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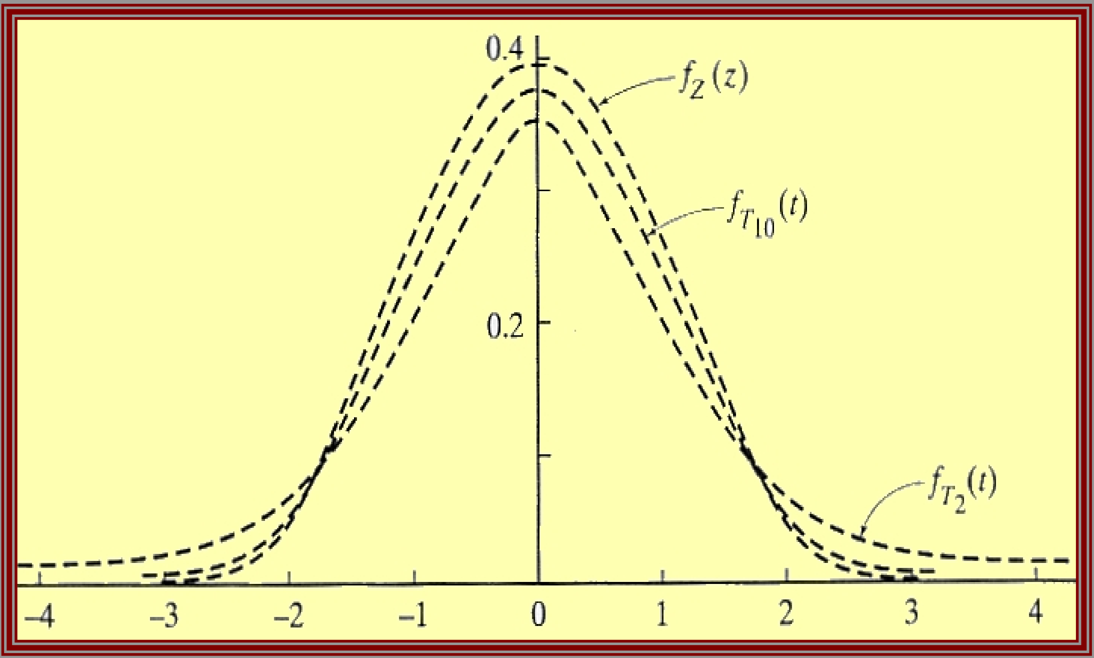
JPSS

A comprehensive journal of probability and statistics
for theorists, methodologists, practitioners, teachers, and others



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JPSS Editorial Board

(continued)

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Aims and Scope The ***Journal of Probability and Statistical Science*** (***JPSS***, ISSN 1726-3328) is a modified version of the ***Journal of Propagations in Probability and Statistics*** (***JPPS***, ISSN 1607-7083). ***JPSS***, like its predecessor ***JPPS***, is a multipurpose and comprehensive journal of probability and statistics that publishes papers of interest to a broad audience of theorists, methodologists, practitioners, teachers, and any other users of probability and/or statistics. The scope of ***JPSS*** is intended to be quite broad, including all the major areas of probability and statistics. Research papers involving probability and/or statistics, either theoretical or applied in nature, will be seriously considered for publication. Additionally, papers involving innovative techniques or methods in teaching probability and/or statistics will also be considered. Papers submitted for publication consideration will be peer reviewed. Initially, we will publish semiannually, one issue each in February and August. Hopefully, as time accrues, we will be able to publish quarterly. It is the goal of ***JPSS*** to publish a wide range of works involving probability and/or statistics (theoretical and/or applied in nature) and provide widespread availability of such to a broad audience of people interested in probability and/or statistics.

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A First Order Autoregressive Asymmetric Laplace Process

K. Jayakumar *University of Calicut*
A. P. Kuttykrishnan *Sir Syed College*
Tomasz J. Kozubowski *University of Nevada*

ABSTRACT We define and study a first order autoregressive time series model with stationary asymmetric Laplace marginal distributions. We derived its basic properties, including estimation, and discuss various extensions following the same construction based on self decomposability of the marginal distribution. We also mention potential applications of this model in stochastic modeling of correlated, peaked, and asymmetric data.

Keywords Autocorrelation; Autoregressive process; Asymmetric Laplace distribution; Class L; Generalized asymmetric Laplace distribution; Geometric stable distribution; Geometric summation; Non-Gaussian time series model; Self-decomposable law; Skew double-exponential model.

