Modeling crash spatial heterogeneity: random parameter versus geographically weighting

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ABSTRACT

The widely adopted techniques for regional crash modeling include the negative binomial model (NB) and Bayesian negative binomial model with conditional autoregressive prior (CAR). The outputs from both models consist of a set of fixed global parameter estimates. However, the impacts of predicting variables on crash counts might not be stationary over space. This study intended to quantitatively investigate this spatial heterogeneity in regional safety modeling using two advanced approaches, i.e. random parameter negative binomial model (RPNB) and semi-parametric geographically weighted Poisson regression model (S-GWPR).

Based on a 3-year data set from the county of Hillsborough, Florida, results revealed that (1) both RPNB and S-GWPR successfully capture the spatially varying relationship, but the two methods yield notably different sets of results; (2) the S-GWPR performs best with the highest value of R_d^2 as well as the lowest mean absolute deviance and Akaike Information Criterion measures. Whereas the RPNB is comparable to the CAR, in some cases, it provides less accurate predictions; (3) a moderately significant spatial correlation is found in the residuals of RPNB and NB, implying the inadequacy in accounting for the spatial correlation existed across adjacent zones.

As crash data are typically collected with reference to location dimension, it is desirable to firstly make use of the geographical component to explore explicitly spatial aspects of the crash data (i.e. the spatial heterogeneity, or the spatially structured varying relationships), then is the unobserved heterogeneity by non-spatial or fuzzy techniques. The S-GWPR is proven to be more appropriate for regional crash modeling as the method outperforms the global models in capturing the spatial heterogeneity occurring in the relationship that is model, and compared with the non-spatial model, it is capable of accounting for the spatial correlation in crash data.

Keywords: Spatial heterogeneity; Regional crash prediction model; Random parameter negative binomial model; Semi-parametric geographically weighted Poisson regression model

INTRODUCTION

Road safety is increasingly considered to be a necessary component in transportation planning. Since SAFETEA-LU (i.e. Safe, Affordable, Flexible, Efficient, Transportation Equity Act-A Legacy for Users) mandates transportation planning agencies engage in proactive safety planning (FHWA 2005), the forecast of crash potentials (measures) for alternative transportation planning schemes, given a number of zone-level characteristics, has not been a mere avenue of safety research, but also a demanded practical application.

The past decade resulted in a fast-growing scope of scientific research in the context of regional/macro-level safety analysis. Various area-wide characteristics were considered, including road characteristics such as intersections density (e.g. Huang et al., 2010), road length with different speed limits (e.g. Abdel-Aty et al., 2011; Siddiqui et al., 2012), road length with different functional classifications (e.g. Quddus, 2008; Hadayeghi et al., 2010; Huang et al., 2010), junctions and roundabouts (e.g. Noland and Quddus, 2004; Quddus, 2008); traffic patterns in terms of volume and speed (Noland and Quddus, 2005; Quddus, 2008; Hadayeghi et al., 2010); trip generation and distribution (Abdel-Aty et al., 2011); environmental and weather conditions (Aguero-Valverde and Jovanis, 2006); land use (e.g. Hadayeghi et al., 2010; Siddiqui et al., 2012; Pulugurha et al., 2013); and socioeconomic factors such as population density (e.g. Hadayeghi et al., 2006; Huang et al., 2010; Siddiqui et al., 2012), age cohorts (e.g.Aguero-Valverde and Jovanis, 2006; Quddus, 2008; Hadayeghi et al., 2010; Huang et al., 2010), household incomes (e.g. Huang et al., 2010; Siddiqui et al., 2012; Xu et al., 2014) and employment (Quddus, 2008; Hadayeghi et al., 2010; Huang et al., 2010; Xu et al., 2014).

Numerous spatial units have been explored in the previous literature, such as states, counties, English wards, census tracts, and traffic analysis zones (i.e. TAZs). Among them, TAZs are now the only traffic-related zone system and are superior in being easily integrated with the transportation planning process, thus having been widely adopted (Hadayeghi et al., 2003; Guevara et al., 2004; Hadayeghi et al., 2006, 2010; Abdel-Aty et al., 2011; Siddiqui et al., 2012; Pulugurha et al., 2013; Wang et al., 2013; Xu et al., 2014).

As crash data are typically collected with reference to location dimension, two problems arise (LeSage, 1999; See: Page 2): (1) spatial dependence exists between the observations, and (2) spatial heterogeneity occurs in the relationships that are modeled. Traditional crash prediction models such as the Poisson lognormal model and the negative binomial model (i.e. NB), have largely ignored the issue of spatial correlation in crash data that violates the Gauss-Markov assumption used in regression modeling. To account for the possible spatial dependence among adjacent zones, by incorporating an error term followed by the conditional autoregressive prior into link function, the Bayesian spatial model with conditional autoregressive prior has been widely employed in existing regional safety analysis (Miaou et al., 2003; Aguero-Valverde and Jovanis, 2006; Quddus, 2008; Huang and Abdel-Aty, 2010; Huang et al., 2010;

Siddiqui et al., 2012; Xu et al., 2014; Zeng and Huang, 2014).

Aforementioned models can be thought of as global or semi-local, as the outputs from these models consist of a set of fixed parameter estimates across the region of analysis. However, the impacts of predicting variables on crash counts might not be stationary over space. In other words, it is possible that some variables have larger impacts in certain spatial zones, but have smaller impacts in others. The possibility of accounting for this spatial heterogeneity by allowing some or all parameters to vary spatially has considerable potential.

Variations in relationships over space could be referred to as ***spatial heterogeneity*** (LeSage and Pace 2009; See: Page 29) or ***spatial non-stationarity*** (Fotheringham et al., 2002; See: Page 9). To address this issue in regional safety modeling, two approaches are promising: the random parameter negative binomial model (i.e. RPNB) and the geographically weighted Poisson regression model (i.e. GWPR). With respect to RPNB, the parameters are drawn from some random distributions, typically the normal, and are assumed to vary randomly from case to case. Relatively recent safety research conducted by Milton et al. (2008), Anastasopoulos and Mannering (2009, 2011), EI-Basyouny and Sayed (2009, 2011), Dinu and Veeragavan (2011), Anastasopoulos et al. (2012), Wu et al. (2013), Venkataraman et al. (2013), Xiong and Mannering (2013) and Chen and Tarko (2014) have empirically demonstrated the applicability of the random parameter approach to explicitly account for the variations in the effects of variables across road segments and intersections. Nevertheless, few existing studies have used RPNB for regional crash modeling. Relative to road sections and intersections, the data in macro-level safety analysis are usually aggregated at a different spatial scale (e.g. TAZs), arising a key issue: the possible spatial correlation across adjacent zones. Therefore, an evaluation on the performance of RPNB particularly for regional crash studies is essential for safety analysts.

