OUTAGE EVALUATION OF ULTRA WIDEBAND SPREAD SPECTRUM SYSTEM WITH RAKE COMBINING IN LOGNORMAL FAINTING MULTIPATH CHANNELS

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Abstract – Ultra wideband (UWB) spread spectrum system capacity is studied. User capacity is evaluated analytically against the conditional outage criterion. The probability of outage varies depending on the multipath channel intensity profile, severity of shadowing, signal-to-interference ratio target, number of users, timing errors, and the number of fingers combined in the rake receiver. The sum of lognormally distributed signals is mainly modelled by a Fenton-Wilkinson approximation. Numerical results show that the outage performance is quite sensitive to the variation of several key parameters.

Keywords – capacity, UWB, diversity, power sum

I. INTRODUCTION

Ultra wideband systems [1] are viable candidates for high-rate short range wireless communication applications. High multipath resolution is one of the inherent UWB system characteristics. Rake reception can be utilised to collect energy from the widely dispersed channel. In this paper, a generic spread spectrum based UWB system (e.g., time hopping (TH) or direct sequence (DS) modulated) is assumed.

Multiuser capacity is dependent on the conditional outage probability due to the various co-channel interference sources. Channel amplitude variations are assumed to follow a lognormal distribution. This leads to the need to calculate power sums of several lognormal signals. In the lack of closed-form expressions numerous approximate solutions have been proposed. Lognormal sum approximations can be extracted, e.g., from the well-known Fenton-Wilkinson [2] and Schwartz-Yeh [3] approaches. Both methods assume that the sum of two or more lognormal random variables is well modelled by another lognormal random variable. Extensions of these approximations to correlated random variables can be found, e.g., in [4] and [5]. Conventionally, these approximations have been applied to narrowband reuse systems (frequency and time division multiple access schemes), where the number of dominant co-channel interference sources is small. This is also the case in high data rate code division based UWB systems.

Focus of this paper is to evaluate multiuser capacity versus conditional outage probability, and to study effects of the key parameters to the performance. Among such parameters are: the severity of shadowing, signal-to-interference ratio, multipath correlation, number of rake fingers, and pulse waveform specific timing error. Section II introduces channel profiles, pulse waveforms, and log-normal sum approximations used in this paper. A procedure for outage probability evaluation is described in Section III. Some numerical examples are shown in Section IV and concluding remarks are made in Section V.

II. SYSTEM DESCRIPTION

A. Multipath Channel Model

Saleh and Valenzuela [6] have proposed a clustered exponentially decaying statistical model for indoor multipath propagation channels. In [7] it has been slightly modified to improve suitability for the UWB applications. According to [7] the measurements indicate that a lognormal distribution fits better than a Rayleigh distribution for the multipath gain magnitudes. This modified model has also been used in [8].

Main properties of the aforementioned channel models are taken into account in this paper. A general exponential multipath intensity profile (MIP) channel model is assumed. It can be implemented as a tapped delay line. Parameters of the profile, such as decay and the number of multipaths, can be easily varied to adapt to different propagation scenarios. Therefore, we can present averaged power coefficients in a single cluster multipath intensity profile with known tap delays as

$$\alpha_l = \alpha_0 e^{-\lambda l} \quad l, \lambda \geq 0$$

(1)

where $\lambda$ is the temporal (delay) decay parameter, $l$ is the multipath delay index. Total energy of the $L$-path MIP is normalised to unity as

$$\sum_{l=0}^{L-1} \alpha_l e^{-\lambda l} = 1.$$  

(2)

For $\lambda = 0$, there would be equal gain in every multipath component, whereas very large $\lambda$ would result in essentially 1-path channel in delay dimension. The number of resolvable multipaths depends on the chip rate and the channel delay spread.

