2.5 years, 41) of a group of children (mean age at the start: 12.9 years). In this study the body cross-sectional area of the children increased by 16%, while no differences in total drag were found (start: 30.1 N ± 2.37 versus end: 30.8 N ± 4.50; both drag values determined at a swimming velocity of 1.25 m•s^{-1}). As we have indicated above, total drag (D) is not only determined by pressure drag (D_p) but also by friction drag (D_f) and wave-making resistance (D_w):

$$D = D_p + D_f + D_w \qquad 8$$

Pressure drag is dominant at the prevailing high Reynold's number R_e of 2.2•10^6-2.5•10^6 (14), and at equal swimming speed is determined mainly by the body cross-sectional area. The friction drag (D_f) being dependent on total surface area (see above) could increase somewhat since the total skin surface will increase due to growth (but see also 42). Hence, changes in D_p and D_f can not explain the lack of increase in total drag at the end of the study.

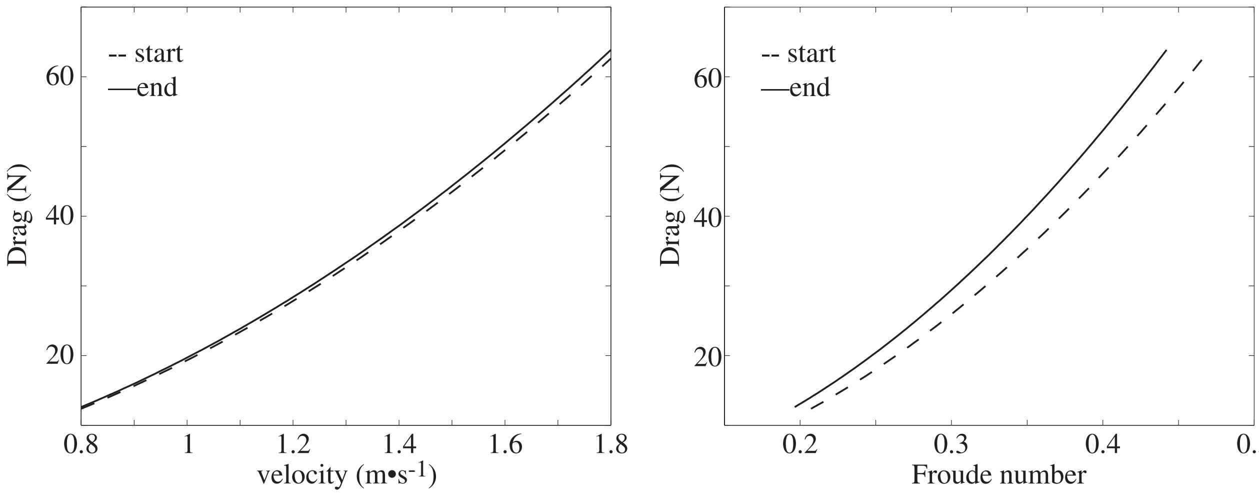

Figure 2: Drag is presented for the children at the beginning (mean age 12.9 years, dotted line) and after 2.5 years (solid line), dependent on velocity (left panel) and dependent on the Froude number (right panel).

The wave-making resistance (D_w) relates to the Froude number. One of the effects of growth is of course increase in height. Thus swimming at the same absolute speed would imply a decrease of the Froude number when height increases (see equation 3), with consequent reduced formation of waves leading to a lower wave-making resistance. Thus in the children the increase of height (from 1.52 to 1.69 m) resulted in a decrease of F_r (from 0.324 to 0.308 at a swimming velocity of 1.25 m s^{-1}). In Figure 2 (left) the mean drag curves for the entire group are related to speed. Again it is clear that the drag has not changed. When the same drag data are presented with respect to the Froude number (Figure 2, right), hence correcting for the change in height, it appears that the drag had increased. If the drag is calculated for a Froude number of 0.324 (v = 1.25 m s^{-1}), it gives a drag of 30.1 N. After 2.5 years drag at the same Froude number yields a value of 34.6. The increase of 15% is about the increase in size of the body cross-sectional area (see also 43). The suggestion that the increase in pressure drag was compensated by a decrease in wave-making resistance is in line with the common notion that taller swimmers seem to have an advantage for a good (front crawl) swimming performance. Furthermore, form indices derived from ship-building technology revealed changes that indicated a more 'streamlined' body. For example the Length-thickness ratio (equal to: $(\text{height})^2$/ body cross-sectional area) significantly increased from 36.5 to 39.4 (41). Therefore, during growth a complex process takes place in which different drag determining factors, such as height, body shape, and body cross-sectional area, change in directions that have opposite effects on drag.

Recently, Vorontsov (9) examined the interesting problem at what depth the wave-making resistance would be negligible. He reasoned that at a certain depth the hydrostatic pressure is higher than the pressure created by the moving swimmer that sets up a wave system. This “wave-equilibrium-depth” appears to be between 0.7-1.2 m (9), depending on the wave-making characteristics of the swimmer. The practical follow-up question is whether total resistance will be less when swimming below the “wave-equilibrium-depth”. According to Vorontsov the wave-making resistance is related to the swimming velocity cubed. Consequently, this component rises sharply and becomes significant at high speeds (>±1.8 m•s^{-1}). Such high velocities are indeed attained after the start and turns. A glide below the wave-equilibrium-depth would evade this high wave-making-resistance, and thus a much smaller propulsive force is required to maintain a high speed. So far, experimental results are controversial. However, some excellent swimmers showed outstanding results in competition by covering up to 50% of the competitive distance under water using the butterfly kick only. This suggest that there may be a performance advantage when the swimmer ‘dives under’ the wave-making-resistance at the short competitive distances where a high swimming speed can be developed.

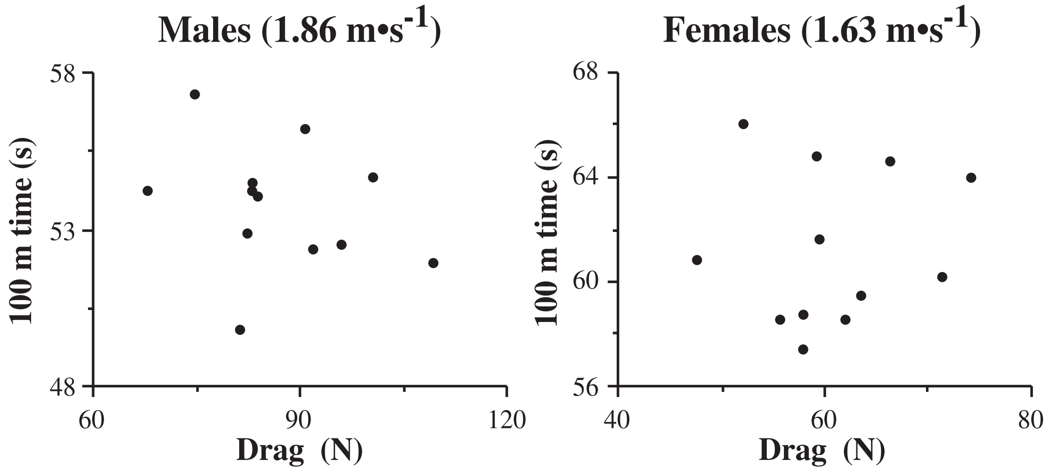

Figure 3: Performance expressed as 100 m time is presented as a function of drag measured at a velocity of 1.63 m s^{-1} for females (n = 12) and 1.86 m s^{-1} for males (n = 12)

Several authors have suggested that the drag forces encountered while swimming at the surface may be diminished by improving the swimming technique (1, 44, 45). If true, this would lead to the hypothesis that elite swimmers have lower resistance at high swimming velocities than poorer ones. To test this hypothesis Hollander *et al.* (46) determined the relationship between drag and maximal swimming performance. Active drag was determined in both a group of 12 male and 12 female elite swimmers at high velocities (mean velocity per group 1.86 and 1.63 m s^{-1} respectively). As no significant relationships were found (males r = -0.27, females r = 0.07, see Figure 3), it was concluded that drag per se is not a determining factor of maximal swimming speed. However, in an experiment in which triathletes were compared to swimmers (47), a considerable difference in drag was found: K (see equation 4) was 30.5 for the competitive swimmers and 41.6 for the triathletes. In part this difference could explain the difference in performance between the two groups of athletes. The different values for K could be interpreted as being a result of the poorer technique of the triathletes causing superfluous body movements in the vertical and horizontal plane perpendicular to the swimming velocity (1, 44, 45). In other words, competitive swimmers seem to be able to use a better stroking pattern resulting in a more stable body position and hence a smaller value of *K*.

The question whether drag is a major performance determining factor cannot be entirely resolved on the basis of the research presently available. It seems as if drag is determined by anthropometric dimensions (e.g. body cross-sectional area and height) in groups of elite swimmers that are homogeneous with respect to swimming technique. Probably a small reduction in drag can be achieved by stretching the arm in the glide phase of the stroke, as was suggested by Holmér (48). The body cross-sectional area is especially reduced when the shoulder is 'stretched behind the arm'. Furthermore, in all descriptions of active drag, it is implicitly assumed that K is constant during the stroke cycle. Most probably this is not the case. This could for example explain why despite the large oscillations in the propulsive force (27, 49) rather small intra-cyclic velocity oscillations are reported (50). It could be conjectured that skilled swimmers are able to synchronize the propulsive peaks with the phases in which the body cross-sectional area (and consequently pressure drag) is at a maximum. Hence, the oscillations in propulsion in drag coincide with oscillations in propulsion, such that the resultant force on the swimmer (about zero at constant speed) shows much less variation. In line with this suggestion is that reduced velocity oscillations are observed in the more proficient swimmers (50, 51) (see also 52). Another as yet unresolved issue is the function of the glide. It could be theorized that the forward stretched arm increases the length of the 'hull', with consequent reduction of the Froude number and wave-making resistance. Also, the gliding arm could reduce the pressure above it and in front of the head, thereby reducing the amplitude of the bow-wave, i.e. similar to function of the cone shaped nose below the water-line in large ships (53). It has been suggested that proficient swimmers have a lower wave making resistance. Whether this is the result of technique or anthropometry is another unresolved issue (see 54).

