An approximate protocol analysis with performance optimization for WDM networks

P.A. Baziana *

School of Electrical and Computer Engineering, Department of Communications Electronic & Information Engineering, National Technical University of Athens, 157 73 Zografou, Athens, Greece

Abstract

In this study, our main goal is to investigate the performance optimization conditions for WDM networks. We introduce a network architecture of passive star topology that uses a Multi-channel Control Architecture (MCA) to avoid both the data channels and the receiver collisions. Especially, we propose a synchronous access scheme that exploits the propagation delay parameter in order to assign the data channels to the stations for successful data packet transmission. Thus, we achieve effective bandwidth utilization. An approximate analysis based on Poisson statistics is developed in order to explore the performance measures optimization. Finally, extensive comparative study is given for various stations populations and number of MCA channels.

1. Introduction

Nowadays, Wavelength Division Multiplexing (WDM) is commonly proposed for the implementation of high speed networks, since it manages to divide the total fiber bandwidth into several channels of lower data rates compatible to the station electronics [1]. In literature, many Wavelength Division Multiplexing Access (WDMA) protocols for Local Area Networks (LANs) of passive star topology have been studied. Most of them aim to reduce the impact of the following performance parameters: the control and the data channels collisions, the destination conflicts, and the propagation delay latency. Especially, an access strategy that optimizes the performance by eliminating the message delay in a WDM network of passive star topology is studied in [2]. Also, modern scheduling algorithms based on the use of clustering techniques that aim to provide performance enhancement in passive star WDM networks are presented in [3]. Moreover, a quality of service (QoS) prediction framework to accommodate different applications with various QoS requirements in WDM networks of passive star topology is proposed in [4].

Some of the WDMA protocols dedicate a specific wavelength to exchange control information prior to the data packet transmission to avoid collisions [5]. Since the maximum control information processing rate is limited by the electronic interface of the station, a fundamental problem rises: this of the disability of a station to receive and process all control packets that are transmitted over the single control WDM channel. This fact causes the electronic processing bottleneck [6]. In order to overcome this problem, the adoption of the Multi-channel Control Architecture (MCA) has been proposed that employs a number of control channels for the control information exchange, while it provides significantly less control information processing overhead [7,8]. Finally, although the propagation delay parameter critically affect to the performance, only few studies consider its influence [9].

In this study, we propose a WDMA protocol suitable for passive star topology that exploits the MCA. In other words, we assume that there are a number of wavelengths that constitutes the MCA and are used for the control information exchange prior to the data packets transmission. Also, we introduce an effective access strategy that is applied at each station in a de-centralized way to totally avoid both the data channel collisions and the destination conflicts [7], by properly exploiting the propagation delay parameter as acknowledgment time period after the end of which we may schedule collisions free transmissions. Also, the proposed WDMA protocol adopts the assignment of the available data channels to the stations whose control packet coordination was successful over the MCA, taking under consideration the receiver collisions avoidance [10]. In this way, it ensures that their data packet transmission is totally collisions-free. This fact consist the strong advantage of our study. Finally, each station is assumed to be a high capacity user that generates and receives traffic and demands

* Fax: +30 210 7722534.
E-mail address: baziana@central.ntua.gr

http://dx.doi.org/10.1016/j.yofte.2014.05.003
1068-5200/ © 2014 Elsevier Inc. All rights reserved.
for maximum bandwidth utilization in a local area scale. For this reason, the possible implementation cost of using a set of optical tunable transceivers per station sounds reasonable. Concluding, the proposed network architecture and the collisions-free WDMA protocol may be applied to optical passive star LANs in which the stations may run several bandwidth-demanding services, such as real-time applications or Internet based telematic services with guaranteed Quality of Service (QoS).

In order to explore the proposed WDMA protocol performance, an approximate protocol analysis based on Poisson statistics is provided. The developed analytical model investigates the performance measures optimization conditions by deriving analytical formulas taking under consideration the effect of receiver collisions phenomenon, while the data packet rejection probability at destination is analytically estimated [11]. Finally, an extensive delay analysis is studied in order to explore the sensitivity of the proposed WDMA protocol on the delay parameters fluctuation.

