Fundamental theoretical inefficiencies in thermo-electric heat recovery applications

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Abstract: The efficiency of thermo-electric (TE) modules is usually specified between two temperatures when powered by a constant temperature source for example electrical heating.
By modelling heat flow and temperatures across the thermal resistances of the heat source as well as the TE module and heat exchangers, this paper shows that the specified efficiencies when operating in applications such as waste heat recovery or with fuel heating are fundamentally incorrect due to the effective thermal resistance of the heating fluid, even in the case of ideal heat exchangers. As the temperature capability of the TE modules increases, the efficiency problem worsens. The low efficiency is independent of the method of heat transfer, whether radiation, convection or conduction as it is dependent only on the properties of the fluid and its source temperature. This phenomenon does not occur with solar, nuclear or heating sources that are able to deliver heat at a constant temperature. When predicting the TE system efficiency in terms of heat in to electricity out, the effective thermal resistance of the source should be taken into account and is described in this paper. For fuel based heating using air as the oxidiser without pre-heating the thermal resistance is 1/Cp/mass flow and efficiency drop (as compared with the TE module specified efficiency) is a function of flame temperature and maximum operating temperature of the TE hot surface. Results from 5 difference modules are presented and show a typical 50% reduction in maximum theoretical efficiency from the manufactures specification when used in combustion or heat recovery applications. To obtain maximum efficiency, the module characteristic have to be matched to the source temperature and maximum heat flow available.

Keywords: Thermo-electric module, waste heat recovery, efficiency reduction

1 Introduction

There is currently much interest in recovering waste heat from processes and heat engines in order to reduce carbon emissions and meet globally set targets [1], [2]. The market size for such systems is mainly driven by the initial capital cost, as the fuel is effectively free. The largest effect on generated income is the efficiency of the heat to generated electricity conversion process [3]. So to minimise electricity cost on a £/kWhr basis, capital cost should be lowered and efficiency maximised. Waste heat is usually low grade, i.e. at a lower temperature than combustion processes. The main technologies for converting this heat to electricity are: Rankine cycle (particularly ORC [4]) for exhaust heat recovery [5], Stirling engines (including thermo-acoustic) and thermo-electric modules (TEM), see Figure 1 [6].

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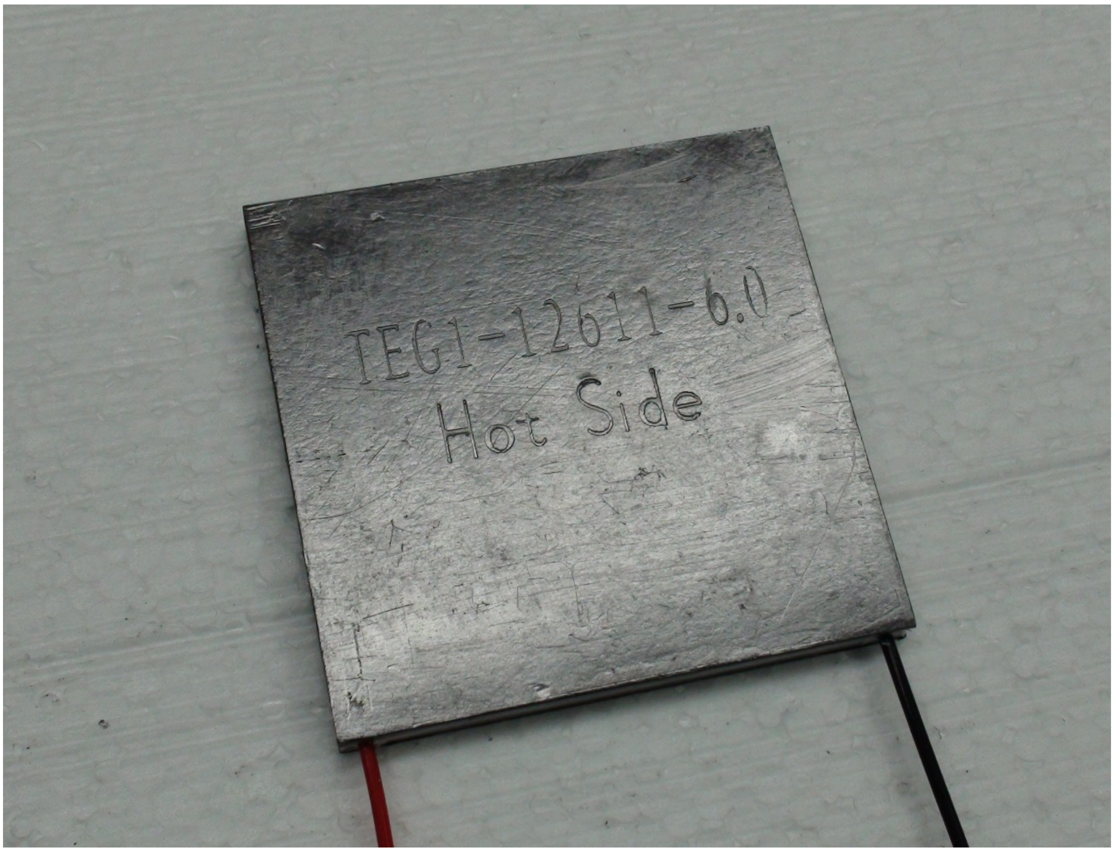