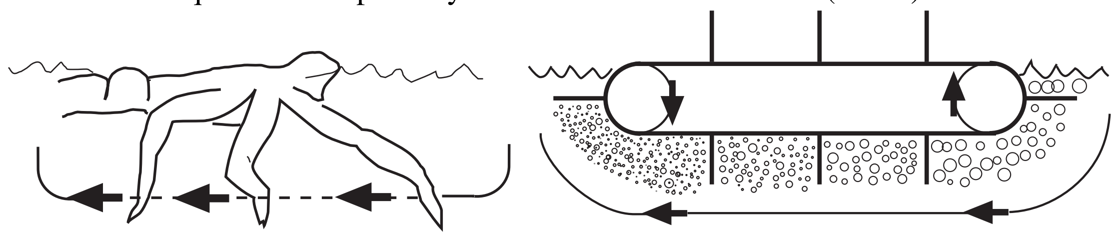

Figure 4: Early view on the mechanics of propulsion: The hand is used as an oar. The hand is pulled straight back, creating a high pressure zone on the palm, and a low pressure zone on the back of the hand. The resulting propulsive drag force would propel swimmers forward corresponding to the caterpillar paddle-wheel (right). However, in reality this form of propulsion is rather ineffective, because turbulence from one blade affects the ability of the following blade to create drag. Also, a relative small mass is given a rather large velocity change, leading to large losses of kinetic energy to the water .

Propulsion in Human Swimming

The mechanics involved in the generation of propulsive forces received scarce attention until the late 1960s when Counsilman (45) published his famous kinematic analysis of the swimming strokes and began to speculate on the fluid dynamic mechanism of propulsion. Previously, propulsive forces on the surface of the hand were thought to be created in a similar fashion to those on the surface of an oar. It was reasoned that the drag forces generated by moving the hand backward would propel swimmers forward as a direct application of Newton's Third Law (action = - reaction, Figure 4). Counsilman modified this view by pointing out that instead of pushing water backwards in a straight line, the hand follows a curvilinear path to continuously find still water to push against and thus gain more resistance than it would by pushing against water that had already been accelerated. In part this description of the involved hydrodynamics could explain the curved paths of actions observed and reported in swimmers and frequently referred to as S-shaped or "inverted question mark" pulls (Figure 5).

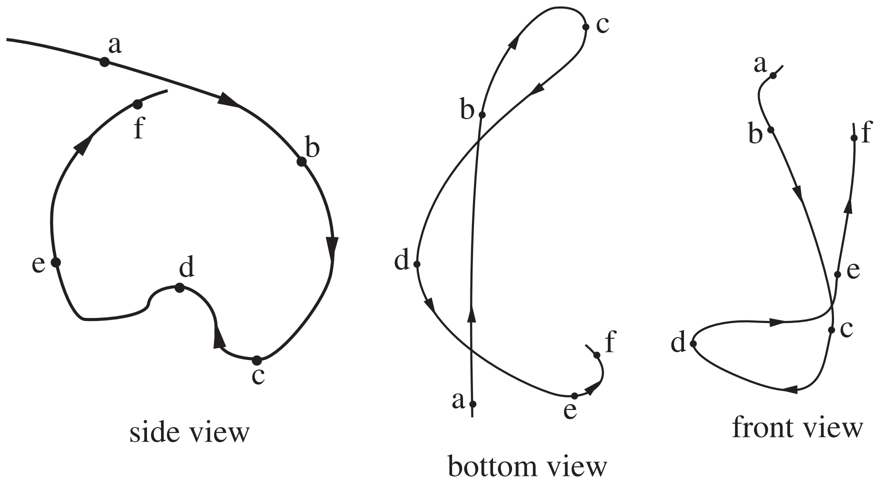

Figure 5: front crawl stroke pattern of the right hand in three dimensions (after Svec (55)) a-b: entry, b-c: entry scull, c-d: inward pull or insweep, d-e: outward pull, e-f: exit or upsweep.

Shortly thereafter Counsilman also drew attention to the importance of lift forces, which act perpendicular to the direction of hand movement (56) and stated that *both* lift and drag forces are important for propulsion. This modified theory could explain the sculling movements during the arm pull observed with underwater cinematography (Figure 5).

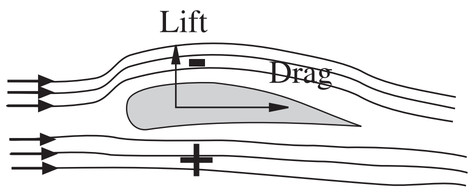

Figure 6 Movement through water is resisted by the water immediately in front and around the hydrofoil. This water resistance is called *drag*. Drag force always acts in the direction of flow. Due to its shape the water speed above the hydrofoil is greater than below. The pressure above the hydrofoil is less than below, resulting in a net upward force called *lift*. By definition, lift acts perpendicular to drag.

The nature of the lift force is similar to that experienced by an aircraft wing. As Counsilman (56) explained: "A wing provides aerodynamic lift through the camber (curvature) of its surfaces. Because the upper surface is more highly cambered than the lower surface, the air moving over the top surface is forced to move more quickly. This results in a lower pressure on the upper surface as compared with the lower surface and results in aerodynamic lift (Bernoulli's Principle)". (p. 61) Thus the pressure differential results in a lift force directed at right angles to the line of motion of the propelling surface (see Figure 6 and 7). Subsequently, numerous papers were published (57-63) to support the notion that the total propelling force acting on the hands is composed of both a lift and drag component.
The suggested hydrofoil behaviour of the hand was investigated in depth by Schleihauf (24, 27, 61, 64-66). According to hydrodynamic theory the drag and lift force can be derived using the following equations (compare equation 2):

$$L = \tfrac{1}{2}\rho\, u_h^2\, C_l\, S \qquad 9$$

$$D = \tfrac{1}{2}\rho\, u_h^2\, C_d\, S \qquad 10$$

where L = lift force, D = drag force, ρ = density of water, u_h = hand velocity, C_l = lift coefficient, C_d = drag coefficient, and S = propelling surface of the hand. Values of C_l and C_d for the human hand as a function of angle of attack and sweep back angle were determined in a fluid lab using hand models (24). Similar data were obtained by Wood (63), who measured the lift properties of plaster casts of swimmers' hands and forearms in a wind tunnel. Both Schleihauf and Wood (and more recently Berger *et al* (49) and Payton (67)) demonstrated that the propulsive forces generated by the hand are indeed composed of both drag and lift forces. The values of the lift coefficient (C_l) and drag coefficient (C_d) are dependent on the *angle of attack* and the *sweep back angle*. The angle of attack is the angle formed by the inclination of the propelling surface to its direction of motion (see Figure 7). The sweep back angle defines the leading edge of the hand (see Figure 8).

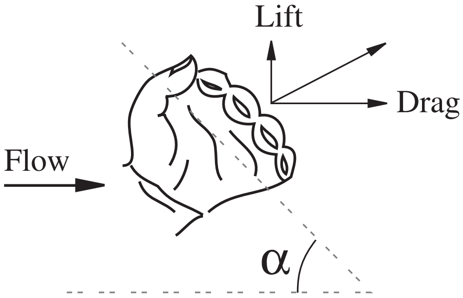

Figure 7 The angle of attack (α) (After Svec (55))

The coefficients of lift and drag are strongly dependent on the angle of attack. Small changes in the angle of attack can significantly change the resulting propelling force, which is the vector product of the lift and drag force. Thus the propelling force can be steered in the desired forward direction by varying the lift and drag component by means of the angle of attack. This implies that instead of one optimal line of motion straight backwards, numerous solutions exist to combine lift and drag force components to provide propulsion in a straight forward direction (see Figure 9). For an optimal propulsive force, the orientation of the hand must be constantly fitted to ever-changing directions of hand movement during the pull (61). In order to select the angle of attack which gives the optimal combination of drag and lift forces at every moment of a pull, the swimmer has to have a “feel” for water. Swimming talent is certainly associated with this feel for water.

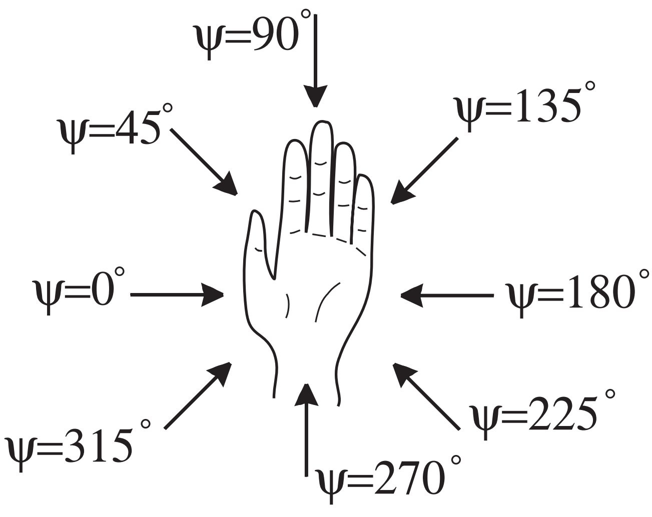

Figure 8: Sweepback angle convention. After Schleihauf ((24)).

