INS/GPS Aided by Frequency Contents of Vector Observations with Application to Autonomous Surface Crafts

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Abstract

This paper presents a high-accuracy, multirate Inertial Navigation System (INS) integrating Global Position System (GPS) measurements and advanced vector aiding techniques for precise position and attitude estimation of Autonomous Surface Crafts (ASCs). Designed to be implemented and tested in the DELFIMx catamaran developed at ISR/IST, the navigation system comprises an advanced INS algorithm to account for coning and sculling motions, combined with an Extended Kalman Filter (EKF) for inertial sensor error compensation. Aiding magnetic and gravitational observations are optimally integrated in the EKF, taking into account the vehicle’s dynamics bandwidth information to properly trace attitude errors and reject measurement disturbances. The proposed aiding techniques and the performance of the navigation system are assessed using experimental data obtained at sea trials with a low-cost hardware architecture installed on-board the DELFIMx platform. It is shown that inertial sensor non-idealities such as bias and noise are effectively compensated for, using the magnetometer measurements and the low frequency information embodied in pendular measurements. The overall improvements obtained with the vector aiding observations are illustrated for the case of GPS signal outage, emphasizing the extended autonomy of the navigation system with respect to position aiding.

Index Terms

Inertial navigation, Kalman filtering, Autonomous vehicles, Marine technology.

I. INTRODUCTION

Autonomous surface crafts (ASCs) are versatile robotic platforms, capable of performing a wide and valuable range of operational tasks in challenging scenarios. Applications of interest for the civilian community include the inspection of coastal areas and the maintenance of large critical semi-submerged infrastructures like bridges and breakwaters, that in general require complex and expensive routine inspection procedures. Most of these structures are exposed to harsh environments and heavy loads and some of them are designed under the proviso that maintenance work will be required during the structure’s life. Performance specifications often demand for...
ultra light weight, high performance, robust navigation systems to provide high resolution position and attitude estimates that characterize accurately the infrastructure status [1], while compensating for the influence of the vehicle motion on the data acquired by the sonars and the LIDAR. However, adopted low-cost sensor units are affected by non-idealities that hinder the required accuracy, and call for advanced filtering techniques that make use of the measurement redundancy and exploit the available information about the kinematics and dynamics of the vehicle. This motivates the Autonomous Vehicles scientific community to develop high accuracy algorithms for strapdown navigation systems, merging the available sensor data, and compensating for disturbances such as bias and noise.

The autonomous catamaran DELFIMx, an ASC built at IST-ISRA and displayed in Fig. 1, was designed for automatic marine data acquisition for risk assessment in semi-submerged structures [2]. This robotic platform allows for the access to remote and confined locations in a systematic way, as required for precise sonar and LIDAR data acquisition. To successfully execute its mission, the ASC is required to have a reliable on-board navigation system based on low-power consumption, inexpensive hardware, capable of efficiently integrating the information from inertial and aiding sensor suites. The problem of poor GPS signal detection due to the proximity to the surveyed structure also demands for a navigation system able to operate under medium term position aiding shortage.

This paper describes the design and implementation of a navigation system with application to autonomous vehicles. A multirate, high accuracy inertial navigation system (INS) is proposed to compute attitude, velocity and position, and is combined with an Extended Kalman Filter (EKF) to integrate GPS position measurements, vector observations and frequency domain characterization of the vehicle. Magnetic and gravitational observations are integrated optimally in the EKF by modeling the sensor readings directly in the filter and by taking into account the vehicle’s dynamics bandwidth information. The direct-feedback configuration of the proposed architecture, illustrated in Fig. 2, is implemented and validated in experiments at sea with the DELFIMx ASC.

The INS is the backbone algorithm that performs attitude, velocity and position numerical integration from rate gyro and accelerometer triads data, rigidly mounted on the vehicle structure (strapdown configuration). For highly maneuverable vehicles, the INS numerical integration must properly address the fast dynamics of inertial
sensors output, to avoid estimation errors buildup. The INS computations adopted in this work account for high frequency attitude, velocity and position motions (denoted as coning, sculling and scrolling respectively), and are based on the algorithm developed in [3], [4]. The inertial algorithm integrates the sensor readings, and hence the results are corrupted by bias and noise, among other error sources. The EKF is adopted to exploit aiding information, to dynamically compensate for the non-ideal sensor characteristics that otherwise would yield unbounded INS errors. Rate gyro and accelerometer biases compensation enhancements are obtained, using magnetometer measurements and selective frequency contents from gravity information, provided by the accelerometer triad readings.

A solution to integrate vector observations such as magnetometer and gravity measurements in the EKF is discussed. Although a snapshot attitude reconstruction can be obtained from the vector measurements using numerically efficient algorithms such as QUEST or TRIAD [5], [6], [7], the magnetometer and pendular readings are fed directly to the Kalman filter. The measurement residual is obtained by comparing the estimated and measured vector observations, and it is modeled in the filter as a function of the attitude estimation error. Consequently, the EKF acts as an attitude determination algorithm, by computing the perturbational attitude term based on vector observations. Vector measurement characteristics, such as sensor noise covariance, are described directly in the filter state model, yielding physical interpretation to the filter design parameters used in the computation of optimal gains.

The proposed vector aiding technique decomposes and optimally integrates gravitational observations in the EKF, taking into account the vehicle dynamics bandwidth information to properly trace inertial motion. Gravity readings are provided by the accelerometer triad, and hence distorted in the presence of linear and angular accelerations. A dynamic compensation of external accelerations is performed using the INS information to estimate angular acceleration, while linear acceleration is characterized in the frequency domain using the filter state space. Using this approach, the low frequency contents of the gravity readings are exploited by the EKF to compute the attitude estimation error. Simulation and experimental results obtained with the DELFIMx at sea validate the frequency domain modeling technique, and illustrate its contribution to attitude and position estimation for the case of sparse/unavailable GPS signals. A preliminary version and study in simulation of this
work have been presented in [8].

Navigation system architectures for oceanic vehicles using Kalman filtering techniques are commonly adopted in the literature [9], [10], [11], [12], [13], [14], [15], and further references can be found in the valuable survey on ocean vehicle navigation [16]. The filter proposed in this work is based on the concept of Multiplicative EKF (MEKF) [17], using the Direction Cosine Matrix (DCM) form as the global attitude parametrization [18]. The linear differential equations of the filter are derived using a perturbational analysis of the vehicle kinematics, see [18] for an introduction on this framework. The adopted perturbational representation for the attitude error is locally linear and non-singular, and consequently can be estimated by the Kalman filter, using the aiding measurements. The estimated attitude error is transferred from the EKF to the INS to update the nonlinear global attitude estimate, see Fig. 2 and reset in the filter. This incremental procedure can be regarded as a storage technique that prevents the filter’s attitude error estimates to fall outside the linearization region. As evidenced in [17], the uncertainty of the estimate, i.e. the estimation error covariance, is unaffected by the reset step. In this work, the attitude error is parameterized using the rotation vector representation in Earth coordinates. Other equivalent frame coordinates and attitude parametrization can be used, such as Gibbs vector and Modified Rodrigues Parameters [17], [19].