1. Introduction

Although the normal distribution is still by far the most widely used probability model in the sciences, it is not adequate to describe fat tailed or asymmetric empirical distributions arising in mathematical finance and other applications. One alternative is the Laplace distribution with its sharp peak at the mode, along with its skew and heavy-tailed generalizations, that are gaining popularity in recent years (see, e.g., [20] and references therein). A simple two-parameter asymmetric Laplace (AL) distribution studied in [23], given by the probability density function (PDF)

$$f(x)=\frac{1}{\sigma}\frac{\kappa}{1+\kappa^2}\begin{cases}\exp(-\frac{\kappa}{\sigma}x) \text{ if } x\geq 0\\ \exp(\frac{1}{\kappa\sigma}x) \text{ if } x<0,\end{cases} \tag{1.1}$$

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The Beta Extended Weibull Family

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ABSTRACT We introduce the beta extended Weibull family of distributions which contains as special models some important distributions discussed in the literature, such as the generalized modified Weibull (Carrasco *et al*. [4]), beta Weibull, beta exponentiated Weibull, beta exponential, beta modified Weibull and Weibull distributions, among several others. New distributions are proposed as members of this family, for example, the beta XTG (Xie *et al*. [37]), beta log-Weibull, beta Chen (Chen [5]) and beta Gompertz distributions. We derive its moments and moment generating function. Maximum likelihood estimation is proposed for estimating the model parameters. We calculate the observed information matrix. Some new distributions are used to improve the analysis of the Aarset's [1] data.

Keywords Beta distribution; Exponentiated exponential; Exponentiated Weibull; Generalized Modified Weibull; Maximum likelihood; Modified Weibull; Observed information matrix; Weibull distribution.

1. Introduction

The Weibull distribution, having exponential and Rayleigh as special models, is a very popular distribution for modeling lifetime data and for modeling phenomenon with monotone failure rates. When modeling monotone hazard rates, the Weibull distribution may be an initial choice because of its negatively and positively skewed density function. However, it does not provide a reasonable parametric fit for modeling phenomenon with non-monotone failure rates such as the bathtub shaped and unimodal failure rates that are common in reliability and biological studies. Such bathtub hazard curves have nearly flat middle portions

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The Beta-Cauchy Distribution

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ABSTRACT In this paper, a four parameter beta-Cauchy distribution is defined and studied. Various properties of the distribution are examined. The distribution is found to be unimodal. Necessary and sufficient conditions are given for the existence of the moments of the beta-Cauchy distribution. Recursive formulae for the non-central moments are obtained. Some relationships between the moments and the parameters of beta-Cauchy distribution are discussed.

Keywords Unimodality; Symmetry; Moments; Beta family of distributions.

1. Introduction

A random variable X has a Cauchy distribution with location parameter θ and scale parameter λ (Johnson *et al*. [13]) if its probability density is defined as

$$f(x) = \left[\pi\lambda\left(1+[(x-\theta)/\lambda]^2\right)\right]^{-1}, -\infty < x < \infty, -\infty < \theta < \infty \text{ and } 0 < \lambda < \infty. \quad (1)$$

The Cauchy distribution has appeared in the literature for over three centuries. Stigler [27] indicated that the curve first appeared in Fermat's work in the middle of the seventeenth century. Subsequently, it was investigated by many researchers during the eighteenth century such as Grandi [12], Poisson [24] and Cauchy [6].

Poisson [24] was the first to notice that the density could provide counterexamples to otherwise general theorems in statistics. In 1810 Laplace discussed a large sample justification for Legendre's principle of least squares using central limit theorem. Poisson noticed that Laplace's argument is violated by the Cauchy density which does not obey the law of large numbers.

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A Generalization of the Waring Disribution

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ABSTRACT A generalization of the Waring distribution has been obtained by compounding the Jain and Consul's [7] generalized negative binomial distribution with the beta distribution of the first kind. The first two moments of this distribution have been obtained and the maximum likelihood estimation of its parameters has been discussed. The distribution has been fitted to some well known data sets to test its goodness of fit.

Keywords Generalized negative binomial distribution; Beta distribution; Compounding; Gaussian hypergeometric function; Goodness of fit.

1. Introduction

Irwin [2] with the help of the Waring expansion

$$c(c-a)^{-1}=\sum_{x=0}^{\infty}\frac{a^{(-x)}}{(c+1)^{(-x)}} \tag{1.1}$$

where $b^{(-n)}=b(b+1)(b+2)\cdots(b+n-1)$; $b^{(0)}=1$ obtained a distribution, known as Waring distribution given by its probability mass function (pmf)

$$P_1(x;a,c)=\frac{(c-a)a(a+1)(a+2)\cdots(a+x-1)}{c(c+1)(c+2)\cdots(c+x)};\quad c>a>0,\ x=0,1,2,\cdots. \tag{1.2}$$

The Yule [16] distribution is a particular case of this distribution and can be obtained by taking a = 1 and c – a = ρ. The Waring distribution (1.2) has been found to be a suitable distribution for the data-sets having extremely long tails. In fact, one can make the tail of this distribution as long as one wishes by letting c and a tend to zero.

The mean and variance of this distribution have been obtained as

$$\mu_1^{/}=a/(c-a-1)\quad\text{and}\quad\mu_2=a(c-a)(c-1)/(c-a-1)^2(c-a-2) \tag{1.3}$$

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Kernel Estimation of Percentiles Obtained from Accelerated Degradation Model

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ABSTRACT A nonparametric kernel estimator for percentiles of the time-to-failure distribution, under normal conditions, obtained from accelerated degradation model is proposed. The proposed kernel estimator is compared to the maximum likelihood and the ordinary least squares estimators. The relative efficiency and the length of the bootstrap confidence interval are used as the criteria of the comparisons. Simulation results show that, in term of the relative efficiency and the length of the bootstrap confidence interval, the proposed kernel estimator is performed well. Furthermore, application to the real data set shows that the performance of the proposed kernel estimator is more efficient than the maximum likelihood and the ordinary least squares estimators.