The propulsive forces generated by the hand during a pull may now be estimated given the values of C_l and C_d for the hand and the velocity and orientation of the hand during the stroke. In his more recent experiments Schleihauf used two battery-operated video cameras (30 Hz) in underwater housings to simultaneously collect front and side views of the swimmer. Eight points on the swimmers body were studied frame by frame from both front and side view. The body landmarks were middle fingertip (I), center of the wrist joint (W), index finger metacarpophalangeal joint (T), little finger distal metacarpeal end (P), elbow joint, shoulder joint, and anterior superior iliac spine. The plane of the hand is defined by the vectors IW and TP. The angle of attack is defined as the angle between the line of motion of the hand and the plane of the hand. The sweep back angle defines the leading edge of the hand (figure 8). See for a more extensive description (27). Using this technique Schleihauf has presented detailed analyses of stroke mechanics and hydrodynamic forces of elite swimmers representing all stroke techniques (24, 66). The analyses reveal that swimmers use complex sculling motions of the hands (compare Figure 5). By changing the angle of attack of the hand they successfully combined lift and drag forces in a resultant propulsive force in the direction of motion (see Figure 9). Further, it was demonstrated that highly skilled swimmers utilized the most rapid hand actions (high velocities *v*, implying high propulsive forces, see Equations 9 and 10) in the side-to-side and up-and-down dimensions of motion, rather than in the front-to-back direction, which would have been expected for purely drag-based propulsion. It may be concluded that lift forces play an important role in propulsion.

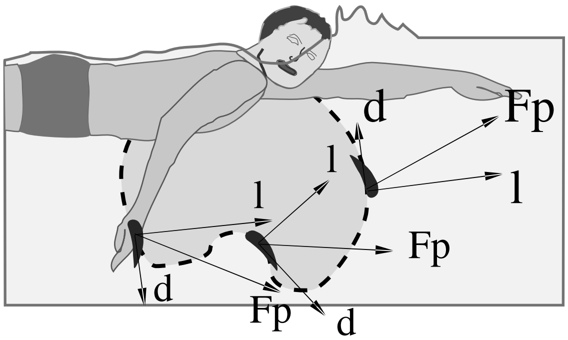

Figure 9: The direction of the hand velocity changes during the pull. The angle of attack is continuously adapted to direct the propulsive force F_p forward.

Although the presented lift-drag approach can explain the complex sculling underwater movements of the hand, recent experiments (31, 68) yield observations suggesting that the "lift-drag" approach sketched above has some deficiencies. In these experiments a comparison was made between the propulsive forces using Schleihauf's approach and those measured directly on the M.A.D.-system (system to Measure Active Drag) (see Figure 1). In general, a reasonable degree of agreement between the two methods was observed ($\overline{F}_{mad} = 34$ $\overline{F}_{Schlei} = 30$ N, r = 0.76). However, Schleihauf's approach yielded on average 10% lower values (31). Different explanations can be given for this discrepancy. An important assumption of Schleihaufs quasi-steady approach is that the coefficient of lift and drag of the underarm and hand obtained using *stationary* flow in a flow tank can be applied for modeling hydrodynamic forces in front crawl swimming. It can be questioned whether the steady flow conditions in the lab can be extrapolated to the flow conditions that are experienced during swimming. The hand in skilled front crawl swimming constantly changes its angle of attack and sweep-back angle with respect to the water and also accelerates and decelerates: the flow conditions are highly *unsteady*. In fluid mechanics there are two important dynamic effects associated with an immersed accelerating segment. These are the vortex shedding and dynamic stall (or delayed stall) effects (69, 70). These effects are ignored, or assumed to be negligible, in the quasi-static approach. Therefore, the, in essence 2-D, quasi-static approach to determine lift and drag coefficients has been questioned (70, 71). Could it be that the quasi-steady assumption fails?

Until recently, a similar situation was present in the study of insect flight. Careful quasi-steady analyses, combining kinematics and wind-tunnel force measurements of isolated wings, similar to Schleihaufs analysis above, led to the paradoxical result that many flying insects cannot generate enough force to carry their own weight (see 72). The implication was that the quasi-steady assumption fails and that unsteady lift-enhancing mechanisms must play an important role. In order

to better understand the problems and peculiarities of unsteady flow, some more fluid dynamic background is needed.
As shown before, a wing will experience an upward lift force when the average air pressure above the wing is lower that the pressure under the wing (see Figure 6). According to Bernoulli's principle (inverse relationship between pressure and velocity), this is the case when the average air velocity at the upper surface of the wing is higher than that beneath the wing. The total airflow around the wing may be decomposed in a circulating flow around the wing with velocity v and a uniform flow at velocity u passing the wing (Figure 10). The velocity of the air above the wing will be $u + v$, and beneath it $u - v$. The pressure difference is then approximated by $\rho(u+v)^2/2-\rho(u-v)^2/2=2\rho uv$. This pressure difference acting across a wing of area A_p, gives a lift-force of $2\rho uvA_p$.

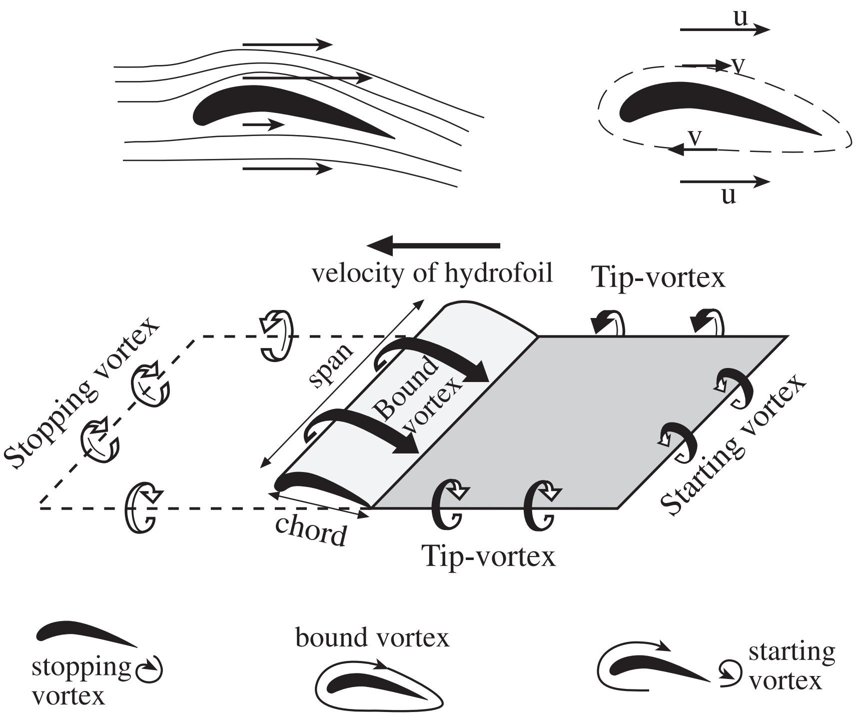

Figure 10: Streamlines round an wing, which is shown in section. The flow past the wing can be decomposed in uniform flow with velocity u with a circulation around the wing with velocity v.
The 3-D vortex wake created by a rapid acceleration of a wing is shown at the top. The shaded area indicates air given a downward impulse. Dashed lines and unshaded vortices indicate future events. Creation of the starting, bound and stopping vortices is shown below in a 2-D view, depicting the vertical plane (after 72)

The circulating flow around the wing is called a 'bound vortex' (72) (see Figure 10). The strength or *circulation* of the bound vortex is proportional to v. Aerodynamics texts such as Prandtl & Tietjens (73) give a formal definition of circulation. In the case presented in Figure 10 it may be approximated by the circumference of the wing (twice the chord, c) multiplied by v, hence $\Gamma=2cv$ (74). The lift on a wing of span s with circulation Γ must be equal to that given in equation 9 and is given by ($c \bullet s = S$):

$$L = \rho u\Gamma s = \tfrac{1}{2}\rho\, u^2\, C_l\, c\, s = \tfrac{1}{2}\rho\, u^2\, C_l\, S \qquad 11$$

Circulation and lift are constant for a wing moving at a constant speed (steady state). However, when the wing accelerates impulsively from rest to a certain speed, the circulation needs time to build up. At the start of the motion, air swirls around the trailing edge and vortices are shed into the wake (Figure 10). This build up of the so-called starting vortex will continue until a constant circulation is reached. During this period the bound vortex will develop to the final steady state value. This gradual build-up of the bound vortex is called the Wagner effect. Even after 6 chord lengths of travel the circulation and lift are only 90% of the final values (72). Since the chord of a hand is about 0.1 m, the steady state value of circulation (requiring > 0.6 m of travel at constant velocity) may not be reached at all during swimming.

It is a fundamental rule of fluid dynamics that no vortex can be created unless a vortex of the opposite sense and strength is set up simultaneously (Kelvin's circulation theorem). Thus the starting vortex is equal in magnitude but opposite in sense of rotation to the bound vortex. 'Tip vortices' join up the starting and bound vortices. They are created as air swirls around the tips from the higher pressure zone below the wing to the lower pressure zone above. Accidental entrapment of air in the water often produces visible evidence of tip vortices trailing behind a swimmer's hands in the early stages of the stroke (52). Thus, the accelerating wing (or a hand) generates a vortex 'ring', enclosing air (water) that has been blown downwards. When the wing motion stops the bound vortex then swirls off the trailing edge and forms the stopping vortex, closing the vortex ring, which subsequently moves downward into the wake (or backward in the case of a swimmer). Whether the water flow associated with swimming propulsion is unsteady or quasi-steady, the reaction force to the lift must impact momentum to the fluid. With a periodic propulsion, as in insect flight and swimming, the formation of start-, tip-, and stop vortices is unavoidable. As argued above, this may well result in a vortex ring left behind in the wake during each halfstroke (in swimming: a pull may be divided in substrokes, dependent on the movement direction of the hand, e.g. change from insweep to upsweep, point d in Figure 5). The momentum of this ring must correspond to the lift force generated during the halfstroke. Thus, in principle, propulsive forces can be estimated by analyzing the wake of the propelling surfaces (75).