Another potential method is GWPR. GWPR is similar in spirit and methodology to local generalized linear regression models (See: Loader, 1999), except that the weights in GWPR are determined by a spatial kernel function instead of a kernel function in the variable space. The geographically weighted approach is one of the most innovative technologies in geography and has been considerably prevalent in spatial econometric, ecology analysis and disease mapping. While in the field of regional crash modeling, it seemingly received less attention. Recently published studies of Hadayeghi et al. (2010) and Li et al. (2013) revealed that the method outperformed the traditional generalized linear model (i.e. Poisson model) in capturing the spatially varying relationship between crash counts and predicting factors. However, the regression coefficients in GWPR calibrated in previous safety literature were all assumed to vary geographically. In some cases, not every parameter in a model has a spatially varying effect, i.e. the degree of variation for some coefficients might be negligible. It is therefore necessary to consider the semi-parametric GWPR (i.e. S-GWPR) in which some coefficients are global.

Clearly, it could be seen that above two methods have intrinsic differences. The local regression coefficients in RPNB are drawn independently from some univariate distributions, and no attention is paid to the location to which the parameters refer, while in GWPR the local coefficients are assumed to be a function of the coordinates in geographical space. Once the local parameter estimates are obtained, they can be mapped and their spatial pattern could be explored. And an empirical comparison of the application of the two models would be worthwhile.

It should be clarified that although it would be beneficial to evaluate the geographically weighted negative binomial regression model, the latest available software–GWR4.0 developed by Nakaya et al. (2012), does not support the calibration of a geographically weighted regression model with a NB structure, and as such, this model could not be developed in this study. As the local models are fitted using a number of vicinity observations that are similar in their characteristics, it is expected that the variance of crash counts will become much closer to the mean during the estimates for local parameters in a GWPR (Li et al., 2013). Meanwhile, it is worth noting that the use of the Poisson regression instead of the negative binomial does not produce much difference in general, since the model's coefficients are similar for the two error distributions (Miaou, 1994; Hadayeghi et al., 2010; Li et al., 2013). This justifies the choice of GWPR for adoption in regional safety modeling.

This study intends to quantitatively investigate the spatial heterogeneity in regional crash modeling. Two advanced approaches, the RPNB and S-GWPR, are employed to account for the locally spatial variations in the relationship between zonal crash frequency and traffic patterns, road network attributes, as well as socio-demographic factors. The performance of the two models is compared with the NB and CAR. It is expected that the results would provide a greater insight into the nature of variations in the relationships over space, as well as better understanding of the factors that influence crash occurrences.

METHODOLOGY

Four model types, the NB, CAR, RPNB, and S-GWPR, were calibrated in this study. They were briefly described in this section, followed by the presentation of the goodness of fit measures for model comparison.

Negative Binomial Model

The Poisson distribution is a useful starting point to model crash outcomes. However, the underlying assumption of the Poisson distribution of variance equal to the mean is often violated in the crash count data. To account for this issue of over-dispersion, NB has been generally used.

Let Y_i denotes the observed number of crashes in areal units $i\,(i=1,\ldots,n)$, EV_i is the exposure variable (e.g. daily vehicle miles traveled) of areal unit i, X_{ik} is the kth explanatory variable for areal unit i. The NB model can be specified as:

$$Y_i \sim Poisson(\lambda_i)$$

$$\ln(\lambda_i) = \beta_0 + \beta_1 \ln(EV_i) + \sum_{k=2}^{p} \beta_k X_{ik} + \theta_i$$

where λ_i is the parameter of Poisson model (i.e. the expected number of crashes in areal unit i), $\beta_0, \beta_1, ..., \beta_p$ are model parameters, $EXP(\theta_i)$ is a gamma-distributed error term with mean 1 and variance α. The addition of this term allows the variance to differ from the mean as $Var(Y_i) = \lambda_i + \alpha\lambda_i^2$.

Bayesian Negative Binomial model with Conditional Autoregressive Prior

Although the NB presented above is capable of capturing unstructured over-dispersion, it largely ignores the possible spatial dependency of traffic crashes among adjacent zones (Huang and Abdel-Aty, 2010). For this purpose, by incorporating an error term followed by the conditional autoregressive prior into the link function, the CAR has widely been applied in regional crash modeling. The model is presented below:

$$\ln(\lambda_i) = \beta_0 + \beta_1 \ln(EV_i) + \sum_{k=2}^{p} \beta_k X_{ik} + \theta_i + \phi_i$$

where ϕ_i is the correlation heterogeneity or spatial correlation, and all other terms are as previously defined.

For the spatial correlation term ϕ_i, the CAR prior proposed by Besag et al. (1991) is adopted:

$$\phi_i \sim N\left(\frac{\sum_{i \neq j} \omega_{ij}\phi_j}{\sum_{i \neq j} \omega_{ij}}, \frac{1}{\sum_{i \neq j} \omega_{ij}\tau_c}\right)$$

in which ω_{ij} is binary entries of proximity matrix, and if i and j are adjacent (i.e. the adjacency-based first order), ω_{ij} =1, otherwise, ω_{ij} =0. τ_c is the precision parameter assumed to be a prior gamma (0.5,0.0005) as suggested by Wakefield et al. (2000) and used by Aguero-Valverde and Jovanis (2006) and Quddus (2008).

Since the maximum likelihood estimation approach was not feasible, a full Bayesian inference using the Markov Chain Monte Carlo (MCMC) algorithm was finally implemented to construct above CAR. Non-informative priors were assigned for the model parameters:

$$\beta_k \sim N(0,1000)$$
$$\ln(\alpha) \sim N(0,0.01)$$

Random Parameter Negative Binomial Model

Despite the local relationship is incorporated into the modeling framework through the covariance structure of the error term in above Bayesian spatial model, the outputs from such models still consist of a set of global parameter estimates. Alternatively, the local variations can be represented by allowing the regression coefficients in above NB to vary randomly from one areal unit to another, which lead to the RPNB:

$$\ln(\lambda_i) = \beta_{i0} + \beta_{i1}\ln(EV_i) + \sum_{k=2}^{p}\beta_{ik}X_{ik} + \theta_i$$

$$\beta_{ik} = \beta_k + \varphi_{ik}$$

where β_{ik} is the coefficient of the kth explanatory variable for areal unit i, and φ_{ik} is a randomly distributed term (e.g. a normally distributed term with mean 0 and variance σ_k^2). In practice, a random parameter β_{ik} is used whenever $\hat{\sigma}_k$ is significantly greater than 0, otherwise the parameter β_k is fixed across the region.