B. Lognormal Sum Approximations

The wireless channel is modelled as a multipath profile with a lognormal distribution in each path. In order to evaluate the mean and variance of sums of multiple independent or correlated lognormal signals, the Fenton-Wilkinson approximation (FWA) and the Schwartz-Yeh approximation (SYA) have been used. Their validity is checked against Monte Carlo simulations. In the simula-
Clearly, the sensitivity increases for higher order derivatives and their derivatives to timing offsets in the receiver. Moreover, susceptible to performance losses. High-order derivative pulses (having narrow main lobes) are rectified. However, when the timing error is accounted for, the performance of all pulse waveforms is identical as long as the receiver sampling (timing) is correct. The performance of all pulse waveforms can be approximated as twice derived waveform with respect to the originally generated. Up to timing errors of 0.05Tp all outputs remain almost within a 2 dB loss margin. The absolute timing requirement depends on the pulse width Tp. In the case of a typical 0.2 ns UWB pulse width 0.05Tp would mean a timing uncertainty of 10 ps.

According to Fig. 1 the mean values (lower set of curves) of all approaches agree well. Standard deviations (upper set of curves) of the Monte Carlo simulations and the SYA are also very close to each other. The FWA tends to give slightly higher variance values than the SYA and the simulations. This result agrees with the previous studies proposing that the SYA gives better estimates on mean and variance than the FWA. However, in tail distributions (like outage) the accuracy of the FWA might be better [5]. Due to a good match of all approaches the simplest of them, being FWA, will be used in the remaining numerical assessments.

C. Pulse Waveforms and Timing Errors

The assumed UWB pulse shapes are Gaussian and their four derivatives, starting from

\[ w(t) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{(t-\mu)^2}{2\sigma^2}\right) \]  

(3)

where μ and σ are the mean and standard deviation of the Gaussian distribution [9]. Pulse waveforms are scaled so that the maximum correlator/matched filter outputs are normalised to one. The performance of all pulse waveforms is identical as long as the receiver sampling (timing) is correct. However, when the timing error is accounted for, the high-order derivative pulses (having narrow main lobes) are more susceptible to performance losses.

Fig. 2 shows the sensitivity of Gaussian pulse waveforms and their derivatives to timing offsets in the receiver. Clearly, the sensitivity increases for higher order derivatives. Due to transmit and receive antennas the received waveform can be approximated as twice derived waveform with respect to the originally generated. Up to timing errors of 0.05T_p all outputs remain within a 2 dB loss margin. The absolute timing requirement depends on the pulse width T_p. In the case of a typical 0.2 ns UWB pulse width 0.05T_p would mean a timing uncertainty of 10 ps.

The impact of timing errors in multipath combining will be included by the following equations

\[ \bar{\alpha}_i(\varepsilon) = R^2(\varepsilon)\bar{\alpha}_i \]  

(4)

\[ \sigma^2_i(\varepsilon) = \sigma^2_i \left(1 - R^2(\varepsilon)\right) \frac{\bar{\alpha}_i}{\bar{\alpha}_i} \]  

(5)

where \( R^2(\varepsilon) \) is the receiver squared correlation function (shown in dB-scale in Fig. 2) with the normalised timing error \( \varepsilon = \frac{t}{T_p} \). Variance in (5) depends on the severity of shadowing per path \( \sigma^2_i \), squared pulse autocorrelation, and the power ratio of multipath components. Error variance, i.e., the second term in (5) is assumed to be inversely proportional to the delay tracking loop signal-to-noise ratio.

III. OUTAGE EVALUATION

In this paper, the capacity is defined as the maximum number of admissible active co-channel interferers satisfying a predefined outage criterion. The probability of outage is given as

\[ P_{out} = P \left( \frac{S}{I} = \frac{S(L_0)}{I_{M_1} + I_{M_1} + I_{L_1}} < \frac{S}{I_{tar}} \right) \]  

(6)

where SI is the signal-to-interference ratio, \( S(L_0) \) is the desired signal power combined by \( L_0 \) rake fingers, and \( SI_{tar} \) is the SI target, also known as a co-channel protection ratio, required for the intended quality-of-service (QoS). Co-channel interference accumulates from other active multiple access users (\( I_{M_1} \)), from multipath propagations.
tion channel ($I_{a,p}$), and from interpath interference ($I_{p}$).