The lift coefficient of a wing is generally roughly proportional to the angle of attack, up to a certain limit. The limit is due to the phenomenon called stalling, and the angle of attack at which stalling occurs is known as the stalling angle. When the angle of attack is less than the stalling angle, the flow is 'attached' to the surface of the wing. When it is greater than the stalling angle, the flow separates from the upper surface of the wing and large eddies form leading to a sudden drop in lift force and hence of $C_l(\alpha)$. It was shown that the lift forces needed to support the weight of insects during hovering is greater than the peak lift forces the wing can generate before it stalls in steady motion (72). The problem is even worse because the quasi-steady analysis ignores past history and therefore the Wagner effect. Circulation and lift will not grow to steady-state values on each half stroke because the wing moves only 3-4 chord lengths during hovering. Thus, the quasi-steady estimate of lift is overly optimistic, and the discrepancy with actual forces produced is even greater. As observed above, this crisis forced insect flight researchers to look for unsteady lift-enhancing mechanisms. Several mechanisms have been proposed (see for a review 72).

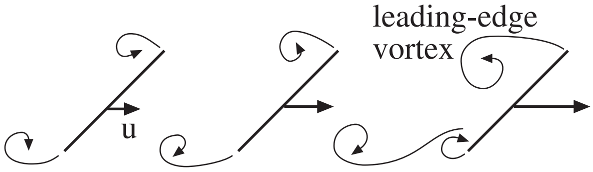

Figure 11: Leading- and trailing-edge vortex shedding from a flat wing moving with increasing speed. Maximum lift is produced when the leading-edge vortex is located above the wing (dynamic stall), (after 72)

Weis-Fogh (76) proposed two novel mechanisms of lift generation that explained the flight mechanism for certain groups of insects. The lift produced by these mechanisms depends on events during the rotational phases (pronation and supination) at either end of the wingbeat, when the wing rapidly rotates about its long axis through about 120° in preparation for the next half stroke. These rotations might be comparable to what happens in the front crawl during the transition from insweep to outward pull (Figure 5). It is hardly surprising that these mechanisms were new to aerodynamics; conventional wings are never operated in this extreme manner (72). Another class of unsteady mechanisms seems more applicable to swimming and is discussed next.

Delayed stall or dynamic stall

We have seen that if the angle of attack is slowly increased to values above the stalling angle, separation of flow from the wing surface occurs. However, if the angle of attack is suddenly increased above the stall angle, or if the wing is accelerated quickly with a high angle of attack, separation of flow is delayed and the wing may move for several chords generating lift that exceeds the maximum steady-state value. The growth of lift beyond the values observed during steady state are associated with the formation of a leading-edge vortex (Figure 11). The leading edge vortex is formed when the flow separates from the wing at the onset of stalling. As long as the leading edge vortex is above the wing, circulation around the wing is enhanced, leading to significantly increased lift forces. However, the problem with accelerating wings in translation is that the leading edge vortex breaks away rather quickly (within 2 chords of travel), and lift will reduce sharply. In other words, the leading edge vortex is rather unstable.

Recently, the airflow around flapping wings was visualized, using smoke released from the leading edge of a scaled-up robotic model of a hawkmoth. Analysis showed that a strong leading-edge vortex was present during the down stroke, which was stabilized by a strong axial flow (i.e. from wing base to wing tip) (77). Thus in a flapping wing which rotates around the “shoulder”, the leading edge vortex was stable enough to remain attached along the wing until approximately three quarters of the wing length during most of the downstroke. At the tip it separates and blends into a wide, tangled tip vortex. The leading edge vortex is stabilized by a strong axial flow, originating from a pressure differential that is caused by the velocity gradient between wing base and wing tip (the tip moves faster than the base). The velocity gradient is due to the rotation of the wing about its ‘shoulder joint’, such that the up- and down velocity at the base is small but increases towards the tip. (In the front crawl stroke a similar situation occurs; the velocity of the hand relative to the water is higher than the velocity of the elbow.)

The extra lift force generated by this vortex alone was sufficient to carry 2/3 of the hawkmoth’s weight, while the impulse of the two ring vortices left in the wake corresponded to about 1.5 times its body weight (78, 79). This new lift-enhancing mechanism seems promising to explain the high lift-forces produced by a range of insects. It also showed that the conventional quasi-steady approach failed for two reasons: 1) dynamic stall cannot occur in (quasi) steady conditions, and 2) the axial flow component, which turned out to be large and crucial for stability of the vortex, is ignored in the 2D quasi-steady approach. A similar leading-edge vortex with strong axial flow is observed in delta-winged airplanes and is responsible for the high lift forces generated by such wings.

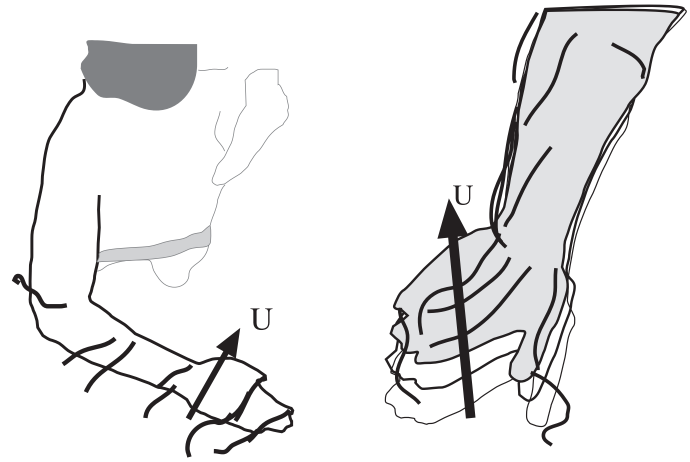

Figure 12: Flow around the forearm and hand visualized using woolen tufts during the insweep (left) and the upsweep (right). Outlines of the swimmer and tufts are traced from an underwater video. The arrow indicates the movement direction of the hand (u). During the insweep a flow pattern compatible with conventional (Schleihaufs) stroke mechanics is observed (left). During the upsweep, however, the tufts on the hand arrange in a direction almost perpendicular to the direction of hand-motion. This suggests a strong axial component in the direction of the fingertips not compatible with Schleihaufs 2-D approach.

Could similar unsteady, lift enhancing mechanisms be operative in the generation of propulsion in competitive swimming? A first step to identify the hydrodynamic mechanisms underlying propulsion is flow visualization. Surprisingly enough, few attempts have been reported (e.g. 80) to visualize the flow around the arms during swimming. We conducted a pilot study in which the direction of the flow around the hand and lower arm was visualized using 20 woolen tufts (10 cm long) attached to the hand and lower arm. In the glide phase, the flow wraps around the arm from tip to base in a gently spiraling fashion. During the transition to the insweep the flow detaches from the surface of the hand and later also from that of the arm. During the insweep the flow is approximately perpendicular to the arm axis, as had been expected from conventional hydrofoil theory (Figure 12). However, during the upsweep the flow over the back of the hand showed a strong axial component (in the direction of the fingertips), which, at the level of the hand, was at more than 45° with the direction of the hand movement. The flow behind the trailing edge (as shown by the twisted tuft on the thumb and the little finger, Figure 12) seemed to show a tip vortex at the finger tips and also at the tip of the extended thumb and was in the general direction of the arm movement. This axial flow over the hand surface is very suggestive in the light of the 3D leading-edge vortex observed in insect flight. It is possible that a similar 3D leading-edge vortex is formed along the leading edge of the hand (i.e. the side of the little finger) during the upsweep. The ulnar abduction of the hand during the upsweep suggests that the hand may be used as a delta-wing in this phase of the stroke. More detailed visualization studies are needed to substantiate this hypothesis. However, whatever the propulsive mechanism, this study clearly demonstrates that the flow pattern during the upsweep is very different than expected.

Energetics

So far, the discussion has focussed on the forces involved with swimming. It will however be obvious that in swimming the aerobic and anaerobic capacities co-determine success. Thus, the question is not simply how to maximize the propulsive force and minimize resistance, but rather how to accomplish this with finite metabolic capacities. Hence, it is also important to analyze the energetics of swimming. We will first discuss the measurement of the total rate of energy expenditure, some of its determining factors like velocity, drag and buoyancy and then look at the measurement of energetics involved with the propulsion mechanics.

Different procedures have been used to determine the energy expenditure of swimming. The measurement of oxygen uptake during and immediately after swimming offers an indirect method to approximate energy expenditure. However, the oxygen uptake is only directly related to the intensity of the effort as long as the swimmer is performing in steady state and at less than 50 - 70% of the maximum oxygen uptake ($\mathring{V}O_2max$). A variety of techniques were developed to assess the oxygen uptake during swimming. Karpovich (81, 82) had the swimmers hold their breath during 50 yd swims. The expired gas of the swimmers was collected for 20 to 40 min after the completion of the swim to determine the so-called 'oxygen-debt' (83). This same procedure was adopted by Adrian (84), and Klissouras (85). Another approach was introduced by Montpetit (86) for swims of longer duration (> 5 min.). The technique relies on the backward extrapolation of the O_2-recovery curve to determine the $\mathring{V}O_2$ during swimming. The swimmers are instructed to take a breath approximately one stroke before the finish of a 400-m swim and to exhale the breath into a breathing mask as soon as it is sealed over the face immediately after the swim. Expired air is continuously collected for the first 20 or 40 s after the swim (2, 87). This procedure has the major advantage that the swimmer can perform without restrictions presented by the instrumentation used (breathing valve and hoses). Direct measurements of energy expenditure have also been performed during tethered swimming (88-90), flume swimming (e.g. 21, 38, 91, 92-95), swimming in an annular pool (e.g. 16, 22, 23), and free swimming (e.g. 47, 89, 96, 97-101).