Figure 1, Typical TEM generator module

Riley [7] has shown that for open-loop waste-heat recovery (i.e. where heat in the exit fluid is lost and not re-circulated) and combustion processes without air pre-heat, the ideal Stirling engine is much less efficient than the Carnot efficiency due to the thermal resistance of the gas supplying the heat. The same theory applies to TEM and is seen as a discrepancy between the efficiency quoted in the specification and that found during waste heat recovery experiments and is the subject of this paper. TE modules convert heat into electricity using the Seebeck effect.

1.1 Applications

Due to their simple design and no moving parts, TEM have been proposed for low power generation from wood burning stoves [8] using a cooling fan for the ambient heat exchanger [9]. Using the exhaust gases stream from an energy efficient mud cooking stove Champier et al. [10] used TEM to generate 2.3 W of electricity from Taihuaxing Co., Ltd. TEP1-12656-0.8, 10.5W modules with a 160C hot heat exchange (HHX) temperature. Bensaid et al. [11] demonstrated electrical generation from hydrogen and methane using a catalytic converter as the heat source with TEM. However, although the theoretical efficiency was predicted to be 5.3% the best achieved during their experiments was 3% efficiency at a 250C HHX temperature. Modules capable of working at higher temperatures, up to 650C are being developed [12] with an expected increase in thermal to electrical efficiency. This paper presents a case that increasing TEM maximum HHX temperature will give diminishing efficiency gains for waste heat and combustion heating processes where there is no pre-heated air; a condition typical in most applications.

2 Heat exchange modelling

Figure 2 shows a typical arrangement for a TEM. A source at temperature Tf transfers heat via a fluid with thermal resistance Rθc, through a heat exchanger (HHX) with resistance Rθh to the hot side of the module, which under steady state conditions reaches a temperature of Th. Where Te is the entry temperature of the HHX. Heat is extracted from the module via an ambient heat exchanger (AHX) with thermal resistance Rθa to ambient temperature Ta.

Electrical output voltage is given by:

$$V = S\delta \mathrm{T} \tag{1}$$

Where S is the Seebeck coefficient and δT the temperature difference across the TEM. At a fixed temperature, the internal electrical resistance of the TEM can be approximated to a fixed resistance R_i and maximum power is obtained when the electrical load resistance $R_L = R_i$.

Maximum electrical power theorem shows that P is a maximum when:

$$\mathrm{P} = \frac{\mathrm{V}^2}{4\mathrm{R_i}} \tag{2}$$

Substituting (2) into (1) gives:

$$P = \frac{(S\partial T)^2}{4R_i} \tag{3}$$

In the ideal case with perfect hot and cold ambient heat exchangers:

Rθh = Rθa = 0 and hence
Te = Th and Tc = Ta

The way most TEM vendors specify performance is by quoting output power at fixed module surface temperatures and hence quote this ideal case.
It should be noted that $R\theta_c$ is a function of the heating fluid and is independent of the heat transfer mechanism, i.e. it applies equally to radiation, conduction or convection mechanisms and where:

$$R\theta_c = \frac{1}{C_{pf}\dot{m}_c} \tag{4}$$

Where C_{pf} is the specific heat and $\dot{m}_c$ is the mass flow rate of the heating fluid.