The paper is organized as follows. Section III briefly discusses the INS algorithm adopted in this work. In Section IV the linear differential equations describing the inertial sensor errors are derived and introduced in the EKF state space model. In Section V the technique to use vector measurements directly in the EKF is derived using perturbational techniques and illustrated for magnetometer measurements. The method is extended for gravity measurements obtained from the accelerometer triad, which require modeling vehicle dynamics bandwidth information in the EKF to compensate for accelerated motion in the frequency domain. The EKF state space model is summarized in Section VI the discrete-time equivalent filter is obtained, and the correction and reset procedures to update the INS states using the EKF estimates are detailed. Simulation results to validate the proposed navigation system prior to experimental testing are shown in Section VII. Namely, the validity and influence of vector measurements and gravity selective frequency contents in the estimation results are studied. Experimental results obtained during the DELFIMx sea trials are presented in Section VIII to assess the navigation system performance in practice. Concluding remarks are found in Section VIII.

NOMENCLATURE

Column vectors and matrices are denoted respectively by lowercase and uppercase boldface type, e.g. $\mathbf{s}$ and $\mathbf{S}$. The transpose of a vector or matrix will be indicated by a prime, and trailing subscripts $\{x,y,z\}$ denote the vector components, $\mathbf{s} = [s_x \ s_y \ s_z]'$. Leading subscripts and superscripts identify the coordinate system of a quantity, e.g. $^E\mathbf{s}$ is represented in coordinate frame $\{E\}$, and $^B\mathbf{R}$ is a rotation matrix that transforms the vector representation $^B\mathbf{s}$ into $^E\mathbf{s}$ by means of the linear operation $^E\mathbf{s} = ^E\mathbf{R}^B\mathbf{s}$. Position, velocity and acceleration are denoted respectively by $\mathbf{p}$, $\mathbf{v}$, and $\mathbf{a}$, and the angular velocity of the vehicle expressed in body coordinates is represented by $\omega$. The measurement and the estimate of quantity $\mathbf{s}$ are denoted by $\mathbf{s}_r$ and $\hat{\mathbf{s}}$, respectively. Discrete time quantities are characterized by the time index $k$ subscript. The $(n \times n)$ identity matrix is denoted by $\mathbf{I}_n$, and $(m \times n)$ zeros and ones matrices are respectively denoted by $\mathbf{0}_{m \times n}$ and $\mathbf{1}_{m \times n}$, where the subscript is omitted whenever clear from the context.
II. INERTIAL NAVIGATION SYSTEM ALGORITHM

In this section, an INS algorithm is briefly introduced, based upon the tutorial work presented in [3], [4] for attitude, velocity, and position computation, where complex angular, velocity and position high-frequency motions, referred to as coning, sculling, and scrolling respectively, are properly accounted for using a multirate integration approach. Interestingly enough, a technique to convert the high accuracy attitude algorithms into its velocity/position counterpart was later proposed in [20].

In this framework, a high-speed, low order algorithm computes dynamic angular rate/acceleration effects at a small sampling interval, and its output is periodically fed to a moderate-speed algorithm that computes attitude/velocity resorting to exact, closed-form equations. Limited operational time and confined mission scenarios for the application at hand allowed to simplify the frame set to Earth and body frames, respectively denoted as \{E\} and \{B\}, and to adopt an invariant gravity model without loss of precision, while equations were derived to the highest accuracy. As depicted in Fig. 3 the inputs provided to the inertial algorithms are the integrated inertial sensor output increments

\[
\begin{align*}
\bm{\nu}(\tau) &= \int_{t_{k-1}}^{\tau} \bm{a}_r \, dt, \\
\bm{\alpha}(\tau) &= \int_{t_{k-1}}^{\tau} \bm{\omega}_r \, dt,
\end{align*}
\]

which correspond to the integral of the inertial sensor readings, obtained using strapdown accelerometer and rate gyro triads, corrupted by white noise and bias errors and modeled as follows

\[
\begin{align*}
\bm{a}_r &= \bm{g} - \bm{b}_a + \bm{n}_a - \dot{\bm{b}}_a, \\
\bm{\omega}_r &= \bm{\omega} + \bm{b}_\omega + \bm{n}_\omega - \dot{\bm{b}}_\omega,
\end{align*}
\]

where \(\bm{g}\) represents Earth’s gravitational field, the sensor biases are denoted by \(\bm{b}_a\) and \(\bm{b}_\omega\), and \(\bm{n}_a \sim \mathcal{N}(0, \Xi_a)\), \(\bm{n}_\omega \sim \mathcal{N}(0, \Xi_\omega)\) are Gaussian white noises.

The attitude moderate-speed integration algorithm detailed in [3] computes body attitude in DCM form

\[
B_{k+1}
B_k^{-1} \bm{R}(\bm{\lambda}_k) = \bm{I}_3 + \frac{\sin(\|\bm{\lambda}_k\|)}{\|\bm{\lambda}_k\|} (\bm{\lambda}_k)_\times + \frac{1 - \cos(\|\bm{\lambda}_k\|)}{\|\bm{\lambda}_k\|^2} (\bm{\lambda}_k)_\times^2,
\]

where \(\{B_k\}\) is the body frame at time \(k\) and \((\bm{s})_\times\) represents the skew symmetric matrix defined by the vector \(\bm{s} \in \mathbb{R}^3\) such that \((\bm{s})_\times \bm{r} = \bm{s} \times \bm{r}, \bm{r} \in \mathbb{R}^3\). Rotation vector dynamics, based on Bortz equation [21], are formulated.
In order to denote angular integration and coning attitude terms $\alpha_k$ and $\beta_k$, respectively

$$\lambda_k = \alpha_k + \beta_k,$$

where $\alpha_k = \alpha(t)|_{t=t_k}$ and the coning attitude term measures the attitude changes due to the effects of angular rate vector rotation. A high-speed attitude algorithm is required to compute $\beta_k$ as a summation of the high-frequency angular rate vector changes using simple, recursive computations [3]. Equations (3) and (4) summarize both the moderate and high-speed attitude dynamics in the DCM format using exact, error-free equations, enabling high-accuracy results.

Exact linear velocity updates can be computed at moderate-speed rate using the equivalence between strapdown attitude and velocity/position algorithms [20], that yields

$$v_k = v_{k-1} + \frac{E_{Bk-1}}{R_{Bk-1}} \Delta v SF_k + \Delta v_{GCor} k,$$

where $\Delta v_{SF} k$ is the velocity increment related to the specific force, and $\Delta v_{GCor} k$ represents the velocity increment due to gravity and Coriolis effects, see [4] for further details. High-speed velocity rotation and high-frequency dynamic variations due to angular rate vector rotation, are likewise accounted for in the high-frequency algorithm and included in the moderate-speed calculations as

$$\Delta v_{rot} k + \Delta v_{scul} k,$$

where $\Delta v_{rot} k$ and $\Delta v_{scul} k$ represent velocity increments due to rotation and sculling, respectively.

The INS algorithm execution rates are set as a trade-off between the available hardware and the performance requirements [3], [4], [22]. Simulation environments and trajectory profiles to tune the algorithm’s repetition rate according to the accuracy requirements are thoroughly described in [23] and algorithm evaluation procedures are presented in [3], [4]. Interestingly enough, high repetition rates can be implemented in a standard low-power consumption Digital Signal Processing (DSP) based hardware architecture. This allows for accurate integration results, that are only diminished by inertial sensor non-idealities such as noise and bias.