Energetics related to buoyancy and drag

Almost all studies (except 82) find that women require a ± 30% lower rate of energy production than men to maintain a given velocity (10, 16, 22, 23, 87, 94, 102, 103). It was suggested that women did not need to expend as much energy in staying afloat, because of their higher mean percentage of fatty tissue (8, 104, 105). Furthermore, the distribution of adipose tissue along the head-feet axis is more favorable in women than in man, so that the tendency for the feet to sink is lower (10, 16, 105). Thus women may need less energy to keep the body in a horizontal position (102), while it is also likely that a more horizontal position reduces drag (48). Thus the question can be raised whether increased buoyancy or reduced drag has the most effect on the energy expenditure.
During constant speed swimming a considerable fraction of the energy expenditure is utilized to overcome drag (10). As was indicated previously, drag is related to the square of the swimming velocity (Equation 4). Consequently, the power to overcome drag (P_d) is related to the velocity cubed and a drag factor K, according to $P_d = K \cdot v^3$ (28). When we related the rate of energy expenditure to the power to overcome drag, no differences between males and females were found (101). In other words, the males do not expend additional energy to stay afloat. The 30% difference in energy expenditure at a velocity of ±1 m•s^{-1} reported by Pendergast *et al.* and di Prampero *et al.* (16, 22) can be explained by the 29% lower resistance reported for female swimmers at this velocity (28). The lower drag values were related to the difference in frontal area (106); see also equation 2. This is in line with the observations of Montpetit *et al.* (102), who demonstrated that the difference in energy expenditure between males and females disappears when correcting for body size. Still it is interesting to study the interrelationship between drag and buoyancy. The effect of a triathlon wet suit on drag was studied in 8 male and 4 female competitive swimmers swimming at a velocity of 1.25 m•s^{-1}. (42). A 14% reduction in drag (from 48.7 N to 41.8 N) was found. The effect of the reduction was probably due to increased buoyancy inducing less frontal resistance. However, since the effect of the suit on the 'lower density' female swimmers was not different from the effect on the 'high density' male swimmers, the true relationship between buoyancy, drag and energy expenditure remains to be determined.

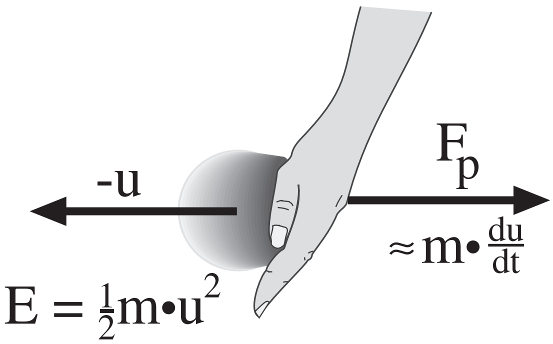

Figure 13: The propelling force F_p is generated by giving a mass of water m a velocity change du/dt. Consequently, the pushed-away mass of water acquires a kinetic energy ($0.5 \cdot m \cdot u^2$). This kinetic energy is the result of the work done by the swimmer on the pushed-away mass of water. Hence, part of the total mechanical work the swimmer delivers is converted into kinetic energy of the water, rather than forward speed of the swimmer.

Energy losses in Propulsion

It seems obvious that the generation of the propulsive force in swimming is different from the propulsion generated during on-land activities. On land the push-off is performed against the earth. Due to the large mass, the earth will endure an undetectable (and therefore negligible) acceleration. Similarly, in the aquatic environment propulsion is generated by accelerating water. However, in this case the acceleration of the water can not be ignored. Or more formally, by Newton's Second Law of Motion, the force F required to give a mass m an acceleration du/dt is given by

$$F = m\, du/dt \qquad 11$$

Since the momentum of a mass of water *m* travelling with velocity *u* is *m•u*, equation 11 states that the force applied to the mass of water equals the rate of change of momentum. Irrespective of the fluid dynamic mechanism of propulsion (drag, lift, vortex, see above), the thrust that propels the swimmer forward is generated by giving a mass of water backward momentum (see Figure 13). Suppose a swimmer obtains the thrust *F* required to swim with velocity *v*, by giving some of the water a backward velocity *–u*. Thus a mass of fluid will acquire a change of momentum (action = -reaction). Consequently, the kinetic energy ($0.5m•u^2$) given to the water in unit time equals the force acting on the mass of water (*F*) times the velocity (*u*): *F•u*. This is the power (P_k) consumed in driving water backwards, in giving it a kinetic energy change, which is wasted power (107). Suppose that the swimmer swims at constant speed. In that case the resistive forces must equal the propulsive forces. The power required to drive the swimmer through the water equals *F•v*. This can be denoted as useful power. Now we can define an efficiency, known as the Froude efficiency, or propelling efficiency (e_p) (107, 108):

$$\text{propelling efficiency} = \frac{\text{useful power}}{\text{useful power} + \text{power lost to water}} = \frac{v}{v+u} \qquad 12$$

From equation 12 it follows (since *u* will not be much smaller than *v*) that the generation of propulsion in a fluid *always* will lead to the loss of mechanical energy of the swimmer that will be transferred in the form of kinetic energy to the fluid. Two aspects of this analysis are important for human swimming: (i) the power losses are considerable (e_p << 100%), and (ii) the power losses to the water are highly dependent on technique.

The first aspect is underlined by the propelling efficiency values observed in fish swimming. For trout values range from e_p = 15% (swimming at 20% of maximum speed) up to e_p = 80% at maximal speed (109). Even if it is assumed that humans could use a swimming technique as efficient as trout, still 20% of the available mechanical power would be 'lost' to moving water backward instead of moving the swimmer forward. Hence, a considerable amount of mechanical power is not transferred into forward speed of the swimmer. In fact, e_p is even lower in human swimmers (see below).

The magnitude of the propelling efficiency depends on the propulsion mechanism. The efficiency is higher if the swimmer accelerates a large mass of water per unit time to a low velocity, than if it obtains the same propulsion by accelerating a small mass to a high velocity (107) (see equation 12). Or, as already stated by Counsilman (56), the technique should be such that the amount of water against which the push-off takes place is as large as possible. Thus swimming *at the same speed* requiring the *same propulsive force* can have different associated energy costs depending on the technique that is employed. Consequently, maximal swimming speed can be achieved by a swimming technique where optimal propelling force is obtained with an optimal propelling efficiency and a minimal body drag (110). Hydrodynamics applied to fish swimming (107, 111) shows that these requirements of a high propulsive force with limited wasted power can be met, in part, by a proper use of lift forces on the propelling surfaces. This implies that the analyses of lift and drag forces acting on the lower arm and hand (27, 49, 67) can be related to propelling efficiency. Again, it is emphasized that basic mechanics of propulsion in swimming reveal that thrust and loss of mechanical energy to pushed-away masses of water are two sides of one coin. For a true understanding of the performance determining factors both aspects should be incorporated in the analysis.

Already in the 1970's, several authors noted that the power lost to water (P_k) should be incorporated in the power bookkeeping or power balance. For example, Holmér (21) noted on the energy for propulsion: "...not all mechanical energy is useful in overcoming drag. *Part of it is lost in the creation of turbulence*, *and in vertical and lateral displacement of water*.". Charbonnier (112) stated: "The speed with which the swimmer can move depends on the energy he can furnish to *push towards the rear masses of water* with his limbs, and the resistance opposing his forward progress in the water". Finally, Miyashita (113) observed: "An efficiency (work output / chemical energy used) in land exercises was approximately 20 to 25%. On the contrary, efficiency in swimming was

0.5 to 2.2%. Therefore, the swimmer must produce more external work than that estimated from the speed variations and water resistance. Propulsive force which drives the swimmer forward is created by the swimmer's arms and legs as they push the water backwards. This means that *some parts of chemical energy may be transformed into kinetic energy of water.*" Nevertheless, in numerous papers on the energetics of swimming P_k was not acknowledged, leading to questionable conclusions with respect to the estimated values for active drag and mechanical efficiency (e.g. 16, 22, 23, 114, 115). See for a more extensive discussion (116).

Measurement of energetics involved with the propulsion mechanics

The total mechanical work a swimmer produces is apportioned to work to overcome the total resistance and work to generate the propulsion. Since in competition the swimming velocity is to be optimized, it is more relevant to look at the time derivative of the work produced by the swimmer, which is the mechanical power production. Power (P) is defined as the rate at which energy is transferred from one system to the other (from swimmer to the aquatic environment), and equals the dot-product of the force-vector **F** and the velocity of the point of application **v**:

$$P = \mathbf{F} \cdot \mathbf{v} \qquad 13$$

where **F** equals a force vector (i.e. drag or propulsion) and **v** the velocity vector of the point of application of the force (i.e. the swimming velocity or hand velocity). The two dominant forces in front crawl swimming (propulsion and drag) are indicated in Figure 14 (ignoring gravity and the buoyant force) with their respective velocity of application. This *free body diagram* underlines that two major power components can be discerned in competitive swimming; power necessary to overcome drag (P_d) and power expended in giving a kinetic energy change to pushed-away mass of water (P_k).

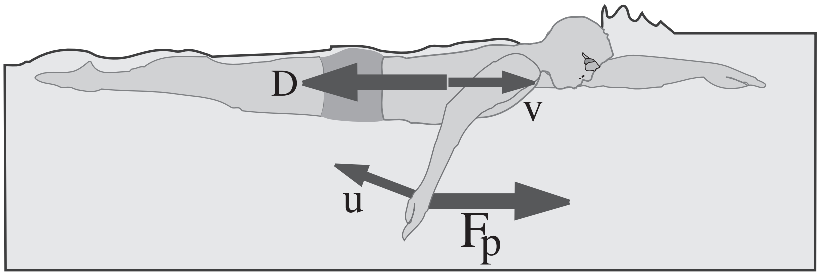

Figure 14: "free-body-diagram" of swimmer: When swimming at constant speed (arms-only), propulsion ($\mathbf{F_p}$) equals drag (**D**). Thus two major forces act on the swimmer in the swimming direction, whereby the point of application of each force has a velocity. Since power equals the dot product of **F** and **v**, the diagram shows that one "energy –flow" relates to drag and one to propulsion.