Our study is carried out as follows: the network model and the assumptions are described in Section 2. In Section 3, the approximate analysis for the proposed protocol performance based on Poisson statistics is investigated, while the protocol performance optimization is derived. Additionally, extensive comparative study is given for diverse numbers of control channels and population. Finally, the concluding remarks are outlined in Section 4.

2. Network model

We assume a passive star network, as Fig. 1 shows. The system uses \( v + N \) (\( v \leq N \)) wavelengths \( \lambda_{c1}, \ldots, \lambda_{cv}, \lambda_{d1}, \ldots, \lambda_{dN} \) to serve a finite number \( M \) of stations. The multi-channel system at wavelengths \( \lambda_{c1}, \ldots, \lambda_{cv} \) forms the MCA and operates as the control multi-channel system, while the remaining \( N \) channels at wavelengths \( \lambda_{d1}, \ldots, \lambda_{dN} \) constitute the data multi-channel system. The proposed MCA network model is described as \([CC]' - TT - [FR]' - [TR]\). This means that there are \( v \) control channels \([CC]'\) and each station has a tunable transmitter \([TT]\) that can be tuned to any of the wavelengths \( \lambda_{c1}, \ldots, \lambda_{cv} \). Also, each station has \( v \) fixed tuned receivers \([FR]'\), each one tuned to one of the control wavelengths of the MCA \( \lambda_{c1}, \ldots, \lambda_{cv} \). Finally, each station has a tunable receiver \([TR]\) that can be tuned to any of the data wavelengths \( \lambda_{d1}, \ldots, \lambda_{dN} \). The out coming traffic of a station is connected to an input of the passive star coupler [12]. Each station uses \( v \) fixed tuned receivers one for each control channel and one tunable receiver to any of data channels \( \lambda_{d1}, \ldots, \lambda_{dN} \). The incoming traffic to a station is splitted into \( v + 1 \) portions by an \( 1 \times (v + 1) \) WDMA splitter. The links through the WDMA splitter are unidirectional.

The transmission time of a fixed size control packet is used as time unit (control slot) and the data packet transmission time normalized in time units is \( L \) (data slot). The control packet includes the source and the destination address of the corresponding data packet. The normalized round trip propagation delay time between any station to the star coupler hub and to any other station is equal to \( R \) data slots \( (R \times L \) time units) for all stations. Both control and data channels use the same time reference which we call cycle. We define as cycle the time interval \( C \) that includes onetime unit for control packets transmission plus the normalized round trip propagation time \( R \times L \) plus the data packet transmission time \( L \). Thus, as Fig. 2 presents, the cycle time duration is:

\[
C = 1 + (R + 1) \times L \text{ time units}
\]

We assume a common clock to all stations. Time axis is divided into contiguous cycles of equal length and stations are synchronized for transmission on the control and data channels during a cycle. At any point in time each station is able to transmit at a given wavelength \( \lambda_{c} \) and simultaneously to receive at a wavelength \( \lambda_{d} \). Finally, for the tunable transceivers, we assume negligible tuning times and very large tuning ranges. Although this assumption is not realistic, the trend in optical transceivers technology is that their tuning time – that depends on their tuning range- consistently decreases. In our analysis, this assumption is made for simplification reasons and gives us the opportunity to adopt an accurate Markovian model to analytically describe the system performance, independently from the changeable technology parameters. Anyway, any possible network real implementation should take under consideration the transmitter tuning time \( t_{t} \) and the receiver tuning time \( t_{r} \) for the selected tuning range and properly extend the cycle \( C \) by: (1) \( t_{t} \) prior to the control packet transmission phase, and (2) the max\(\{t_{r}, t_{t}\} \) prior to the data packet transmission phase.