### III. INERTIAL ERROR DYNAMICS

In a stand alone INS, bias and inertial sensor errors compensation is usually performed offline. The usage of filtering techniques in navigation systems, such as the EKF, allows for the dynamic estimation of inertial sensor non-idealities, bounding the INS errors. The EKF error equations adopted in this work are based on perturbational rigid body kinematics, and were brought to full detail in [18]. The nominal rigid body kinematics are given by

$$\dot{p} = v, \quad \dot{v} = R^B a, \quad \dot{R} = R (\omega) \times, \quad \dot{b}_a = n_{b_a}, \quad \dot{b}_\omega = n_{b_\omega},$$

where $R$ is the shorthand notation for $E_B R$, the inertial sensor biases are modeled as random walk processes, and $n_{b_a} \sim \mathcal{N}(0, \Xi_{b_a})$, $n_{b_\omega} \sim \mathcal{N}(0, \Xi_{b_\omega})$ are Gaussian white noises. The position, velocity and bias estimation errors are defined by the difference of the estimated and nominal quantities,

$$\delta p := \hat{p} - p, \quad \delta v := \hat{v} - v, \quad \delta b_a := \hat{b}_a - b_a, \quad \delta b_\omega := \hat{b}_\omega - b_\omega.$$

November 12, 2008 DRAFT
and the attitude error, denoted as $\delta \lambda$, is parametrized by an unconstrained rotation vector representation in Earth coordinates, which can be assumed locally linear and non-singular, for details and equivalent attitude parametrizations, see [17], [19]. Define the rotation error matrix as $R(\delta \lambda) := \hat{R}R'$, the attitude error rotation vector $\delta \lambda$ is described by the first order approximation

$$R(\delta \lambda) \approx I_3 + (\delta \lambda)_x \Rightarrow (\delta \lambda)_x \approx \hat{R}R' - I_3,$$

(6)

that is valid for “small-angle” attitude errors [18]. The rigid body coordinates are estimated using the available inertial sensor information

$$\dot{\mathbf{p}} = \dot{\mathbf{v}}, \quad \dot{\mathbf{v}} = \hat{R}a_r + \mathbf{g}, \quad \hat{R} = \hat{R}(\omega_r)_x, \quad \dot{\mathbf{b}}_a = 0, \quad \dot{\mathbf{b}}_w = 0,$$

(7)

Combining (5) and (7), the attitude, velocity, and position error kinematics are obtained by retaining the first-order terms of Taylor’s series expansions or by using perturbation algebraic techniques [18], producing

$$\delta \dot{\mathbf{p}} = \delta \dot{\mathbf{v}}, \quad \delta \dot{\mathbf{v}} = \hat{R}(a_r - a_{SF}) - (\hat{R}a_r)_x \delta \lambda, \quad \delta \dot{\lambda} = \hat{R}(\omega_r - \omega), \quad \delta \dot{\mathbf{b}}_a = -\mathbf{n}_b, \quad \delta \dot{\mathbf{b}}_w = -\mathbf{n}_w,$$

(8)

where $a_{SF} = B a - B g$ is the specific force, defined as the nominal reading of an accelerometer. The terms $(a_r - a_{SF})$ and $(\omega_r - \omega)$ represent the non-idealities of the accelerometer and rate gyro readings (1) and (2) respectively, and are described by

$$(a_r - a_{SF}) = -\delta \mathbf{b}_a + \mathbf{n}_b, \quad (\omega_r - \omega) = -\delta \mathbf{b}_w + \mathbf{n}_w.$$  

(9)

Combining (8) and (9), the error state space model is

$$\delta \dot{\mathbf{p}} = \delta \dot{\mathbf{v}}, \quad \delta \dot{\mathbf{v}} = -\hat{R}\delta \mathbf{b}_a - (\hat{R}a_r)_x \delta \lambda + \hat{R}\mathbf{n}_b, \quad \delta \dot{\lambda} = -\hat{R}\delta \mathbf{b}_w + \hat{R}\mathbf{n}_w,$$

$$\delta \dot{\mathbf{b}}_a = -\mathbf{n}_b, \quad \delta \dot{\mathbf{b}}_w = -\mathbf{n}_w.$$  

(10)

The continuous-time error state space model $\delta \dot{\mathbf{x}} = \mathbf{F}(\hat{\mathbf{x}}, \mathbf{u})\delta \mathbf{x} + \mathbf{G}(\hat{\mathbf{x}})\mathbf{n}_x$ is described by

$$\delta \dot{\mathbf{x}} = \begin{bmatrix} \delta \dot{\mathbf{p}}' & \delta \dot{\mathbf{v}}' & \delta \dot{\lambda}' & \delta \dot{\mathbf{b}}_a' & \delta \dot{\mathbf{b}}_w' \end{bmatrix}', \quad \mathbf{n}_x = \begin{bmatrix} \mathbf{n}_p' & \mathbf{n}_v' & \mathbf{n}_w' & \mathbf{n}_b_a' & \mathbf{n}_b_w' \end{bmatrix}',$$

$$\mathbf{F}(\hat{\mathbf{x}}, \mathbf{u}) = \begin{bmatrix} 0 & I & 0 & 0 & 0 \\ 0 & 0 & - (\hat{R}a_r)_x & -\hat{R} & 0 \\ 0 & 0 & 0 & 0 & -\hat{R} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{G}(\hat{\mathbf{x}}) = \text{blkdiag}(I_3, \hat{R}, \hat{R}, -I_3, -I_3),$$

(11)

where $\mathbf{x} = (\mathbf{p}, \mathbf{v}, \hat{\mathbf{R}})$ are the quantities computed by the INS algorithm, $\mathbf{u} = (a_r, \omega_r)$ are the inertial measurements, blkdiag(·) denotes a block diagonal matrix obtained by the concatenation of its matrix arguments, $\mathbf{n}_w = \mathbf{n}_b$, and $\mathbf{n}_p \sim \mathcal{N}(0, \Xi_p)$ is a Gaussian white noise that accounts for linearization and modeling errors and is used in the practical tuning of the filter.

The adopted attitude error parametrization is locally linear and hence can be integrated in the EKF estimation algorithm without requiring attitude normalization procedures. After each filter update step, the EKF estimated inertial errors are transferred to the INS, as illustrated in Fig. 4. This technique preserves the small error assumption underlying the linearized model (11), combining local parametrization update in the EKF with a global representation propagation and storage in the INS [17].

November 12, 2008 DRAFT
IV. VECTOR AIDING TECHNIQUES

The EKF relies on aiding sensor readings to successfully estimate the error states. The physical coupling between attitude and velocity errors enables the use of GPS position readings to partially estimate attitude errors. As convincingly argued in [24], for observability analysis purposes a GPS based navigation system with bias estimation can be approximated by a concatenation of piece-wise time-invariant systems and, under that assumption, full observability is met by performing specific maneuvers along the desired trajectory. Recent work has been directed towards replacing the necessity for alignment maneuvers by equipping the filter with additional information sources, namely aiding sensors or vehicle dynamic model information, see [13], [25], [26], [27].