The mechanical power required to overcome drag will thus equal (101, 117)

$$P_d = \mathbf{D} \bullet \mathbf{v} = K \cdot v^2 \cdot v = K \cdot v^3 \qquad 14$$

The mechanical power lost in the generation of propulsion (P_k) is less obvious to calculate. At least three approaches can be used: (1) estimate the transfer of kinetic energy to the water on the basis of measured kinematics and estimated kinetics of the propelling surfaces, whereby the propelling force ($\mathbf{F_p}$) and its velocity of application (**u**) are estimated (68), hence

$$P_k = \mathbf{F_p} \bullet \mathbf{u} \qquad 15$$

(2) quantify the rate of kinetic energy change of the water (see below), and (3) estimate the difference in rate of total energy expenditure between swimming whereby the push-off is made against water, and swimming whereby the push-off is made against fixed points (P_k=0) (108). This approach is visualized in Figure 15. When the push off is made against fixed push-off points swimming on the M.A.D.-system, no kinetic energy is lost to the water (P_k=0). Hence, the difference in rate of energy expenditure, when swimming at the same velocity, is proportional to P_k. The magnitude of P_k is estimated by multiplying the difference in rate of energy expenditure ($\mathring{E}$) with the gross-efficiency. This is the ratio of the mechanical power produced swimming on the

M.A.D.-system (**D•v**) and Ė (see 101). With this approach it was established that P_k is a considerable part of the total mechanical power output (P_o) of the swimmer. The ratio between the useful mechanical power spent to overcome drag and the total mechanical power output (P_o) is defined as the propelling efficiency e_p (see equation 12):

$$e_p = \frac{P_d}{P_o} = \frac{P_d}{P_d + P_k} \qquad 16$$

For a group of highly trained swimmers e_p values of 61% were found (108). Hence, even in highly skilled swimmers (among them an Olympic Champion) still 39% of the total mechanical power is lost to P_k. In well-trained, but not so skilled swimmers (triathletes) a value for e_p of 44% was found underlining the importance of technique (i.e. optimizing the propelling efficiency) as a performance determinant (47).

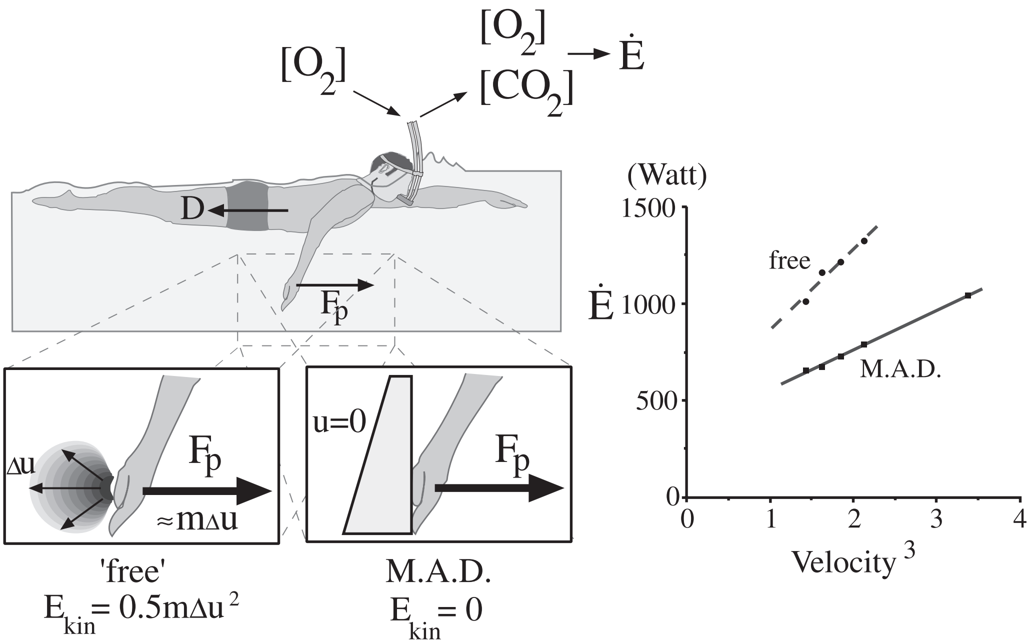

Figure 15: The power lost in the generation of propulsion is estimated by comparing the oxygen uptake during free swimming to the oxygen uptake during 'M.A.D.' swimming at the same velocity. In the free swimming condition the rate of oxygen uptake reflects the power necessary to overcome drag plus the power necessary to generate the propulsive force, whereas during M.A.D.-swimming the oxygen uptake reflects the power necessary to overcome drag only.

Values for propelling efficiency quantified on the basis of measured kinematics and estimated kinetics of the propelling surfaces yielded values for e_p of about 35% versus an e_p of 56% for the same group of swimmers using oxygen uptake measurements (68). Berger (68) could not give a satisfactory explanation for the large difference in values. However, as indicated above, the 'kinematics' method is based on a 2-D, quasi-static approach to determine lift and drag forces. As indicated above this approach fails at least in part of the stroke (i.e. the upsweep).

Factors determining propelling efficiency

Propulsion mechanics

The propulsive force is caused by momentum flow. This fluid momentum is convected by vortex action as was explained above. Irrespective of the kind of propulsion (i.e. lift or drag), the propulsive system generates wake vortices. The strength or circulation (Γ) of the wake vortex is determined in a similar way as the strength of the bound vortex around the wing and is roughly proportional to the rotation velocity (ω) times the radius of the vortex. In 3D, the center of a vortex forms a line. The strength of a vortex line is constant along its length and a vortex line cannot end in a fluid. It must extend to the boundaries of the fluid or form a closed path e.g. ring vortex. (Helmholtz's vortex theorem). As explained above, vortex rings transport momentum. The magnitude of the momentum carried by a ring vortex is proportional to the product of its area and the vortex strength (circulation). This implies that the vortex ring will also carry kinetic energy. Thus energy is transferred from the swimmer to the water in the form of kinetic energy whenever a vortex is shed. The amount of energy of a vortex lost to the water is proportional to the strength squared divided by the square root of the vortex area. Thus, the most efficient vortex with given momentum is large in size and has low vortex strength. According to Lighthill (118) vortices of approximately circular shape carry a large amount of momentum in relation to their energy. This type of vortex is created by propelling surfaces (tails) with a special lunate shape. The visualization of flow in swimming propulsion might shed more light on this interesting issue, as was indicated in the preceding and as was previously suggested by Ungerechts (119, 120). Hence, a true quantification of the kinetic energy of the pushed away masses of water might resolve the divergent values obtained for e_p using the methods described previously. As a first approximation to solve this problem it is interesting to note that a ring vortex moves at a so-called self-induced velocity, which is proportional to its kinetic energy. Hence, a first order estimate of the kinetic energy of the wake may be obtained if the diameter and the self-induced velocity of the ring vortex can be estimated (cf. 78).

Hand surface area

A factor that will affect P_k and thereby e_p is the size of the propelling surfaces. It can be conjectured that small propelling surfaces (small hand area) will push-off from a small mass of water, while larger hands will push-off from larger masses. This implies that for the same propulsive force and with the same technique the swimmer with smaller hands must give a larger velocity (u) to the water. Consequently, the loss of kinetic energy to the pushed-away water will be larger when pushing off with smaller hands. This suggests that large propelling surfaces (a large hand area) give a performance advantage in swimming (e.g., one can swim much faster with 'flippers' on the feet). This can be derived mathematically by expanding equation 12:

$$e_p = \frac{P_d}{P_d + P_k} = \frac{\mathbf{D} \bullet \mathbf{v}}{\mathbf{D} \bullet \mathbf{v} + \mathbf{F_p} \bullet \mathbf{u}} \qquad 17$$

If, for the sake of simplicity, only drag forces are generated by the hand, then $\mathbf{D} = \mathbf{F_p}$. Equation 17 can be developed using equations 2 and 4 into (a similar, but slightly more complicated formula may be derived if lift forces are also taken into consideration):

$$e_p = \frac{1/2 \cdot \rho \cdot v^3 \cdot C_{db} A_p}{1/2 \cdot \rho \cdot v^3 \cdot C_{db} \cdot A_p + 1/2 \cdot \rho \cdot u^3 \cdot C_{dh} \cdot S} = \frac{1}{1 + \sqrt{\dfrac{C_{db} \cdot A_p}{C_{dh} \cdot S}}} \qquad 18$$

with C_{db} the coefficient of drag of the body, and A_p the frontal area of the whole body, and C_{dh} the coefficient of drag of the hand, and S the frontal area of the hand. In this formula an increased propelling surface size S directly leads to an increase of the propelling efficiency e_p. To test whether swimming with an enlarged propelling surface will increase e_p, $\mathring{E}$ was measured at equal speeds swimming with and without paddles (attached to the hands) (121). At the same average velocity the effect of swimming with paddles was an increase of e_p of 8%. This is in line with other studies that showed that swimmers of a high performance level have a significantly larger hand and arm surface than swimmers of lower performance level (122, 123). This finding has some practical implications for swim training. Although for obvious reasons the use of hand paddles in competition is not allowed, they may be useful during training. If maximal performance is taken into account, at an equal power output a higher swimming velocity can be attained, due to the higher propelling efficiency. Hence, a higher propulsive force must be applied, since the drag forces will increase at higher swimming speed, and again at constant speed the average propulsion must equal the average drag. In this sense paddle swimming might be a rather specific form of strength training. However, paddles significantly reduce backward hand velocity and reduce the rotation of the upper arm (124). If both movement pattern and movement speed specificity are deemed important, the efficacy of paddle swimming as a specific resistance training exercise can be questioned. Apart from the practical implications for training, the paddle study did show that P_k and e_p are important and may be changed by improving technique.