Furthermore, additive white Gaussian thermal noise with power spectral density $N_0$ is present. However, the contribution of thermal noise in interference limited systems can be assumed to negligible in comparison to the co-channel interference ($N_0$ << $I_0$), and thereby it will be ignored.

In spread spectrum channels, the channel signal-to-interference ratio can be related to the baseband bit energy-to-interference power spectral density ratio by the following expression

$$
\frac{S}{I} = \frac{R_s E_b}{R_c I_0} = \frac{E_b / I_0}{PG}
$$

where $PG = R_s / R_c$ denotes the processing gain, achieved through the despreading in the receiver matched filter or correlator. $R_s$ is the chip rate of the spread data stream and $R_c$ is the bit rate of the original data stream. From (7) we see that a link level QoS requirement (e.g., bit error rate = $10^{-2}, \ldots, 10^{-6}$) can be mapped into system level in a straightforward manner. Therefore, link level performance of the desired user with respect to one interfering user can be simulated in detail including data modulation, pulse shaping/filtering, channel coding, channel estimation, antenna imperfections, power control, etc. Realistic $E_b / I_0$ requirements for different services can be taken from link level simulations and according to (7) they can be directly converted to the corresponding $S/I$ targets.

Fig. 3 shows the model for $S/I$ and outage evaluation used in this paper. It can be seen that the desired signal power $S$ is composed of $I_0$ combined fingers in the selective rake combiner. Interference is the sum signal of multiaccess signals through $L$ multipath channels and interpath interference of the desired user through $L_d(L-1)$ paths.

Mean and variance of the desired signal, multiple access, multipath, and interpath interference contributions are derived from the lognormal sum approximation separately. Then, these statistics are used in the outage calculation. As a result, (6) can be conditioned on $k$ interferers and $L_0$ rake fingers, leading to the slightly modified expression from [5]

$$
P_{out}(k, L) = 1 - Q\left(\frac{\ln(S/I)_{\text{uar}} - m_d(I_0) + m_z(k)}{\sigma_d^2(I_0) + \sigma_z^2(k) - 2 r_d \sigma_d(I_0) \sigma_z(k)}\right)
$$

where $Q(x) = \frac{1}{2} \text{erfc}\left(\frac{x}{\sqrt{2}}\right)$ is a zero mean, unit variance Gaussian complementary distribution function, $m_d(I_0)$ is the area mean desired signal power with $L_0$ rake fingers, $m_z(k)$ is the area mean joint interference power of $k$ interferers, $\sigma_d(I_0)$ is the standard deviation of the desired signal with $L_0$ rake fingers, $\sigma_z(k)$ is the standard deviation of the joint interference from $k$ interferers and $r_d$ is the correlation coefficient of the desired signal and joint interference.

Overall interference statistics in (8) can be divided into multiaccess, multipath, and interpath components as

$$
m_z(k) = m_{\text{MAI}}(k) + m_{\text{MPI}}(L) + m_{\text{IP}}(I_0(L-1))
$$

$$
\sigma_z(k) = \sigma_{\text{MAI}}(k) + \sigma_{\text{MPI}}(L) + \sigma_{\text{IP}}(I_0(L-1)).
$$

Aggregate interference is calculated with respect to index $k = 1, \ldots, K$, i.e., the number of active multiple access interference sources. All multipath profiles include $L$ components. The number of interpath interference components depends on the diversity order $L_0$ in the rake combiner in addition to the number of multipaths.