The vector observation technique enhances the system accuracy by providing attitude observations and vehicle dynamics bandwidth information to the EKF. For example, a self-contained TRIAD-like algorithm was previously adopted by the authors in [28], to compute an attitude matrix observation $R_{MPS}$ using the magnetometer triad readings and the Earth’s gravitational field available from processing the accelerometer triad measurements. A straightforward but naive method to introduce the attitude measurement $R_{MPS}$ in the filter is obtained by defining the attitude measurement residual $z_{MPS}$ after the attitude error $\theta$, yielding

$$(z_{MPS})_\times = \hat{R}R'_{MPS} - I_3,$$

and modeling it in the filter as

$$z_{MPS} = \delta \lambda + n_{MPS},$$

using a noise term $n_{MPS}$ to compensate for the effect of magnetometer, accelerometer, rate gyro and INS disturbances on the attitude computation $R_{MPS}$. Nonetheless, characterizing $n_{MPS}$ as white noise can degrade the filter performance because it does not properly model the non-linear influence of pendular/magnetic sensors errors in $R_{MPS}$ computations. Also, the aiding attitude matrix $R_{MPS}$ is computed using the vector measurements available at each time instant (snapshot algorithm), and hence dynamic disturbances in the vector observation readings are not accounted for.

In this work, vector observations are embedded in the EKF, as depicted in Fig. 2. The magnetometer reading and gravity selective frequency contents provided by the accelerometer triad are modeled directly in the filter, bearing a more clear and accurate stochastic description of the measurement errors and disturbances.

The EKF implicitly computes the attitude based on the vector observations, presenting an alternate solution to the Wahba’s problem [5], [29] that encloses system dynamics, without external attitude determination algorithms and using optimality criteria. Sensor error characteristics other than just white noise are properly modeled in the filter, using the EKF covariance matrices and the structure of the error state space model. The algorithm presented herein can be generalized to any number of vector observations, devising a straightforward procedure to enhance the accuracy of the navigation system results, which also reinforces the EKF linearization assumption.

A. Vector Measurement Residual Model

The attitude measurement residual is obtained by comparing the estimated and the measured vectors. The considered vector measurement model is

$$s_r = R^{FE} s + n_s,$$  \hspace{1cm} (12)
where $n_s \sim \mathcal{N}(0, \Xi_s)$ is a Gaussian white noise. The attitude measurement residual in Earth frame coordinates is described by

$$E_{z_s} = E_s - \hat{R} n_s.$$  

Using the sensor model (12) and replacing the INS attitude estimate $\hat{R}$ by the attitude error approximation (6) yields

$$E_{z_s} = E_s - (I + (\delta \lambda)_x) E_s - \hat{R} n_s = -(\delta \lambda)_x E_s - \hat{R} n_s,$$

which relates the EKF measurement residual $E z_r$ with the attitude error $\delta \lambda$, producing the linearized model

$$E_{z_s} = (E_s) \times \delta \lambda - \hat{R} n_s.$$  \hspace{1cm} (13)

The measurement residual can be represented in Earth or in body frame coordinates, which are related by a rotation transformation and hence contain the same information. Repeating the same algebraic manipulations, the linearized model of the measurement residual in body coordinates is given by

$$B_{z_s} := \hat{R}' E_s - s_r \approx \hat{R}' (E_s) \times \delta \lambda - n_s.$$  \hspace{1cm} (14)

Although the measurement residuals (13) and (14) describe the same attitude information, the linearized measurement matrix for (13) is constant and the components of $\delta \lambda$ can be related directly with those of $E z_s$. For example, the measurement model (13) for the vector $E_s = [0 \ 0 \ 1]'$ is given by $E z_s = [-\delta \lambda_y \ \delta \lambda_x \ 0]' - \hat{R} n_s$, that contains information solely about the rotation error along the x-axis and y-axis, and illustrates the fact that the yaw angle error, i.e. $\delta \lambda_z$, cannot be determined by gravity readings.

In general, the vector reading $s_r$ can be corrupted by other additive sensor disturbances, namely biases $b_s$, and dynamic disturbances $d_s$, as follows

$$s_r = \hat{R}' E_s + n_s - \delta b_s + d_s,$$  \hspace{1cm} (15)

where $\delta b_s$ is the bias compensation error term, and $d_s$ is the output of a process modeled in the state space form. The linearized measurement residual representations for the sensor reading (15), in Earth and in body coordinates, are respectively described by

$$E_{z_s} = (E_s) \times \delta \lambda + \hat{R} \delta b_s - \hat{R} d_s - \hat{R} n_s, \quad B_{z_s} = \hat{R}' (E_s) \times \delta \lambda + \delta b_s - d_s - n_s.$$  \hspace{1cm} (16)

Using the measurement model (16), vector observations obtained by sensors such as pendulums and magnetometers can be introduced directly in the EKF. The sensor non-idealities are modeled in the filter, as opposed to using intermediate attitude reconstruction, which allows for the integration of any number of vector measurements, at different sampling rates, and compensating for dynamic disturbances. The observation noise covariance matrix, used in the computation of the optimal feedback gains, is directly given by the sensor noise variance $\Xi_s$.

### B. Magnetic and Pendular Measurements Integration

The magnetometer model considered in this work is given by

$$m_r = \hat{R}' m + n_m,$$  \hspace{1cm} (17)
where \( \mathbf{m} \) denotes Earth’s magnetic field, \( \mathbf{n}_m \sim \mathcal{N}(0, \Xi_m) \) is a Gaussian white noise, and magnetic distortions such as soft iron and hard iron are compensated offline using calibration algorithms available in the literature [30], [31]. The sensor description (17) is identical to the vector reading description (12). As illustrated in Fig. 4 the measurement residual (13) is adopted to integrate the magnetometer information in the EKF, yielding

\[
\mathbf{z}_m := \mathbf{E}_m - \hat{\mathbf{R}}_m \mathbf{r} = (\mathbf{E}_m) \times \delta \lambda - \hat{\mathbf{R}}_m \mathbf{n}_m.
\]

A gravity vector measurement is obtained from the accelerometer reading (1), which can be decomposed in Coriolis and linear acceleration components

\[
a_r = \frac{d}{dt} \mathbf{v} + \omega \times \mathbf{v} - B \mathbf{g} - \delta \mathbf{b}_a + \mathbf{n}_a. \tag{18}
\]