Note that this random parameter formulation is equivalent to a random effects model if only the constant term is a random parameter.

The full Bayesian approach and simulation-based maximum likelihood method are both available for the development of RPNB. We calibrated the model utilizing both methods and found that the results revealed little significant difference. Meanwhile, as the S-GWPR presented below could only be estimated by the maximum likelihood method currently, for the purpose of comparison, the later using Halton draws was adopted eventually.

The software–LIMDEP 10 (Greene, 2012) was employed to estimate the NB and RPNB.

Semi-Parametric Geographically Weighted Poisson Regression Model

Another potential approach to account for this spatial heterogeneity is GWPR. An important extension of GWPR is its semi-parametric formation by mixing fixed and geographically varying coefficients. To present the S-GWPR, we would first introduce the specification of the basic GWPR. The model's framework is as follows:

$$\ln(\lambda_i) = \beta_0(u_i, v_i) + \beta_1(u_i, v_i)\ln(EV_i) + \sum_{k=2}^{p}\beta_k(u_i, v_i)X_{ik}$$

where (u_i, v_i) denotes the two-dimensional coordinates of the centroid of areal unit i. Note that in GWPR, the $\beta_k(u_i, v_i)$ is not assumed to be random as in RPNB, but rather to be a function of the centroid of areal unit i, which could be estimated by:

$$\hat{\boldsymbol{\beta}}(u_i, v_i) = (\mathbf{X}^T\mathbf{W}(u_i, v_i)\mathbf{X})^{-1}\mathbf{X}^T\mathbf{W}(u_i, v_i)\mathbf{Y}$$

where $\hat{\boldsymbol{\beta}}(u_i, v_i) = (\beta_0(u_i, v_i), \beta_1(u_i, v_i), \ldots, \beta_p(u_i, v_i))^T$ is the vector of the $p+1$ local regression coefficients for the areal unit i, $\mathbf{X}$ is the design matrix of explanatory variables, $\mathbf{X}^T$ is the transposed one of $\mathbf{X}$, $\mathbf{Y}$ is the $n \times 1$ vector of dependent variables, and $\mathbf{W}(u_i, v_i)$ denotes an $n \times n$ spatial weight matrix which can be expressed as:

$$\mathbf{W}(u_i,v_i)=\begin{bmatrix} w_{i1} & 0 & \cdots & 0 \\ 0 & w_{i2} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & w_{in} \end{bmatrix}$$

where $w_{ij}\,(j=1,2,...,n)$ is the weight given to areal unit j in the calibration of model for the areal unit i.

In GWPR, a regression equation is estimated for each areal unit based on the observations in nearby ones. The estimation process is repeated for all regression points. Each areal unit is weighted by its distance from the nearby regression points. Hence, the observations closer to areal unit i have more influence on the estimation of $\hat{\boldsymbol{\beta}}(u_i,v_i)$. This influence around i is described by the weight function w_{ij}, which could be calculated by the commonly used fixed Gaussian and adaptive bi-square kernel function:

Fixed Gaussian kernel: $w_{ij}=\exp(-d_{ij}^2/b^2)$

Adaptive bi-square kernel: $w_{ij}=\begin{cases}(1-d_{ij}^2/b_{i(k)}^2)^2 & d_{ij}\le b_{i(k)} \\ 0 & d_{ij}>b_{i(k)}\end{cases}$

where d_{ij} is Euclidean distance between the centroid of areal unit i and j, b is the fixed bandwidth defined by a distance metric measure, and $b_{i(k)}$ is an adaptive bandwidth size defined as the kth nearest neighbor distance.

The selection of optimal spatial kernel and consequent bandwidth could be based on the Corrected Akaike Information Criterion (i.e. AICc; Hurvich et al., 1998). The model with lower AICc is preferred (Fotheringham et al., 2002; Nakaya et al., 2005; Hadayeghi et al., 2010; Li et al., 2013).

In several empirical applications, not every regression coefficient in a model varies geographically. Alternatively, there may be no significant geographical variations in some of the local parameter estimates. In such a case, the S-GWPR, which allows some local parameters to vary spatially and others to be held constant, could be considered:

$$\ln(\lambda_i)=\beta_0(u_i,v_i)+\beta_1(u_i,v_i)\ln(EV_i)+\sum_{j=2}^{l}\beta_j X_{ij}+\sum_{k=l+1}^{p}\beta_k(u_i,v_i)X_{ik}$$

where β_j is the jth global coefficient. For testing the geographical variability of the kth coefficient, a model comparison is carried out between the fitted GWPR and a model (i.e. the switched GWPR) in which only β_k is set fixed, while the other coefficients are kept as they are in the fitted GWPR. If the fitted GWPR is better than the compared switched GWPR by a model comparison criterion, such as AICc, we can judge that the β_k is certainly varying over space. The test routine repeats this comparison for each geographically varying coefficient.

Clearly, it could be seen that if there were no global coefficients, the S-GWPR would be equivalent to the basic GWPR. Combining geographically local scoring and back-fitting algorithms, the estimates of coefficients and indicators for model diagnostics, including the standard errors of coefficients, degree of freedom and AICc, can be computed. Computational details for the calibration procedures of S-GWPR could be found in Nakaya et al. (2005).

Measures of Goodness of Fit

In response to compare above model performance, three common measures, i.e. R_d^2, mean absolute deviance (MAD) and Akaike Information Criterion (AIC) are used.

As the conditional mean function is nonlinear, and moreover, the regression is heteroscedastic, the Poisson model and its extensions produce no natural counterpart to R^2 in a linear regression model as usual. To evaluate the overall goodness of fit, a measure based on the standardized residuals is suggested (Cameron and Windmeijer, 1993):

$$R_d^2 = 1 - \frac{\sum_{i=1}^{n} \left(Y_i - \hat{\lambda}_i\right)^2 / \hat{\lambda}_i}{\sum_{i=1}^{n} \left(Y_i - \overline{Y}\right)^2 / \overline{Y}}$$

where $\hat{\lambda}_i$ is the expected crash number obtained by above regional crash prediction models, and $\overline{Y}$ is the average of crash frequency. Model associated with lager values of R_d^2 towards 1 fits better to the data.

To provide a measure of model prediction performance, the MAD is adopted:

$$MAD = \frac{1}{n} \sum_{i=1}^{n} \left| \hat{\lambda}_i - Y_i \right|$$

A smaller value of MAD suggests that on average the model predicts the observed crash data better.