We assume that each station is equipped with a transmitter buffer with capacity of one packet of application data flow. If the buffer is empty the station is said to be free, otherwise it is backlogged. If a station is backlogged and generates a new packet, the packet is lost. Free stations that unsuccessfully transmit on the control channels or in case of rejection at destination due to receiver collisions during a cycle are getting backlogged on the next cycle. A backlogged station is getting free at the next cycle if it manages to retransmit without collision over a control channel and its data packet retransmission is not aborted due to receiver

---

**Fig. 1.** Network architecture.
collisions. Packets are generated independently at each station following a geometric distribution, i.e. a packet is generated at each cycle with birth probability \( p \). A backlogged station retransmits the unsuccessfully transmitted packet following a geometric distribution with probability \( p \).

The proposed WDMA protocol consists of the following Access and Reception Modes.

### 2.1. Access Mode

At the beginning of each cycle if a station attempts a data packet transmission, it informs the other stations by sending a control packet over the MCA, choosing randomly one of the \( v \) control channels. The control packets compete according to the Slotted Aloha scheme to gain access. The station continuously monitors the MCA with its fixed tuned receivers. The outcome of its control packet will be known \( R \times L \) time units later (acknowledgement time period). After the end of this period, the station is informed about the transmission claims of the stations whose control packets have been successfully transmitted. At that time instant, a data packet will be known over the MCA. We can imagine several assignment rules, as the correspondence algorithm for the assignment of an available data channel to each station number to the first available data channel number). The control channels are less or equal than the data channels as the time percentage of a cycle period that is exploited for successful transmissions over the control channels during a cycle. It means that each station runs in a decentralized way an appropriate channel assignment (DCA) algorithm is applied at all stations. This means that each station runs in a decentralized way an appropriate algorithm for the assignment of an available data channel to each station whose control packet has been successfully transmitted (we can imagine several assignment rules, as the correspondence of the station number to the first available data channel number).

Since, the control channels are less or equal than the data channels \( (v \leq N) \), it is evident that all the stations whose control packets have been successfully transmitted gain access to the data channels system and an available data channel is assigned to them. Assuming the receiver collisions, if two or more packets from different data channels are addressed to the same destination only one of them can be correctly received according to the specified arbitration rules (we can imagine several rules, such as the packet priority or age etc.) while the others are aborted. Thus, only the data packet that wins the receiver collisions avoidance competition is finally transmitted over the selected data channel.

### 2.2. Reception Mode

After the data packet transmission, the destination waits \( R \times L \) time units while the data packet is transmitted from the source to the destination. Then it adjusts its tunable receiver to the data channel specified by the DCA algorithm for reception.

### 3. Analysis

In this Section, we explore the performance optimization conditions for the proposed protocol. The conditional rate of successful transmissions \( S_v(i) \) over the MCA is given by [13]:

\[
S_v(i) = [p_i \left(1 - \frac{1}{p_i}\right)]^{-1} \left(1 - \frac{p_i}{M}\right)^{M-i} + (M-i)p_i \left(1 - \frac{p_i}{p_v}\right)^{M-i-1} \left(1 - \frac{p_v}{p_i}\right)^i
\]  

(2)

We consider that the system population is infinite, i.e. \( M \rightarrow \infty \). Also, we assume that:

\[
(p = p_i, = \sigma)
\]  

(3)

From (2) and (3), we get:

\[
S_v(i) = M\sigma \left(1 - \frac{\sigma}{p_v}\right)^{M-1}
\]  

(4)

From (4), we obtain that \( S_v(i) \) is independent from \( i \). Thus, the average successful transmissions rate \( S_v \) from the MCA in steady state is given by:

\[
S_v = M\sigma \left(1 - \frac{\sigma}{p_v}\right)^{M-1}
\]  

(5)

We consider that \( \sigma \rightarrow 0 \). Let \( G \) be the mean offered traffic from the new generated packets and the retransmitted packets over the \( v \) control channels during a cycle. It is:

\[
G = M\sigma
\]  

(6)

Substituting (6) in (5) and assuming that \( M \rightarrow \infty \) and \( \sigma \rightarrow 0 \), we get:

\[
S_v = G \left(1 - \frac{1}{v} \frac{G}{M} \right)^{M-1}
\]  