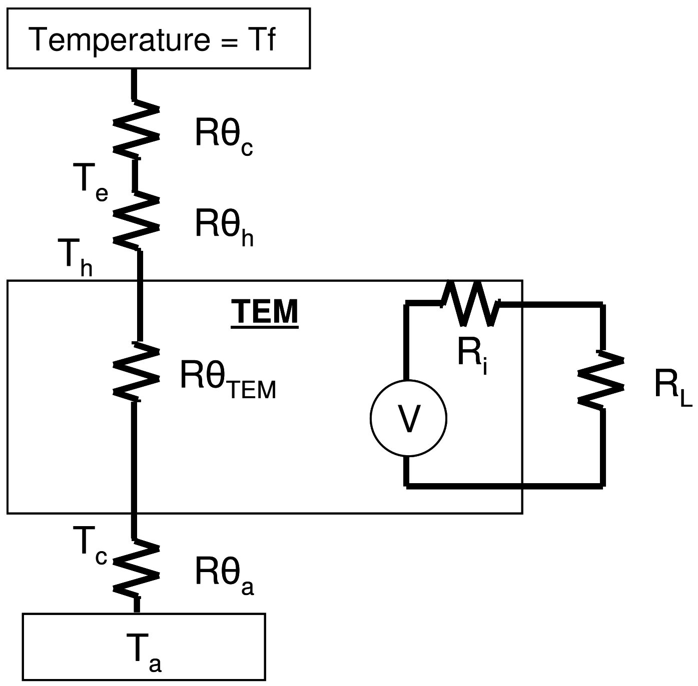

Figure 2, model for TEM

The heating fluid at temperature T_f passes heat through the thermal resistances to the surrounding temperature T_a. So that heat available from the fluid is:

$$Q_c = \frac{T_f - T_a}{R\theta_c + R\theta_{TEM}} \tag{5}$$

And heat supplied to the (TEM) is:

$$Q_{TEM} = \frac{T_f - T_h}{R\theta_c} \tag{6}$$

The difference between Q_c and Q_{TEM} being the heat lost in the exit fluid.

Inspecting equations (5) and (3) show that when $T_h \rightarrow T_f$ heat available to the TEM approaches zero and when $T_h \rightarrow T_c$ the TEM output drops to zero. In both cases the overall TEM efficiency $\eta_{TEM} \rightarrow 0$.

2.1 Heating with waste heat recovery

Waste heat recovery systems can be divided into two types:

1. Where the heat source is at a relatively constant temperature, i.e. where extracting heat does not reduce the temperature
2. Extracting heat requires a temperature difference, for example, where a fluid is discharged from a process and heat can be extracted before the fluid is discarded. Examples are:
    a. Car exhaust heat recovery
    b. Liquid waste containing low grade heat
    c. Wood burning cooking stoves

For type 2 systems, the fluid discarded contains useful heat that is not recovered and so, in the ideal case, heat efficiency is reduced.

$$\eta_{QTEM} = \frac{Q_{TEM}}{Q_c} \tag{7}$$

Substituting 5 and 6 into 7

$$\eta_{QTEM} = \frac{(T_f - T_h)(R\theta_c + R\theta_{TEM})}{R\theta_c(T_f - T_a)} \tag{8}$$

Simplifying and setting $\frac{\partial\eta}{\delta T_h} = 0$ gives maximum heat into the TEM when:

$$R\theta_{TEM} = R\theta_c \tag{9}$$

At this condition and assuming no material temperature limits are exceeded:
Heat into the TEM = 50% of the available heat, and

$$T_h = \frac{T_f + T_a}{2} \tag{10}$$

Overall system efficiency is given by:

$$\eta_s = \eta_{QTEM}\eta_{TEM} \tag{11}$$

Where

$$\eta_{TEM} = \frac{Q_{TEM}}{P} \tag{12}$$

2.2 Heating through combustion

Where the heat is supplied via a combustion process and inlet air is at ambient temperature (i.e. not pre-heated), in the general case where the fuel may contain water (for example wood), mass flow of the combustion products is the sum of its constituents:

$$\dot{m}_c = \dot{m}_f + \dot{m}_a + \dot{m}_w \tag{13}$$

Where $\dot{m}_f$, $\dot{m}_a$ and $\dot{m}_w$ are the mass flow rates of the fuel, air and water.