Meanwhile, the penalized goodness of fit measure–AIC is also employed to take into account the model complexity:

$$AIC = D + 2K$$

where D and K denote the deviance and the number of parameters estimated in the model, respectively. In the case of conventional Poisson regression, deviance as defined by Greene (2011; See: Page 804) is given by:

$$D = 2\sum_{i=1}^{n}\left[Y_i \ln\left(Y_i / \hat{\lambda}_i\right) + \left(\hat{\lambda}_i - Y_i\right)\right]$$

where, by convention, $0\ln(0) = 0$.

In the non-parametric framework of GWPR, the concept of number of parameters is fairly meaningless. Alternatively, the related concept of effective number of parameters is considered, which could be defined straightforwardly as:

$$K = trace(\mathbf{S})$$

where matrix $\mathbf{S}$ is known as the hat matrix. Please see Nakaya et al. (2005) and Fotheringham et al. (2002; See: Page 91-92) for more detailed descriptions about the calculation of K.

DATA PREPARATION

Data sets were collected from Hillsborough County, Florida during the years 2005–2007. Hillsborough County is a county located in the western part of central Florida. It is the largest county in the Tampa–St. Petersburg–Clearwater Metropolitan Area, with Tampa forming the region's hub.

As mentioned above, TAZs are currently the only traffic-related zone system and are superior in being easily integrated with the transportation planning process. Compared with census geographic units (e.g. block groups and census tracts), TAZs are thought to have better homogeneity as they are special areas delineated by state, metropolitan planning officials or local transportation officials particularly for tabulating traffic-related data (Abdel-Aty et al., 2013). Therefore, TAZs were adopted as the base units, and the following data sets were aggregated at the TAZ-level. Hillsborough County contains 738 TAZs in total. The shape file of the TAZ boundary was collected from the Florida Department of Transportation (FDOT) District 7's Intermodal Systems Development Unit.

The crash data were obtained from the FDOT Crash Analysis Reporting System. During the period of 2005–2007, a total of 57,694 crashes were recorded, among which 4,854 (8.41%) were severe crashes with fatalities and severe injuries. Road and traffic-related data were collected primarily from two sources: FDOT's Roadway Characteristics Inventory and the geographical information system maps with Hillsborough. These include total roadway length, road segment length per TAZ with 25/35/45/55/65 mph speed limits, intersections and daily vehicle miles traveled (DVMT). Moreover, a number of demographic and socioeconomic factors were also investigated, which were downloaded from the United States Census report. These data include the geographical area of each TAZ, population and incomes.

The variables used for model development, as well as their descriptive statistics, are shown in Table 1. The numbers of total crashes and severe crashes were selected as the dependent variables, respectively. DVMT was utilized as the exposure variable. The explanatory variables were the predictors commonly used in previous regional safety analyses. Meanwhile, the multi-collinearity was investigated in order to ensure the non-inclusion of highly correlated independent variables in the final models. It should be noted that the percentage of segments with a speed limit of 25 mph and the percentage of segments with a speed limit of 35 mph were found to have some kind of collinearity. The Pearson product-moment correlation was 0.79 and the variance inflation factor values for the two variables were 7.85 and 5.20, respectively (A common rule of thumb is that if the VIF is larger than 5, multicollinearity is high). Considering the fact that the countermeasure of the speed limit of 25 mph was widely adopted in Hillsborough (the percentage of the road segment length with a speed limit of 25 mph is 76%, while it is only 14% for 35 mph), the percent of segments with a speed limit of 35 mph was finally omitted from the models.

TABLE 1 Summary of variable and descriptive statistics

Variables	Definition	Mean	S.D.	Min	Max
Predictor Variable					
Total_cra	Total number of crashes per TAZ	78.18	72.89	0.00	481.00
Severe_cra	Total number of fatal and severe injury crashes per TAZ	6.58	7.02	0.00	47.00
Exposure Variable					
DVMT	Daily vehicle miles traveled (in thousands)	95.07	110.24	0.06	788.77
Explanatory Variables					
Inter_density	Number of intersections/road length	3.17	5.61	1.00	66.12
P_Seglen25	Percent of road segment length with 25 mph speed limit (%)	72.01	20.80	0.00	100.00
P_Seglen45	Percent of road segment length with 45 mph speed limit (%)	2.10	5.32	0.00	43.78
P_Seglen55_65	Percent of road segment length with 55–65 mph speed limit (%)	5.10	10.47	0.00	83.27
POP_density	Population/area (per acre)	3.76	3.38	0.00	19.01
MHINC	Median household income (in thousands)	40.14	20.24	0.00	115.66

Note: S.D. represents the abbreviation of standard deviation; Min and Max refer to the minimum and maximum values of variable, respectively.

RESULTS

Based on above methodology and data availability, four model types were developed to explain the observed variation in total/severe crash counts given a number of zone-level factors, and as such, a total of eight models were estimated. The performance of these models was compared in this section firstly, followed by the presentation and discussion of parameter estimates.

Model Comparison

With respect to the CAR, the model was run with three chains started from different points. The first 5,000 iterations in each chain were discarded as burn-ins, then, 5,000 iterations were performed for each chain resulting in a sample distribution of 15,000 for each parameter. Convergence of the model was monitored by visual examination of the MCMC chains, autocorrelation plots as well as Gelman-Rubin statistic plots.

The RPNB was estimated by specifying a functional form of the parameter density function and using simulation-based maximum likelihood with 200 Halton draws. Consideration was given to normal, lognormal, uniform and triangular distributions for the density function of RPNB. For all parameters found to be random, the normal distribution was revealed to provide the best statistical fit, as similar in prior research (Milton et al., 2008; Anastasopoulos and Mannering 2009, 2011; EI-Basyouny and Sayed 2009, 2011; Dinu and Veeraragavan 2011; Anastasopoulos et al. 2012; Venkataraman et al. 2013).

For the development of S-GWPR, the fixed and adaptive kernels were both applied. For all cases, the models with adaptive kernel provided lower AICc values. The optimal number of areal units included in the adaptive kernel was 56 for both the total crashes model and severe crashes model.

The basic GWPR was also calibrated for the purpose of comparison. The difference of AICc values was 1,661 between GWPR and S-GWPR (i.e. 12,036 versus 10,375) for the total crashes model. With respect to the severe crashes model, the AICc values were 1,887 and 1,533 for GWPR and S-GWPR, respectively. Above results revealed that compared with the basic GWPR, the S-GWPR could significantly reduce model complexities and enhance its predictable performance. Given the space constraints and based on the fact that the S-GWPR obviously ruled out, the results for basic GWPR were not presented.