(7)

Using in (7) the approximation: \((1 - x)^y \approx e^{-xy}\) for small \( x \) (i.e. large population \( M \)), we get:

\[
S_v \approx Ge^{-\frac{M-1}{M}} \approx Ge^{-\frac{1}{M}}
\]  

(8)

Based on the results of (8), we obtain that the average successful transmissions rate \( S_v \) from the MCA in steady state is similar to the case that the mean offered traffic \( G \) from the new generated and retransmitted packets over the MCA during a cycle obeys to Poisson statistics. It is obvious that the average transmission rate \( S_v \) of data packets during a cycle in steady state is:

\[
S_v = S_v
\]  

(9)

We define the average throughput \( S \) of data packets from the \( N \) data channels as the time percentage of a cycle period that is exploited for successful transmissions over the \( N \) data channels in steady state. It is:

\[
S = \frac{L}{C}S_v = \frac{L}{C}Ge^{-\frac{1}{M}}
\]  

(10)

Thus, the average throughput per data channel \( S_d \) is defined as:

\[
S_d = \frac{S}{N} = \frac{L}{C} \frac{G}{N} e^{-\frac{1}{M}}
\]  

(11)

### 3.1. Performance optimization

The optimum value \( G_{opt} \) of offered traffic that maximizes the normalized transmission rate per data channel is determined by
differentiating the eq. (11) with respect to $G$ and setting it equal to zero. It is:

\[ \frac{\partial S_d}{\partial G} = 0 \Rightarrow G_{\text{opt}}(\nu) = \nu \]

(12)

So, the maximum value of normalized transmission rate per data channel $S_{d,\text{max}}$ is given by:

\[ S_{d,\text{max}} = \frac{L \nu}{C N \bar{e}} \]

(13)

3.2. Receiver collisions analysis

In order to explore the system performance under the effect of receiver conflicts, we assume that the approximation that the number of stations $M$ is finite.

We consider the mean offered traffic $G_v$ over the $i$-th control channel $i \in \{1, 2, \ldots, \nu\}$. It is:

\[ G_v = \frac{G}{\nu} \]

(14)

given that each control channel is chosen with equal and constant probability $P_c = 1/\nu$.

The probability $P_v$ of one successful transmission over a control channel during a cycle is defined as:

\[ P_v = \frac{G}{\nu} e^{-\frac{G}{\nu}} \]

(15)

Let $B_n$ be a random variable of the number of successfully transmitted control packets over the MCA during a cycle, $0 \leq B_n \leq \nu$. The probability of finding $B_n = k$ control channels, each with one packet arrival during a cycle, obeys the binomial probability law:

\[ \Pr[B_n = k] = \binom{\nu}{k} P_c^k (1 - P_c)^{\nu - k} \]

(16)

Let $A_n(k)$ be a random variable representing the number of correctly received data packets at destination, given that $k$ successful transmissions occurred over the MCA (that means that $k$ data packets are successfully transmitted over the $N$ data channels) during a cycle, $0 \leq A_n(k) \leq B_n$ for every $B_n \geq 1$.

The probability $\Pr[A_n(k) = r]$ of finding $r$ correctly received data packets at destination, given that $k$ successful transmissions occurred over the $\nu$ control channels during a cycle, is in [10]:

\[ \Pr[A_n(k) = r] = \binom{M}{r} \frac{1}{r!} \left( \frac{G}{\nu} \right)^r \left( \frac{1 - G}{\nu} \right)^{M-r} \]

(17)

The probability $P_{d,c}(r)$ of finding $r$ correctly received data packets at destination during a cycle in steady state is given by:

\[ P_{d,c}(r) = \sum_{k=r}^{\min(M,\nu)} \Pr[B_n = k] \Pr[A_n(k) = r] \]

(18)

We define as average throughput $S_{d}$, the average number of correctly received data packets at destination during a cycle in steady state and it is:

\[ S_{d,c} = E[P_{d,c}(r)] = \sum_{r=1}^{\min(M,\nu)} r P_{d,c}(r) \]

(19)