Power balance applied to swimming

Several aspects of the mechanics and energetics of swimming are hotly debated. For example, discussion persists whether the rate of oxygen uptake is linearly related to velocity (87, 102) or that it is related to the cube of the velocity (101). Another issue is whether active drag differs from passive drag (17). It is also debated whether the mechanical efficiency of proficient swimmers is much better than their less speedy counterparts (10, 16, 35, 94). Is there a way to more or less resolve the debated issues? A fundamental approach is to use the first law of thermodynamics: energy can be neither created nor destroyed, it can only change in form (125). The swimmer converts metabolic energy to heat, to energy for internal organs (heart, lungs, brain, etc.) and to mechanical energy necessary to generate the propulsion and overcome the resistive forces of the water. Energy conservation implies that a balance must exist among these various forms of energy. Also, the rate of energy conversion must be in balance. Hence, a power balance should also exist. If a consistent power balance can be formulated, it provides, albeit circumstantial, evidence for the correctness of the incorporated elements.

The power balance during steady state swimming can be described as:

$$P_o = e_g \cdot \dot{E} \qquad 19$$

where e_g equals the gross efficiency and reflects that not all metabolic power is converted to mechanical power; part of it is converted to heat or is necessary to support other body functions. In the transformation process part of the chemical power present in foodstuff is converted to heat. e_g is quantified by the total mechanical power output (P_o) divided by the rate of energy expenditure. The average regression equation describing the process of converting ($\dot{E}$) into P_o while swimming was estimated with the M.A.D. system and appears to be (47, 101):

$$P_o = 0.093 \bullet \dot{E} - 16 \qquad 20$$

Since P_o equals Pd / e_p (see equation 17), equation 19 develops into (using equation 14):

$$\dot{E} = \frac{P_d}{e_p \cdot e_g} = \frac{K \cdot v^3}{e_p \cdot e_g} \qquad 21$$

This suggests that the rate of energy expenditure is determined by the swimming velocity, the drag factor K, and the gross- and propelling efficiencies. This equation holds as long as the energy spent in accelerating the body is negligible. Holmér (48) showed that this is the case in front and back crawl. In general, data in the literature provide support for the presented relationship: The energy expenditure rate was shown to increase with velocity cubed when swimming at velocities above ± 1 $m \bullet s^{-1}$ (82, 84, 85, 94, 96). At the same speed, the more proficient swimmer (higher e_p) swims at a lower $\dot{E}$ than the less skilled (16, 22, 23, 47, 82, 93, 94, 96). This is only true if the gross efficiency is similar for the groups that are compared. This seems to be the case, as no differences in gross- or mechanical efficiency are observed when comparing elite and less skilled swimmers (47, 101, 112, 123).

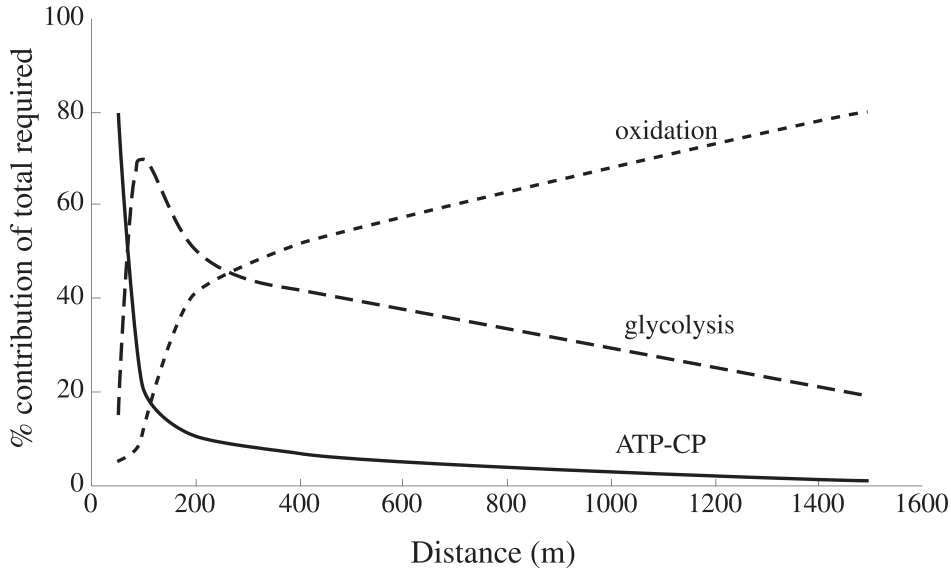

Figure 16: The relative contribution of the ATP-CP, glycolytic, and oxidative energy system dependent on race distance (after 126)

Equation 21 can more or less be substantiated from results in the literature. All of these studies used oxygen uptake measures to quantify Ė. However, the majority of the competitive distances is equal to or shorter than 200 m, hence lasting less than ± 2'30" (200 m breast stroke women). Therefore, a considerable part of the total power production will be from the immediate (ATP-CP) and the short term (anaerobic glycolysis) rather than the long term (oxidative or aerobic system) power production systems (127). Each of these systems has its own time constant and therefore, the rate at which Ė, or metabolic power, is produced, is dependent on the duration of the exercise. For example, Costill (126) (see also 128) estimates the relative contribution of the different power contribution systems dependent on the distance swum (Figure 16). The *ATP-CP system* relies on the phosphocreatine and ATP stores which are present in the muscle. The amount of ATP and phosphocreatine in the muscle is rather limited and at intense levels of exercise this power production system is exhausted in a few seconds.

Modeling of the aerobic and anaerobic power production in swimming

Studies of ice skating (129, 130) and running (131) have shown that the power production of the aerobic and anaerobic systems can be modeled and related to the mechanical power output. It seems justified to assume that the metabolic power production in swimming (both anaerobic and aerobic system) will obey a power-time curve similar to those modeled for speed-skating or running:

$$P_{aer} = P_{aer,\max}(1 - e^{-\lambda t}) \text{ and } P_{an} = P_{an,\max}e^{-\lambda t} \qquad 22$$

where P_{aer} equals the metabolic power liberated aerobically, and P_{an} the metabolic power liberated anaerobically. In words, the anaerobic system immediately has its peak activity and activity decays exponentially over time, while the aerobic system kicks in gradually and approaches its peak asymptotically in time. $P_{aer,max}$ and $P_{an,max}$ equal the maximal aerobic and anaerobic power, while λ denotes a time constant defining the rate of increase and decrease of aerobic respectively anaerobic power. Following Margaria (132) and Ward-Smith (133) it is assumed that the power production of the anaerobic and aerobic pathways can be described with one time constant, suggesting a causal relation between them. The total amount of aerobic and anaerobic energy over a certain amount of time τ can be calculated by integration of the P_{aer}- and P_{an}-functions:

$$E_{aer} = \int_0^{\tau} P_{aer,\max}(1 - e^{-\lambda t})dt = P_{aer,\max}\tau + \frac{P_{aer,\max}}{\lambda}(e^{-\lambda\tau} - 1) \qquad 23$$

$$E_{an} = \int_0^{\tau} P_{an,\max}e^{-\lambda t}dt = \frac{P_{an,\max}}{\lambda}(1 - e^{-\lambda\tau}) \qquad 24$$

An energy balance applied to front crawl swimming

The energy production of the aerobic and anaerobic system is used to swim a specific event. The required metabolic power ($\mathring{E}$) swimming at a specific velocity can be quantified using the power balance equation (equation 21). Integration of this equation gives the energy expenditure necessary to swim a certain distance (*d*) at a specific constant speed (*v*) or in a certain time (τ):

$$E = \frac{K \cdot v^2 \cdot d}{e_p \cdot e_g} = \frac{K \cdot d^3}{\tau^2 \cdot e_p \cdot e_g} \qquad 25$$

The energy generated should balance the energy necessary to swim a distance in a certain time. In other words (E_{an} + E_{aer}) at time *t* must equal the total metabolic energy expended to swim a distance *d* in a time τ (i.e. with velocity *d/t*). This equality can be used to predict the performance times over different distances, hence

$$\frac{K \cdot d^3}{\tau^2 \cdot e_p \cdot e_g} = P_{aer,\max}\tau + \frac{P_{aer,\max}}{\lambda}(e^{-\lambda\tau} - 1) + \frac{P_{an,\max}}{\lambda}(1 - e^{-\lambda\tau}) \qquad 26$$

The magnitude of λ can be estimated from repeated oxygen uptake measurements during one maximal exercise bout. Data were available from a group of 8 male college swimmers (weight : 65.74 ± 8.23 kg, and $\mathring{V}O_2max_{(swimming)}$: 3.54 ± 0.67 l•min^{-1}) that had performed a 60 s all out swim in a swimming flume. Every 10 seconds the oxygen uptake was determined. The data were least square fitted to the function $P_{aer} = P_{aer,max}$ (1 – $e^{-\lambda t}$), where $P_{aer,max}$ was set equal to the individual power equivalent of the $\mathring{V}O_{2max}$.