To obtain a gravity measurement reading \( \mathbf{g}_r \), adequate modeling is adopted to remove the acceleration terms in (18). Typical maneuvers of autonomous oceanic vehicles involve mostly short term linear accelerations, and hence the \( \frac{d}{dt} \mathbf{v} \) term is modeled in the filter state model as a high-frequency process. The Coriolis term \( \omega \times \mathbf{v} \) occurs in transient but also in trimming maneuvers such as helicoidal paths, and is compensated for using the linear and angular velocities information provided by the INS. The gravity vector measurement \( \mathbf{g}_r \) is given by

\[
\mathbf{g}_r = B \mathbf{g} - \mathbf{a}_{LA} + \delta (\omega \times \mathbf{v}) + \delta \mathbf{b}_a - \mathbf{n}_a, \tag{19}
\]

where \( \delta (\omega \times \mathbf{v}) = \hat{\omega} \times \mathbf{v} - \omega \times \mathbf{v} \) is the error of the centripetal acceleration removal, and \( B \mathbf{v} = \hat{\mathbf{R}}^E \mathbf{v} \) is the estimated velocity in body coordinates. The gravity reading (19) is modeled as

\[
\mathbf{g}_r = B \mathbf{g} - \mathbf{a}_{LA} + \delta (\omega \times \mathbf{v}) + \delta \mathbf{b}_a - \mathbf{n}_a, \tag{20}
\]

where \( \mathbf{a}_{LA} = \begin{bmatrix} a_{LAx} & a_{L Ay} & a_{LAz} \end{bmatrix} \) represents the linear acceleration estimate. Each of the \( \mathbf{a}_{LA} \) components is modeled as a band pass signal whose bandwidth is shaped according to the vehicle characteristics, often to filter out high-frequency INS acceleration jitter and to simultaneously avoid the influence of erroneous low-frequency accelerometer bias. The state model dynamics for the x-axis component, is generically represented in Fig. 5 and can be written as

\[
x_{LAx} = \begin{bmatrix} 0 & 1 \\ -\alpha_h \alpha_l & -\alpha_l + \alpha_l \end{bmatrix} x_{LAx} + \begin{bmatrix} 0 \\ \alpha_l \end{bmatrix} n_{LAx}, \tag{21}
\]

\[
a_{LAx} = \begin{bmatrix} 0 & 1 \end{bmatrix} x_{LAx}.
\]
where $\alpha_h$ and $\alpha_l$ are the high-frequency and low-frequency cutoff frequencies, respectively, and $n_{LAx}$ is modeled as a zero-mean, Gaussian white noise process with variance $\sigma_{LAx}^2$.

Using the results for the vector reading model (15), the measurement residual for the gravity reading (19) is defined as $E_{zg} = E_g - \hat{R}g_r$, and the first-order formulation is given by

$$E_{zg} \approx (E_g) \times \delta \lambda - \hat{R} \delta b_a - \hat{R} \delta (\omega \times B \hat{v}) + \hat{R} a_{LA} + \hat{R} n_a.$$  

Using (2) and (6), the centripetal acceleration compensation term is given by

$$\delta (\omega \times B \hat{v}) \approx (\hat{\omega}) \times \hat{R} \delta \hat{v} + (\dot{\hat{v}}) \times (\hat{R} \delta \lambda) + (\dot{\hat{v}}) \times (\hat{b}_o - n_o),$$

and the observation equation of the gravity measurement residual equation is

$$E_{zg} = - (\hat{R} \dot{\omega}) \times \delta \hat{v} + \left( (E_g) \times (\hat{R} \dot{\omega}) \times (\hat{v}) \right) \delta \lambda - \hat{R} \delta b_a - (E \hat{v}) \times \hat{R} \delta b_o + \hat{R} a_{LA} + \hat{R} n_a + (E \hat{v}) \times \hat{R} n_o,$$

where $a_{LA}$ is the output of triaxial generalization of the state model dynamics (21), integrated in the EKF, and tuned according to the maneuverability characteristics of the vehicles.

Fig. 4 illustrates the computation of the gravity measurement residual $E_{zg}$, which is fed to the filter using the observation model (22). While the INS calculates the body attitude estimates using high-precision algorithms described in Section II, the EKF estimates the attitude, velocity, position errors using the aiding sensor measurements. The INS estimates are corrected using the errors estimated by the EKF, which are then reset without influencing the estimation error covariance (17), thus keeping the first order approximation of the filter model valid.
V. IMPLEMENTATION

The continuous-time state space model $\dot{x}_C = F_C(\dot{x}, u)x_C + G_C(\dot{x})n_{zC}$ adopted in the filter is described by

$$x_C = \begin{bmatrix} \delta x' & x'_{LAz} & x'_{LAG} & x'_{LAy} \end{bmatrix}^T, \quad n_{zC} = \begin{bmatrix} n'_x & n_{La}x & n_{La}y & n_{La}z \end{bmatrix}^T,$$

$$F_C(x, u) = \text{blkdiag}(F(x, u), F_{LA}, F_{LA}, F_{LA}), \quad G_C(x) = \text{blkdiag}(G(x), G_{LA}, G_{LA}, G_{LA}),$$

$$F_{LA} = \begin{bmatrix} 0 & 1 & 0 \ -\alpha_l\alpha_h & -(\alpha_l + \alpha_h) \end{bmatrix}, \quad G_{LA} = \begin{bmatrix} 0 \ \alpha_h \end{bmatrix},$$

where $\delta x$, $n_z$, $F(x, u)$ and $G(x)$ are defined in [11]. The measurement model $z = H(x, u)x_C + n_z$ can be written as

$$z = \begin{bmatrix} \zeta'_{GPS} & E\zeta'_m & E\zeta'_g \end{bmatrix}^T, \quad n_z = \begin{bmatrix} -n'_{GPS} & -\hat{R}n'_m & -\hat{R}n'_u \ -\hat{R}n'_u + (E\hat{v})_{x} & \hat{R}n'_w + \hat{R}n'_g \end{bmatrix},$$

$$H(x, u) = \begin{bmatrix} \text{blkdiag} \left( \begin{array}{cccc} I_3 & 0 & 0 & 0 \\ 0 & (E^T m)^T & 0 & 0 \\ 0 & \hat{R}\omega_x & \hat{R}\omega_x & \hat{R} \end{array} \right) \end{bmatrix}, \quad H_{LA} = \begin{bmatrix} 0 & 1 & 0 & 0 \ 0 & -1 & 0 & 0 \ 0 & 0 & -1 & 0 \ 0 & 1 & 0 & 0 \end{bmatrix},$$

where $n_g$ is a fictitious white noise associated with $z_g$ observation, and $z_{GPS}$ is the GPS measurement residual, classically defined by the difference between the position estimated by the INS and that measured by the GPS [32], that is

$$z_{GPS} := \tilde{p} - p_{GPS} = \delta p - n_{GPS},$$

where $n_{GPS} \sim \mathcal{N}(0, \Xi_{GPS})$ is a Gaussian white noise that models the GPS measurement noise.