Also, we define the throughput per data channel $S_{d,\text{rc}}$ as:

\[ S_{d,\text{rc}} = \frac{L S_{d,c}}{C N} \]

(20)

Substituting (19) to (20):

\[ S_{d,\text{rc}} = \frac{L}{C N} \sum_{r=1}^{\min(M,\nu)} r P_{d,c}(r) \]

(21)

3.3. Packet rejection probability

The probability of packet rejection at destination $P_{\text{rej}}$ is defined as the ratio of the expected rate of packets rejections per cycle due to the receiver collisions phenomenon, to the expected rate of successful transmissions over the $N$ data channels system per cycle in steady state. So, we get:

\[ P_{\text{rej}} = \frac{S_d - S_{d,\text{rc}}}{S_d} \]

(22)

3.4. Approximate receiver collisions analysis

We assume that $B_n = k$ control packets are successfully transmitted over the $\nu$ control channels during a cycle. This means that $k$ data packets are also successfully transmitted over the $N$ data channels during a cycle. We assume that the $k$ data packets are uniformly distributed among the $M$ stations (for sake of simplicity of the analysis, we consider that a station may send packets to itself). Thus, the random distribution in $M$ stations gives $M^k$ arrangements each with probability $M^{-k}$.

Let $P_{\text{rej}}(k)$ be the probability that no one from the $k$ successfully transmitted data packets has as destination the $j$-th station, $j \in \{1, 2, \ldots, M\}$. Thus, the $k$ data packets are destined to the remaining $(M - 1)$ stations in $(M - 1)^k$ different ways. $P_{\text{rej}}(k)$ can be written as [10]:

\[ P_{\text{rej}}(k) = \frac{1}{M} (M - 1)^k = \left( 1 - \frac{1}{M} \right)^k \]

(23)

In steady state it is:

\[ E[B_n = k] = S_{v} \]

(24)

In steady state (23) is written as:

\[ P_{\text{rej}}(k) = \left( 1 - \frac{1}{M} \right)^{S_{v}} \]

(25)

Using in (25) the approximation: $(1 - x)^y \approx e^{-xy}$ for small $x$ (i.e. large population $M$), we get:

\[ P_{\text{rej}} = e^{-\frac{S_{v}}{M}} \]

(26)

We define the probability $P_{d}$ that one data packet with destination the $j$-th station, $j \in \{1, 2, \ldots, M\}$ is received correctly in a cycle in steady state as:

\[ P_{d} = 1 - P_{\text{rej}} = 1 - e^{-\frac{S_{v}}{M}} \]

(27)

Let $H_m(S_d) = y$ be the random variable representing the number of different stations selected as destination, given that $S_v$ is the rate of successful transmitted control packets over the $\nu$ control channels, during a cycle in steady state. Also, we define the $\Pr[H_m(S_d) = y]$ as the probability that $y$ different stations have been selected as destination during a cycle in steady state. It is:

\[ \Pr[H_m(S_d) = y] = \binom{M}{y} (P_{d})^{y} (1 - P_{d})^{M-y} \]

(28)

Finally, we define the average throughput $S_{d}$ at destination as the average number of correctly received data packets at destination. It is:

\[ S_{d} = E[\Pr[H_m(S_d) = y]] = P_{d} M \]

(29)

Substituting (27)–(29):

\[ S_{d} = M \left( 1 - e^{-\frac{S_{v}}{M}} \right) \]

(30)

Finally, the throughput per data channel $S_{d,\text{rc}}$ from (21), is written as:
\[ S_{d,rc} = \frac{L M}{C N} \left( 1 - e^{-\frac{K}{C}} \right) \]  (31)

3.5. Delay analysis

Stations participating in unsuccessful transmissions, defer their retransmissions for a random time. We consider that the random time delay between two consecutive retransmissions is uniformly distributed from 1 to \( K \) time units, with average value \( D_b \) given by:

\[ D_b = \frac{(K + 1)}{2} \]  (32)

Also, we assume that the total offered traffic from new generated and retransmitted packets obeys Poisson statistics. Let’s assume the following steady state notations:

- \( D \) = the time interval between the data packet generation time and the time instant of successful reception at destination.
- \( D_w \) = the waiting delay from the generation time of a data packet (or from the time instant that a backlogged station decides to retransmit) until the beginning of the next cycle.
- \( D_r \) = the delay from the transmission time of a control packet until the beginning of the corresponding data packet reception.