In order to compare model performance, R_d^2, MAD and AIC were calculated. As shown in Table 2, results revealed that: (1) the S-GWPR performs best with the highest value of R_d^2 as well as lowest MAD and AIC, indicating that by accounting for the spatial heterogeneity in data, the variability in crash frequency is better captured by S-GWPR; (2) with regard to the total crashes models, the RPNB provides a slightly better overall fit than does the CAR. While in severe crashes models, the CAR definitely rules out the RPNB as the overall fit of goodness and average prediction accuracy is improved by 11.59% and 15.91% according to the R_d^2 and MAD results; (3) the traditional NB expectedly performs worst in both cases.

TABLE 2 Measures of model goodness of fit

	Total crashes models			Severe crashes models		
	R_d^2	MAD	AIC	R_d^2	MAD	AIC
NB	0.59	35.14	18923	0.52	3.86	2259
CAR	0.75	28.42	—	0.77	2.59	—
RPNB	0.76	27.86	12385	0.69	3.08	1755
S-GWPR	**0.80**	**25.23**	**10237**	**0.81**	**2.36**	**1428**

Note: The bold values refer to the best in terms of three different model performance measures. As the CAR could only be constructed in the context of the Bayesian inference method, AIC values were specifically calculated for the NB, RPNB and S-GWPR, respectively.

Previous studies (Aguero-Valverde and Jovanis, 2006; Huang et al., 2010; Quddus, 2008; Siddiqui et al., 2012; Xu et al., 2014) reported that the spatial correlation of traffic crashes existed widely across adjacent spatial zones. The four model types developed assume the error term to be independent spatially distributed. If the spatial correlation existed in the residuals, the underlying model assumption would be violated and biased estimates may be produced (Anselin, 2001). To quantify the slope of spatial autocorrelation, Moran's *I* statistics were calculated for the number of total/severe crashes, as well as the model residuals. The results were presented in Table 3.

As illustrated in Table 3, it could be seen that: (1) crash counts are revealed to be statistically significant spatially correlated at a 99.9% confidence level; (2) with respect to the total crashes models, there appears to be no significant spatial autocorrelation among all of the model residuals. This may be due to the fact that the spatial correlation of total crash counts has been well explained by the specific effects of predicting variables included in the model form, thus the residuals show no significant spatial correlation at all; (3) for severe crashes models, a moderately significant spatial correlation is found in the residuals of NB and RPNB at the 90% confidence level. This finding may be not surprising to some extent, as the NB and RPNB are essentially ***non-spatial models***. For example, the local regression coefficients in RPNB are drawn independently from normal distributions and no attention is paid to the location to which the parameters refer. Above results suggest that regarding regional safety analyses, it is essential to make use of the geographical component to explicitly explore spatial aspects of the crash data. And the CAR and S-GWPR employed in this study could appropriately take into account the issue of spatial correlation.

TABLE 3 Moran's *I* statistics for model residuals

Moran's *I*	Total crashes models		Severe crashes models	
	Z Score	*p*-value	Z Score	*p*-value
Total crashes	**6.42**	**0.00**	—	—
Severe crashes	—	—	**9.71**	**0.00**
NB	0.44	0.66	**1.76**	**0.08**
CAR	-0.68	0.50	-0.44	0.66
RPNB	0.45	0.65	**1.82**	**0.07**
S-GWPR	-0.81	0.42	-1.04	0.30

Note: The bold number means that the calculated Moran's I value is statistically significant at the 90% confidence level.

Parameter Estimation

Model estimation results were summarized in Table 4 and Table 5. While only estimated means of coefficients in the NB and CAR models were provided, the local parameters in RPNB and S-GWPR were also described by the minimum, lower quartile, median, upper quartile, and maximum of values.

TABLE 4 Models with total crashes frequency as the dependent variable

Total crashes	**NB**	**CAR**	**RPNB**						**S-GWPR**					
			Mean	Min	Lwr	Med	Upr	Max	Mean	Min	Lwr	Med	Upr	Max
Intercept	**4.15**	**4.10**	**4.10**						4.11	2.08	3.89	4.18	4.40	6.03
LnDVMT	**0.62**	**0.63**	**0.68**	0.40	0.67	0.69	0.71	0.85	0.68	0.15	0.56	0.65	0.76	1.38
s.d._LnDVMT			**0.18**											
Inter_density	***0.05***	***0.07***	**0.06**						0.26	-2.13	-0.02	0.23	0.57	2.70
P_seglen25	-0.05	-0.03	-0.03	-0.21	-0.04	-0.03	-0.02	0.11	0.02	-0.55	-0.10	0.01	0.17	0.49
s.d._P_seglen25			**0.14**											
P_seglen45	**0.06**	**0.07**	***0.05***						0.06					
P_seglen55_65	-0.01	-0.02	-0.03						-0.08	-4.70	-0.17	-0.04	0.13	1.06
POP_density	**0.26**	**0.20**	**0.26**						0.12	-0.81	0.04	0.15	0.25	1.09
MHINC	**-0.11**	**-0.08**	**-0.15**	-0.36	-0.17	-0.15	-0.13	0.15	-0.16	-0.76	-0.30	-0.16	-0.01	0.34
s.d._MHINC			**0.19**											
over-dispersion	**0.40**	**0.30**	**0.32**											
CAR effects		**0.35**												

Note: The italicized bold numbers mean statistically significant at 90% significance level in the NB, CAR and RPNB; while the bold ones mean statistical significant at 95% significance level; Min, Lwr, Med, Upr and Max refer to the minimum, lower quartile, median, upper quartile and maximum of values in the local parameters, and all other abbreviations are defined as in Table 1.

TABLE 5 Models with severe crashes frequency as the dependent variable

Severe crashes	**NB**	**CAR**	**RPNB**						**S-GWPR**					
			Mean	Min	Lwr	Med	Upr	Max	Mean	Min	Lwr	Med	Upr	Max
Intercept	**1.66**	**1.52**	**1.62**						1.53	-0.92	1.10	1.59	1.93	2.76
LnDVMT	**0.67**	**0.61**	**0.70**	0.11	0.69	0.70	0.72	0.85	0.65	0.18	0.52	0.63	0.77	1.36
s.d._LnDVMT			**0.17**											
Inter_density	**-0.21**	0.03	**-0.21**						0.06	-2.38	-0.20	0.06	0.39	2.37
P_seglen25	-0.01	0.00	-0.01						0.02	-0.84	-0.13	0.05	0.18	0.81
P_seglen45	0.01	**0.07**	0.00						0.06					
P_seglen55_65	-0.03	0.01	-0.05						-0.07	-5.41	-0.21	0.00	0.17	1.85
POP_density	**0.18**	**0.14**	**0.17**	0.09	0.16	0.17	0.17	0.28	0.12					
s.d._POP_density			**0.12**											
MHINC	**-0.09**	**-0.08**	**-0.11**	-0.25	-0.12	-0.11	-0.09	0.19	-0.16	-0.91	-0.33	-0.14	0.02	0.77
s.d._MHINC			**0.18**											
over-dispersion	**0.47**	**0.19**	**0.39**											
CAR effects		**0.60**												

Note: The bold ones mean statistically significant at 95% significance level in the NB, CAR and RPNB; Min, Lwr, Med, Upr and Max refer to the minimum, lower quartile, median, upper quartile and maximum of values in the local parameters, and all other abbreviations are defined as in Table 1.