The air to fuel ratio A_f is defined as:

$$A_f = \frac{\dot{m}_f}{\dot{m}_a} \tag{14}$$

Water content R_w is:

$$R_w = \frac{\dot{m}_w}{\dot{m}_f} \tag{15}$$

The specific heat of the combustion products is a function of fuel, air and fuel moisture content given by:

$$C_{pc} = C_{pf} + C_{pa} X_a A_f + C_{pw} R_w \tag{16}$$

The heat available from combustion is reduced due to the evaporation of water content in the fuel, so:

$$Q_c = \dot{m}_f C_{vf} - m_w L_h \tag{17}$$

Assuming no dissociation of component gasses, the idealised flame temperature is:

$$T_f = \frac{\dot{m}_f C_{vf} - m_w L_h}{C_{pc} \dot{m}_c} + T_a \tag{18}$$

Making substitutions from ($C_{pc} = C_{pf} + C_{pa} X_a A_f + C_{pw} R_w$ (16) and dividing by fuel mass flow rate gives:

$$T_f = \frac{C_{vf} - R_w L_h}{C_{pf} + X_s A_f C_{pa} + R_w C_{pw}} + T_a \tag{19}$$

Maximum power output from the TEM is given by

$$P = Q_{TEM} \eta_{TEM} \tag{20}$$

Let

$$K = \frac{C_{pf} + X_s A_f C_{pa} + R_w C_{pw}}{C_{vf} - R_w L_h} \tag{21}$$

For maximum power and efficiency from combustion:

$$T_h = \frac{K+T_a}{2} \qquad (22)$$

Hence any TEM module that is capable of operating above T_h will not give any improvement in either efficiency or power output from a module whose maximum temperature is limited to T_h, However, modules with higher maximum operating temperatures do give better performance than those with lower maximum temperatures.

3 Results

The specified characteristics of some common TEM modules is given on Table 1 and are compared with the maximum theoretical efficiencies for the waste heat application are shown on Table 2. Note that these efficiencies only occur at the temperature specified on the table, efficiency will decrease at higher fluid temperatures.

Using equation 22 with a typical wood burning stove conditions of 200% excess air and a 15% moisture content gives a Tf of 1024C. The figures below show operating conditions for various waste heat fluid temperatures up to this value.

Note that these are maximum theoretical conditions assuming perfect heat exchangers, real applications are less than these figures.

Table 1, selection of TEM manufacturer's specifications

		M1	M2	M3	M4	M5
Rθ	C/W	0.39	0.71	0.71	0.65	1.06
Ri	Ω	0.42	3.89	20.91	2.40	2.25
η_{TEM}	#	5.0%	4.7%	4.2%	4.2%	6.4%
Max power	W	28.3	19.511	9.539	21.6	11.5
Matched current	A	8.2	2.24	0.675	3	2.3
Matched voltage	V	3.45	8.708	14.124	7.2	5
Max Hot	C	250	260	150	300	220
AHX	C	30	20	30	30	30
Heat flow(hot side)	Wth	566	415	226	415	180

	Part Number	Manufacturer
M1	GM250-127-28-10	European Thermodynamics (supplied by RS Comps)
M2	TEG 241-260-35	Thermal Force
M3	TEG450-200-45	Thermal Force
M4	TGPR-22W-7V	Tegmart
M5	TGPR-22W-7V (de-rated)	Tegmart (maximum efficency condition)

Table 2, conditions for maximum theoretical efficiency

		M1	M2	M3	M4	M5
Tf at max efficiency	C	470	500	270	570	410
Max Efficiency		2.5%	2.3%	2.1%	2.1%	3.2%

When Tf is below the maximum allowed TEM temperature, the theoretical power available from the module is less than the specified conditions as shown on Figure 3.