It is obvious that:

\[ D = D_w + D_r + C \]  (33)

The average delay is:

\[ E[D] = E[D_w] + E[D_r] + C \]  (34)

The average \( E[D_w] \) time is:

\[ E[D_w] = \frac{C}{2} \]  (35)

For the average \( E[D_r] \) time evaluation, we work as follows:

We assume that \( B_n = k \) data packets are successfully transmitted over the \( N \) data channels. We define the random variable \( U(k) \) that represents the number of successful data packets transmissions with destination the \( j \)-th station, \( j \in \{1, 2, \ldots M\} \) conditional that \( k \) data packets are successfully transmitted over the \( N \) data channels. Let’s suppose that \( U(k) = r \). For sake of simplicity, we consider that a station may send packets to itself. Thus, the random distribution of \( k \) data packets in \( M \) stations gives \( M^k \) arrangements each with probability \( M^{-k} \). The probability that \( r \) data packets are destined to the \( j \)-th station, \( j \in \{1, 2, \ldots M\} \) is found assuming that: the \( r \) data packets can be chosen in \( \binom{N}{r} \) ways and the remaining \( (k - r) \) data packets are destined to the remaining \( (M - 1) \) stations in \( (M - 1)^{k-r} \) ways.

Thus, the probability \( \Pr(U(k) = r) \) of finding \( r \) data packets with destination the \( j \)-th station, \( j \in \{1, 2, \ldots M\} \) is given by:

\[ \Pr(U(k) = r) = \binom{k}{r} \left( \frac{1}{M} \right)^r \left( 1 - \frac{1}{M} \right)^{k-r} \]  (36)

Also, we consider the probability \( P_{col}(m) \) that \( m \) data packets destined to \( j \)-th station, \( j \in \{1, 2, \ldots M\} \) are aborted at destination due to the receiver collisions phenomenon. It is:

\[ P_{col}(m) = \sum_{i=m}^{\min(M-1)} \Pr(B_n = i + 1) \Pr(U(i + 1) = 1 + m) \]  (37)

The mean probability \( P_{col} \) that a successfully transmitted data packet is aborted by destination \( j \)-th station, \( j \in \{1, 2, \ldots M\} \) in steady state is given by:

\[ P_{col} = \sum_{m=1}^{\min(M-1)} mp_{col}(m) \]  (38)

The average rate \( S_M \) of correctly received data packets per station per cycle in steady state is:

\[ S_M = \frac{S_0}{M} (1 - P_{col}) \]  (39)

The probability \( F_r \) of successful reception at destination of a data packet in steady state is given by:

\[ F_r = \frac{M S_M}{C} \]  (40)

Substituting (38) and (39)–(40):

\[ F_r = e^{\frac{C}{2} (1 - P_{col})} \]  (41)

Let \( Q(n) \) be the probability of successful transmission of a data packet after \( n \) trials. Assuming that the probability of success is the same on any trial, \( n \) has a geometric distribution, i.e.:

\[ Q(n) = F_r (1 - F_r)^{n-1} \]  (42)

The average number \( Z \) of trials for a successful transmission of a data packet is:

\[ Z = E[n] = \sum_{n=1}^{\infty} n Q(n) = \frac{1}{F_r} \]  (43)

Substituting (41)–(43), we get:

\[ Z = \frac{e^{\frac{C}{2} (1 - P_{col})}}{1 - P_{col}} \]  (44)

The average retransmission delay per packet is given by:

\[ E[D_r] = (Z - 1)E[D_w] + C + D_b \]  (45)

From (34), we get:

\[ E[D] = \frac{C}{2} + (Z - 1) \left( \frac{C}{2} + C + K + 1 - K/2 \right) + C \]  (46)

Finally, it is:

\[ E[D] = \frac{3C}{2} + \left( \frac{e^{\frac{C}{2} (1 - P_{col})}}{1 - P_{col}} - 1 \right) \left( \frac{3C}{2} + K + 1 - K/2 \right) \]  (47)

4. Performance evaluation

In this Section, we investigate the numerical solution of the proposed protocol and analytical study. The performance measures in Figs. 3–7 are derived by running the relative equations of the Analysis Section in a programming environment, such as C. In the following numerical results, we assume that \( L = 10 \) time units.