Several general observations are worth noting: (1) unlike NB and CAR which have a constant parameter for each variable, the RPNB and S-GWPR allow parameters to vary spatially. Thus, using the NB and CAR, one crash prediction model is developed across the region. While employing the RPNB and S-GWPR, different crash prediction models are developed for each individual TAZ; (2) with regard to the variables with varying magnitude, the parameters estimated in NB and CAR always fall into the range of corresponding counterparts in RPNB and S-GWPR, indicating that the parameters estimated in the global models generally represent the average effects of the factors on crash counts; (3) concerning the total crashes models, three variables (i.e. *the logarithm of DVMT*, *P_seglen25*, and *MHINC*) are found to produce statistically significant random parameters (i.e. the standard deviation of the parameter's distribution is significantly different from 0), whereas in the S-GWPR, all variables except *P_seglen45* have geographically varying coefficients. Meanwhile, with respect to the severe crashes models, *the logarithm of DVMT*, *POP_density*, and *MHINC* resulted in a random parameter with varying magnitude, while five variables (i.e. *the logarithm of DVMT*, *Inter_density*, *P_seglen25*, *P_seglen55_65*, and *MHINC*) are reported to have spatially varying coefficients in S-GWPR. The distributions of local coefficient estimates for total/severe crashes models were plotted in Figures 1 and 2, respectively, and their spatial patterns were subsequently investigated.

As illustrated in Figures 1 and 2, the parameters revealed obvious patterns of spatial variations. And the RPNB and S-GWPR yield notably different sets of results. First, as mentioned above, the list of covariates with geographically varying coefficients is not identical between the two models. Second, the mapped patterns for the RPNB coefficients apparently exhibit less smoothness than do the S-GWPR counterparts. This result is unsurprising given the fact that the S-GWPR provides estimates using a mechanism that is essentially based on spatial smoothing, while the RPNB makes no spatial assumptions and allows more 'noise' to introduce roughness into the local coefficient estimates. Third, it is interesting to find that the magnitude of local estimates in the RPNB seems to 'shrink' towards a global mean value, falling into the range of parameters of the same variable in the S-GWPR.

Meanwhile, there are also some similarities between the two processes. The signs of local coefficients in the RPNB are in accordance with the counterparts in S-GWPR. For example, as the most significant variable, *the logarithm of DVMT* always has a positive sign for all local estimates in both RPNB and S-GWPR. However, it is also seen that several local coefficients vary from positive to negative values, which sometimes seem to be unexpected. For instance, the *MHINC* was pointed out to have significantly negative effects on crash occurrence in previous research (Huang et al., 2010; Li et al., 2013; Xu et al., 2014), as well as in our NB and CAR models (See: Tables 4 and 5), implying that affluent areas are relatively safer. Nevertheless, Figures 5 and 6 show that in some of the TAZs, the sign of the *MHINC* coefficient is positive.

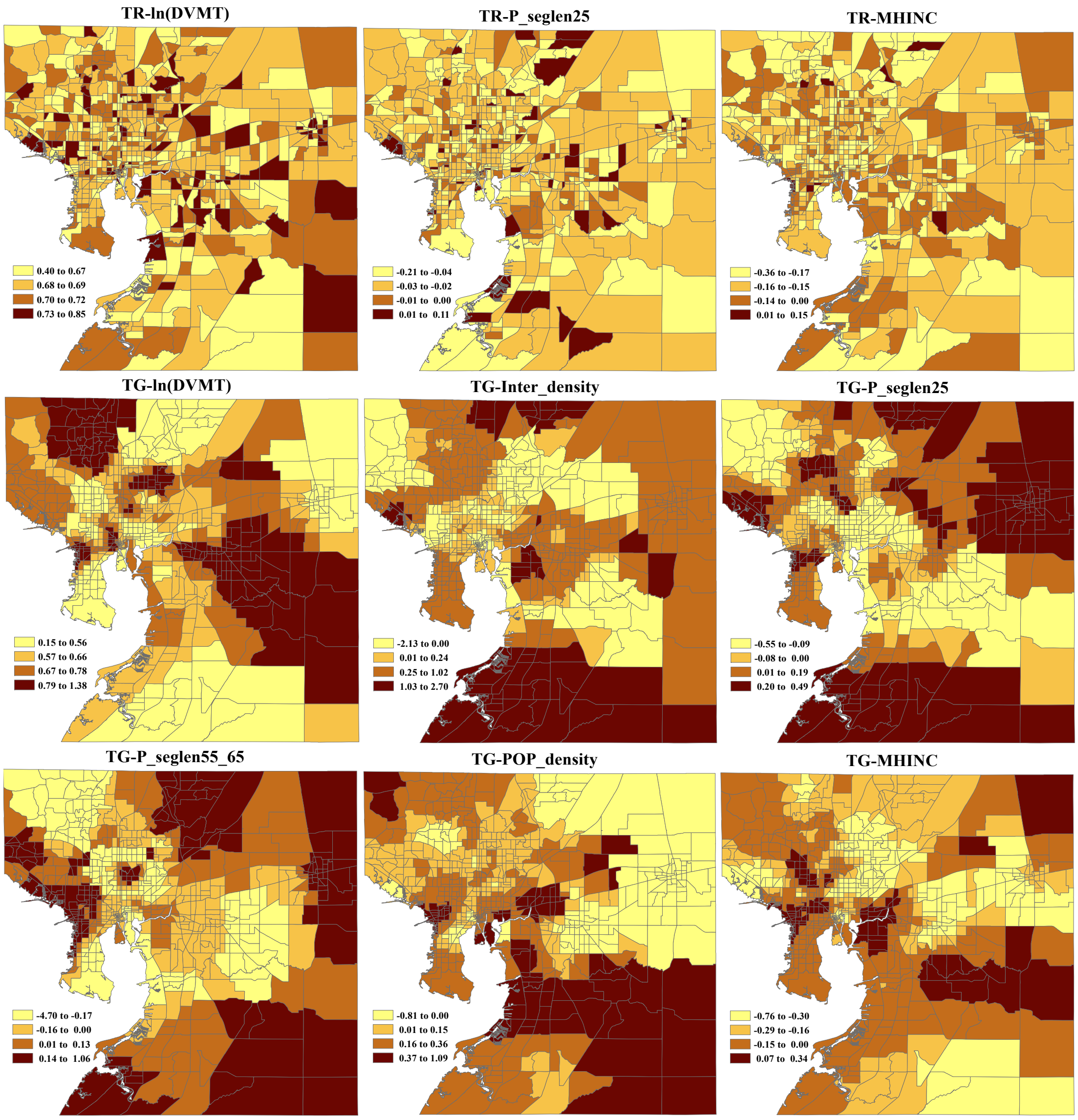

FIGURE 1 Local estimates of predicting variables in RPNB and S-GWPR with total crashes frequency as the dependent variable

Note: TR/TG denotes the RPNB/S-GWPR with total crashes frequency as the dependent variable.