Keywords Accelerated Degradation; Kernel Density Estimation; Maximum Likelihood Estimator; Ordinary Least Squares Estimator.

1. Introduction

In analyzing the reliability of a product, the quantity of interest is usually the time to failure. However many product may degrade before failure. Some time it may takes a long time to determine the time to failure under normal conditions, so accelerated degradation model is used to get the results in relatively short amount of time.

Accelerated degradation test usually collects the degradation data at higher levels of stress, then using interpolations, we can estimate the time to failure distribution and consequently its percentiles under normal conditions.

In the literature, Meeker *et al*. [9] gave an overview of modeling and analyzing accelerated degradation tests and used the maximum likelihood estimation (MLE) method to

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Some Results on Maximum Likelihood Estimators of Parameters of Geometric Distribution under Grouped and Ungrouped Progressive Type-I Censoring

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ABSTRACT In this paper the study of bias, exact variance and mean square error of the maximum likelihood estimators of the geometric distribution under Type-I progressive censoring with different parameters is considered. Further a minimum mean square error estimator for the parameter at each stage is derived. Estimation in case of group censoring is also considered. Optimal spacing time of censoring and total expected waiting time of the test are derived. The results obtained are applied to a real life data. The numerical evaluation of the estimators' relative performance is made.

Keywords Geometric distribution; Maximum likelihood estimator; Minimum mean square error estimator; Optimal spacing; Total expected waiting time.

1. Introduction

In life testing experiment, it is a common practice to terminate the experiment when certain number of items have failed (Type II censoring) or a stipulated time has elapsed (Type I censoring). Progressively censored samples frequently occur in life and fatigue tests, where individual observations are time ordered and where at various times during a test, some of the survivals are removed (i.e., censored) from further observations. The idea of removing some items at every stage of censoring stamps from the fact that these items might be required for use somewhere else for related experimentation. In such experiments after first stage, the experiment is continued with the remaining surviving items. Samples of this type from normal

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Computing Waiting-time Distribution of M/D$_N$/1 Queue: An Alternative Approach

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ABSTRACT Recently, Shortle *et al*. [5] carried out the analysis of waiting-time distribution of M/D$_N$/1 queue wherein arrivals follow the Poisson process and service- time of each customer is constant and takes one of the N values with probability p_i $(1 \leq i \leq N)$. They proposed a recursive method of second order and carry out a thorough comparison of this method with other methods such as Fourier, Euler, Laguerre, Gaven- Stehfest methods. They claim that the recursive method performs well in most of the examples provided that each service-time is an integer multiple of small step size ($h > 0$). As the service-time distribution is discontinuous and this poses numerous difficulties in inverting the Laplace transforms associated with this queue, we propose an alternative method for computing waiting-time distribution for this queue which is based on the roots of the so-called characteristic equation of the Laplace-Stieltjes transform (LST) of the waiting-time distribution. Using Padé approximation of the LST of service-time distribution, we obtain a closed-form expression of waiting-time distribution from which we can calculate the value of the distribution function for any given $t \in [0, \infty)$ directly whereas the recursive method works only for those values of t that can be expressed as a multiple of step size h. Numerical aspects have been tested for a variety of arrival and service time parameters and the method works for all values.

Keywords Deterministic; Queueing-time; Roots; System-time; Padé approximation.

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Asymptotic Distribution of the Number of Isolated Nodes in Wireless Ad-hoc Network

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ABSTRACT In this paper, we derive the weak law for the number of isolated nodes in wireless ad-hoc networks in a unit area square region. Here, we assume that the nodes are uniformly distributed in a unit area square and follow the poisson point process. We prove that for sufficiently large n and edge distance $r_n = \sqrt{(\log n + \xi)/(n\pi)}$, the number of isolated nodes in an n node network is distributed according to Poisson with mean $\exp(-\xi)$.

Keywords Random networks; Poisson point process; Transmission radius.

Introduction

An Ad-hoc network is characterized by a set of autonomous nodes that communicate with each other by forming a multi-hop network and maintaining connectivity in a decentralized manner. Several important characteristics of ad hoc wireless networks and sensor networks are closely related with the transmission radius of nodes, like connectivity, maximum and minimum vertex degree, number of components in the network etc. There is another important question related to the presence of isolated nodes in the network. For prefixed transmission radius and compact space, the number of isolated nodes decreases with the increase of the number of nodes. In this paper, we are interested to find the distribution of the number of isolated nodes in a network, when the transmission radius is a function of the number of nodes. Here we derive a weak law result for the number of isolated vertices distributed in a compact space $C \subseteq \mathbb{R}^2$. The basic object of study is the graphs G_n with vertex set $\mathcal{X}_n = \{X_1, X_{2,} \cdots, X_n\}$, n = 1, 2, …, where the vertices are independently and uniformly distributed.

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