In order to study the proposed protocol performance and explore the benefits provided, we choose to compare it with a relative protocol presented in [10]. The protocol of [10] uses the same network architecture but it employs a different access strategy. Especially, the protocol of [10] uses the MCA for the control packets transmission and utilizes the data channel system for the data packets transmission, taking into account the receiver collisions phenomenon. In opposition to the proposed protocol which assigns the data channels to the stations whose control packet transmission was successful over the MCA in order to avoid the data channels collisions, the protocol of [10] suffers from collisions over the data channels system. Thus, in [10] the data packets loss due to the data channels collisions is considering and is analytically determined. In other words, the protocol of [10] faces packet loss over the data channels system, while the proposed protocol totally avoids it grace to the adopted DCA algorithm. This is the main difference between the two protocols. The performance improvement achieved by the DCA adoption is studied in Fig. 3 that presents the throughput per data channel \( S_{d,rc} \) curves versus the offered traffic \( G \), for \( M = 100 \) stations, \( R = 5 \) time units, \( K = 10 \) time units, and various number \( v \) of control channels in the MCA, \( v = 20, 30, 40, \ldots \).
for both the proposed protocol and the protocol of [10]. As it is shown, the proposed protocol performance is significantly higher as it is compared with the protocol of [10] for a large offered load range. For example, the maximum throughput obtained by the proposed protocol as it is compared with that of [10] is higher about: 18% for \( v=20 \), 18.5% for \( v=30 \) and 19% for \( v=40 \).

In the followings, in order to representatively study the packet rejection probability at destination and the receiver collisions effect, the throughput per data channel \( S_{d-rc} \) curves are compared with the corresponding \( S_d \) curves that ignore the receiver collisions.

Fig. 4 presents the throughput per data channel \( S_d \), \( S_{d-rc} \) curves versus the offered traffic \( G \), for \( M=100 \) stations, \( R=5 \) time units, \( K=10 \) time units, and various number \( v \) of control channels in the MCA, \( v=20, 30, 40, 50 \). Also, the \( S_{d-rc} \) curves are comparatively given to the throughput per data channel \( S_d \) curves that ignore the receiver collisions phenomenon. It is noticeable that the system that takes into account the receiver collisions reaches lower efficiency. This is an immediate result coming from the fact that the system that considers the receiver collisions takes under evaluation the rejection probability at destination, giving a more realistic performance estimation. Also, as it is observed in Fig. 4, for low values of traffic \( G \) the \( S_{d-rc} \) increase almost linearly with \( G \), while we obtain low \( S_{d-rc} \) values for all \( v \). As \( G \) increases approaching the optimum value \( G_{opt}(v) \), the throughput is getting higher. It is remarkable that in the case where the receiver collisions are ignored, for offered traffic \( G=G_{opt}(v) \) that is given by (12), the throughput reaches the \( S_{d-max} \) value that is given by (10). Although the \( G_{opt}(v) \) value is a function of the number of control channels (actually is equal to \( v \), as (9) denotes), the corresponding \( S_{d-max} \) value is the same for all values of \( v \). This is understood since we assume that \( N=v \). Thus, as (10) denotes,
the $S_{d_{\text{max}}}$ value is independent of $\nu$ and equals to $S_{d_{\text{max}}} = \frac{1}{2} \frac{1}{\nu}$ for all values of $\nu$. As offered traffic values reach $G_{\text{opt}}(\nu)$, the system gradually reaches saturation and the throughput increases slowly towards $S_{d_{\text{max}}}$ value. For offered traffic values higher than $G_{\text{opt}}(\nu)$, the throughput decreases. The explanation is based on the fact that as $G$ grows, the throughput from the MCA is getting lower since the control channels collisions increase.