### IV. NUMERICAL EXAMPLES

A generic spread spectrum UWB system is assumed (e.g., TH-UWB or DS-UWB), targeted for $S/I = -14$ dB. Desired user and interfering multiple access user signals are equally strong. Selective rake combined desired signal is formed as an incoherent lognormal power sum of $L_0$ strongest paths. Key assumptions and initial reference parameters, used in the comparative analysis, have been gathered in Table 1.

| Number of multipaths $L$ | 8 |
| Number of rake fingers $L_0$ | 1, ..., 8 |
| MIP decay parameter $\lambda$ | 0, 0.5, 1 |
| $m_d(I_0)$ [dB] | $-9, -4, -2$ |
| $\sigma_d(I_0)$ [dB] | $2, \ldots, 3, \ldots, 7$ |
| $S/I_{\text{uar}}$ [dB] | $-15, \ldots, -14, \ldots, -10$ |
| Correlations $r_{\text{MAI}}, r_{\text{MPI}}, r_d$ | $0, 0, 0, 1, 0$ |
| Number of interfering users $k$ | 1, ..., 15 |
| Timing error $\varepsilon$ [ns] | $0, 0, 1, 0.1$ |

Figs. 4–6 demonstrate differences of the multipath intensity profiles at various rake finger allocations and multiuser interference loads. In Fig. 4 the channel profile is flat. Hence, the performance improves gradually as more multipaths are combined. In all 3-D illustrations a reference plane is plotted at $P_{\text{uar}} = 10^{-1}$, in order to emphasise surface cross-sections and outage regions.

Fig. 5 illustrates the corresponding case for exponentially decaying MIP with $\lambda = 0.5$. Here the conditional outage probability is high both at the low-end and at the high-end of diversity orders. The optimal number of rake fingers

![Fig. 3. Model of $S/I$ and outage calculation.](image-url)
in this particular case is $L_0 = 5$. A reason for this kind of behaviour is that the weakest combined multipath components gain more interference than useful signal energy.

The same effect is seen even more drastically in Fig. 6, where the MIP is steeply decreasing with $\lambda = 1$. Now, only two or three fingers are enough for the optimal performance.

Sensitivity to the lognormal signal standard deviation variation is plotted in Fig. 7. It can be noticed that $P_{\text{out}}$ depends very strongly on this parameter that physically reflects the propagation environment. As small as half a decibel change in the source standard deviation is significant for the outcome. In the outage probability the impact of 13 interfering users with $\sigma = 2$ dB roughly equals the effect of one interfering user with $\sigma = 6$ dB.

The conditional outage probability is very sensitive for the changes in targeted $S/I$ range, which depends mainly on the ratio of total bandwidth and data rate (spreading ratio, processing gain). From Fig. 8 we can see that a 5 dB difference in $(S/I)_{\text{tar}}$ scales the outage probability up or down several orders of magnitude.

The initial assumption has been that the multipath components are uncorrelated. However, in certain propagation scenarios they may be correlated. Fig. 9 demonstrates the performance degradation in situations, where the multipath components are equally correlated with correlation coefficient $r_{\text{cor}} = 0, \ldots, 1$. From Fig. 9 it can be noted that rather moderate multipath correlations are resulting in significant increases in outage probabilities. Higher the correlation less the diversity is available in the MIP.
In the one interfering user case of the first and fourth derivative Gaussian pulse waveforms, the outage probability is more sensitive to timing error. For the fourth derivative, the edge of the surface is much steeper, indicating greater sensitivity.

Figs. 10 and 11 demonstrate how the pulse width normalised timing error affects the outage probability in the case of the first and fourth derivative Gaussian pulse waveforms. In the one interfering user case $P_{\text{out}}$ lies below the reference plane throughout the timing error scale. For the fourth derivative, the edge of the surface is much steeper, indicating greater sensitivity.

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V. Conclusions

Some simple analyses were conducted in order to evaluate UWB system capacity versus conditional outage probability in various interference scenarios in lognormal fading multipath channels. The results presented in the paper show indicative and relative effects of the key system parameter variations. The number of useful rake fingers was seen to be strongly dependent on the multipath intensity profile. High sensitivity was observed to the propagation parameter changes (severity of shadowing, target $S/I$). High-order Gaussian derivative pulse waveforms were found to be quite susceptible to timing jitter. Multipath correlation showed also detrimental effects to the outage performance.

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References