The state and observation noise covariance matrices are

$$Q_C = \text{blkdiag}(\Xi_p, \Xi_u, \Xi_w, \Xi_{b_a}, \Xi_{b_a}, \Xi_{LA}),$$

$$R_C(x) = \text{blkdiag}(\Xi_{GPS}, \Xi_m, \hat{R}\Xi_u \hat{R}', -(E\hat{v})_{x} \hat{R}\Xi_w \hat{R}' (E\hat{v})_{x} + \hat{R}\Xi_g \hat{R}'),$$

where $\Xi_{LA} = \sigma^2_{LA}I_3$. The discrete-time state space model

$$x_{k+1} = \Phi_kx_k + w_k, \quad z_k = H_kx_k + v_k,$$

is obtained by sample-and-hold of the inputs [32] and is given by

$$\Phi_k = e^{F_kT}, \quad H_k = H(x, u)|_{t = t_k},$$

and the discrete-time noise covariance matrices are [33]

$$Q_k \approx G_k Q_C G_k^T, \quad R_k \approx \frac{R_{Ck}}{T},$$

where $T$ is the sampling period, $F_k = F_C(x, u)|_{t = t_k}$, $G_k = G_C(x)|_{t = t_k}$, $R_{Ck} = R_C(x)|_{t = t_k}$ and $\Phi_k = \Phi(t_{k+1}, t_k)$ denotes the state transition matrix.
The gravity measurement residual $Ez_{g}$ introduces state and measurement noise correlation matrix \([32]\)

$$C_{C}(\dot{x}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\hat{R}_{E} & -(\hat{E}v)_{E} & \hat{R}_{E} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$C_{k} = \frac{1}{T} \int_{t_{k-1}}^{t_{k}} \phi(t_{k}, \tau) G(\tau) C_{C}(\tau) d\tau \simeq (I_{3} + \frac{F_{k}T}{2})G_{k}C_{k},$$

where $C_{C}(x)$ is the continuous state and measurement noises correlation matrix and $C_{CK} = C_{C}(x)|_{t=t_{k}}$. The discrete-time equivalent matrix $C_{k}$ is computed using a first order approximation similar to those discussed in \([32]\) for $Q_{k}$ and $R_{k}$. The following Kalman gains and error covariance matrix equations are modified to include the state and measurement noises correlation matrix

$$K_{k} = (P_{k}^{-1}H_{k}^{T} + C_{k}) [H_{k}P_{k}^{-1}H_{k}^{T} + R_{k} + H_{k}C_{k} + C_{k}H_{k}^{T}]^{-1}, \quad P_{k}^{-1} = (I_{n} - K_{k}H_{k})P_{k}^{-1} - K_{k}C_{k},$$

and the filter covariance matrix is updated using $P_{k+1}^{-} = \Phi_{k}P_{k}^{-}\Phi_{k}^{T} + Q_{k}$.

After each EKF update, error estimates are fed into the INS error correction routines as depicted in Figs. 2 and 3 where the quantities predicted by the INS are denoted by the superscript $-$. It is important to stress that linearization assumptions are kept valid during the algorithm execution since the EKF error estimates are reset after being used to compensate the corresponding variables. The error correction procedures are specific to the INS algorithms and error state space representations. For the INS described in Section II error routines are detailed next.

The attitude estimate is compensated using the rotation error matrix $R(\delta\lambda)$ definition, which yields

$$\hat{R}_{k}^{-} = R(\delta\lambda_{k})\hat{R}_{k}^{-}, \quad (24)$$

where matrix $R(\delta\lambda_{k})$ is described exactly as

$$R(\delta\lambda_{k}) = I_{3} - \frac{\sin(\|\delta\lambda_{k}\|)}{\|\delta\lambda_{k}\|} \times \left(\delta\lambda_{k}\right) + \frac{1 - \cos(\|\delta\lambda_{k}\|)}{\|\delta\lambda_{k}\|^{2}} \times \left(\delta\lambda_{k}\right)^{2},$$

and is computationally implemented using power series expansion of the scalar trigonometric terms up to an arbitrary accuracy \([23]\). In the case where few computational resources are available, $R(\delta\lambda_{k})$ can be approximated to first order by $R(\delta\lambda_{k}) \simeq I_{3} - \left(\delta\lambda_{k}\right)^{2}$ that, nonetheless, introduces DCM orthogonalization problems in $\hat{R}_{k}^{-}$ whose compensation usually requires considerable computational effort \([34]\). The remaining state variables are simply compensated using

$$\hat{p}_{k}^{-} = \hat{p}_{k}^{-} - \delta\hat{p}_{k}, \quad \hat{v}_{k}^{-} = \hat{v}_{k}^{-} - \delta\hat{v}_{k}, \quad \hat{b}_{w_{k}}^{-} = \hat{b}_{w_{k}}^{-} - \delta\hat{b}_{w_{k}}, \quad (25)$$

The INS block structure with EKF corrections is depicted in Fig. 3 where the error compensation and bias update routines, (24) and (25) respectively, are executed after the INS outputs have been fed to the EKF and errors estimates are available. Note that the EKF sampling rate is synchronized with the moderate-speed INS output rate and that no corrections are involved in the high-speed computation algorithms. After the error correction procedure is completed, the EKF error estimates are reset $\delta\hat{x}_{k} = 0$. The INS error correction and EKF estimate reset do not influence the uncertainty of the estimated quantities, and hence the estimation error covariance is unaffected by this procedure \([17]\). At the start of the next computation cycle ($t = t_{k+1}$), the INS attitude and velocity/position updates presented in Section II are performed on the corrected estimates ($\hat{R}_{k}^{-}, \hat{v}_{k}^{-}, \hat{p}_{k}^{-}$) to provide new inputs ($\hat{R}_{k+1}^{-}, \hat{v}_{k+1}^{-}, \hat{p}_{k+1}^{-}$) to the EKF.
TABLE I

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Bias</th>
<th>Noise Variance</th>
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</thead>
<tbody>
<tr>
<td>Rate Gyro</td>
<td>$5 \degree/s$</td>
<td>$(0.02 \degree/s)^2$</td>
</tr>
<tr>
<td>Accelerometer</td>
<td>$12 \text{ mg}$</td>
<td>$(0.6 \text{ mg})^2$</td>
</tr>
<tr>
<td>Magnetometer (calibrated)</td>
<td></td>
<td>$(60 \mu \text{G})^2$</td>
</tr>
<tr>
<td>GPS</td>
<td>-</td>
<td>$10 \text{ m}^2$</td>
</tr>
</tbody>
</table>

VI. SIMULATION RESULTS

This section presents a simulation study of the proposed navigation system, prior to the practical implementation of the algorithm in the autonomous surface craft. The impact of the vector observation in the estimation results is analyzed, by considering three case study simulations. In the first case, the navigation system is initialized with large estimation errors to evidence how the estimation results can be enhanced by the use of vector measurements, and a standard rigid body trimming trajectory with constant centripetal acceleration is generated to demonstrate the necessity of centripetal acceleration removal in the pendular measurements. The linear acceleration model (20) is validated in the second case study, by presenting the response of the navigation system when the vehicle is subject to a step acceleration with damping. In the last case study, poor GPS signal detection is simulated to illustrate how the position estimates remain within acceptable bounds by means of the pendular measurements.

The INS high-speed algorithm is executed at 100 Hz and the normal-speed algorithm is synchronized with the EKF discrete time frequency of 50Hz. The GPS position measurements are obtained at the nominal frequency of 1Hz. The characteristics of the simulated inertial and aiding sensors are presented in Table I.