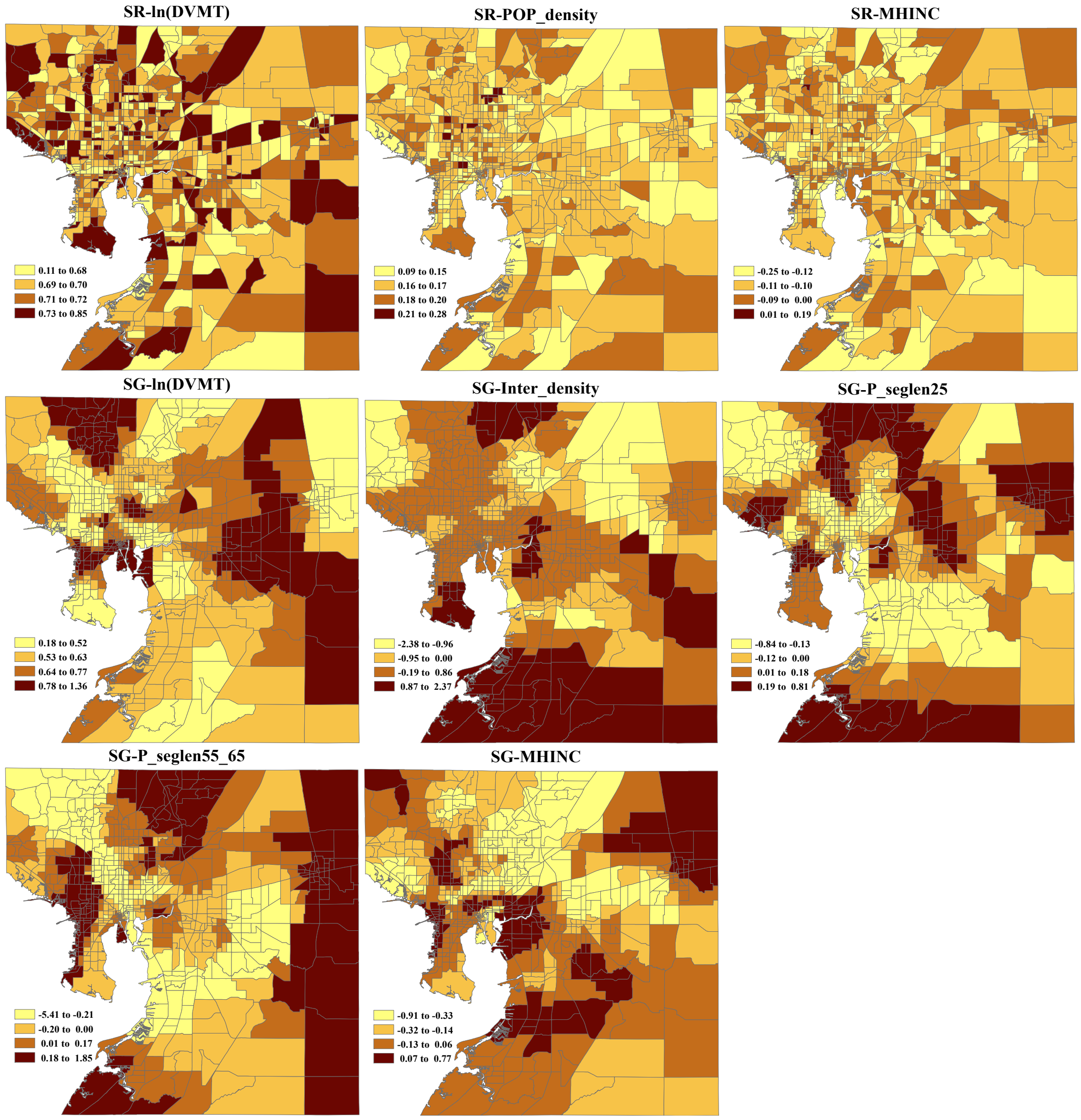

FIGURE 2 Local estimates of predicting variables in RPNB and S-GWPR with severe crashes frequency as the dependent variable

Note: SR/SG denotes the RPNB/S-GWPR with severe crashes frequency as the dependent variable.

Above problem with the counterintuitive signs is not uncommon in GWR or GWPR models and has been reported in many studies (Chow et al., 2006; Hadayeghi et al., 2010; Pirdavani et al., 2013). One explanation for this problem is the existence of multicollinearity among some of the explanatory variables for some locations. It is quite possible that the variables were correlated locally, although no global multicollinearity was observed. Up till now, there was a lack of feasible diagnostic tools to directly examine this local correlation among explanatory variables (Hadayeghi et al., 2010). As a result, above hypothesis could not be effectively confirmed in the present study.

On the other hand, Wheeler and Tiefelsdorf (2005) indicated that the strong correlation among the local parameter estimates could also lead to the problematic interpretation of the individual coefficients. To quantify the extent of multicollinearity, the bivariate correlation coefficient was calculated for each pair of local coefficients in the RPNB and S-GWPR. The results revealed that the maximum value of the Pearson product-moment correlation was -0.47 between *P_seglen25* and *POP_density* in the total crashes model, implying that the multicollinearity among local coefficient estimates is not a problem in any of the developed models.

Another reason may be due to the basis of calibrating S-GWPR. Presumably, the variables with unexpected signs may be less significant or even insignificant for some locations. To examine this possibility, local *t*-statistics can be computed in order to determine where the relationships are significant and where they are not. Finally, the presence of over-dispersion in crash data may also play some role in the unexpected coefficient signs with significant *t*-values (Note: the coefficient of over-dispersion in the NB, CAR, and RPNB is statistically significant in this study). Although the use of the Poisson regression instead of the negative binomial does not produce many inaccurate estimates in general, it could largely underestimate the variance of the parameters (Lord and Mannering 2010), thus producing more significant variables.