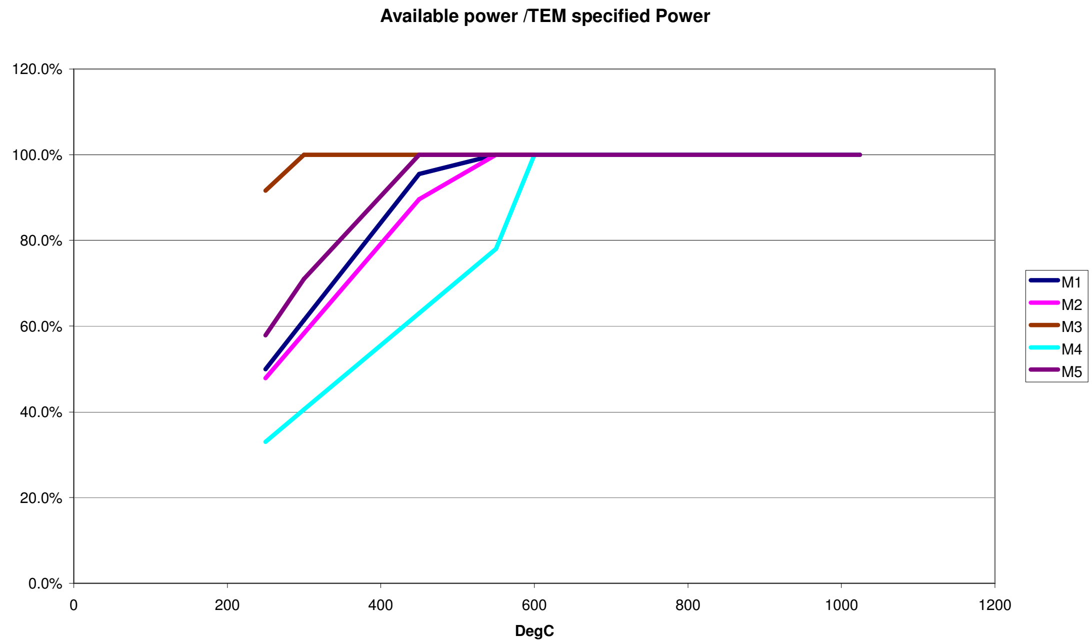

Figure 3

The effect of operating Tf above a temperature that would exceed the module maximum operating temperature is shown on Figure 4. In this case $R\theta_{TEM} < R\theta_c$ so as to keep Th at the maximum temperature.

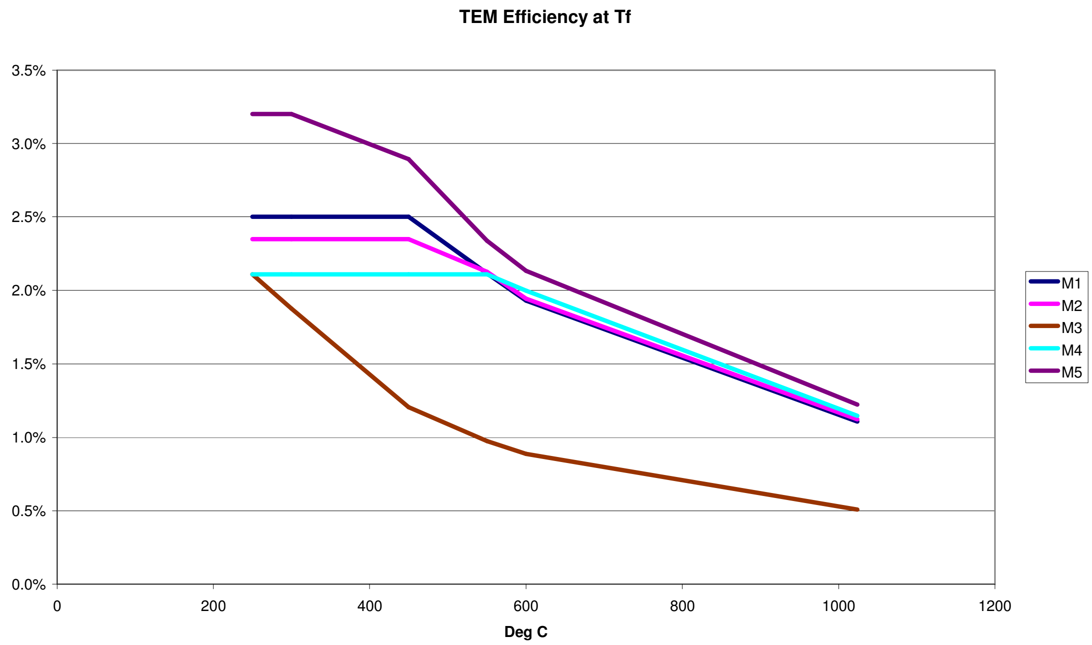

Figure 4

4 Discussion of results

The formulations used in this paper assume that the module parameters (thermal resistance and Seebeck constant) do not vary with temperature, which is a valid assumption for the explanations given. However, the parameters do have a second order effect that can be used for more accurate calculations, an example can be found in module specification sheets [13].