A. Initial Calibration Error

The contribution of the magnetic and pendular measurements to the accuracy of the estimates is studied for the initial estimation error of $5\degree$ in the roll angle, and calibration errors in the rate gyro and accelerometer bias given by $\delta b_{\omega x} = 0.57\degree/s$ and $\delta b_{ax} = 1 \text{ mg}$, respectively. The rigid body describes the ascending helix depicted in Fig. 6(a) which is a standard trimming trajectory subject to constant centripetal acceleration. The convergence of the estimation errors shown in Figs. 6(b), 6(c) and 6(d) evidences that the pendular readings improve on the GPS and magnetometer aiding, by enhancing the observability of errors such as the gyro bias, roll angle and vertical accelerometer bias, as expected from physical intuition and analysis of the observability matrix for trimming trajectories. Interestingly enough, the obtained estimation results were stable and accurate although the small error assumption underlying the EKF derivation was not verified by the initial estimation errors.

B. Linear Accelerated Motion

The impact of the vector aiding in the navigation system results is analyzed for the case of a straight line trajectory. The vehicle is subject to a constant acceleration input that is progressively compensated by the linear drag effects, as depicted in Fig. 7(a) and linear uniform motion is attained. Fig. 7(b) validates the assumption.
that vehicle’s linear acceleration component in $z_g$ can be modeled as a band pass signal (21), and hence the low frequency contents of $z_g$ are used to estimated the inertial system errors. Numerical results obtained with the proposed technique are presented in Table II where improvements due to the inclusion of aiding vector observations are evidenced.
C. Trimming Trajectory

The medium term navigation system behavior is assessed for the trimming trajectory with standard initial estimation errors. Fig. 8 demonstrates the performance enhancements introduced by the magnetometer readings and the selective frequency contents of the accelerometers measurements. As presented in Table III the magnetometer readings smooth out yaw errors, and pendular observations enhance the estimation of roll and pitch. Due to the position and attitude errors correlation expressed in (10), x-axis and y-axis position errors are improved by the magnetic and pendular observations, respectively. Also, the constant centripetal acceleration of the trimming trajectory is successfully compensated for.

Simulation results for a GPS signal with output frequency of 0.2 Hz are depicted in Fig. 9, with and without pendular measurement aiding. The figure shows that x- and y-axes position estimates are enhanced by the selective frequency contents of the accelerometers measurements. The filter exploits the pendular measurements, limiting the position estimate divergence when the GPS signal is sparse, and extending the navigation system autonomy with respect to the GPS aiding source.

VII. EXPERIMENTAL RESULTS

The proposed navigation system is validated using a low-power hardware architecture enclosing low-cost sensors and mounted on-board the DELFIMx catamaran. This section details the characteristics of the DELFIMx oceanic vessel and introduces the hardware architecture adopted to collect and process experimental data. Experimental results obtained at sea illustrate the performance of the navigation system in practice for standard ASC trajectories, emphasizing its robustness characteristics. Namely, the compensation of the pendular measurements disturbances in the frequency domain is validated, and the autonomy of the navigation system with respect to GPS measurements is demonstrated.
TABLE III
FILTER RESULTS, TRIMMING TRAJECTORY

| $z = \bar{z}_{\text{GPS}}$ | 1.35 | 1.77 | 0.91 | 12.96 | 0.26 | 0.17 |
| $z = \left[ \bar{z}_{\text{GPS}}^T \bar{z}_{m} \right]^T$ | 0.66 | 1.79 | 0.91 | $3.17 \times 10^{-3}$ | 0.20 | 0.17 |
| $z = \left[ \bar{z}_{\text{GPS}}^T \bar{z}_{m} \bar{z}_{g} \right]^T$ | 0.65 | 1.51 | 0.91 | $3.16 \times 10^{-3}$ | 0.18 | 0.14 |

Fig. 8. Trimming trajectory results.

Fig. 9. Position results for sparse GPS signal, sampled at 0.2 Hz.
A. DELFIMx ASC and Sensor Characteristics

The DELFIMx craft, depicted in Fig. 1, is a small Catamaran 4.5 m long and 2.45 m wide, with a mass of 300 Kg. Propulsion is ensured by two propellers driven by electrical motors, and the maximum rated speed of the vehicle with respect to the water is 6 knots. For integrated guidance and control, a path-following control strategy was adopted due to its enhanced performance, which translates into smoother convergence to the path and less demand on the control effort [35]. The vehicle has a wing shaped, central structure that is lowered during operations at sea. At the bottom of this structure, a low drag body is installed that can carry acoustic transducers. For bathymetric operations and sea floor characterization, the wing can be equipped with a Tritech Super SeaKing mechanically scanned pencil beam sonar or a RESON 8125 multibeam sonar. On top of this structure it is installed a SICK LD-LRS3100 laser range finder, to survey the emerged part of semi-submerged infra-structures like breakwaters.

The DELFIMx hardware architecture developed by the ISR-IST is a self-contained system mounted on three cases which can be fit into and removed from the ASC. The most sensitive parts are vibration isolated from the hull using a soft suspension mechanism, which acts as a low pass mechanical filter that provides further attenuation of the ASC vibration on the electronics. The hardware architecture is built around the low-cost low-power floating point Digital Signal Processor (DSP) TI TMS320C33, displayed in Fig. 10(a) which is connected to the data acquisition hardware through a dual port RAM expansion board developed by IST-ISR. Special care was taken during the electronics development in order to implement measures that improve the Electromagnetic Compatibility (EMC). The data acquisition distributed architecture was built around the CAN (Controller Area Network) Industrial Real Time Network, for control and navigation purposes and on 100MBits/sec Ethernet for payload data interface. A series of very low-power boards designed at ISR using the Phillips XAS3 16 bit microcontroller, and the ATME9 AT90CAN128 8-bit AVR® Flash microcontroller with extended CAN capabilities are used to interface all sensors and exchange data through the CAN Bus. In this architecture the TMS320C33 schedules all Guidance, Control, and Navigation tasks to meet their deadlines. Finally, a PC104 board connected to the CAN Bus and to Ethernet runs the mission control system and implements a blackbox where relevant data generated by the ASC are properly saved in a solid state disk for post-mission analysis.

The Inertial Measurement Unit (IMU) on-board the DELFIMx craft is a strapdown system comprising a
triaxial XBOX CXL02LF3 accelerometer and three single axes Silicon Sensing CRS03 rate gyros mounted along three orthogonal axes. These sensors are attached orthogonally to a custom made stand that is presented in Fig. 10(b) with the sensors assembled. The inertial sensors are sampled at 56 Hz using six Texas ADS1210 directly connected to a microcontroller board. The ADS1210 is a high precision, wide dynamic range, delta-sigma analog-to-digital converter with 24-bit resolution operating from a single +5V supply. The differential inputs are ideal for direct connection to transducers guaranteeing 20 bits of effective resolution which is a suitable accuracy for the set of inertial sensors used in the present application.

The hardware architecture is also equipped with a Honeywell HMR3300 magnetometer, interfaced by a serial port connection with a sampling rate of 8 Hz. The GPS receiver installed on board the DELFIMx is a Thales Navigation DG14 receiver which presents an accuracy of 3.0 m Circular Error Probable (CEP) in autonomous mode and 0.40 m in differential mode. In the present work, the GPS works in autonomous mode and the measurements are provided at a 4 Hz sampling rate. The diverse frequency rates of the aiding sensors, i.e. GPS, magnetometer, and pendular measurement, are easily handled in the filter by selecting the rows of the measurement matrix (23) according to the available measurement at each time instant.