DVMT reveals an obvious pattern of spatial non-stationarity. All of the parameter signs are positive, indicating that the increase of DVMT always increases the crash frequency. Similar findings were also found in previous safety literature, such as Traynor (2008), Huang et al. (2010) and Li et al. (2013).

Intersections are well-known as hazardous locations in a road network. Given equal road length, more intersections are expected to be associated with a higher crash frequency. In our study, *Inter_density* is found to be positively associated with total crashes in the NB, CAR and RPNB, whereas it has a significantly negative effect on severe crashes in NB and RPNB. As mentioned above, the residuals in NB and RPNB for severe crashes models were reported to be moderate spatially correlated across the study area, which may result in biased estimates of the parameter. This may be the reason for the existence of the unexpected sign for *Inter_density* in NB and RPNB. With regard to the S-GWPR, the variable results in a geographically varying coefficient with a magnitude ranging from positive values to negative. The local *t*-statistics were computed and results revealed that 87.53% and 84.76% of the TAZs with negative coefficients are insignificant at the 95% confidence level for the total crashes model and the severe crash model, respectively.

Concerning the percentage of roadways with various speed limits, only *P_seglen45* is found to be significantly positive in the NB, CAR, and RPNB. The same variable is held constant in S-GWPR, indicating no apparent spatial variations across TAZs. The other two variables (i.e. *P_seglen25*, and *P_seglen55_65*) result in spatially varying coefficients in the S-GWPR.

POP_density is positively related to the total/severe crashes frequency in NB and CAR, suggesting that more residents in an area have more activities that could result in more traffic crashes. The variable in S-GWPR has varying coefficients ranging from -0.81 to 1.09 for total crashes models. An examination of the *p*-value for *POP_density* revealed that the majority of the TAZs with negative signs (i.e. 86.32%) are insignificant. Furthermore, the variable is found to be fixed in the S-GWPR for severe crashes models, while resulting in a random parameter in the RPNB that is normally distributed, with a mean 0.17 and standard deviation 0.12.

With regard to the *MHINC*, the variable produces a random parameter that is normally distributed with a mean -0.15 and a standard deviation 0.19 for the total crashes model. Given these distributional parameters, 97.87% of the distribution indicates a negative effect on the total crash occurrence. While in S-GWPR, the magnitude of this parameter ranges from -0.76 to 0.34 for the total crashes model. The inspection of the local *t*-statistics suggests that 87.53% of the TAZs with positive coefficients are insignificant. Similar conclusions could be also found in the severe crashes models.

CONCLUSIONS

This study intended to quantitatively investigate the spatial heterogeneity in regional crash modeling. Two advanced approaches, the RPNB and the S-GWPR, were employed to account for the spatially varying relationship between the zonal total/severe crash counts and the traffic patterns, road network attributes, and socio-demographic factors.

The possible reasons why we expect a measurement of relationship to vary over space may be that (1) the relationship is intrinsically different across regions, and (2) the estimated relationship is a gross misspecification of reality and that one or more relevant variables are either omitted or represented by an inappropriate functional form. Because of the limited resource, the development of a fully specified model seems to be impossible and unrealistic. To deal with this challenge, considerable efforts have been made either from acquiring new data from advanced data collection technology (e.g. the naturalistic driving), or developing potential models with existing datasets (e.g. the emergence of random parameter model). Both benefit a better understanding of the factors that influence the crash occurrence.

Therefore, it raises an interesting and yet unsolved puzzle. If we observe spatial variation in relationships, are they simply due to model misspecification or are they due to intrinsically different local spatial behavior? Is the role of spatial locations simply a surrogate for individual-level effects which we cannot recognize or measure? If the nature of the model misspecification due to omitted variables could be identified and corrected, would the local variations in relationship disappear? We can never be completely confident that the calibrated models are correct representation of reality because of our lack of theoretical understanding of the processes governing human driving behavior. In such a case, the local modeling then serves the purpose of allowing these otherwise omitted effects to be included through locally varying parameter estimates.

The random parameter approach outperforms in the ability of incorporating unobserved heterogeneity (i.e. any type of unobserved factors that can vary systematically across the observation; Mannering and Bhat, 2014) into modeling process, thus has been widely adopted in current safety research (Milton et al. 2008;

Anastasopoulos and Mannering 2009, 2011; EI-Basyouny and Sayed 2009, 2011; Dinu and Veeraragavan 2011; Anastasopoulos et al. 2012; Wu et al. 2013; Venkataraman et al. 2013; and Xiong and Mannering 2013). However, the present study demonstrated that the method is inadequate in accounting for the spatial correlation existed across adjacent zones. The major explanation may be that as the data in regional safety analyses typically have observations in close spatial proximity, it is very likely that ***these unobserved factors are also correlated over space***. Note that ignoring this spatial correlation may result in bias parameter estimates and incorrect inference (e.g. the variable of intersection density was found to have a significantly negative effect on severe crashes in RPNB, which seemed to be counterintuitive).

With respect to the geographically weighted approach, the coefficients are not assumed to be random, but rather they are deterministic functions of the locations in space. The calibration of this model based on the mechanism that ***all attribute values on a geographic surface are related to each other, but closer values are more strongly related than are more distant ones***. From this point of view, the S-GWPR seems to be more appropriate for regional crash modeling as the method outperforms the global models in successfully capturing the spatially heterogeneity between the zonal crash frequency and the potential transportation planning predictors, and compared with the non-spatial models, the S-GWPR is capable of accounting for the spatial correlation in crash data.

It should be clarified that the original intention of this comparison study was not to attack the advancement and effectiveness of random parameter approach in crash prediction. Instead, we want, by this comparison, to advocate that as crash data are typically collected with reference to location dimension, we should firstly make use of the geographical component to explore explicitly spatial aspects of the crash data (i.e. the spatial heterogeneity, or the spatially structured varying relationships), then is the unobserved heterogeneity or unstructured random errors by non-spatial or fuzzy techniques such as the random parameter model.

Although the S-GWPR seems to be an excellent technique for regional crash modeling, the models calibrated in this research are not spatially transferable since they produce a set of local parameters for a specific geographic region. As a consequence, most jurisdictions need to develop their own models for local regions. Meanwhile, as the results presented in the study were based on a single dataset, future research with different datasets is also required to confirm the paper’s findings. Besides, considering the fact that the S-GWPR may suffer from the issue of over-dispersion commonly existed in crash data, further efforts are needed to the calibration of geographically weighted regression model with a NB structure as well.

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