When the available temperature from waste heated fluid is low, the power output from TEM modules with a lower maximum operating temperature is closer to the specified value. As the cost of the modules is a function of maximum specified power then choosing the TEM specification closer to the expected operating conditions will lower the cost per generated watt.

For higher grade heat, where the fluid temperature Tf would cause Th to exceed the maximum specified module temperature, heat flow into the module is limited to keep Th at the max temperature limit. However, under these circumstances overall efficiency drops with increasing Tf.

For applications where efficiency is not the limiting design consideration but simplicity of installation is more important, then TE generators do have advantages over other more complicated solutions. However, as with Stirling engines [7], where the efficiency of converting thermal energy to electrical power is a key design parameter, then other thermodynamic cycles perform better and hence the current interest in Organic Rankine Cycles (ORC) for waste heat recovery. ORC engines can operate effectively with low grade heat.

5 Conclusions

The ideal performance of TEM generators when using open loop waste heat or non-pre-heated combustion systems is much lower than the manufacturer's TEM specifications in terms of available power and thermal to electrical conversion efficiencies.

Module performance can be improved by selecting the most appropriate TEM for the expected heat flow and temperature operating conditions.

The theoretical degradation in performance compared with manufacturer's specifications is due to the effective thermal resistance of the heating fluid and is not a function of the TEM itself.

The performance figures shown in this paper are theoretical maxima, performance under real world conditions will be considerably worse due to the effects of the thermal resistance of the hot and ambient heat exchangers.

6 References

[1] United Nations. Adoption of the Paris Agreement. Conf Parties Its Twenty-First Sess 2015;21932:32.

[2] McGlade C, Ekins P. The geographical distribution of fossil fuels unused when limiting global warming to 2 °C. Nature 2015;517:187–90. doi:10.1038/nature14016.

[3] Evans A, Strezov V, Evans TJ. Assessment of sustainability indicators for renewable energy technologies. Renew Sustain Energy Rev 2009;13:1082–8. doi:10.1016/j.rser.2008.03.008.

[4] Wang J, Yan Z, Wang M, Li M, Dai Y. Multi-objective optimization of an organic Rankine cycle (ORC) for low grade waste heat recovery using evolutionary algorithm. Energy Convers Manag 2013;71:146–58. doi:10.1016/j.enconman.2013.03.028.

[5] Sprouse C, Depcik C. Review of organic Rankine cycles for internal combustion engine

exhaust waste heat recovery. Appl Therm Eng 2013;51:711–22. doi:10.1016/j.applthermaleng.2012.10.017.
[6] Gerardtv. TEM module picture 2010:CC BY – SA 3.0. https://commons.wikimedia.org/w/index.php?curid=23208759.
[7] Riley PH. The Myth of the High-Efficiency External-Combustion Stirling Engine. Engineering 2015;Vol.7 No.1:789–95. doi:10.4236/eng.2015.712068.
[8] O'Shaughnessy SM, Deasy MJ, Doyle J V., Robinson AJ. Performance analysis of a prototype small scale electricity-producing biomass cooking stove. Appl Energy 2015;156:566–76. doi:10.1016/j.apenergy.2015.07.064.
[9] Mal R, Prasad R, Vijay VK, Verma AR. The Design, Development and Performance Evaluation of Thermoelectric Generator(TEG) Integrated Forced Draft Biomass Cookstove. Procedia Comput Sci 2015;52:723–9. doi:10.1016/j.procs.2015.05.085.
[10] Champier D, Bedecarrats JP, Rivaletto M, Strub F. Thermoelectric power generation from biomass cook stoves. Energy 2010;35:935–42. doi:10.1016/j.energy.2009.07.015.
[11] Bensaid S, Brignone M, Ziggiotti A, Specchia S. High efficiency Thermo-Electric power generator. Int J Hydrogen Energy 2012;37:1385–98. doi:10.1016/j.ijhydene.2011.09.125.
[12] Edwards M, Brinkfeldt K. Thermo-mechanical modelling and design of SiGe-based thermo-electric modules for high temperature applications. 2013 14th Int. Conf. Therm. Mech. Multi-Physics Simul. Exp. Microelectron. Microsystems, EuroSimE 2013, 2013. doi:10.1109/EuroSimE.2013.6529990.
[13] European Thermodynamics. GM250-127-28-10 Data Sheet n.d. http://www.europeanthermodynamics.com/products/datasheets/GM250-127-28-10 (2).pdf.