B. Experimental Results Analysis

This section evaluates the navigation system for a set of experimental data. The results presented in this work were obtained for a DELFIMx sea-trial conducted on October 2007 on the coast of Sesimbra, Portugal, located at $38^\circ 26'\ N$, $9^\circ 6'\ W$. The trajectory described by the catamaran was obtained using the path-following preview controller proposed in [36], and was designed to demonstrate the maneuverability of the vehicle in challenging applications by comprising straight lines, curves and oscillatory trajectories generated by coning motion, as shown in Fig. 11.

The parameters of the EKF were tuned as follows. The covariances of the inertial and aiding sensors were computed by processing sensor data obtained with the DELFIMx at rest on the harbor facilities. The pendular model (21) was characterized by the covariance $\sigma^2_{LA} = 10^{-5}$ and the poles $\alpha_l = 3.64$ Hz and $\alpha_h = 27$ Hz, which describe a high frequency process given that the sampling frequency of the navigation system is 56 Hz. The classical technique of system robustification by inflating the noise covariances was adopted [37], namely the covariance of the pendular observation noise $n_g$ was defined as $\Xi_g = 10^{-5}I_3$ to account for second order terms in the observation model (22), and the velocity error state covariance was set as $\Xi_v = \Xi_a + 7 \times 10^{-4}I_3$, to balance the influence of the GPS aiding and the IMU computations in the estimated position and velocity. To validate the adopted covariances, it will be shown that the navigation system successfully merges the available information and is in fact robust to GPS outage. To better illustrate the qualities of the proposed solution, navigation system results with GPS signal blockage are also considered in this section.

The navigation system was initialized using attitude and position estimates provided by the aiding sensors. The initial attitude guess was obtained using the QUEST attitude reconstruction algorithm [7] to process the first magnetometer and accelerometer measurements, and the position estimate was acquired directly from the first good quality GPS measurement available.

The position and attitude estimation results are presented in Figs. 11, 12 and 13 and are consistent with the trajectory outlined by the GPS measurements. The estimated position smoothly tracks the trajectory described...
by the DELFIMx catamaran. The estimated yaw is according to the described trajectory, and to the heading measurement provided by the GPS, which is depicted only for comparison purposes. The average estimated pitch and roll angles are according to the installation angles of the IMU architecture in the DELFIMx platform.

The estimated angular and linear velocities of the catamaran are shown in Fig. 14. The angular velocity is consistent with the vehicle maneuvers. The linear velocity is represented in body fixed coordinates because the velocity variations occur naturally in the body axis. As expected, $\beta v_x$ is positive and characterized roughly by concatenation of forward velocities, while the lateral and vertical velocities fluctuate around zero.

Although the navigation system was stable in extensive simulation studies where large initial bias estimation error was considered, offline calibration was adopted in practice to guarantee that the small error assumption of the EKF perturbational model was kept valid from the start. An initial guess of the accelerometer and rate gyro biases was obtained offline and after warming up the IMU. The initial covariance of the filter was set to compensate for small errors of the offline calibration, and to account for the bias fluctuations between the time instants of the calibration procedure and the navigation system initialization. The filter covariances $\Xi_{b_a}$, $\Xi_{b_\omega}$ were designed small enough to compensate for the slow variations of the bias in the course of the mission, $\Xi_{b_a} = \Xi_{b_\omega} = 10^{-12} I_3$. As shown in Fig. 15 the bias estimate is approximately constant, which endorses the
The vector aiding technique described in Section IV was adapted to the application at hand. Analyzing the measurement model (22) for \( \mathbf{E}_g = \begin{bmatrix} 0 & 0 & g \end{bmatrix} \), it is straightforward to verify that the z-axis measurement residual \( E_z \) does not relate to the attitude error \( \delta \lambda \), i.e. it is uninformative for the purpose of attitude determination. Also, the collected magnetometer data were roughly planar and hence enough to calibrate only the soft iron and hard iron distortions in the \( xy \) plane of the magnetometer. Consequently, the vertical components of the measurement residuals \( E_z \) and \( E_m \) were put aside in the course of the filtering algorithm, by omitting the corresponding rows of the measurement matrix \( \mathbf{H} \). The aiding measurements components can be easily selected, which shows the flexibility of the present navigation solution.

The modeling of the pendular vector measurements described in Section IV-B is validated using frequency domain analysis of the measured and estimated signals. The power spectral density (PSD) of the desired signals...
Fig. 15. Bias estimation results (DELFImx trajectory).

Fig. 16. Frequency contents of the measured acceleration and estimated linear acceleration.

was obtained using Matlab’s `pwelch` function, i.e. Welch’s averaged modified periodogram method of spectral approximation. Fig. 15 presents the frequency contents of the pendular reading $g_r$, defined in (19), and that of the linear acceleration estimate $a_{LA}$, defined in (21). The PSDs of $g_r$ and $a_{LA}$ are very similar in the medium and high frequency regions, and diverge in the low frequency domain where the PSD of $a_{LA}$ is smaller than the PSD of $g_r$. This shows that the filter exploits in fact the low frequency contents of $g_r$ for attitude estimation, while the medium and high frequency linear acceleration disturbances are associated with the signal $a_{LA}$, as desired. The PSD of the signals in the low frequency region is shown in detail in Fig. 17.

The dependency of the navigation system with respect to the aiding measurements is studied by disabling the GPS measurements at selected time intervals when the vehicle turns or enters in long straight paths. The nominal and estimated trajectories are shown in Figs. 18 and 19 and a zoom of the trajectories at the GPS outage time intervals is presented in Fig. 20. The position and attitude estimates track the curve and straight line paths in the short term, which shows that the performance of the system without GPS aiding is adequate.
for practical applications. It also evidences that the navigation system acts according to the concept of filtering, by merging the IMU and aiding measurements without relying solely on the GPS data.

The tests in the presence of GPS outage also illustrate the necessity of pendular measurements, as shown in Table IV, where the position drift is approximated by the first measurement residual $z_{GPS}$ when the GPS is successfully reacquired. The performance of the navigation system is clearly enhanced by the pendular measurements, which extend the autonomy of the unit with respect to GPS measurements.

VIII. CONCLUSION

An advanced Global Positioning System/Inertial Navigation System (GPS/INS) using Extended Kalman Filter (EKF) and integrating vector observations was described. The navigation system comprised an high-accuracy, multirate INS algorithm, combined with an EKF in a direct-feedback configuration to compensate for inertial sensors non-idealities. An aiding technique that directly integrates vector measurements in the filter was detailed,
allowing for the use of a frequency domain model of the vehicle motion in the filter. The experimental results obtained at sea with the DELFIMx ASC showed that the proposed navigation system can accurately estimate position and attitude. The compensation of sensor non-idealities such as bias and noise effects, and the autonomy with respect to GPS aiding by exploiting the vector measurement directly in the filter, were evidenced in practice.

REFERENCES

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<tr>
<th>Time Interval (s)</th>
<th>With Pendular Aiding</th>
<th>Without Pendular Aiding</th>
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<tbody>
<tr>
<td></td>
<td>Final Position Drift (m)</td>
<td>Average Position Drift (m/s)</td>
</tr>
<tr>
<td>[370–380]</td>
<td>1.76</td>
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