The estimation of $P_{an,max}$ relies on a method to measure the total anaerobic energy production, which was described by Medbø (134) for running. This method was applied in the context of swimming by Troup *et al* (135) and Ogita *et al.* (136, 137). In a series of submaximal swims the relation between oxygen consumption and the cube of the swimming velocity was determined. From this regression equation the oxygen consumption was estimated ($\mathring{V}O_{2(100m)}$) to swim at the speed equal to the individual 100 m best performance. The subject was then invited to swim at this specific speed for 60 s in the swimming flume. During this (maximal) exercise all oxygen consumed is measured by integrating the measured $\mathring{V}O_2$. The accumulated oxygen deficit can then be calculated from the integrated amount of oxygen necessary to swim at this speed (duration • $\mathring{V}O_{2(100m)}$) minus the total oxygen consumed. This deficit reflects the total anaerobic energy production (134, 135). Data derived from the literature for *K*, e_g, and e_p (28, 101, 108) were used to estimate *E* of front crawl dependent on *v*. The results of the model were compared to the actual swimming times. The coefficients of correlation were for the 50 yard 0.94, (adjusted coefficient of determination: 0.86) and for the 100 yard 0.91 (adjusted R^2: 0.80) (138). The coefficients of correlation indicate that reasonable predictions of individual swim performances can be made with a power equation model based on the individual kinetics of the anaerobic and aerobic pathways. Thus, although for all subjects the same mean values were used for the gross efficiency (9.2%) and propelling efficiency (60%), and drag was estimated on the basis of subject mass, more than 80% of the variance in 'sprint' performance could be explained with the model. Even better predictions can be expected when individual determined values for these parameters are incorporated in the model. Apart from the mass dependent drag coefficient, the subject specific input of the model related only to the metabolic factors. This suggests that these factors determine sprint performance for reasonably skilled swimmers (i.e., a propelling efficiency equal to ± 60%) given their drag profile. The success of the model in explaining variance of sprint performance provides support for the validity of the model. However, as stated before, individual data concerning *K*, e_g, e_p, and performance times at the longer distances were not available for the group, thus the final verification of this approach remains to be determined (see also 139). The outcome also underlines the importance for the training practice of determining the capacity of the aerobic and anaerobic energy systems.

Another practical test to determine the capacity of the aerobic and anaerobic metabolic pathways was developed by Wakayoshi et al. (140, 141). The test estimates the swimming velocity that theoretically can be maintained forever without exhaustion. This so-called critical swimming velocity is the slope of the regression line between swimming distance and time. This equation (Distance = "critical velocity" • time + "anaerobic swimming capacity") is used to determine the capacity of the aerobic and anaerobic metabolic pathways. There is, however, uncertainty that "critical velocity" and "anaerobic swimming capacity" are fitness measures that truly separate aerobic and anaerobic components. Therefore, equation 26 was used to predict the "critical velocity" and the "anaerobic swimming capacity" (see Figure 17) (139). The effect of a 20% increase and decrease for $P_{aer,max}$ and $P_{an,max}$ on the performance for the 50, 100, 200, 400, 800, and 1500 m and on the regression equation of distance on time was determined. The 20 % decrease and increase in $P_{aer,max}$ resulted in a 5.7% decrease respectively 6.2% increase of the critical velocity. This suggest that changes in $P_{aer,max}$ are reflected in critical velocity albeit to a considerable lesser extent than the 20% variation in $P_{aer,max}$.. The changes in $P_{an,max}$ had no influence on the critical velocity, but strongly affected the y-intercept of the regression equation (-54.7% respectively 48.5%). This is in line with Hill *et al.* (142) who suggested that the y-intercept is a measure of the "anaerobic capacity". However, it also became apparent that besides $P_{an,max}$, variations in $P_{aer,max}$ have an influence on the y-intercept. Thus the model calculations suggest that variations in the y-intercept can not be attributed to variations in $P_{an,max}$ only. This agrees with previous observations in which the test-retest reliability of the "anaerobic capacity" was qualified as ambiguous (143).

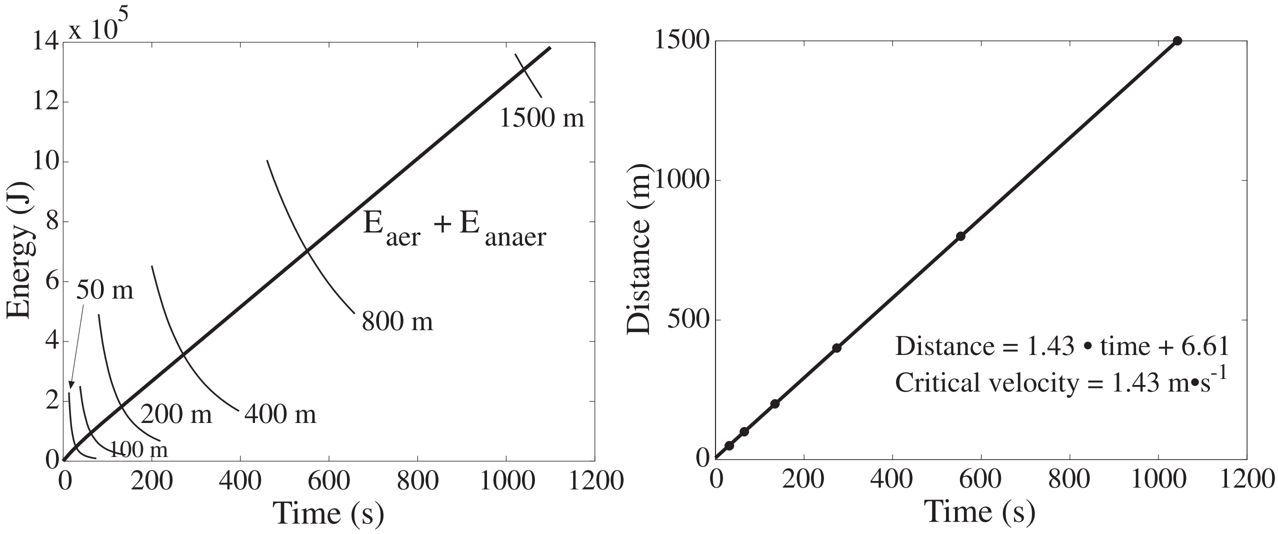

Figure 17: Energy balance in swimming (left panel): The energy required to swim the 50, 100, 200, 400, 800, and 1500 m front crawl is plotted dependent on time (thin lines) together with the combined energy production of the anaerobic and aerobic system (E_{aer} + E_{anaer}, heavy line). The intersection of the curves shows the time where the energy necessary to swim that distance is in balance with the maximal energy produced by the swimmer during that time and thus represents the best time attainable for that distance.
right panel: The predicted best times for the 50, 100, 200, 400, 800, and 1500 m together with the linear fit between swimming distance and time

In conclusion, the model calculations illustrate that the "critical swimming velocity" is a measure of swimming endurance and, thus, reflects the aerobic capacity of the swimmer. However, the y-intercept of the regression line between swimming distance and time was influenced by both $P_{aer,max}$ and $P_{an,max}$. This suggests that the 'anaerobic swimming capacity' does not truly reflect $P_{an,max}$.
Equation 21 can also be used to answer questions such as 'What if the aerobic power increases 10%? or the anaerobic power or technique (propelling efficiency) improves by 10%. Results of model predictions regarding these questions are presented in Figure 18.

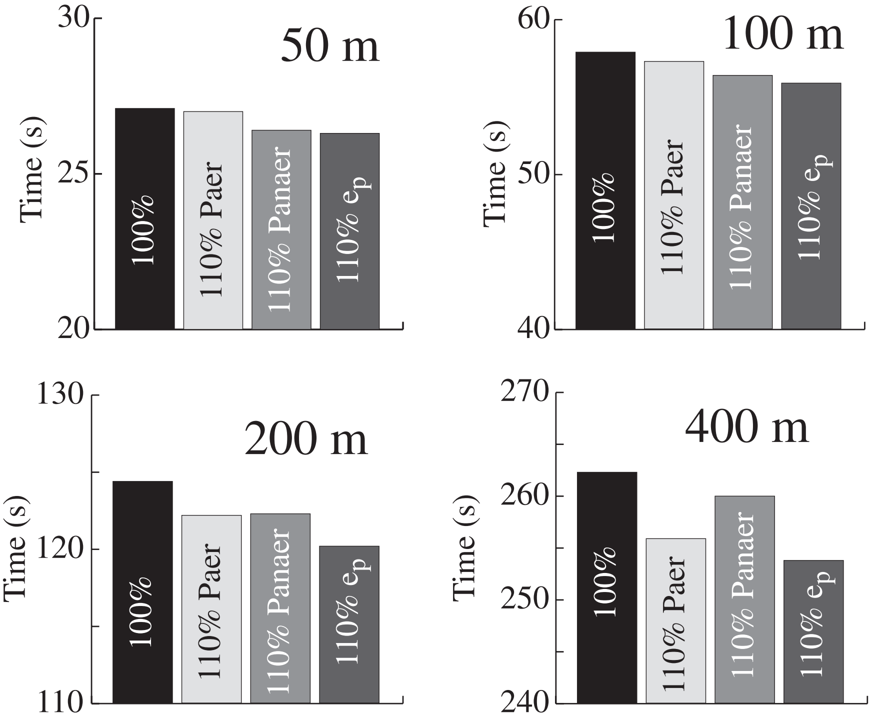

Figure 18: Estimated best times for the 50, 100, 200, 400 m front crawl. The distances were adapted to incorporate the effect of the dive, turns and the final touch with the hands (116). The first bar (in black) gives the result if P_{aermax}, P_{anmax} and e_p are set to 100%.

For the shorter distances (50 and 100 m) performance is predicted to benefit from gain in the anaerobic capacity, whereas on the 400 m a clear effect is seen when improving the aerobic power. For all distances a 10% improvement of technique gives the highest performance gain. This underlines the importance of technique improvement in training. This underlines the need for a better understanding of the fluid dynamics of swimming propulsion such that guidelines aimed at improving thrust while reducing energy losses (P_k) can be given. In our opinion this is the true challenge of swimming research into the 21st century (52).

In conclusion, for an optimal use of training time and for an optimal use of the capacities of the swimmer, it seems important to determine both the mechanical parameters (technique, drag) and the parameters describing the energy production ($P_{aer,max}$, $P_{an,max}$, and the time constant λ). The analysis techniques described here can help to estimate these parameters for each swimmer. From such an inventory of weak and strong points, it can be decided what the optimal distance is to train for, and also, what performance factors are the weakest, and are most likely to improve with training.

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