This result is validated in Fig. 5 that depicts the packet rejection probability $P_{\text{req}}$ at destination curves versus the offered traffic $G$, for $M = 100$ stations, $R = 5$ time units, $K = 10$ time units, and various number $\nu$ of control channels in the MCA, $\nu = 20, 30, 40, 50$. Indeed it is shown that, for low offered traffic, the $P_{\text{req}}$ remains low enough. As $G$ increases towards to $G_{\text{opt}}(\nu)$, the $P_{\text{req}}$ increases reaching the maximum value for $G = G_{\text{opt}}(\nu)$. For offered traffic higher than $G_{\text{opt}}(\nu)$, the $P_{\text{req}}$ decreases, similar to the throughput.

Also it is observed that, for $G < G_{\text{opt}}(\nu)$ i.e. before reaching saturation, the $S_{d_{\text{rc}}}$ is a decreasing function of $\nu$, for fixed $M$. For example, for $G = 10$ control packets/cycle, the $S_{d_{\text{rc}}}$ is: 0.035 data packets/cycle for $\nu = 20$, 0.032 data packets/cycle for $\nu = 30$, 0.030 data packets/cycle for $\nu = 40$ and 0.028 data packets/cycle for $\nu = 50$. This is because as $\nu$ increases the number of successfully transmitted control packets over the MCA and consequently the number of transmitted data packets over the $N$ data channels increases too, giving rise to the $S_{d_{\text{rc}}}$. Also, this fact causes the increase of the number of data packets that are rejected at destination due to the receiver collisions. This behaviour is presented in Fig. 4, where the difference between the curves that take into account the receiver collisions and those that ignore them is getting wider as $\nu$ increases.

These results are validated in Fig. 6 that depicts the delay $D$ versus $S_{d_{\text{rc}}}$, for $M = 100$ stations, $R = 5$ time units, $K = 10$ time units.

![Fig. 5. Packet rejection probability $P_{\text{req}}$ versus offered traffic $G$, for $M = 100$, $R = 5$, $K = 10$, and $\nu = 20$, 30, 40, 50.](image)

![Fig. 6. Delay $D$ versus $S_{d_{\text{rc}}}$, for $M = 100$, $R = 5$, $K = 10$, and $\nu = 20$, 30, 40, 50.](image)
and $v = 20, 30, 40, 50$. Thus as $v$ increases, the $S_{d-rc}$ is getting lower, while the system delay significantly increases.

For further evaluation, Fig. 7 presents the delay $D$ curves versus the $S_{d-rc}$, for $M = 100$ stations, $R = 5$ time units, $v = 30$ control channels, and various number $K$ of time units that a station has to wait between two consecutive retransmissions, $K = 10, 20, 30$ time units. As it is observed, as traffic increases approaching its optimum value $G_{opt}$ (30) which corresponds to $S_{max}$, and the average number $Z$ of retransmissions given by (38) becomes significant, the average delay $D_b = \frac{(K+1)}{2}$ between two successive retransmissions begins to essentially contribute to the average delay $E[D]$, as (41) denotes.

5. Conclusions

In this paper, we study a synchronous transmission WDMA protocol of passive star network topology that exploits the MCA to avoid the data channel and the receiver collisions. The presented work novelty is that the proposed access strategy properly assigns the available data channels to those stations whose transmission is successful over the MCA, providing maximum bandwidth utilization, while it exploits the whole number of data channels for collisions-free transmission. An approximate analysis based on Poisson statistics is adopted to analytically derive the performance measures and the optimization conditions. Extensive numerical results show that the protocol performance depends on the following parameters: the number of control channels in the MCA, the system population, and the propagation delay latency value. The presented work could be a useful analytic tool for the development and evaluation of new access schemes that consider the above performance parameters.

References